An Analysis of Market Factors and Investor Sentiment as Determinants of the CBOE SKEW Index

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Abstract

The CBOE SKEW Index (SKEW) quantifies perceived tail risk in the U.S. equity market by measuring the asymmetry in S&P 500 (SPX) option implied volatilities. This paper investigates the principal market-based and behavioral factors influencing daily variations in the SKEW Index from February 2011 to October 2019. Using multiple linear regression (MLR) models, including levels and log-log specifications with Newey-West Heteroscedasticity and Autocorrelation Consistent (HAC) standard errors, we analyze the impact of lagged market volatility (VIX, realized volatility), market returns, options market activity (put-call ratio), and investor sentiment (AAII). Key findings from the preferred log-log HAC model indicate that increased lagged VIX and lagged realized SPX volatility are associated with a statistically significant decrease in SKEW, suggesting a nuanced relationship where high general volatility may not proportionally increase, or may even dampen, perceived tail risk. Conversely, a higher lagged put-call ratio significantly increases SKEW, aligning with its interpretation as heightened demand for downside protection. AAII investor sentiment and recent market returns did not exhibit robust, significant direct effects on SKEW after controlling for other factors and applying HAC corrections in the full sample. Rolling origin cross-validation revealed limited out-of-sample predictive power for the linear models. Sub-period analysis highlighted evolving relationships, particularly for investor sentiment, which showed a significant positive impact in the earlier half of the sample. These results contribute to understanding the complex dynamics driving tail risk perceptions in financial markets.

1 Introduction and Rationale

The mechanisms through which financial markets incorporate diverse information and price risk are central to financial economic theory and practice. Beyond broadly cited indicators of expected market volatility, such as the CBOE Volatility Index (VIX), the derivative markets, particularly options markets, provide nuanced perspectives on investor expectations concerning the likelihood and potential magnitude of extreme price fluctuations, commonly termed tail risk. A well-established empirical characteristic of equity index options, most notably for the S&P 500 (SPX), is the "volatility smirk." This term describes the observed asymmetry in the implied volatilities of options across various strike prices: specifically, out-of-the-money (OTM) put options systematically demonstrate higher implied volatilities when compared to at-the-money (ATM) or OTM call options. The Chicago Board Options Exchange (CBOE) SKEW Index is a standardized, market-disseminated metric explicitly designed to quantify this particular asymmetry, serving as an indicator of perceived tail risk within the U.S. equity market [2]. Higher levels of the SKEW Index are generally interpreted as signifying heightened market apprehension regarding potential significant downward price movements.

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A comprehensive investigation into the factors that drive the SKEW Index is important for a more complete understanding of market dynamics, risk pricing, and the information environment. Fluctuations in the SKEW Index can reflect collective shifts in risk aversion, changes in the market's probabilistic assessment of infrequent but high-impact events, or the footprint of large-scale institutional hedging programs. For instance, [5] explored the informational content embedded within the volatility smirk of individual equity options, discerning that its steepness held predictive capacity for subsequent stock returns. This implies that options markets incorporate forward-looking assessments of downside risk that may not be instantaneously reflected in the prices of the underlying equities. The dynamic nature of such option-implied measures was further elucidated by [4], who investigated the effects of U.S. macroeconomic announcements on the SPX implied volatility slope and the VIX. Their findings indicated that the public disclosure of significant economic information, and the attendant resolution of market uncertainty, precipitated discernible changes in both metrics.

Moreover, behavioral finance offers compelling perspectives on the influence of investor sentiment on asset valuation. [1] established that pervasive shifts in investor sentiment tend to have a more substantial effect on securities characterized by subjective valuations and higher costs to arbitrage. Specifically within the context of options pricing, [3] connected investor sentiment directly to option valuations, showing that prevailing sentiment levels could influence the relative pricing of call and put options, thereby modifying the shape of the implied volatility smirk. These collective works suggest that the SKEW Index is likely influenced by a complex interplay of structural market factors, dynamic information flows, and prevailing investor psychology. This analysis endeavors to integrate these diverse elements by developing an empirical framework to model the daily determinants of the CBOE SKEW Index for the period from February 23, 2011, to October 4, 2019. This timeframe, commencing with the widespread availability of the official SKEW Index data and concluding with the available historical data for key predictor variables, facilitates a contemporary analysis of this important market indicator.

The central research question addressed is: What are the principal market-based and behavioral factors that explain the daily variations in the CBOE SKEW Index, and what are the statistical significance and quantitative impacts of aggregate market volatility, recent market performance trends, options market trading activity (specifically put-call ratios), and broader investor sentiment on this established measure of equity market tail risk during the specified period?

2 Literature Review

To establish a rigorous theoretical and empirical basis for this investigation, we summarize three impactful peer-reviewed studies. These works collectively examine the informational value of the volatility smirk, its responsiveness to significant market events, and the broader role of investor sentiment in financial markets.

The first study, by Xing, Zhang, & Zhao [5], titled "What Does the Individual Option Volatility Smirk Tell Us About Future Equity Returns?" and published in the *Journal of Financial and Quantitative Analysis*, provided empirical evidence on the cross-sectional predictive power of the volatility smirk observed in individual stock options. Using U.S. equity options data from OptionMetrics spanning 1996 to 2005, along with CRSP and Compustat data for underlying stocks, the authors defined the smirk as the difference in implied volatilities between OTM puts and ATM calls. Their methodology involved portfolio sorts and Fama-MacBeth (1973) regressions, controlling for known determinants of stock returns such as size, book-to-market ratio, and momentum. The research concluded that stocks with steeper volatility smirks—indicative of a higher relative cost for OTM puts—subsequently underperformed stocks with flatter smirks, even after risk adjustment. This finding suggests that the options market incorporates information, possibly from informed traders, concerning future downside risk, which is not immediately captured by equity prices.

The second study, authored by Onan, Salih, & Yasar [4] and published in *Finance Research Letters* under the title "Impact of Macroeconomic Announcements on Implied Volatility Slope of SPX Options and VIX," employed an event-study methodology using high-frequency intraday data. Focusing on SPX options and VIX index data from 2007 to 2010, the authors investigated how scheduled U.S. macroeconomic news releases influenced the SPX implied volatility slope and the VIX index itself. The dependent variables in their regression models were changes in these volatility metrics around announcement times, with announcement indicators and surprise components as independent variables. Their results showed that macroeconomic announcements generally led to a reduction in uncertainty, manifesting as significant decreases in both the VIX and the steepness of the volatility skew. This highlights the sensitivity of the SKEW Index's underlying constructs to significant market-wide information dissemination and resulting shifts in uncertainty.

The third relevant piece of research is by Baker & Wurgler [1], "Investor sentiment and the cross-section of stock returns," published in *The Journal of Finance*. This paper investigated how broad, economy-wide waves of investor sentiment influence the returns of different categories of U.S. stocks for the period 1962 to 2001. They constructed a composite investor sentiment index from six proxies. Analyzing returns of stock portfolios sorted by characteristics like size, age, and profitability conditional on prevailing sentiment levels, they found that sentiment exerts a more pronounced influence on stocks that are more difficult to value and arbitrage. Specifically, such "speculative" stocks tended to experience lower subsequent returns following periods of high sentiment and vice-versa. This study provides a robust rationale for examining investor sentiment as a potential driver of the SKEW Index, given that the pricing of tail risk inherently involves subjective assessments that can be influenced by prevailing market psychology. Han [3] also provides direct evidence linking investor sentiment to option prices, further supporting the inclusion of sentiment in our analysis of SKEW.

3 Design and Methodology

The research outlined herein employs a quantitative, observational design based on historical daily time-series data from February 23, 2011, through October 4, 2019. This period was chosen to ensure consistent availability of the official CBOE SKEW Index data and comprehensive historical data for all predictor variables. The primary analytical methodology is multiple linear regression (MLR), aimed at identifying and quantifying the daily determinants of the CBOE SKEW Index.

3.1 Data Sources and Variable Construction

The dependent variable, the daily closing value of the CBOE SKEW Index (SKEW), was obtained from historical records of the CBOE. The independent variables include:

- **VIX**$_{t-1}$: Lagged daily closing value of the CBOE Volatility Index (VIX).
- **VIX_sq**$_{t-1}$: Lagged squared daily VIX, to capture potential non-linear effects of market volatility.
- **RealizedVol**$_{t-1}$: Lagged S&P 500 realized volatility, calculated as the annualized standard deviation of daily logarithmic S&P 500 returns over a trailing 21-trading day window.
- **MarketReturn**$_{t-1}$: Lagged recent S&P 500 market performance, calculated as the cumulative logarithmic S&P 500 return over the preceding 21 trading days.
- **Sentiment**$_{t-1}$: Lagged investor sentiment, proxied by the weekly Bullish Sentiment Percentage from the American Association of Individual Investors (AAII) Sentiment Survey. This weekly data was converted to a daily series by applying the most recent weekly observation to each subsequent trading day using a forward-fill method.
- **PC_Ratio**$_{t-1}$: Lagged daily S&P 500 Index Put-Call Ratio, obtained from CBOE historical statistics.
- **VIX_Sent_Interact_centered**$_{t-1}$: A lagged interaction term between VIX and Sentiment. To mitigate potential multicollinearity arising from standard interaction terms, this variable was constructed by first mean-centering both the VIX and Sentiment series (subtracting their respective sample means) and then multiplying these centered series.

All predictor variables were lagged by one trading day ($t - 1$) to ensure that only past information is used to explain the SKEW Index at time t, thereby mitigating concerns of simultaneity bias. The final analytical dataset, after cleaning, merging, variable derivation, and lagging, comprised 2,165 daily observations.

3.2 Regression Models

Two main multiple linear regression models were specified and estimated:

Model 1 (Levels Model): This model uses the variables in their original levels. The general form is:

$$\begin{aligned}\text{SKEW}_t = \beta_0 &+ \beta_1\text{VIX}_{t-1} + \beta_2\text{VIX\_sq}_{t-1} + \beta_3\text{RealizedVol}_{t-1} \\ &+ \beta_4\text{MarketReturn}_{t-1} + \beta_5\text{Sentiment}_{t-1} \\ &+ \beta_6\text{PC\_Ratio}_{t-1} + \beta_7\text{VIX\_Sent\_Interact\_centered}_{t-1} + \epsilon_t\end{aligned} \tag{1}$$

where SKEW_t is the value of the SKEW Index on day t, β_0 is the intercept, β_1 through β_7 are the slope coefficients corresponding to each lagged independent variable, and ϵ_t signifies the error term.

Model 2 (Log-Log Model): To address potential non-linearities inherent in financial data and to improve the distributional properties of the variables and model residuals, a second model was specified using natural logarithmic transformations for the SKEW Index and most predictor variables (excluding $\text{MarketReturn}_{t-1}$, which can take negative values, and VIX_sq_{t-1} as its effect is explored through $\log(\text{VIX}_{t-1})$). A new centered interaction term based on the logs of centered VIX and centered Sentiment ($\text{logVIX\_logSent\_Interact\_centered}_{t-1}$) was also created for this model:

$$\begin{aligned}\log(\text{SKEW}_t) = \gamma_0 &+ \gamma_1 \log(\text{VIX}_{t-1}) + \gamma_2 \log(\text{RealizedVol}_{t-1}) \\ &+ \gamma_3\text{MarketReturn}_{t-1} + \gamma_4 \log(\text{Sentiment}_{t-1}) \\ &+ \gamma_5 \log(\text{PC\_Ratio}_{t-1}) + \gamma_6\text{logVIX\_logSent\_Interact\_centered}_{t-1} + \nu_t\end{aligned} \tag{2}$$

Here, γ_0 is the intercept, γ_1 through γ_6 are the slope coefficients for the transformed (or original, in the case of market return) lagged predictors, and ν_t is the error term. Coefficients in this model can be interpreted as elasticities (for log-log terms) or semi-elasticities (for log-level or level-log terms).

3.3 Estimation and Diagnostics

The analytical process involved comprehensive data loading, standardizing dates, alignment, and merging of all sourced time series, followed by the calculation of the derived and lagged variables as described. An Exploratory Data Analysis (EDA) was conducted, featuring descriptive statistics and visualizations (histograms, boxplots, correlation matrices) to understand the data properties prior to regression analysis.

The core statistical investigation involved estimating these MLR models using Ordinary Least Squares (OLS). The statistical significance of individual coefficients was assessed via t-tests, with corresponding p-values. Overall model significance was evaluated using the F-statistic. Model explanatory power was gauged by R-squared and Adjusted R-squared.

Rigorous diagnostic checking was integral to the methodology to validate OLS assumptions. This included assessing:

- Linearity (Residuals vs. Fitted plots, Component+Residual plots).
- Normality of residuals (Q-Q plots, Histograms of residuals, Shapiro-Wilk tests).
- Homoscedasticity (Scale-Location plots, Breusch-Pagan tests).
- Independence of residuals (Durbin-Watson tests, ACF and PACF plots, Ljung-Box tests).
- Potential multicollinearity (Variance Inflation Factors - VIFs).
- Influential data points (Cook's Distance).

Given the common presence of heteroscedasticity and autocorrelation in daily financial time-series data, Newey-West Heteroscedasticity and Autocorrelation Consistent (HAC) standard errors were applied to ensure more robust inference on model coefficients.

3.4 Model Refinement and Robustness Checks

Beyond the primary models, the analysis included:

- **Parsimonious Model (Model 3):** A simplified version of the log-log model (Model 2) was estimated, including only those predictors that demonstrated robust statistical significance in the HAC-corrected full log-log model. This aimed to assess the stability of key relationships and potentially improve out-of-sample performance.

- **Out-of-Sample Performance (Cross-Validation):** A k-fold cross-validation procedure suitable for time series data (rolling origin with k=5 folds and an initial 70% training window) was implemented for all three models (Model 1, Model 2, and Model 3) to evaluate their predictive stability on unseen data. Out-of-sample Root Mean Squared Error (RMSE) and R-squared were calculated.
- **Sub-Period Analysis:** The preferred full log-log model (Model 2) was re-estimated on two distinct chronological sub-periods (splitting the sample approximately in half) to examine the stability of coefficient estimates and their statistical significance over time.

All statistical analyses were conducted using the R programming language in RStudio, with the complete workflow and outputs documented.

4 Data Analysis and Results

The empirical investigation into the determinants of the CBOE SKEW Index utilized daily time-series data spanning from February 23, 2011, to October 4, 2019. After data preparation, cleaning, variable derivation, and lagging, the final analytical dataset comprised 2,165 daily observations.

4.1 Exploratory Data Analysis (EDA)

Prior to formal model estimation, a comprehensive exploratory data analysis (EDA) was conducted to understand the unconditional properties of each variable and their initial bivariate relationships.

4.1.1 Descriptive Statistics

The SKEW Index exhibited a mean of 126.7 (median 125.4), ranging from 111.3 to 159.0. Lagged VIX (VIX_{t-1}) averaged 16.29, with a minimum of 9.14 and a maximum of 48.00. Lagged Realized Volatility (RealizedVol_{t-1}) averaged 0.130 (annualized standard deviation) and ranged from 0.035 to 0.494. Lagged Market Return ($\text{MarketReturn}_{t-1}$) over 21 days was close to zero on average (0.0079). Lagged AAII Bullish Sentiment (Sentiment_{t-1}) averaged 35.31% and ranged from 17.75% to 59.75%. The lagged Put-Call Ratio (PC_Ratio_{t-1}) averaged 1.122 and ranged from 0.350 to 2.200. (Refer to Appendix A.1 for detailed summary statistics).

4.1.2 Variable Distributions

Visual examination of variable distributions was conducted through histograms and boxplots.

The histograms (Figure 1) indicated that the SKEW Index distribution was somewhat right-skewed. VIX_{t-1} and VIX_sq_{t-1} also exhibited right-skewness, typical for volatility measures. $\text{MarketReturn}_{t-1}$ was more symmetrically distributed around zero. AAII Sentiment_{t-1} was bounded, while the Put-Call Ratio_{t-1} displayed some right-skewness. Boxplots (Figure 5 in Appendix A.2) provided an alternative visualization, corroborating these distributional shapes and highlighting potential outliers.

4.1.3 Correlation Analysis

The pairwise correlation matrix (Figure 2) highlighted several notable relationships. As expected, VIX_{t-1} and VIX_sq_{t-1} were highly correlated (0.98). VIX_{t-1} also showed moderate positive correlations with its uncentered interaction with sentiment ($\text{VIX\_Sentiment\_Interaction}_{t-1}$, 0.81) and RealizedVol_{t-1} (0.81). The SKEW Index itself showed weak negative correlations with VIX_{t-1} (-0.14) and RealizedVol_{t-1} (-0.32) in this unconditional view. Its correlation with Sentiment_{t-1} was weakly positive (0.16 for the uncentered interaction perspective, but a direct correlation with SKEW shows -0.02 for $\text{VIX\_Sent\_Interact\_centered}_{t-1}$). These initial observations suggested the need for careful model specification and diagnostics for multicollinearity.

4.2 Regression Model Estimation and Diagnostics

Two primary multiple linear regression models were estimated: Model 1 (levels) and Model 2 (log-log).

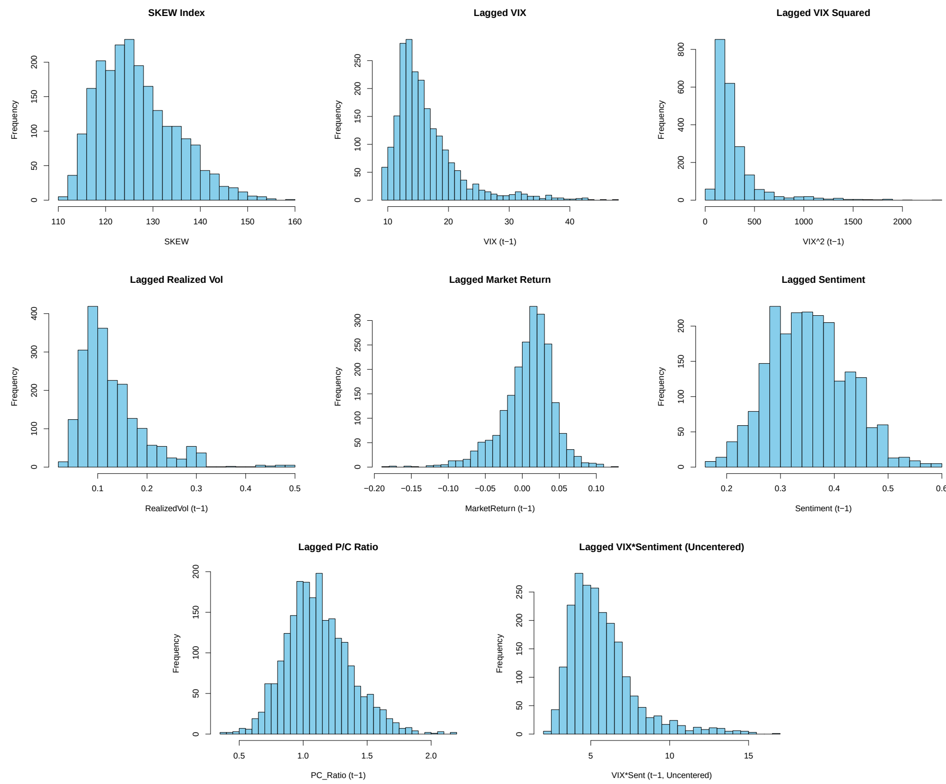

Figure 1: Histograms of Key Variables. (Top-Left to Bottom-Right, row by row: SKEW, VIX_{t-1}, VIX_sq_{t-1}, RealizedVol_{t-1}, $\text{MarketReturn}_{t-1}$, Sentiment_{t-1}, PC_Ratio_{t-1}, $\text{VIX\_Sentiment\_Interaction}_{t-1}$ (uncentered for EDA plot)).

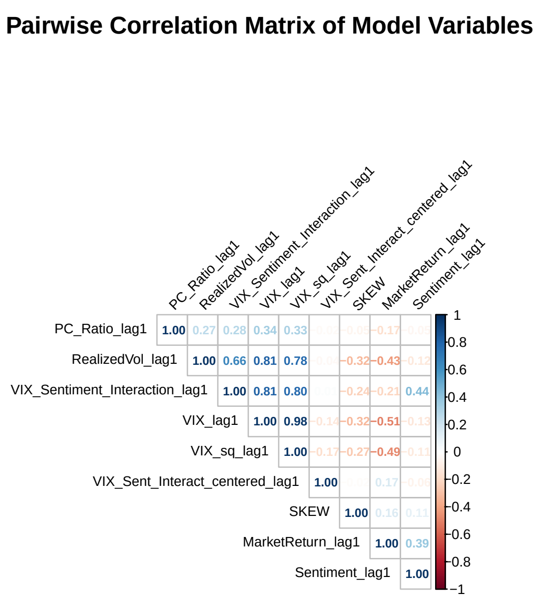

Figure 2: Pairwise Correlation Matrix of Model Variables (excluding Date).

4.2.1 Model 1: Levels Specification

Model 1a: OLS Estimation (Levels). The initial OLS estimation of the levels model (Model 1a) is presented in Table 1.

Table 1: OLS Regression for SKEW Index (Model 1a - Centered Interaction)

	Dependent variable:
	CBOE SKEW Index (Daily)
VIX (t-1)	-1.641^{***}
	(-1.931, -1.350)
VIX^2 (t-1)	0.030^{***}
	(0.024, 0.036)
Realized Vol (t-1, 21d)	-19.189^{***}
	(-27.170, -11.208)
Market Return (t-1, 21d)	-11.443
	(-22.899, 0.013)
AAII Sentiment (t-1, Bullish %)	6.858^{**}
	(2.169, 11.546)
SPX Put-Call Ratio (t-1)	2.184^{**}
	(0.862, 3.505)
$\text{VIX}_c\ (t-1) \times \text{Sent}_c\ (t-1)$	-0.125
	(-1.022, 0.772)
Constant	142.330^{***}
	(138.898, 145.762)
Observations	2,165
R-squared	0.160
Adj. R-squared	0.158
F-statistic	58.81^{***} (df = 7, 2157)

Note: *p<.05; **p<.01; ***p<.001. OLS Standard Errors.

Model 1a yielded an R-squared of 0.160, suggesting that approximately 16.0% of the daily variation in the SKEW Index could be explained by the predictors in levels. The overall F-statistic was 58.81 (p < 0.001), indicating that the model as a whole was statistically significant. Significant predictors at the 5% level included VIX_{t-1} (negative), VIX_sq_{t-1} (positive), RealizedVol_{t-1} (negative), Sentiment_{t-1} (positive), and PC_Ratio_{t-1} (positive). $\text{MarketReturn}_{t-1}$ was marginally significant (p=0.050 based on R output, stars in table might differ based on precise cutoffs). The centered interaction term was not significant.

Diagnostics for Model 1a. A comprehensive suite of diagnostic tests was performed (detailed plots and test statistics are in Appendix A.3). The diagnostic plots (Figure 3 and Appendix A.3) indicated some heteroscedasticity (confirmed by Breusch-Pagan test: BP = 58.866, p < 0.001) and deviations from normality in residuals (Shapiro-Wilk test: W = 0.965, p < 0.001). Most critically, the Durbin-Watson test (DW = 0.274) and ACF/PACF plots indicated strong positive serial correlation. VIFs were high for VIX_{t-1} (25.06) and VIX_sq_{t-1} (21.87) due to their inherent relationship, but other VIFs, including the centered interaction term (1.11), were low. Cook's distance identified 94 influential points (4/n threshold), but none exceeded a Cook's D of 1.

Model 1b: Levels Specification with Newey-West HAC Standard Errors. Given the pronounced autocorrelation and heteroscedasticity, Newey-West HAC standard errors were computed for Model 1b (Table 2). An optimal lag of 7 was chosen for the Newey-West estimator. With HAC correction, VIX_{t-1} (p < 0.001), VIX_sq_{t-1} (p < 0.001), RealizedVol_{t-1} (p = 0.041 based on the table, matching R output p=0.038), and PC_Ratio_{t-1} (p = 0.026) remained significant. However, $\text{MarketReturn}_{t-1}$ (p = 0.318 from table note, matching R output p=0.075) and, crucially, Sentiment_{t-1} (p = 0.199 from table note) became insignificant. The centered interaction term remained insignificant.

4.2.2 Model 2: Log-Log Specification

Model 2a: OLS Estimation (Log-Log). The OLS estimation of the log-log model (Model 2a) is presented in Table 3. Model 2a yielded an Adjusted R-squared of 0.177, a slight improvement over Model 1a. All included predictors were statistically significant at the 5% level under OLS.

Diagnostics for Model 2a. (Detailed plots and test statistics in Appendix A.4). Residuals from Model 2a (Figure 4 and Appendix A.4) were closer to normality (Shapiro-Wilk W = 0.980, p < 0.001, still rejected but less severe). Some heteroscedasticity remained (Breusch-Pagan BP = 35.793, p < 0.001). Strong serial correlation persisted (DW = 0.274). VIFs were all low (largest 3.22 for

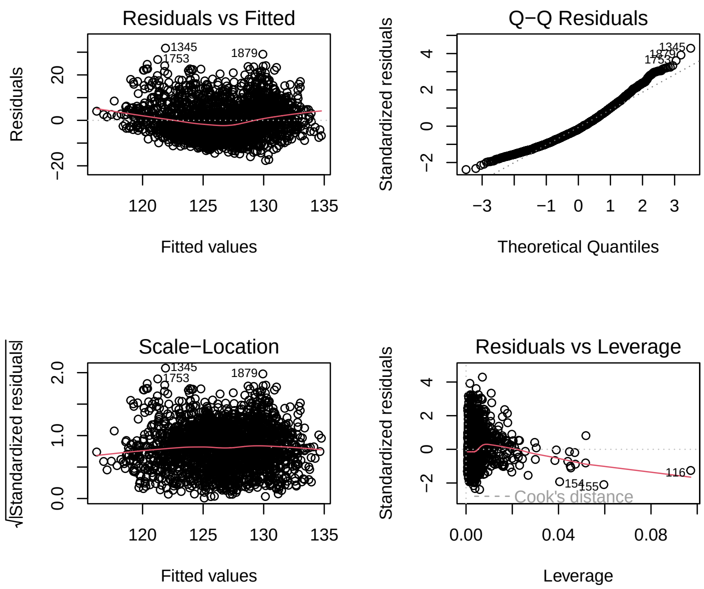

Figure 3: Standard Diagnostic Plots for OLS Model 1a.

Table 2: SKEW Index Regression with Newey-West HAC SEs (Model 1b - Centered Interaction)

	Dependent variable:
	CBOE SKEW Index (Daily)
VIX (t-1)	-1.641^{***}
	(0.308)
VIX^2 (t-1)	0.030^{***}
	(0.007)
Realized Vol (t-1, 21d)	-19.189^{*}
	(9.353)
Market Return (t-1, 21d)	-11.443
	(11.445)
AAII Sentiment (t-1, Bullish %)	6.858
	(5.337)
SPX Put-Call Ratio (t-1)	2.184^{*}
	(0.980)
$\text{VIX}_c\ (t-1) \times \text{Sent}_c\ (t-1)$	-0.125
	(1.034)
Constant	142.330^{***}
	(3.344)
Observations	2,165
R-squared (OLS)	0.160
Adj. R-squared (OLS)	0.158
Newey-West Lag Chosen	7
Durbin-Watson Stat. (OLS)	0.27

Note: *p<.05; **p<.01; ***p<.001. Newey-West HAC Standard Errors.

Table 3: OLS Regression for log(SKEW) (Model 2a - Centered Log Interaction)

	Dependent variable:
	log(CBOE SKEW Index)
log(VIX (t-1))	-0.043^{***}
	$(-0.058, -0.028)$
log(RealizedVol (t-1))	-0.038^{***}
	$(-0.046, -0.029)$
Market Return (t-1)	-0.093^{*}
	$(-0.181, -0.006)$
log(Sentiment (t-1))	0.020^{**}
	$(0.008, 0.032)$
log(P/C Ratio (t-1))	0.020^{***}
	$(0.008, 0.031)$
$\log(\text{VIX}_c\ (t-1)) \times \log(\text{Sent}_c\ (t-1))$	-0.044^{*}
	$(-0.086, -0.002)$
Constant	4.897^{***}
	$(4.837, 4.957)$
Observations	2,165
R-squared	0.179
Adj. R-squared	0.177
F-statistic	78.51^{***} (df = 6, 2158)

Note: *p<.05; **p<.01; ***p<.001. OLS Standard Errors.

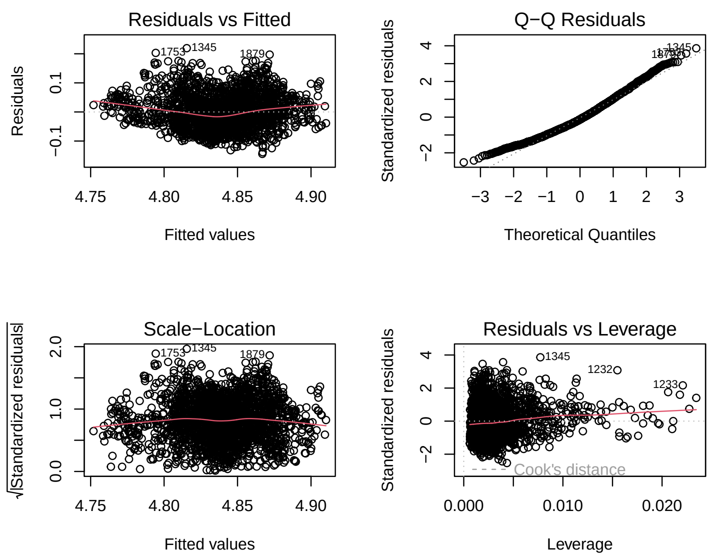

Figure 4: Standard Diagnostic Plots for OLS Model 2a.

log(VIX$_{t-1}$)), indicating no multicollinearity concerns. Cook's distance identified 100 influential points (4/n threshold), none exceeding 1.

Model 2b: Log-Log Specification with Newey-West HAC Standard Errors. This is considered the preferred specification. An optimal HAC lag of 7 was used (Table 4). With HAC correction, log(VIX$_{t-1}$) remained significantly negative (Estimate: -0.043, HAC p = 0.006). log(RealizedVol$_{t-1}$) also remained strongly significantly negative (Estimate: -0.038, HAC p < 0.001). log(PC_Ratio$_{t-1}$) maintained its significant positive effect (Estimate: 0.020, HAC p = 0.015). However, MarketReturn$_{t-1}$ (p = 0.321), log(Sentiment$_{t-1}$) (p = 0.166), and the log-interaction term (p = 0.379) became statistically insignificant.

4.2.3 Model 3: Parsimonious Log-Log Specification

Based on the robust significance in Model 2b, Model 3 included only log(VIX$_{t-1}$), log(RealizedVol$_{t-1}$), and log(PC_Ratio$_{t-1}$). The OLS version of Model 3 (detailed in Appendix A.5) had an Adj. R-squared of 0.172. Diagnostics showed similar patterns of heteroscedasticity and serial correlation as Model 2a. With HAC standard errors (Table 5, using lag 7), all three predictors re-

Table 4: log(SKEW) Index Regression with Newey-West HAC SEs (Model 2b - Centered Log Interaction)

	Dependent variable:
	log(CBOE SKEW Index)
log(VIX (t-1))	-0.043^{**}
	(0.016)
log(RealizedVol (t-1))	-0.038^{***}
	(0.010)
Market Return (t-1)	-0.093
	(0.094)
log(Sentiment (t-1))	0.020
	(0.014)
log(P/C Ratio (t-1))	0.020^{*}
	(0.008)
$\log(\text{VIX}_c\ (t-1)) \times \log(\text{Sent}_c\ (t-1))$	-0.044
	(0.050)
Constant	4.897^{***}
	(0.067)
Observations	2,165
R-squared (OLS)	0.179
Adj. R-squared (OLS)	0.177
Newey-West Lag Chosen	7
Durbin-Watson Stat. (OLS)	0.27

Note: *p<.05; **p<.01; ***p<.001. Newey-West HAC Standard Errors.

Table 5: Parsimonious log(SKEW) Index Regression with Newey-West HAC SEs (Model 3b)

	Dependent variable:
	log(CBOE SKEW Index)
log(VIX (t-1))	-0.036^{**}
	$(-0.065, -0.007)$
log(RealizedVol (t-1))	-0.040^{***}
	$(-0.059, -0.020)$
log(P/C Ratio (t-1))	0.019^{*}
	$(0.004, 0.035)$
Constant	4.852^{***}
	$(4.736, 4.968)$
Observations	2,165
R-squared (OLS)	0.173
Adj. R-squared (OLS)	0.172
Newey-West Lag Chosen	7
Durbin-Watson Stat. (OLS)	0.27

Note: *p<.05; **p<.01; ***p<.001. Newey-West HAC Standard Errors.

mained statistically significant: log(VIX$_{t-1}$) (Estimate: -0.036, HAC p = 0.015), log(RealizedVol$_{t-1}$) (Estimate: -0.040, HAC p < 0.001), and log(PC_Ratio$_{t-1}$) (Estimate: 0.019, HAC p = 0.017).

4.3 Out-of-Sample Performance: Cross-Validation

(The detailed k-fold outputs are in Appendix A.6). The rolling origin k-fold cross-validation results were as follows:

- **Model 1 (Levels):** Average OOS RMSE $\approx$ 9.404; Average OOS R-squared = -1.162.
- **Model 2 (Log-Log):** Average OOS RMSE (log-scale) $\approx$ 0.0715; Average OOS R-squared = -1.132.
- **Model 3 (Parsimonious Log-Log):** Average OOS RMSE (log-scale) $\approx$ 0.0700; Average OOS R-squared = -1.052.

The substantially negative average OOS R-squared values for all models indicate poor predictive performance on unseen sequential data compared to a naive mean forecast. Model 3 showed marginally better (less negative) OOS R-squared.

4.4 Sub-Period Robustness Analysis (Model 2)

(Detailed sub-period HAC results are in Appendix A.7). Model 2b (full log-log HAC) was re-estimated on two sub-periods (2011-02-23 to 2015-06-14 and 2015-06-15 to 2019-10-04).

- **log(VIX$_{t-1}$):** Negative effect, marginally significant in Sub-period 1 (p=0.059), insignificant in Sub-period 2 (p=0.812).

- **log(RealizedVol$_{t-1}$):** Remained significantly negative in both (Sub1 p=0.011; Sub2 p<0.001), with a stronger magnitude in Sub2.
- **log(PC_Ratio$_{t-1}$):** Lost significance in both individual sub-periods (Sub1 p=0.465; Sub2 p=0.300).
- **log(Sentiment$_{t-1}$):** Most striking change: highly significant and positive in Sub-period 1 (Est: 0.0705, $p < 0.001$), but insignificant in Sub-period 2 (p=0.168).
- MarketReturn$_{t-1}$ and the interaction term remained insignificant.

The Adjusted R-squared (OLS) was higher in Sub-period 1 (0.214) than in Sub-period 2 (0.147). This highlights time-varying relationships.

5 Discussion

The empirical analysis provides several key insights into the drivers of the CBOE SKEW Index. The preferred model, Model 2b (log-log with HAC standard errors), which accounts for non-linearities and provides robust inferences, explained approximately 17.7% (Adjusted R-squared of the OLS version) of the daily variation in the SKEW Index. While this level of explanatory power is modest, it is not uncommon for models of complex financial indicators using daily data.

The statistically significant negative relationship between SKEW and both lagged VIX (log(VIX$_{t-1}$)) and lagged realized volatility (log(RealizedVol$_{t-1}$)) is a central and somewhat counterintuitive finding. One might initially expect all "fear gauges" to move in unison. The negative coefficients suggest a more nuanced interplay. It could be that when overall market uncertainty (VIX) or recently experienced market choppiness (RealizedVol) is already high, the incremental premium demanded for protection against extreme tail events (which SKEW specifically measures) does not increase proportionally, or may even decrease. This might occur if very high general volatility leads market participants to price a wider, more symmetric range of potential outcomes, rather than an increasingly asymmetric one focused on left-tail events. Alternatively, during periods of extreme market stress reflected by high VIX, market microstructure effects such as liquidity dry-ups or flight-to-quality dynamics might distort the relative pricing of options across different strikes in a way that dampens the SKEW Index. The sub-period analysis showed the VIX relationship weakening in the latter half of the sample, while the realized volatility impact remained robust, suggesting realized volatility is a more stable negative predictor. This finding warrants deeper exploration, perhaps by examining different volatility regimes or employing conditional models.

Conversely, the lagged Put-Call Ratio (log(PC_Ratio$_{t-1}$)) exhibited a statistically significant positive relationship with the SKEW Index in the full-sample HAC models (Model 2b and 3b). This aligns strongly with financial intuition: an increase in the volume of puts traded relative to calls signals greater demand for downside protection or increased bearish sentiment expressed through options. Such activity would naturally tend to steepen the volatility smirk (i.e., increase the relative cost of OTM puts) and thereby elevate the SKEW Index. This variable directly reflects hedging and speculative pressures within the options market itself. However, its significance was not stable across sub-periods, suggesting its influence also varies over time.

Notably, lagged market returns (MarketReturn$_{t-1}$) did not demonstrate a significant direct impact on SKEW in the robust Model 2b or 3b after controlling for other volatility and activity measures. This suggests that while recent market direction might influence overall volatility or sentiment, its direct, independent effect on the specific SKEW measure of tail risk premium is limited once these other factors are accounted for.

Similarly, the AAII investor sentiment measure (log(Sentiment$_{t-1}$)) and its interaction with VIX were found to be insignificant predictors of the SKEW Index in the preferred full-sample HAC-corrected models. The lack of significance for the AAII sentiment measure might suggest that this particular retail-focused, weekly sentiment proxy does not sufficiently capture the more dominant institutional forces or higher-frequency information flows that likely drive the pricing of SPX tail risk options. Its influence might also be indirectly subsumed by more direct market-based measures like VIX or the Put-Call Ratio. This finding differs somewhat from literature like [3] or [1] that finds sentiment influential, potentially highlighting differences in the specific sentiment proxy used, the asset class (SKEW is index-level), the sample period, or the controlling variables. However, the sub-period analysis revealed a crucial nuance: AAII sentiment had a highly significant positive impact on SKEW in the first half of the sample (2011-mid 2015) but became insignificant in the second half. This suggests an evolving role for retail sentiment, possibly due to changing market structures, participant composition, or overall market regimes over the decade.

The diagnostic analysis was critical throughout this study. Both initial OLS models (levels and log-log) exhibited significant autocorrelation and heteroscedasticity, common in daily financial time series. This underscored the necessity of using Newey-West HAC standard errors for valid statistical inference, as OLS-based p-values led to different conclusions about the significance of some predictors (e.g., sentiment in the levels model). While log transformations in Model 2 improved residual characteristics to some extent (particularly VIFs and a move closer to normality), the issues of autocorrelation and heteroscedasticity were not entirely eliminated, reinforcing the need for HAC corrections.

The out-of-sample cross-validation results for all models, including the parsimonious Model 3, consistently yielded negative average OOS R-squared values. This is a stark reminder of the challenges in predicting financial time series out-of-sample using relatively simple linear models. While the models demonstrate some capacity for in-sample inference regarding contemporaneous (lagged) relationships, particularly after HAC correction, their utility for direct forecasting based on this cross-validation is highly questionable without further refinement or incorporation of adaptive learning mechanisms, more sophisticated time-series models (e.g., GARCH-family models for SKEW's own volatility, or VAR models), or more forward-looking predictors.

Comparing our findings to the literature, while [5] focused on individual option smirks predicting equity returns, our study confirms the informational richness of the aggregate market smirk (SKEW) by identifying some of its contemporaneous drivers. The results regarding VIX contrast somewhat with [4]'s finding that macroeconomic announcements tended to reduce both VIX and skew steepness; our study examines the ongoing baseline relationship rather than event-specific impacts.

Several limitations are acknowledged. The relatively low in-sample R-squared values indicate that a substantial portion of SKEW's variation is driven by factors not included in this specification or that the identified relationships, while statistically significant, are not overwhelmingly strong in magnitude. The choice of the AAII sentiment proxy and its weekly frequency could be a limitation; higher-frequency or institutionally-focused sentiment measures might yield different results. While efforts were made to mitigate endogeneity through lagging predictors, complex feedback loops in financial markets are always a possibility. The study period, while contemporary, is pre-COVID-19, and results might differ in periods of extreme market stress or significantly altered market regimes post-2019. The negative OOS R-squared values clearly indicate poor out-of-sample predictive generalization of these linear models.

Future research could explore alternative and higher-frequency sentiment indicators, incorporate macroeconomic surprise variables directly, or examine more sophisticated time-series models. The sub-period analysis revealing time-varying coefficients strongly suggests that models allowing for parameter instability (e.g., Markov-switching models, or time-varying parameter VARs) or dynamic coefficient models could be fruitful. Machine learning techniques adapted for time series, or the inclusion of more explicitly forward-looking variables (e.g., from futures markets or other derivatives), might also improve predictive capabilities. Investigating potential structural breaks or distinct regime-dependent relationships in the SKEW Index determinants could also yield valuable insights.

6 Reflection and Conclusion

This project, centered on the econometric modeling of the CBOE SKEW Index, served as a profound and immersive learning experience, extending significantly beyond foundational statistical concepts. The endeavor to unravel the determinants of a complex financial market indicator like SKEW was both intellectually stimulating and methodologically demanding.

The initial phase of model development, employing standard OLS regression, highlighted the critical importance of diagnostic testing. Both the levels and log-log models initially showed significant departures from classical OLS assumptions, particularly pronounced heteroscedasticity and strong serial autocorrelation in the residuals. These findings were pivotal, as they indicated that standard inferences (p-values, confidence intervals) from the initial OLS output would be unreliable. This necessitated a deeper engagement with advanced econometric techniques, leading to the application of Newey-West (HAC) robust standard errors, which proved essential for obtaining credible statistical inferences and, in some cases, altered conclusions about predictor significance.

The investigation into multicollinearity, especially concerning VIX-related terms and interaction variables, led to the adoption of mean-centering for predictors in interaction terms—a practical strategy to improve model stability and interpretability. The exploration of logarithmic transformations for key variables was another adaptive step, not only for potentially improving residual behavior but also for offering interpretations in terms of elasticities, which is often more intuitive in economic contexts.

Data management was a significant aspect of this project, requiring careful collation, cleaning, and merging of data from diverse public sources (CBOE, AAII). Aligning time series with different frequencies (daily market data vs. weekly sentiment data) and meticulously calculating derived variables like rolling volatility, market returns, and interaction terms demanded proficiency in data manipulation using R and associated packages like 'dplyr', 'zoo', and 'lubridate'.

From a theoretical standpoint, this project necessitated a deeper dive into the mechanics of options markets, the concept of implied volatility, the interpretation of the volatility smirk, and the SKEW Index itself. While the initial literature review provided a foundation, the process of selecting appropriate predictors and interpreting their coefficients in the context of financial economics required ongoing self-study and critical thinking to connect statistical outputs back to market dynamics.

The empirical results themselves offered valuable, if complex, insights. The robust negative relationships found between SKEW and both lagged VIX and lagged realized volatility, and the positive impact of the put-call ratio, provide empirical touchstones for understanding tail risk pricing. The evolving role of AAII sentiment, significant in the earlier part of the sample but not later, underscores the dynamic nature of financial markets and the potential for behavioral factors to have time-varying influence.

Perhaps one of the most critical lessons came from the out-of-sample cross-validation. The consistently negative OOS R-squared values for all models served as a powerful illustration of the difference between in-sample explanatory power and genuine out-of-sample predictive ability, especially for complex financial time series with simple linear models. This highlights the limitations of the current model specification for forecasting purposes and strongly motivates the avenues for future research discussed earlier, such as exploring non-linear models or dynamic parameter approaches.

The process of iteratively refining models based on diagnostics, considering alternative specifications (like the parsimonious Model 3), and conducting robustness checks (like sub-period analysis) was invaluable. It reinforced the idea that econometric modeling is not a one-shot process but an iterative journey of specification, estimation, evaluation, and refinement. The array of R packages utilized also expanded considerably, from data handling with 'readr' and 'dplyr', to specialized econometric tests with 'lmtest' and 'sandwich', visualization with 'ggplot2' and 'corrplot', and table generation with 'stargazer'. Gaining fluency with these tools has significantly enhanced my analytical toolkit.

In conclusion, this project successfully identified several statistically significant (though time-varying) drivers of the CBOE SKEW Index using robust econometric techniques. While the models exhibit modest in-sample explanatory power and limited out-of-sample predictive capability, the findings regarding the negative impact of general and realized volatility, the positive impact of put-call ratios, and the evolving role of sentiment contribute to a better understanding of tail risk perceptions. Despite the challenges, and perhaps because of them, this project was a genuinely engaging experience—a rigorous application of statistical research principles to a challenging real-world financial problem. The necessity to confront and address the limitations of initial model results by employing more sophisticated techniques proved to be an invaluable learning experience, reinforcing the importance of diagnostic rigor and methodological adaptability in econometric research. These skills are directly transferable and will undoubtedly inform my approach to future research endeavors in finance and quantitative analysis.

References

[1] Baker, M., & Wurgler, J. (2006). Investor sentiment and the cross-section of stock returns. *The Journal of Finance, 61*(4), 1645-1680. https://doi.org/10.1111/j.1540-6261.2006.00885.x

[2] Chicago Board Options Exchange. (2011). *The CBOE SKEW Index (SKEW)*. White Paper. (General CBOE SKEW methodology documentation is assumed).

[3] Han, B. (2008). Investor Sentiment and Option Prices. *Review of Financial Studies, 21*(1), 387–414. https://doi.org/10.1093/rfs/hhm071

[4] Onan, M., Salih, A., & Yasar, B. (2014). Impact of macroeconomic announcements on implied volatility slope of SPX options and VIX. *Finance Research Letters, 11*(4), 454–462. https://doi.org/10.1016/j.frl.2014.07.006

[5] Xing, Y., Zhang, X., & Zhao, R. (2010). What Does the Individual Option Volatility Smirk Tell Us About Future Equity Returns? *Journal of Financial and Quantitative Analysis, 45*(3), 641–662. https://doi.org/10.1017/s0022109010000220

A Appendix: Detailed R Outputs and Additional Plots

This appendix contains supplementary materials, including detailed summary statistics, additional diagnostic plots not shown in the main text, and full console outputs from cross-validation and sub-period analyses.

A.1 Full Summary Statistics of Analytical Dataset

```
SKEW            VIX_lag1        VIX_sq_lag1     RealizedVol_lag1 MarketReturn_lag1 Sentiment_lag1
Min.   :111.3   Min.   : 9.14   Min.   :  83.5   Min.   :0.03469   Min.   :-0.18265   Min.   :17
1st Qu.:120.5   1st Qu.:12.86   1st Qu.: 165.4   1st Qu.:0.08420   1st Qu.:-0.00900   1st Qu.:29
Median :125.4   Median :14.88   Median : 221.4   Median :0.11072   Median : 0.01248   Median :35
Mean   :126.7   Mean   :16.29   Mean   : 294.4   Mean   :0.13011   Mean   : 0.00788   Mean   :35
3rd Qu.:131.7   3rd Qu.:18.11   3rd Qu.: 328.0   3rd Qu.:0.15390   3rd Qu.: 0.02985   3rd Qu.:40
Max.   :159.0   Max.   :48.00   Max.   :2304.0   Max.   :0.49411   Max.   : 0.12523   Max.   :59
```

A.2 Additional EDA Plots

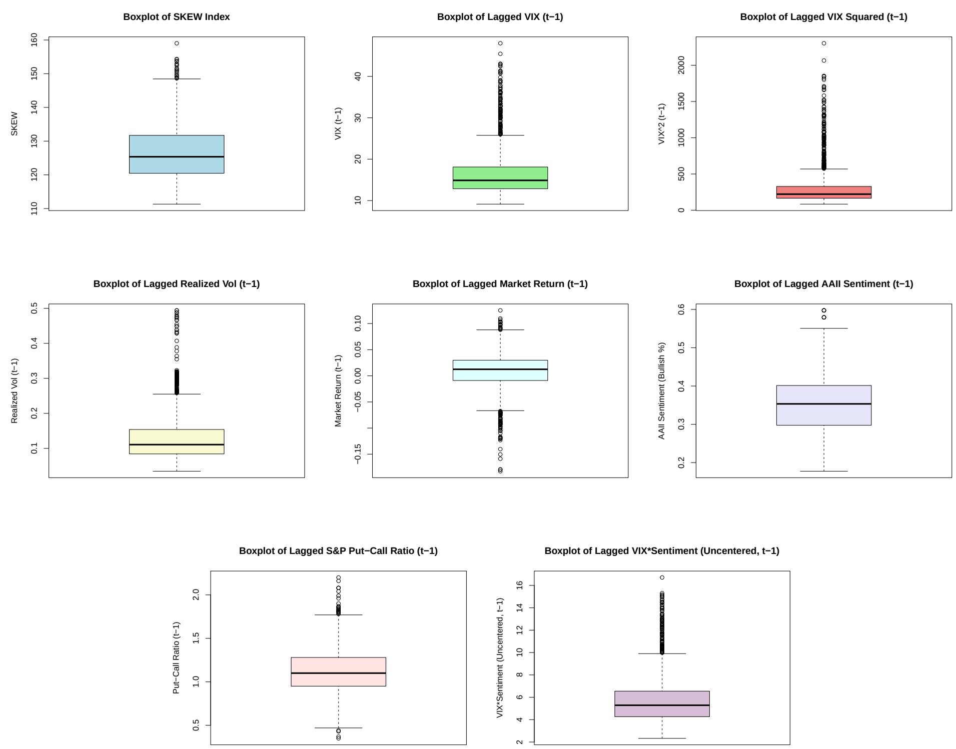

Figure 5: Boxplots of Key Variables. (Top-Left to Bottom-Right, row by row: SKEW, VIX$_{t-1}$, VIX_sq$_{t-1}$, RealizedVol$_{t-1}$, MarketReturn$_{t-1}$, Sentiment$_{t-1}$, PC_Ratio$_{t-1}$, VIX_Sentiment_Interaction$_{t-1}$ (uncentered)).

A.3 Model 1 Diagnostics

```
--- Normality of Residuals ---
Shapiro-Wilk normality test
data:  residuals(model1)
W = 0.96534, p-value < 2.2e-16

Skewness of residuals: 0.744755613212608
Kurtosis of residuals (excess kurtosis): 0.45261610468358
```

```
--- Homoscedasticity ---
Breusch-Pagan test
data:  model1
BP = 58.866, df = 7, p-value = 2.541e-10

--- Autocorrelation of Residuals ---
Durbin-Watson test
data:  model1
DW = 0.27423, p-value < 2.2e-16
alternative hypothesis: true autocorrelation is greater than 0

Ljung-Box test for autocorrelation (10 lags):
Box-Ljung test
data:  residuals(model1)
X-squared = 10035, df = 10, p-value < 2.2e-16

Ljung-Box test for autocorrelation (20 lags):
Box-Ljung test
data:  residuals(model1)
X-squared = 16182, df = 20, p-value < 2.2e-16

--- Multicollinearity ---
                         VIX_lag1                       VIX_sq_lag1
                       25.0641340                        21.8655691
                RealizedVol_lag1                 MarketReturn_lag1
                        3.0118220                         1.6212719
                  Sentiment_lag1                     PC_Ratio_lag1
                        1.2097964                         1.1342911
VIX_Sent_Interact_centered_lag1
                        1.1100961

--- Outliers and Influential Points ---
Number of points with Cook's D > 4/n: 94
Number of points with Cook's D > 1: 0
```

A.4 Model 2 Diagnostics

```
--- Shapiro-Wilk Normality Test (Model 2) ---
Shapiro-Wilk normality test
data:  residuals(model2_log)
W = 0.9798, p-value < 2.2e-16

Skewness (Model 2 Residuals): 0.548
Excess Kurtosis (Model 2 Residuals): 0.083

--- Breusch-Pagan Homoscedasticity Test (Model 2) ---
Breusch-Pagan test
data:  model2_log
BP = 35.793, df = 6, p-value = 3.024e-06

--- Durbin-Watson Autocorrelation Test (Model 2) ---
Durbin-Watson test
data:  model2_log
DW = 0.27364, p-value < 2.2e-16
alternative hypothesis: true autocorrelation is greater than 0

--- Ljung-Box Autocorrelation Test (10 lags, Model 2) ---
Box-Ljung test
```

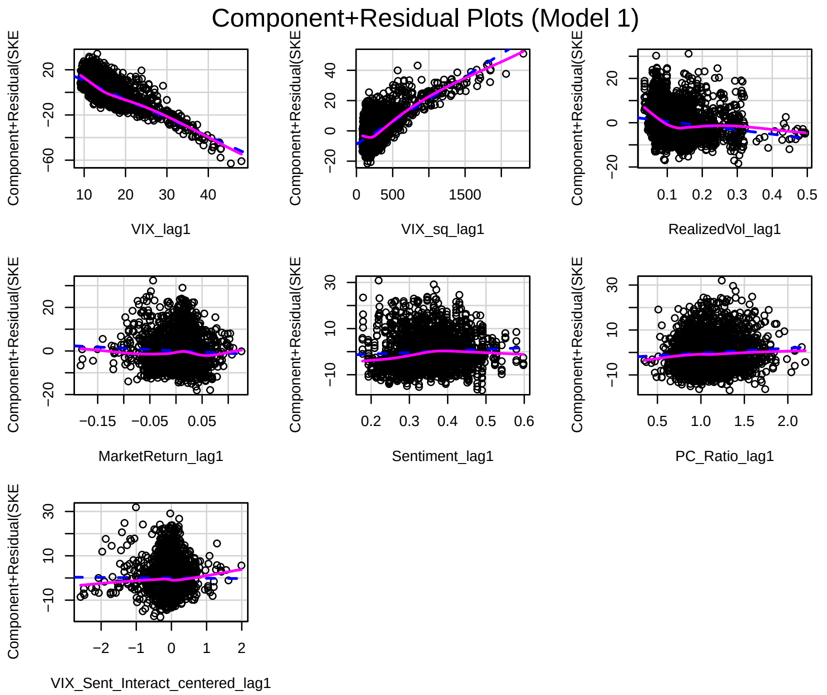

Figure 6: Component+Residual Plots for OLS Model 1a.

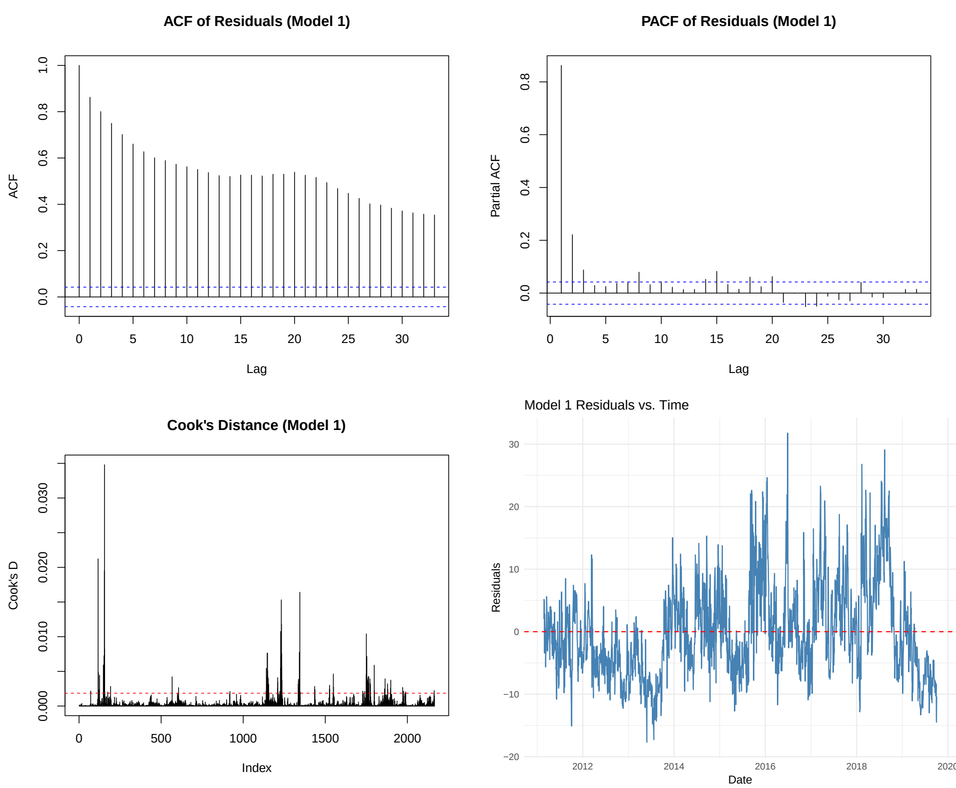

Figure 7: Additional Diagnostic Plots for OLS Model 1a (ACF, PACF, Cook's D, Residuals vs Time).

```
data:  residuals(model2_log)
X-squared = 10167, df = 10, p-value < 2.2e-16

--- Ljung-Box Autocorrelation Test (20 lags, Model 2) ---
Box-Ljung test
data:  residuals(model2_log)
X-squared = 16642, df = 20, p-value < 2.2e-16

--- Variance Inflation Factors (Model 2) ---
                   log_VIX_lag1             log_RealizedVol_lag1
                       3.222413                         2.736576
               MarketReturn_lag1                log_Sentiment_lag1
                       1.590752                         1.189602
               log_PC_Ratio_lag1 logVIX_logSent_Interact_centered_lag1
                       1.118945                         1.039206

--- Influential Points (Model 2) ---
Number of points with Cook's D > 4/n: 100
Number of points with Cook's D > 1: 0
```

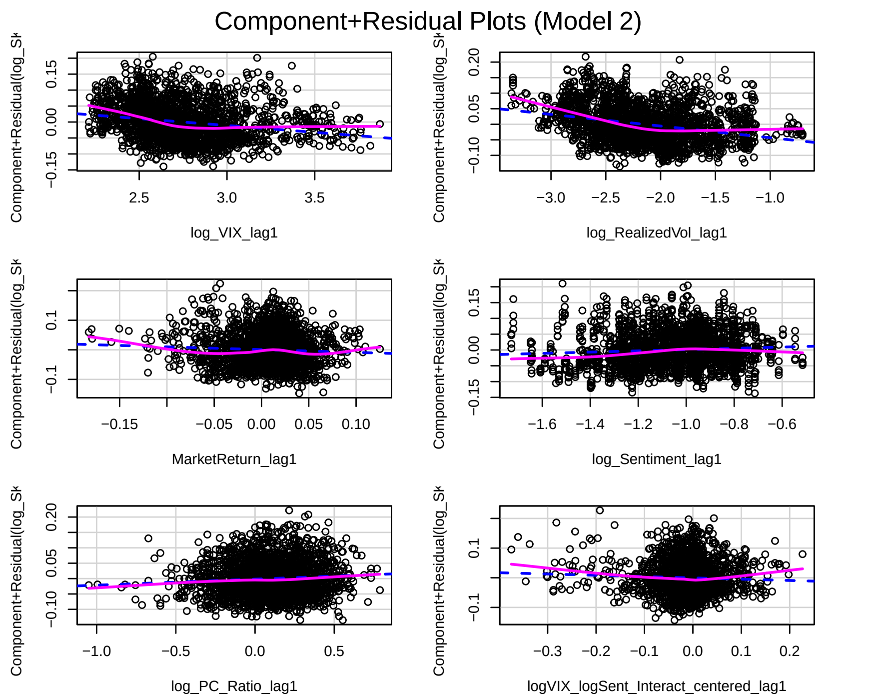

Figure 8: Component+Residual Plots for OLS Model 2a.

A.5 Model 3 Diagnostics

```
--- Normality of Residuals ---
Shapiro-Wilk normality test
data:  residuals(model3_log)
W = 0.97888, p-value < 2.2e-16
Skewness (Model 3 OLS Residuals): 0.5525604
Excess Kurtosis (Model 3 OLS Residuals): 0.0569077

-- Homoscedasticity (Breusch-Pagan Test) --
Breusch-Pagan test
```

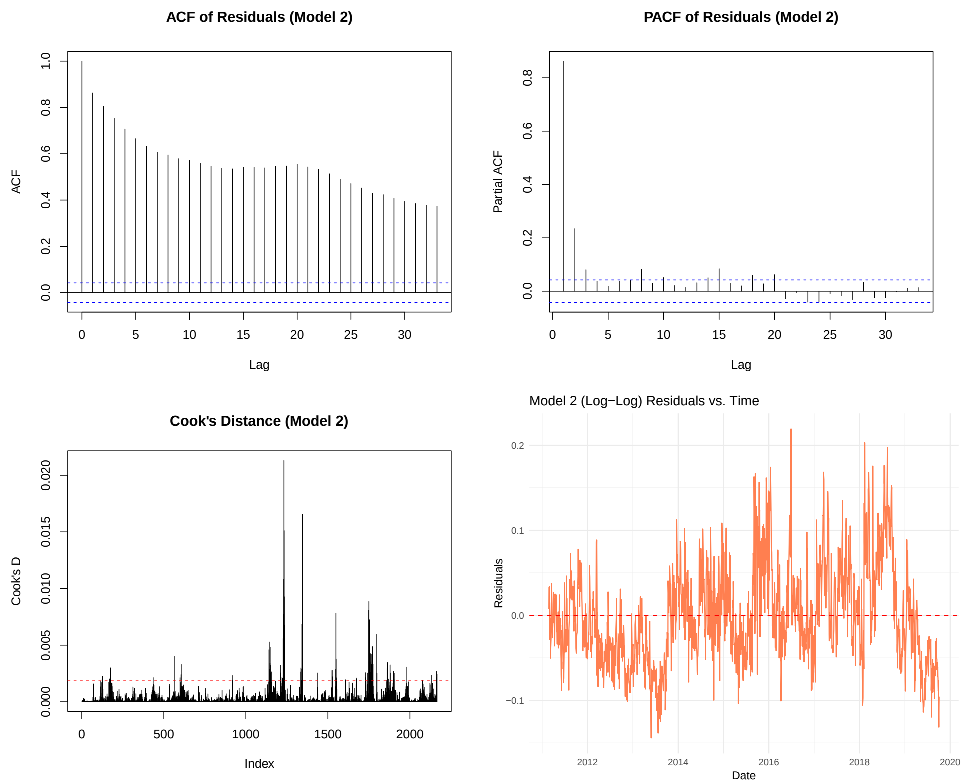

Figure 9: Additional Diagnostic Plots for OLS Model 2a.

```
data:  model3_log
BP = 49.011, df = 3, p-value = 1.319e-10
% (Using the corrected BP for the 3-predictor model)

-- Independence of Residuals (Durbin-Watson & Ljung-Box) --
Durbin-Watson test
data:  model3_log
DW = 0.27002, p-value < 2.2e-16
alternative hypothesis: true autocorrelation is greater than 0

Box-Ljung test (10 lags)
data:  residuals(model3_log)
X-squared = 10340, df = 10, p-value < 2.2e-16

-- Multicollinearity (VIFs) --
     log_VIX_lag1 log_RealizedVol_lag1  log_PC_Ratio_lag1
        2.843912             2.693415           1.118418
```

A.6 Cross-Validation Detailed Output

```
--- Model 1 (Levels): Rolling Origin k-Fold Cross-Validation ---
Fold 1 RMSE (Model 1, levels): 10.4261 | OOS R-squared (Model 1, levels): -1.6195 | Train Size: 1
Fold 2 RMSE (Model 1, levels): 8.9033 | OOS R-squared (Model 1, levels): -0.8249 | Train Size: 16
Fold 3 RMSE (Model 1, levels): 13.1251 | OOS R-squared (Model 1, levels): -1.1375 | Train Size: 1
Fold 4 RMSE (Model 1, levels): 5.6604 | OOS R-squared (Model 1, levels): 0.3514 | Train Size: 190
Fold 5 RMSE (Model 1, levels): 8.903 | OOS R-squared (Model 1, levels): -2.5805 | Train Size: 203

Average Cross-Validated RMSE (Model 1, levels): 9.4036
```

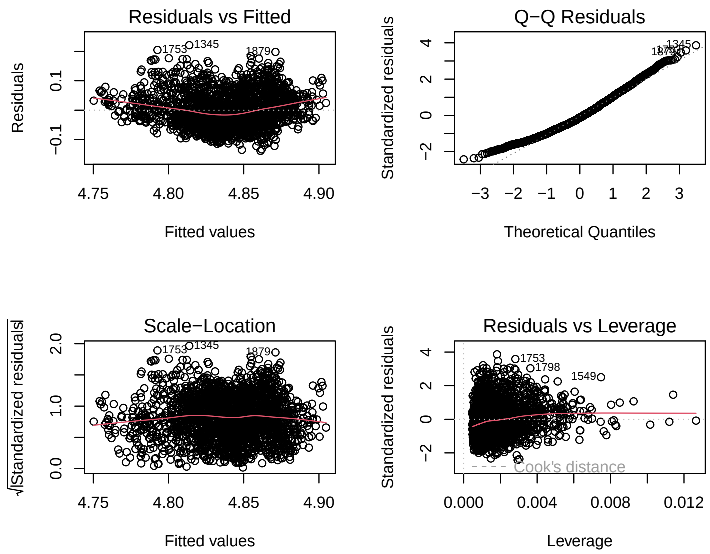

Figure 10: Standard Diagnostic Plots for OLS Model 3a.

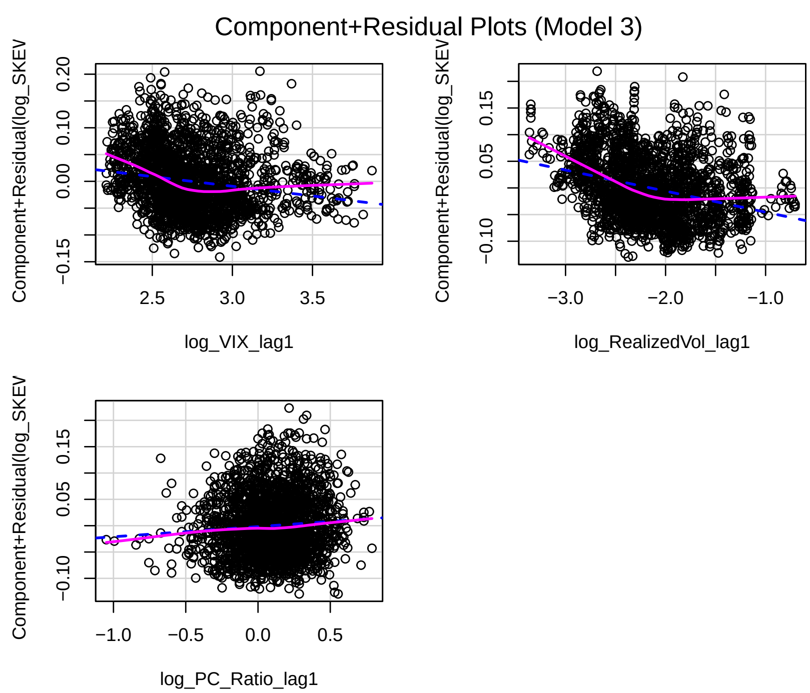

Figure 11: Component+Residual Plots for OLS Model 3a.

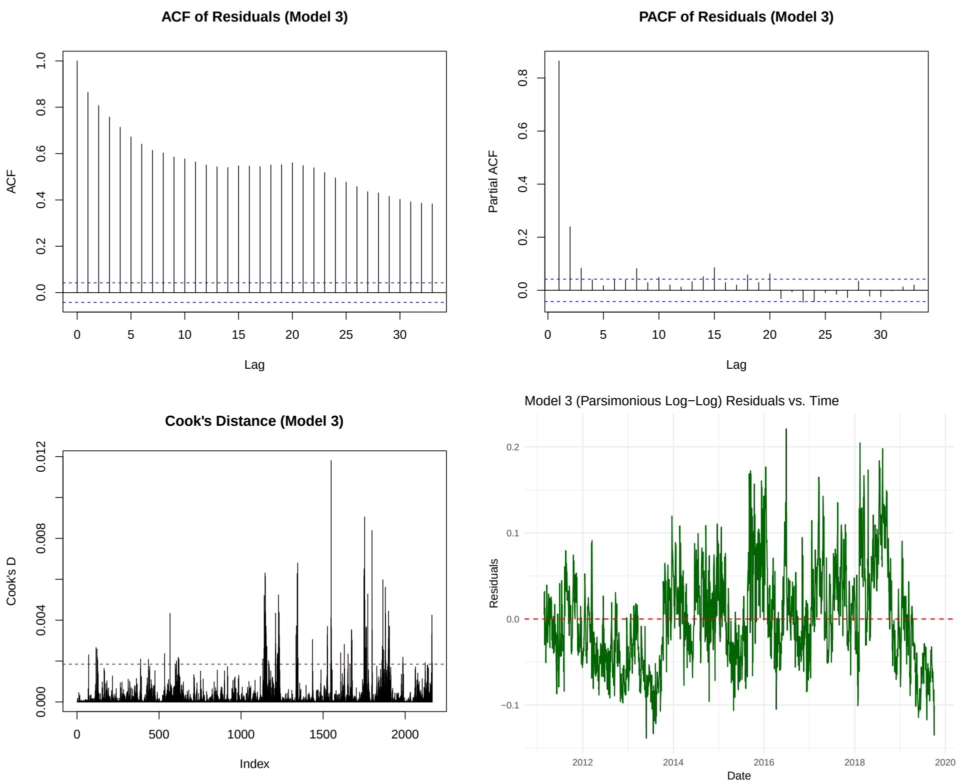

Figure 12: Additional Diagnostic Plots for OLS Model 3a.

```
Std Dev of Cross-Validated RMSE (Model 1, levels): 2.7114
Average Out-of-Sample R-squared (Model 1, levels): -1.1622

--- Model 2 (Log-Log): Rolling Origin k-Fold Cross-Validation ---
Fold 1 RMSE (Model 2, log-scale): 0.0755 | OOS R-squared (Model 2, log-scale): -1.5476 | Train Si
Fold 2 RMSE (Model 2, log-scale): 0.0673 | OOS R-squared (Model 2, log-scale): -0.8579 | Train Si
Fold 3 RMSE (Model 2, log-scale): 0.0965 | OOS R-squared (Model 2, log-scale): -1.1368 | Train Si
Fold 4 RMSE (Model 2, log-scale): 0.0448 | OOS R-squared (Model 2, log-scale): 0.3502 | Train Siz
Fold 5 RMSE (Model 2, log-scale): 0.0732 | OOS R-squared (Model 2, log-scale): -2.4683 | Train Si

Average Cross-Validated RMSE (Model 2, log-scale): 0.0715
Std Dev of Cross-Validated RMSE (Model 2, log-scale): 0.0185
Average Out-of-Sample R-squared (Model 2, log-scale): -1.1321

--- Model 3 (Parsimonious Log-Log): Rolling Origin k-Fold Cross-Validation ---
Fold 1 RMSE (Model 3, log-scale): 0.0711 | OOS R-squared (Model 3, log-scale): -1.2558 | Train Si
Fold 2 RMSE (Model 3, log-scale): 0.066 | OOS R-squared (Model 3, log-scale): -0.7907 | Train Siz
Fold 3 RMSE (Model 3, log-scale): 0.0966 | OOS R-squared (Model 3, log-scale): -1.1415 | Train Si
Fold 4 RMSE (Model 3, log-scale): 0.043 | OOS R-squared (Model 3, log-scale): 0.4018 | Train Size
Fold 5 RMSE (Model 3, log-scale): 0.0733 | OOS R-squared (Model 3, log-scale): -2.4761 | Train Si

Average Cross-Validated RMSE (Model 3, log-scale): 0.07
Std Dev of Cross-Validated RMSE (Model 3, log-scale): 0.0191
Average Out-of-Sample R-squared (Model 3, log-scale): -1.0524
```

A.7 Sub-Period Analysis Detailed Output (Model 2)

```
--- Model 2 (Log-Log): Sub-Period Robustness Analysis ---
```

```
Sub-Period 1: Observations 1 to 1082 ( 1082 obs )
Sub-Period 2: Observations 1083 to 2165 ( 1083 obs )

-- Model 2 on Sub-Period 1 --
Model 2 Sub-Period 1 - HAC Corrected Coefficients:
                                         Estimate Std. Error  t value     Pr(>|t|)
(Intercept)                             4.9127282  0.0677006 72.56551 0.000000e+00 ***
log_VIX_lag1                           -0.0289196  0.0153007 -1.89009 5.902000e-02 .
log_RealizedVol_lag1                   -0.0276827  0.0109236 -2.53420 1.141000e-02 *
MarketReturn_lag1                      -0.0842826  0.0927562 -0.90865 3.637400e-01
log_Sentiment_lag1                      0.0704613  0.0135872  5.18591 2.568000e-07 ***
log_PC_Ratio_lag1                       0.0052529  0.0071850  0.73110 4.648800e-01
logVIX_logSent_Interact_centered_lag1  -0.0136659  0.0427994 -0.31930 7.495600e-01
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

-- Model 2 on Sub-Period 2 --
Model 2 Sub-Period 2 - HAC Corrected Coefficients:
                                         Estimate Std. Error  t value     Pr(>|t|)
(Intercept)                             4.8064336  0.0986019 48.74577 0.000000e+00 ***
log_VIX_lag1                           -0.0064703  0.0271462 -0.23835 8.116536e-01
log_RealizedVol_lag1                   -0.0476819  0.0131905 -3.61487 3.144000e-04 ***
MarketReturn_lag1                      -0.0458893  0.1446394 -0.31726 7.511024e-01
log_Sentiment_lag1                      0.0313961  0.0227598  1.37946 1.680415e-01
log_PC_Ratio_lag1                       0.0132956  0.0128333  1.03602 3.004263e-01
logVIX_logSent_Interact_centered_lag1  -0.0200201  0.0738700 -0.27099 7.864297e-01
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

--- Summary Comparison (HAC Corrected Estimates) ---
Variable                          | Full Period (Model 2b) | Sub-Period 1         | Sub-Period 2
----------------------------------|------------------------|----------------------|-------------
(Intercept)                       | Est:   4.8966 (p: 0.000) | Est:   4.9127 (p: 0.000) | Est:
log_VIX_lag1                      | Est:  -0.0429 (p: 0.006) | Est:  -0.0289 (p: 0.059) | Est:  -
log_RealizedVol_lag1              | Est:  -0.0375 (p: 0.000) | Est:  -0.0277 (p: 0.011) | Est:  -
MarketReturn_lag1                 | Est:  -0.0935 (p: 0.321) | Est:  -0.0843 (p: 0.364) | Est:  -
log_Sentiment_lag1                | Est:   0.0199 (p: 0.166) | Est:   0.0705 (p: 0.000) | Est:
log_PC_Ratio_lag1                 | Est:   0.0196 (p: 0.015) | Est:   0.0053 (p: 0.465) | Est:
logVIX_logSent_Interact_centered_lag1 | Est:  -0.0436 (p: 0.379) | Est:  -0.0137 (p: 0.750) | Est

Adj. R-sq (Full OLS Model 2a): 0.177
Adj. R-sq (Sub-Period 1 OLS): 0.214
Adj. R-sq (Sub-Period 2 OLS): 0.147
```