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Data Structures Using C

Module 5 - PART 1

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01

Introduction to Graph

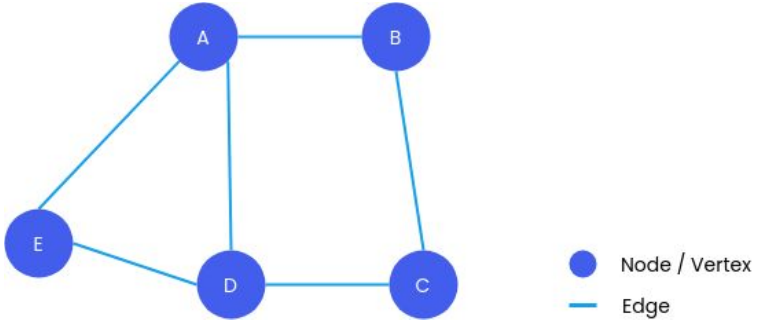
Introduction, Definition, Basic Terminology & Types.

Definition

A **Graph** is a non-linear data structure consisting of nodes and edges.

Definition : A graph **G=<V,E>** consist of two sets: V and E, where

- V is the set of **nodes** (vertices or points)
- E is the set of **edges**, identified with a unique pair of nodes in V, denoted by e=[u, v]**.**



Terminology

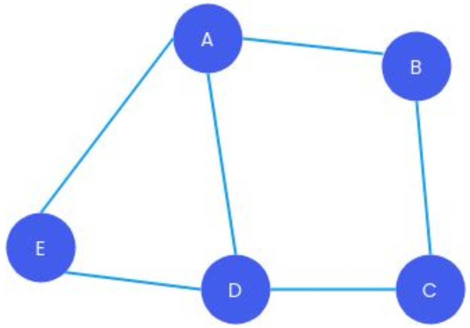
- Suppose e = [u, v]. Then nodes u and v are called **endpoints** of e, and u and v are called **adjacent nodes** or neighbours.
- The **degree** of node u, is the number of edges containing u.
- If u does not belong to any edge(degree 0)-then u is called **isolated edge**.
- A **path** in a graph is a finite sequence of edges which joins a sequence of vertices.
- **Path length** is equal to the number of edges present in the path.
- A path is said to be **closed** if it starts and ends at the same node.
- A path is said to be **simple** if all the nodes are distinct with exception that first node may equal last node .
- A **cycle** is a closed simple path with length 3 or more.
- A graph is said to be **connected** if and only if there is a path between any two of its nodes.
- A graph G is said to be **complete** if every node u in G is adjacent to every other node v in G.
- An edge e is called a **loop** if it has identical endpoints, that is, if e = [u,u].

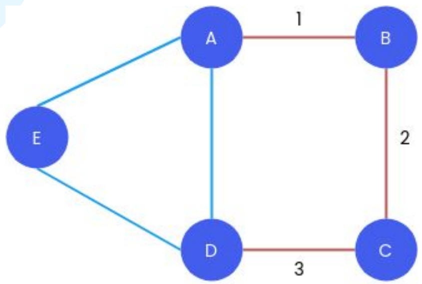
Terminology

- If there are multiple edges between the same pair of nodes in a graph, then the edges are called **parallel edges**.
- A graph containing self loop or parallel edges or both is called a **multigraph**.

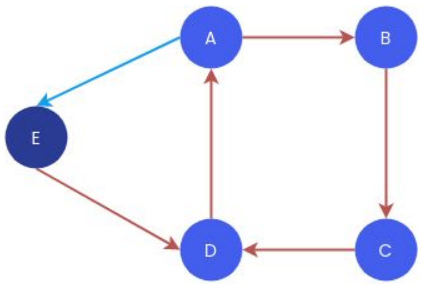
Example:

- A, B, C, D, E - **Nodes**.
- A and B are **adjacent nodes.**
- A-B, B-C, C-D, D-C, D-E, E-A - **Edges**.
- **Degree of A** = 3.
- A-B-C-D-E - **Path from A to E** with **length 4.**
- A-B-C - **Path from A to C** with **length 2.**
- B-C-D-A-B - **Closed path from B to B**.
- E-A-B-C-D-E is a **cycle**.

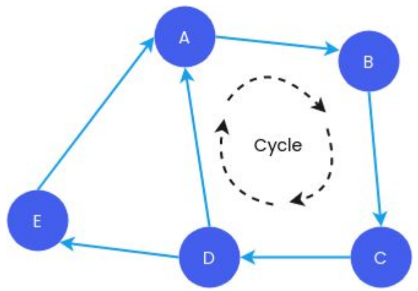


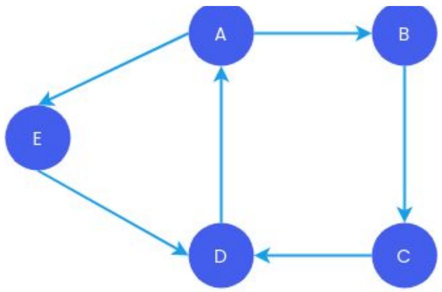


Path from A to D with length 3

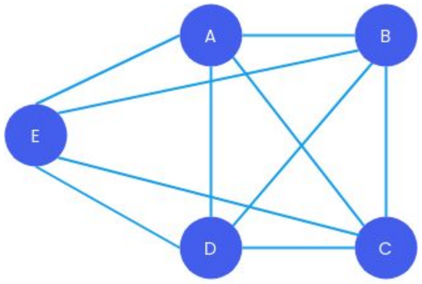


Not a simple path (E-D-A-B-C-D-A)

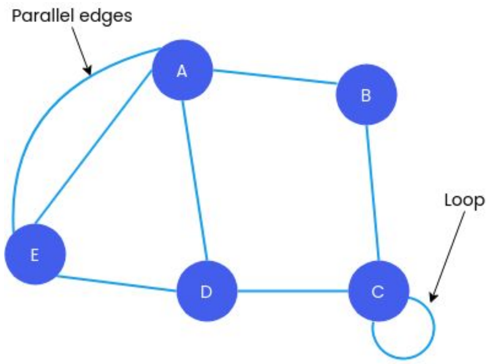




Connected Graph



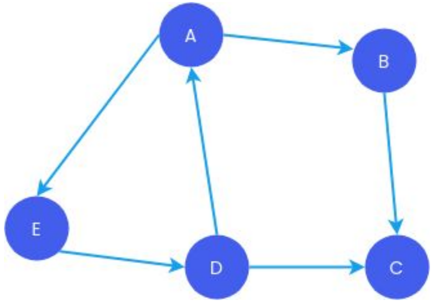
Complete Graph



Multigraph

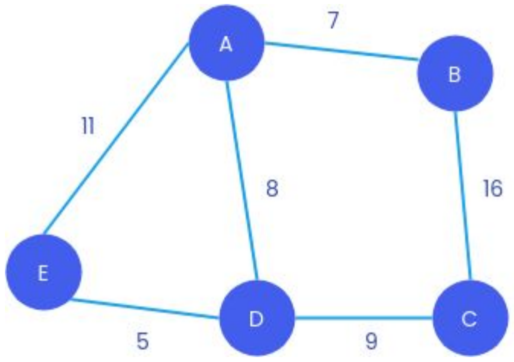
Directed Graph

- A **directed graph** is a graph with the property that its **edges** have **directions**.
- Each edge e is identified with an **ordered pair** [u, v].
- They are also called a digraph.
- Suppose a digraph has a directed edge e = [u, v]
  - e begins at u, and ends at v.
  - u is the origin or initial point of e, and v is the destination or terminal point of e.
- The **outdegree** of a node u is the number of edges beginning at u.
- The **indegree** of a node u is the number of edges ending at u.
- A node u is called a **source** if it has a positive outdegree but zero indegree.
- A node u is called a **sink** if it has a zero outdegree but a positive indegree.
- A node v is said to be reachable from a node u if there is a directed path from v to u.
- A directed graph G is said to be connected(strongly connected) if for each pair u, v of nodes in G there is a path from u to v and there is also a path from v to u.



Weighted Graph

- A **weighted graph** is a graph with the property that its edges have a number associated with it.
- The number is called **weight** of the edge.
- A weighted graph can be undirected or directed.
- The **weight of a path** P is the sum of the weights of the edges along the path P.



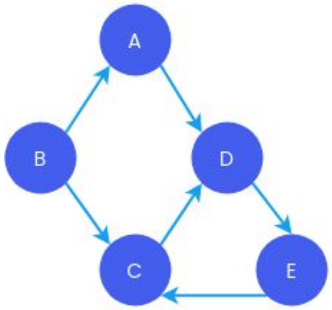
02

Graph Representation

Sequential & Linked list representation of graphs.

Sequential Representation : Adjacency Matrix

- A graph having N nodes can be represented using a NxN square matrix where rows and column numbers represent the vertices.
- A cell at the cross-section of the vertices can have only values 0 or 1.
- An entry $(v_i, v_j) = 1$ only when there exists an edge from v_i to v_j.
- In case of weighted graph, the edge weight is stored instead of 1.
- Adjacency matrix will be a sparse, hence a great deal of space will be wasted.
- It may be difficult to add or delete new nodes.



	A	B	C	D	E
A	0	0	0	1	0
B	1	0	1	0	0
C	0	0	0	1	0
D	0	0	0	0	1
E	0	0	1	0	0

Path Matrix

- For a directed graph G with adjacency matrix A, the element A_{ij} of matrix $\mathbf{A^k}$ gives the **number of paths of length k** from vertex v_i to v_j.
- For example, an element A_{ij} of A^2 will denote the number of paths of length 2 from vertex v_i to v_j.
- Now we define a matrix B_r ,

  $\boldsymbol{B_r = A+A^2+A^3+ \ldots +A^r}$
- The ij entry of the matrix B, gives the number of paths of lengths r or less from node v_i to v_j.
- If G has m nodes, a simple path from any v_i to v_j must have length m-1 or less.
- Similarly a cycle must have length m or less.
- That is if there is a non-zero ij entry in the matrix B_m , there is a simple path from v_i to v_j.
- A **path matrix** is obtained by replacing all non-zero elements in B_m by 1.
- Then P_{ij} = 1 if and only if there is a path from v_i to v_j.

Path Matrix

	A	B	C	D	E
A	0	0	0	1	0
B	1	0	1	0	0
C	0	0	0	1	0
D	0	0	0	0	1
E	0	0	1	0	0

A

	A	B	C	D	E
A	0	0	0	0	1
B	0	0	0	2	0
C	0	0	0	0	1
D	0	0	1	0	1
E	0	0	0	1	0

A^2

	A	B	C	D	E
A	0	0	1	0	0
B	0	0	0	0	1
C	0	0	1	0	0
D	0	0	0	1	0
E	0	0	0	0	1

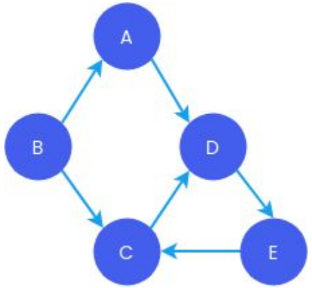
A^3

	A	B	C	D	E
A	0	0	0	1	0
B	0	0	2	0	0
C	0	0	0	1	0
D	0	0	0	0	1
E	0	0	1	0	0

A^4

	A	B	C	D	E
A	0	0	0	1	0
B	0	0	0	2	0
C	0	0	0	0	1
D	0	0	1	0	0
E	0	0	0	1	0

A^5



	A	B	C	D	E
A	0	0	1	3	1
B	1	0	3	4	1
C	0	0	1	2	2
D	0	0	2	1	3
E	0	0	2	2	0

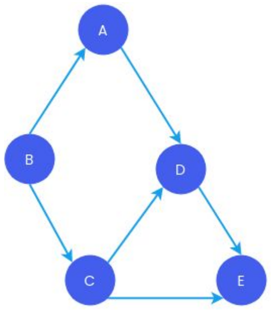
$B=A+A^2+A^3+A^4+A^5$

	A	B	C	D	E
A	0	0	1	1	1
B	1	0	1	1	1
C	0	0	1	1	1
D	0	0	1	1	1
E	0	0	1	1	0

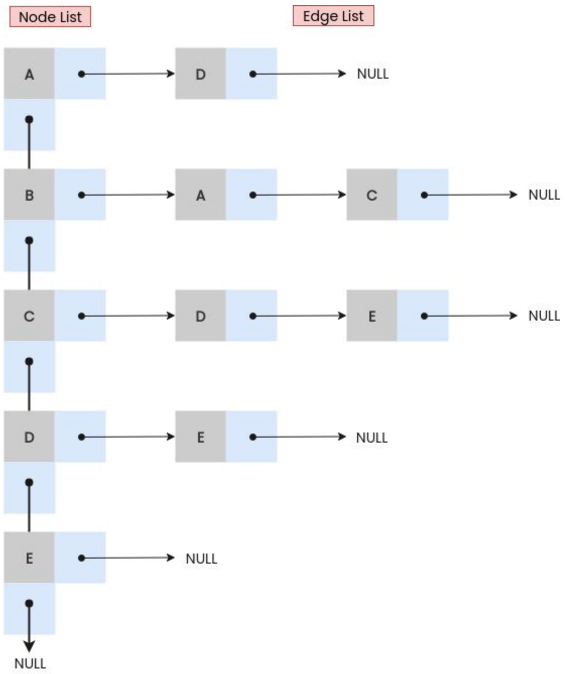
P

Linked Representation

- The linked representation contains two lists, a **NODE** list and an **EDGE**(Adjacency) list.
- Each element in the NODE list will correspond to a node in the graph.
  - **Name** of the Node.
  - Pointer to **adjacent nodes**.
  - Pointer to **next node**.
- Each element in the edge list will represents an edge of the graph.
  - **Destination node** of the edge.
  - **Link** that link together the edges with the same initial node.



Node	Adjacency List
A	D
B	A, C
C	D, E
D	E
E	-



03

Traversal of Graph

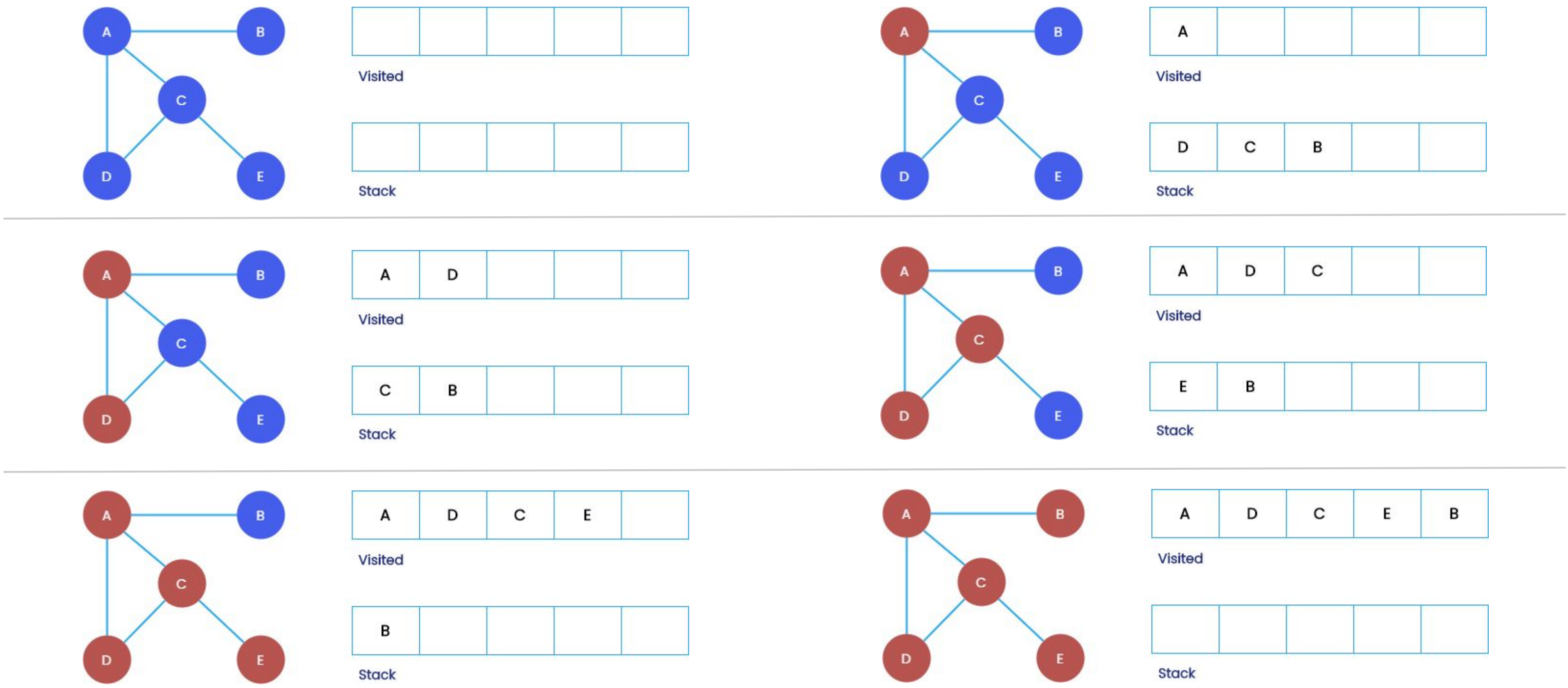
Depth First Search(DFS) and Breadth First Search(BFS).

Depth First Search (DFS)

- In DFS method, the travel starts from a node then follows its outgoing edges.
- Within successors, it travels their successors and so on.
- The search goes deeper into the search space till no successor is found.
- DFS implementation puts each nodes of the graph in following lists
  - Visited
  - Not Visited (Stack)
- This marking is used to avoid cycles while traversing.

```
Algorithm: DFS
 1.   Push Starting Node A on the Stack.
 2.   Repeat steps 3 and 4 until stack is empty.
 3.   Take the top item N from the Stack and add it to the
      Visited List.
 4.   Push all unvisited neighbours of N onto Stack.
 5.   Exit.
```

Depth First Search (DFS): An Example

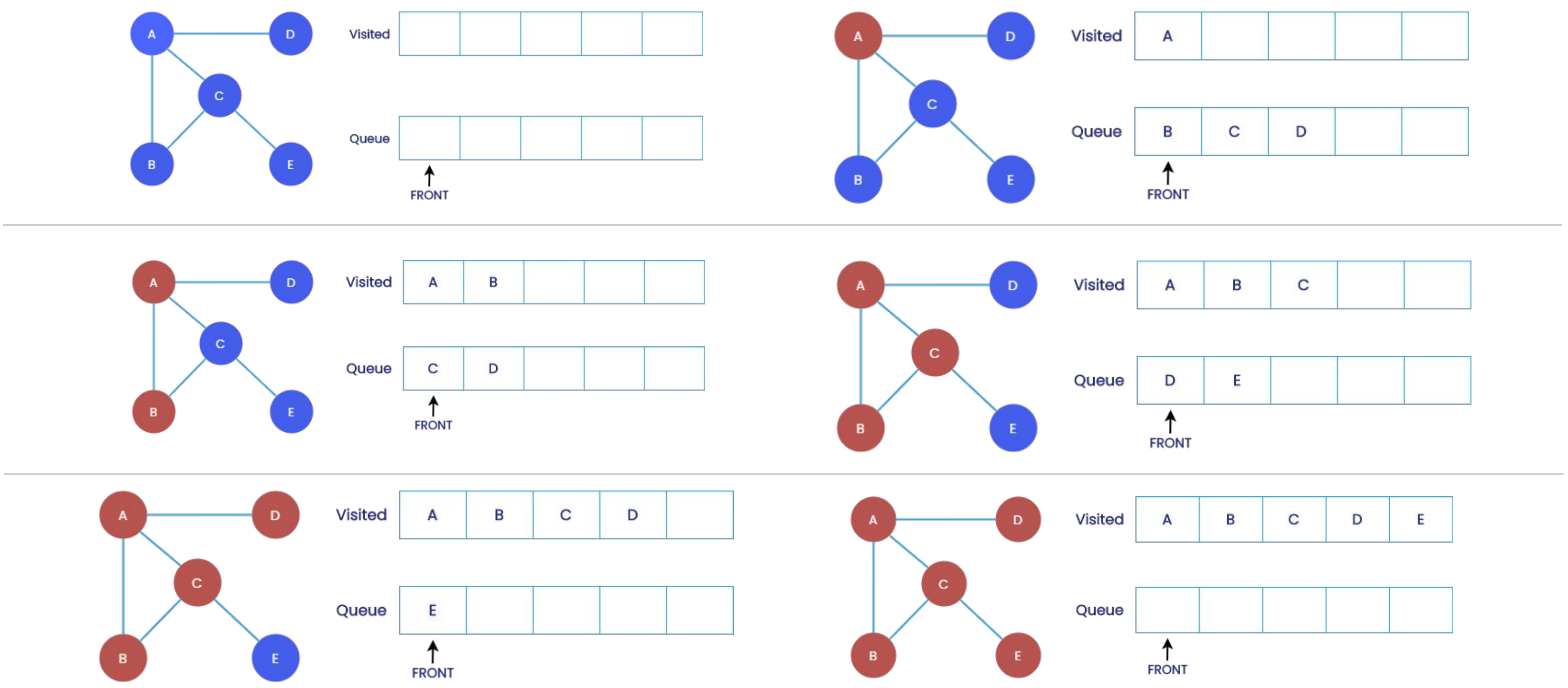


Breadth First Search (BFS)

- In BFS method, the travel starts from a node then carries onto its adjacent nodes.
- It traverse all the sibling nodes within a level and then moves to next level.
- This continues until all the vertices are visited.
- DFS implementation puts each nodes of the graph in following lists
  - Visited
  - Not Visited (Queue)
- This marking is used to avoid cycles while traversing.

```
Algorithm: BFS
 1.   Put the starting node A to the back of the Queue.
 2.   Repeat Steps 3 and 4 until Queue is empty.
 3.   Remove the front node N of Queue and add it to the
      Visited List.
 4.   Add all unvisited neighbours of N to the rear of the
      Queue.
 5.   Exit
```

Breadth First Search (BFS): An Example

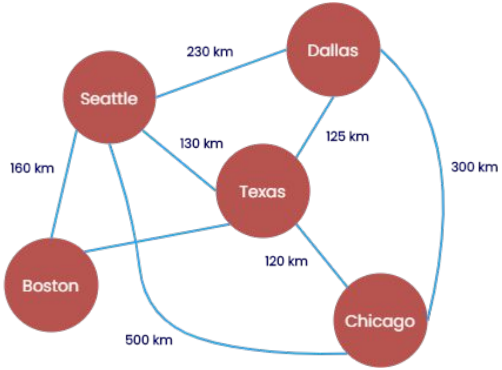


03

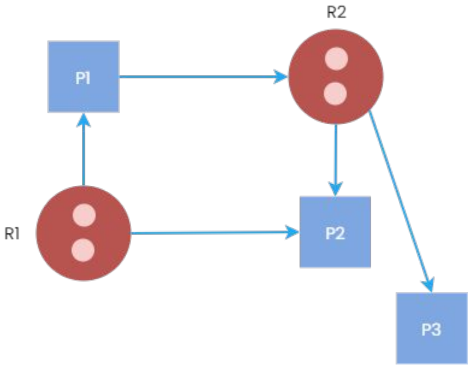
Applications

Applications of Graph data structure.

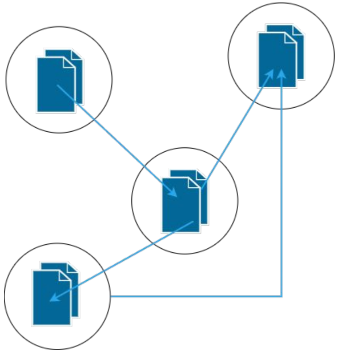
Applications



Google Map as a Graph



Resource Allocation Graph



World Wide Web as a Graph

- In Facebook, Users, Pages, Groups, etc. - nodes and edges - connection/relation.
- Path optimization Algorithms.
- Scientific Computations.
- Recommendation Algorithms.

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