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AXS 4465

CSE 5311 - HW2

Ex 2.2-1 Express the function $n^3/1000 - 1000n^2 - 1000n + 3$ in terms of Θ notation

Ans $n^3/1000 = \Theta(n^3)$

$1000n^2 = \Theta(n^2)$

$1000n = \Theta(n)$

$3 = \Theta(1)$

Therefore, $n^3/1000 - 1000n^2 - 1000n + 3 = \Theta(n^3)$

Q1 Return C (C) { // C = 0, 1, 2 .....

```
        X = C
        y = 0
    While (X > 0) {
        X = X-1
        y = y+1
    }
    return y
}
```

Ans 1 $x = c$, where $c \geq 0$
$y = 0$, loop runs while $x > 0$

Loop invariant Guess : $x + y = c$

Loop invariant Proof :

- Initialization :

$x = c$, $y = 0$

$x + y \Rightarrow c + 0 = c$ ✓

- Maintenance :

$x = x - 1$, $y = y + 1$

$x + y \Rightarrow (x-1) + (y+1)$

$= x + y \cancel{-1} \cancel{+1} = x + y$

Therefore, $x + y = c$

- Termination :

$x = 0$ (loop termination)

$x + y \Rightarrow 0 + y = c \Rightarrow y = c$

Since, program returns y, hence it returns C

Find a formula for $T(n) = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} 1$ & by reducing the Σ down to well known formulas. (Hint: use summation formula)

c) Given $T(n) = \sum_{j=2}^{n} 1$ for the above alg. Find a formula for $T(n)$ by reducing ~~the~~ Σ down to well known formulas.

Ans 3

a)

1	2	3	4	5	
5	1	4	2	8	- iteration 1

5 5 5

1	4	2	5	8	- iteration 2

4

1	2	4	5	8	- iteration 3

↓

algorithm exits because swapped = false

3 b) ~~Re~~ Re-written as:

$$T(n) = \sum_{j=2}^{n} \sum_{i=1}^{j-1} 1$$

$$\Rightarrow \sum_{j=2}^{n} j-1$$

$$\Rightarrow T(n) = \left[(n-1) \times \frac{n}{2}\right] - \left[(1-1) \times \frac{1}{2}\right]$$

$$\Rightarrow T(n) = (n-1) \times \frac{n}{2}$$

3c) When summing a constant from i to j
$\rightarrow (j-i+1) \times c$

Applying:
$$(n-2+1) \times 1 \Rightarrow (n-1) \times 1 = n-1$$

Q4 Selection sort is another simple but ineffecient algorithm

SelectionSort (A)	Cost	Time
for i = 1 to A.len - 1	c_1	n
min = i	c_2	$n-1$
for j = i+1 to ~~m~~ n {	c_3	$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n+1} 1$

```
            if A[j] < A[min] {
                  min = j
            }
      }
      Swap (A[min], A[i])
   }
```

a) Demonstrate how it sorts (64, 25, 12, 22, 11)

b) Fill in the remaining lines with cost and time

c) using part "b" find a formula for T(n). Simplify your summations

Ans B4

a)

1	2	3	4	5
64	25	12	22	11

iter 1 → set min to index 5 and swap with first position

1	2	3	4	5
11	25	12	22	64

iter 2 → set min to index 3 and swap with second position

1	2	3	4	5
11	12	25	22	64

iter 3 → set min to index 4 and swap with position 3

1	2	3	4	5
11	12	22	25	64

iter 4 → set min to index 4 but no swap takes place and loop breaks

Selection Sort (A)	Cost	Time
For $i=1$ to A.len -1 {	c_1	n
min $=1$	c_2	$n-1$
For $j=i+1$ to n {	c_3	$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n+1} 1$
if $A[j] < A[min]$ {	c_4	$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n+1}(1-1)$
min $= j$	c_5	$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n+1}(1-1)$
}		
}		
swap $(A(min), A(i))$	c_6	$n-1$
}		

4b)

4c) $\cancel{T(n) = c_1 n + c_2(n-1) + c_3\sum_{i=1}^{n-1}\sum_{j=i+1}^{n+1} 1 + c_6(n-1)}$

$\Rightarrow$

4c) First Solving for, $\sum_{i=1}^{n-1}\sum_{j=i+1}^{n+1} 1$

$$\Rightarrow \sum_{i=1}^{n-1} (n+1)-(i+1)+1$$

$$\Rightarrow \sum_{i=1}^{n-1} n-i+1$$

$$\Rightarrow (n+1)\left[\sum_{i=1}^{n-1} 1\right] - \left(\sum_{i=1}^{n-1} i\right)$$

$$\boxed{\left(\sum_{i=1}^{n} i = \frac{n(n+1)}{2}\right)}$$

$$\Rightarrow (n+1)\left((n-1)\cancel{-i}+1\right) - \frac{(n-1)(n-1\cancel{+1})}{2}$$

4b)

Selection Sort (A)	Cost	Time
For i = 1 to A.len - 1 {	c_1	n
min = 1	c_2	$n-1$
For j = i+1 to n {	c_3	$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n+1} 1$
if A[j] < A[min] {	c_4	$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n+1} (1 \cancel{-1})\ n-1$
min = j	c_5	$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n+1} (1 \cancel{-1})\ n-1$
}		
}		
swap (A(min), A(i))	c_6	$n-1$
}		

4c) $\cancel{T(n) = c_1 n + c_2(n-1) + c_3 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n+1} 1 + c_6(n-1)}$

$\Rightarrow$

4c) First Solving for, $\sum_{i=1}^{n-1} \sum_{j=i+1}^{n+1} 1$

$$\Rightarrow \sum_{i=1}^{n-1} (n+1) - (i+1) + 1$$

$$\Rightarrow \sum_{i=1}^{n-1} n - i + 1$$

$$\Rightarrow (n+1) \left[\sum_{i=1}^{n-1} 1 \right] - \left(\sum_{i=1}^{n-1} i \right)$$

$$\boxed{\sum_{i=1}^{n} 1 = \frac{n(n+1)}{2}}$$

$$\Rightarrow (n+1)((n-1) \cancel{-i} + 1) - \frac{(n-1)(n-1 \cancel{+1})}{2}$$

$$\Rightarrow (n-1)\left[(n+1) - \frac{(n-2)}{2}\right]$$

$$\Rightarrow (n-1)\left(\frac{2n+2-n+2}{2}\right)$$

$$\Rightarrow (n-1)\left(\frac{n+4}{2}\right)$$

$$\Rightarrow \frac{n^2+4n-n-4}{2}$$

$$\Rightarrow \frac{n^2+3n-4}{2}$$

From degree of highest polynomial, $O(n^2)$