Control Chart for Standard Deviation

SESSION 5

Structure

Prerequisite

- Lab Sessions 1, 3 and 7 of MSTL-001 (Basic Statistics Lab).
- Lab Sessions 3 and 4 of MSTL-002 (Industrial Statistics Lab).
- Unit 2 of MSTE-001 (Industrial Statistics-I).

5.1 INTRODUCTION

You have learnt how to construct control charts for process mean, i.e., $\bar{X}$-chart in Lab Sessions 1 to 3. We have also described one control chart for process variability, i.e., R-chart in Lab Session 4. R-chart is easy to calculate and explain. But for large sample sizes (usually greater than 10), it does not give the correct picture of the process variability since range depends only on maximum and minimum values of the data.

In such situations, we prefer to use standard deviation as discussed in Sec. 2.6 of Unit 2 of MSTE-001 (Industrial Statistics-I). In this Lab Session, you will learn how to construct the S-chart using MS Excel 2007.

Objectives

After performing the activities of this session, you should be able to:

- prepare the spreadsheet in MS Excel 2007;
- determine the control limits for control chart for standard deviation;
- construct the control chart for standard deviation;
- obtain the revised control limits for control charts for standard deviation and mean; and
- interpret the control charts.

5.2 PROBLEM DESCRIPTION

Suppose a bulb manufacturing company wants to check whether the variation in the life of bulbs produced by a particular machine is due to chance causes or due to assignable causes. For this purpose, a quality control inspector at this company selects 35 samples each of size 12 and measures the life of each bulb (in hours).

Table 1: Life of bulbs

Sample Number	Life of Bulbs (in hours)						
	1	2	3	4	5	6	7
Obs. 1	1152	854	876	725	900	1080	952
Obs. 2	834	822	987	1150	926	870	1167
Obs. 3	1170	954	1267	927	1334	951	924
Obs. 4	788	875	1067	1208	984	928	1154
Obs. 5	1145	1129	1242	902	1309	926	1181
Obs. 6	724	712	985	1040	816	760	1057
Obs. 7	1060	1044	1157	817	1224	841	1096
Obs. 8	678	765	957	1098	874	818	1044
Obs. 9	754	742	1015	1070	846	790	1087
Obs. 10	1090	1074	1187	847	1254	871	1126
Obs. 11	708	795	987	1128	904	848	1074
Obs. 12	1065	1049	1162	822	1229	846	1101

Table 1: Life of bulbs (continued…)

Sample Number	Life of Bulbs (in hours)						
	8	9	10	11	12	13	14
Obs. 1	741	1241	1194	1200	745	926	781
Obs. 2	1066	1124	1085	950	1044	1015	956
Obs. 3	658	987	694	939	1130	1210	1061
Obs. 4	1124	1158	1143	1008	1102	1124	1014
Obs. 5	917	1067	955	914	1105	987	1036
Obs. 6	956	1034	975	840	934	1142	846
Obs. 7	548	1110	875	829	1020	1024	951
Obs. 8	1014	957	1033	898	992	1068	904
Obs. 9	986	1064	1005	870	964	1172	876
Obs. 10	578	1140	614	859	1050	1054	981
Obs. 11	1044	1078	1063	928	1022	1098	934
Obs. 12	837	987	575	834	1025	1175	956

Table 1: Life of bulbs (continued…)

Sample Number	Life of Bulbs (in hours)						
	15	16	17	18	19	20	21
Obs. 1	1120	880	1138	674	862	711	886
Obs. 2	1430	1055	716	906	1127	1136	912
Obs. 3	830	848	1156	1140	987	913	1320
Obs. 4	1488	1113	774	664	1137	1194	970
Obs. 5	805	823	1131	1115	894	888	1295
Obs. 6	1320	945	857	696	1004	1026	802
Obs. 7	720	738	1046	1030	1143	803	1210
Obs. 8	1378	1003	664	754	1027	1084	860
Obs. 9	1350	975	636	826	1034	1056	832
Obs. 10	750	768	1076	1060	1173	833	1240
Obs. 11	1408	1033	694	584	1057	1114	890
Obs. 12	725	743	1051	1035	1148	808	1215

Table 1: Life of bulbs (continued…)

Sample Number	Life of Bulbs (in hours)						
	22	23	24	25	26	27	28
Obs. 1	1066	938	727	991	1227	1186	731
Obs. 2	856	1153	1052	1067	1071	936	1030
Obs. 3	937	1192	644	1206	878	925	1116
Obs. 4	914	1047	1110	1048	1129	994	1088
Obs. 5	912	1167	903	1181	758	900	1091
Obs. 6	746	1043	942	957	961	826	920
Obs. 7	827	1082	534	1096	768	815	1006
Obs. 8	804	937	1000	938	1019	884	978
Obs. 9	776	1073	972	987	991	856	950
Obs. 10	857	1112	564	1126	798	845	1036
Obs. 11	834	967	1030	968	1049	914	1008
Obs. 12	832	1087	823	1101	678	820	1011

Table 1: Life of bulbs (continued…)

Sample Number	Life of Bulbs (in hours)						
	29	30	31	32	33	34	35
Obs. 1	912	1124	1106	866	1124	660	848
Obs. 2	1238	942	1416	1041	1067	738	974
Obs. 3	967	1047	658	834	1142	1126	1239
Obs. 4	1064	1000	1570	1099	760	864	937
Obs. 5	1117	1022	791	809	1117	1101	1214
Obs. 6	1128	832	1306	931	957	628	957
Obs. 7	1156	937	548	724	1032	1016	1129
Obs. 8	954	890	1460	989	650	754	827
Obs. 9	1158	862	1336	961	987	658	1314
Obs. 10	1186	967	578	754	1062	1046	1159
Obs. 11	984	920	1490	1019	680	784	857
Obs. 12	1037	942	711	729	1037	1021	1134

The quality control inspector of this company needs to construct a control chart for standard deviation as well as mean to infer whether the production is under control or not. If it is out-of-control, he/she also computes the revised control limits.

Therefore, the problem for this session is to construct the control charts for standard deviation and mean for the data given in Table 1.

5.3 PROCEDURE FOR THE CONSTRUCTION OF S-CHART

You have already learnt how to calculate control limits for S-chart in Sec. 2.7 of Unit 2 of MSTE-001. The main steps involved in the construction of S-chart for the data given in Table 1 are as follows:

When the value of σ is known

Step 1: The control limits for S-chart for known σ are given by

- ✓ Centre line (CL) $= c_4\sigma$ …(1)
- ✓ Upper control limit (UCL) $= B_6\sigma$ …(2)
- ✓ Lower control limit (LCL) $= B_5\sigma$ …(3)

where c_4, B_5 and B_6 are constants and depend on the size of the sample. These constants have been tabulated for various sample sizes in the Appendix given at the end of this lab course.

Step 2: Interpretation of the S-chart.

When the value of σ is unknown

In practice, the value of σ is not known. Therefore, it is estimated from the samples, which are taken when the process is assumed to be under control. In S-chart, σ is estimated by sample standard deviation (S).

Step 1-2: The first two steps are the same as Steps 4 and 5 of Sec. 3.3 of Lab Session 3.

Step 3: The control limits for S-chart when σ is estimated by $\bar{S}/c_4$ are given by

- ✓ Centre line (CL) $= \bar{S}$ …(4)
- ✓ Upper control limit (UCL) $= B_4\bar{S}$ …(5)
- ✓ Lower control limit (LCL) $= B_3\bar{S}$ …(6)

where B_3 and B_4 are constants and depend on the size of the sample. These constants have been tabulated for various sample sizes in the Appendix given at the end of this lab course.

Step 4: Interpretation of the S-chart.

5.4 STEPS INVOLVED IN THE CONSTRUCTION OF S-CHART IN EXCEL 2007

In order to calculate the control limits of S-chart and to plot the control chart in Excel 2007 for the given data, we follow the steps given below:

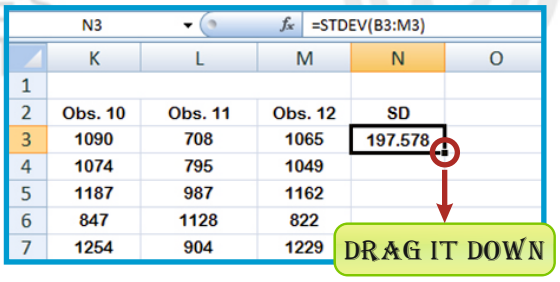
Step 1: We enter the given data in Excel sheet as shown in Fig. 5.1.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1							Life of Bulbs (in hours)						
2	Sample No.	Obs. 1	Obs. 2	Obs. 3	Obs. 4	Obs. 5	Obs. 6	Obs. 7	Obs. 8	Obs. 9	Obs. 10	Obs. 11	Obs. 12
3	1	1152	834	1170	788	1145	724	1060	678	754	1090	708	1065
4	2	854	822	954	875	1129	712	1044	765	742	1074	795	1049
5	3	876	987	1267	1067	1242	985	1157	957	1015	1187	987	1162
6	4	725	1150	927	1208	902	1040	817	1098	1070	847	1128	822
7	5	900	926	1334	984	1309	816	1224	874	846	1254	904	1229
8	6	1080	870	951	928	926	760	841	818	790	871	848	846
9	7	952	1167	924	1154	1181	1057	1096	1044	1087	1126	1074	1101
10	8	741	1066	658	1124	917	956	548	1014	986	578	1044	837
11	9	1241	1124	987	1158	1067	1034	1110	957	1064	1140	1078	987
12	10	1194	1085	694	1143	955	975	875	1033	1005	614	1063	575
13	11	1200	950	939	1008	914	840	829	898	870	859	928	834
14	12	745	1044	1130	1102	1105	934	1020	992	964	1050	1022	1025
15	13	926	1015	1210	1124	987	1142	1024	1068	1172	1054	1098	1175
16	14	781	956	1061	1014	1036	846	951	904	876	981	934	956
17	15	1120	1430	830	1488	805	1320	720	1378	1350	750	1408	725
18	16	880	1055	848	1113	823	945	738	1003	975	768	1033	743
19	17	1138	716	1156	774	1131	857	1046	664	636	1076	694	1051
20	18	674	906	1140	664	1115	696	1030	754	826	1060	584	1035
21	19	862	1127	987	1137	894	1004	1143	1027	1034	1173	1057	1148
22	20	711	1136	913	1194	888	1026	803	1084	1056	833	1114	808
23	21	886	912	1320	970	1295	802	1210	860	832	1240	890	1215

S Chart

Fig. 5.1: Partial screenshot of the spreadsheet for the given data.

Step 2: We calculate the standard deviation of the first sample by typing "=Stdev(B3:M3)" in Cell N3. We then drag down Cell N3 up to Cell N37 as shown in Fig. 5.2.



	K	L	M	N	O
1					
2	Obs. 10	Obs. 11	Obs. 12	SD	
3	1090	708	1065	197.578	
4	1074	795	1049	143.633	
5	1187	987	1162	125.175	
6	847	1128	822	157.103	
7	1254	904	1229	200.582	
8	871	848	846	85.071	
9	1126	1074	1101	79.021	
10	578	1044	837	197.382	
11	1140	1078	987	81.686	
12	614	1063	575	204.506	
13	859	928	834	102.787	
14	1050	1022	1025	101.597	
15	1054	1098	1175	85.778	
16	981	934	956	80.437	
17	750	1408	725	317.527	
18	768	1033	743	128.421	
19	1076	694	1051	206.361	
20	1060	584	1035	198.013	
21	1173	1057	1148	101.567	

Fig. 5.2

Step 3: In Cell N38, we find the average of standard deviations obtained from 35 samples as shown in Fig. 5.3.

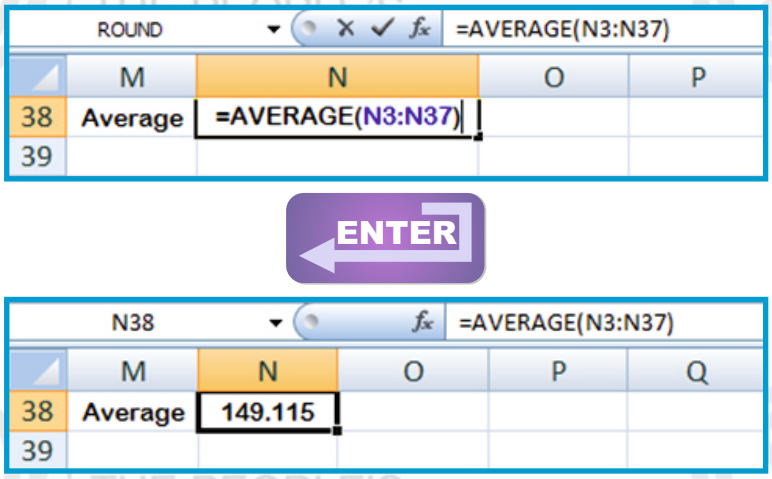


Fig. 5.3

Step 4: We type the values of k and n in Cells N40 and N41, respectively. We also type the values of B_3 and B_4 for n = 12 in Cells N42 and N43, respectively, from the Appendix given at the end of this lab course (Fig. 5.4).

	L	M	N
40		k	35
41		n	12
42	B_3 Value from Table		0.354
43	B_4 Value from Table		1.646

Fig. 5.4

Step 5: We now use the method for computing the centre line and both control limits described in Sec. 5.3. Here we use Columns O, P and Q for putting the values of the centre line, upper and lower control limits, respectively. We compute the centre line, upper and lower control limits as follows:

i) From equation (4), the centre line is CL = $\bar{S}$ and $\bar{S}$ is given in Cell N38 (see Fig. 5.3). So we type "=$N$38" in Cell O3 to obtain the centre line as shown in Fig. 5.5a.

The formula with dollar sign ($) is used for an absolute reference.

ii) From equation (5), the upper control limit is $UCL = B_4\bar{S}$. The values of B_4 and $\bar{S}$ are given in Cells N43 and N38, respectively (see Figs. 5.4 and 5.3). So we type "=$N$43*$N$38" in Cell P3 as shown in Fig. 5.5b.

iii) Similarly, we calculate the lower control limit $LCL = B_3\bar{S}$ (see equation (6)). The values of B_3 and $\bar{S}$ are given in Cells N42 and N38, respectively (see Figs. 5.4 and 5.3). So we type "=$N$42*$N$38" in Cell Q3 as shown in Fig. 5.5c.

(a)

O3	=$N$38		
	O	P	Q	R
1		Control Limits		
2	Centre Line	UCL	LCL	
3	149.115			
4				

(b)

P3	=$N$43*$N$38		
	P	Q	R	S
1	ntrol Limits			
2	UCL	LCL		
3	245.443			
4				

(c)

Q3	=$N$42*$N$38		
	Q	R	S	T
1	ts			
2	LCL			
3	52.787			
4				

Fig. 5.5

Step 6: For plotting control limits on chart using Excel 2007, we first select Cells O3:Q3 and drag them down up to Row 37 as shown in Fig. 5.6.

	N	O	P	Q	R
1		Control Limits			
2	Standard Deviation	Centre Line	UCL	LCL	
3	197.578	149.115	245.443	52.787	
4	143.633				
5	125.175				
6	157.103				
7	200.582				
8	85.071				
9	79.021				
10	197.382				

DRAG THEM DOWN

	N	O	P	Q	R
1		Control Limits			
2	Standard Deviation	Centre Line	UCL	LCL	
3	197.578	149.115	245.443	52.787	
4	143.633	149.115	245.443	52.787	
5	125.175	149.115	245.443	52.787	
6	157.103	149.115	245.443	52.787	
7	200.582	149.115	245.443	52.787	
8	85.071	149.115	245.443	52.787	
9	79.021	149.115	245.443	52.787	
10	197.382	149.115	245.443	52.787	
11	81.686	149.115	245.443	52.787	
12	204.506	149.115	245.443	52.787	
13	102.787	149.115	245.443	52.787	
14	101.597	149.115	245.443	52.787	
15	85.778	149.115	245.443	52.787	
16	80.437	149.115	245.443	52.787	
17	317.527	149.115	245.443	52.787	
18	128.421	149.115	245.443	52.787	
19	206.361	149.115	245.443	52.787	
20	198.013	149.115	245.443	52.787	
21	101.567	149.115	245.443	52.787	
22	157.103	149.115	245.443	52.787	
23	200.582	149.115	245.443	52.787	

S Chart

Fig. 5.6

Step 7: To obtain the S-chart in Excel 2007, we refer to Fig. 5.7. It means that we

1. select Cells N2:Q37,
2. click on the ***Insert*** tab,
3. select the ***Line*** option, and
4. choose the chart subtype.

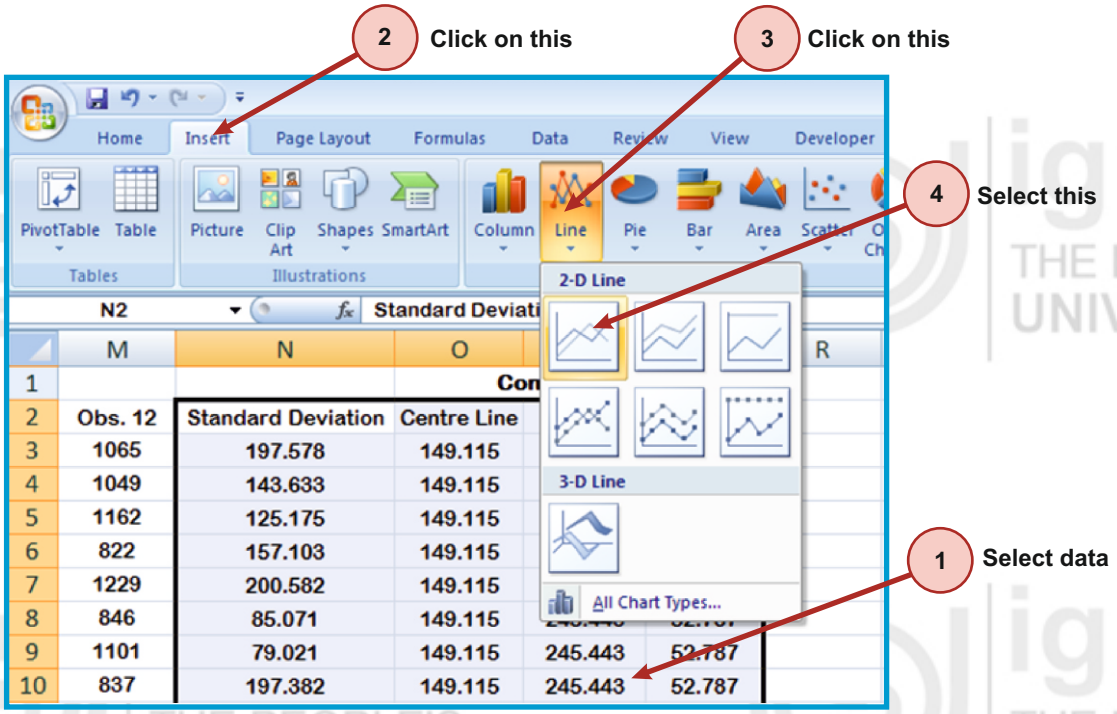


Fig. 5.7

Step 8: Columns O, P and Q provide horizontal lines on the chart representing the centre line, UCL and LCL, respectively. The values in Column N provide the sample standard deviations for 35 samples on the chart. We format the chart as explained in Sec.1.4 of Lab Session 1 and obtain the S-chart shown in Fig. 5.8.

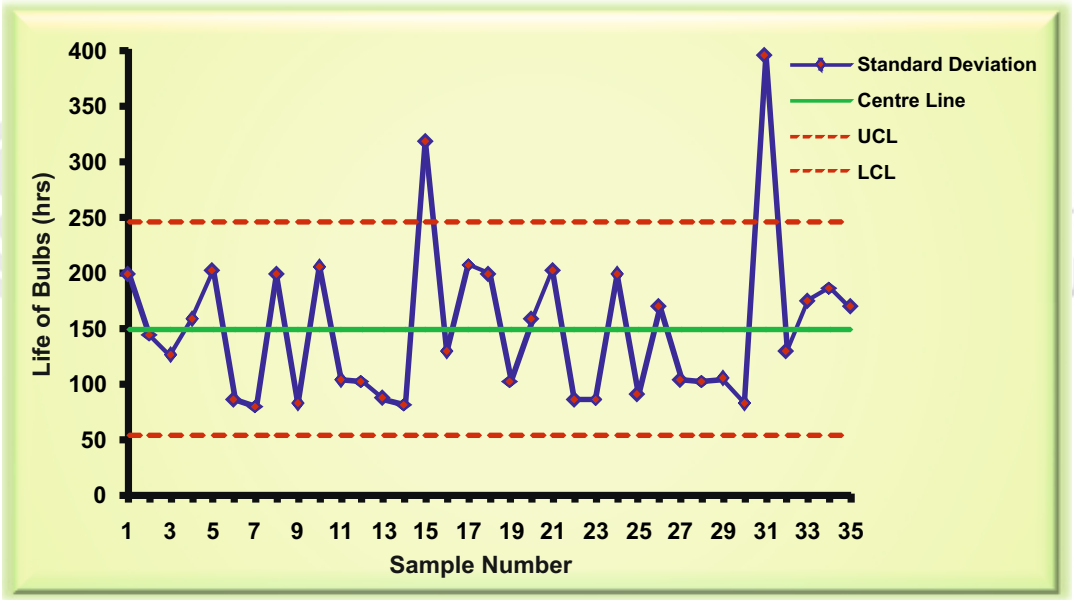


Fig. 5.8

Interpretation

We note from Fig. 5.8 that two points corresponding to Samples 15 and 31 lie outside the UCL. Therefore, the control chart indicates that the process is **not under statistical control**. Some **assignable causes** are present in the process. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them.

5.5 REVISED S-CHART

After eliminating the assignable causes, we revise the S-chart given in Fig. 5.8. For this purpose, we delete the out-of-control samples and compute revised control limits for S-chart using only the remaining samples as described below:

Step 1: For revised limits for S-chart, we first calculate new $\bar{S}$ as follows:

$$\bar{S}_{new} = \frac{\sum_{i=1}^{k} S_i - \sum_{j=1}^{d} S_j}{k-d} \quad \ldots(7)$$

where d – Number of discarded samples

$\sum_{j=1}^{d} S_j$ – Sum of the ranges of discarded samples

Step 2: After finding the new $\bar{S}$, we reconstruct the centre line and control limits the chart by replacing $\bar{S}$ by $\bar{S}_{new}$ as given below:

- ✓ Centre Line (CL) = $\bar{S}_{new}$ …(8)
- ✓ Upper control limit (UCL) = $B_4\bar{S}_{new}$ …(9)
- ✓ Lower control limit (LCL) = $B_3\bar{S}_{new}$ …(10)

Step 3: Interpretation of the S-chart.

Steps in Excel

The steps for obtaining the revised centre line and control limits for the S-chart in Excel 2007 using the remaining samples are as follows:

Step 1: We highlight the samples which lie outside the control limits, i.e., the 15th and 31st samples, with light orange colour as shown in Fig. 5.9.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1		Life of Bulbs (in hours)													Control Limits		
2	Sample No.	Obs. 1	Obs. 2	Obs. 3	Obs. 4	Obs. 5	Obs. 6	Obs. 7	Obs. 8	Obs. 9	Obs. 10	Obs. 11	Obs. 12	Standard Deviation	Centre Line	UCL	LCL
3	1	1152	834	1170	788	1145	724	1060	678	754	1090	708	1065	197.578	149.115	245.443	52.787
4	2	854	822	954	875	1129	712	1044	765	742	1074	795	1049	143.633	149.115	245.443	52.787
5	3	876	987	1267	1067	1242	985	1157	957	1015	1187	987	1162	125.175	149.115	245.443	52.787
6	4	725	1150	927	1208	902	1040	817	1098	1070	847	1128	822	157.103	149.115	245.443	52.787
7	5	900	926	1334	984	1309	816	1224	874	846	1254	904	1229	200.582	149.115	245.443	52.787
8	6	1080	870	951	928	926	760	841	818	790	871	848	846	85.071	149.115	245.443	52.787
9	7	952	1167	924	1154	1181	1057	1096	1044	1087	1126	1074	1101	79.021	149.115	245.443	52.787
10	8	741	1066	658	1124	917	956	548	1014	986	578	1044	837	197.382	149.115	245.443	52.787
11	9	1241	1124	987	1158	1067	1034	1110	957	1064	1140	1078	987	81.686	149.115	245.443	52.787
12	10	1194	1085	694	1143	955	975	875	1033	1005	614	1063	575	204.506	149.115	245.443	52.787
13	11	1200	950	939	1008	914	840	829	898	870	859	928	834	102.787	149.115	245.443	52.787
14	12	745	1044	1130	1102	1105	934	1020	992	964	1050	1022	1025	101.597	149.115	245.443	52.787
15	13	926	1015	1210	1124	987	1142	1024	1068	1172	1054	1098	1175	85.778	149.115	245.443	52.787
16	14	781	956	1061	1014	1036	846	951	904	876	981	934	956	80.437	149.115	245.443	52.787
17	15	1120	1430	830	1488	805	1320	720	1378	1350	750	1408	725	317.527	149.115	245.443	52.787
18	16	880	1055	848	1113	823	945	738	1003	975	768	1033	743	128.421	149.115	245.443	52.787
19	17	1138	716	1156	774	1131	857	1046	664	636	1076	694	1051	206.361	149.115	245.443	52.787

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
32	30	1124	942	1047	1000	1022	832	937	890	862	967	920	942	81.754	149.115	245.443	52.787
33	31	1106	1416	658	1570	791	1306	548	1460	1336	578	1490	711	394.919	149.115	245.443	52.787
34	32	866	1041	834	1099	809	931	724	989	961	754	1019	729	128.421	149.115	245.443	52.787
35	33	1124	1067	1142	760	1117	957	1032	650	987	1062	680	1037	173.761	149.115	245.443	52.787
36	34	660	738	1126	864	1101	628	1016	754	658	1046	784	1021	185.803	149.115	245.443	52.787
37	35	848	974	1239	937	1214	957	1129	827	1314	1159	857	1134	168.538	149.115	245.443	52.787

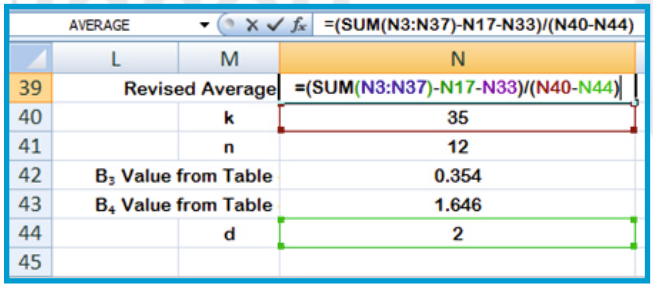
Fig. 5.9

Step 2: We have two samples outside the UCL. So we put d = 2 in Cell N44 as shown in Fig. 5.10.

	L	M	N
40		k	35
41		n	12
42	B_3 Value from Table		0.354
43	B_4 Value from Table		1.646
44		d	2
45	A_3 Value from Table		0.886

Fig. 5.10

Step 3: In Cell N39, we use the formula "=(Sum(N3:N37)-N17-N33)/(N40-N44)" and press ***Enter*** to calculate the revised $\bar{S}$, i.e., $\bar{S}_{new}$ as shown in Fig. 5.11.



	L	M	N
39		Revised Average	136.563

Fig. 5.11

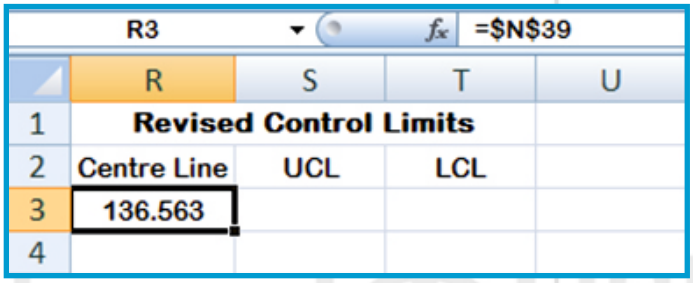
Step 4: As explained in Step 5 of Sec. 5.4 of Lab Session 5, we use Columns R, S and T for typing the values of revised centre line, upper and lower control limits, respectively.

i) From equation (8), the revised centre line is (CL) = $\bar{S}_{new}$ and $\bar{S}_{new}$ is given in N39 (see Fig. 5.11). So we type "=$N$39" in Cell R3 to find the centre line as shown in Fig. 5.12a.

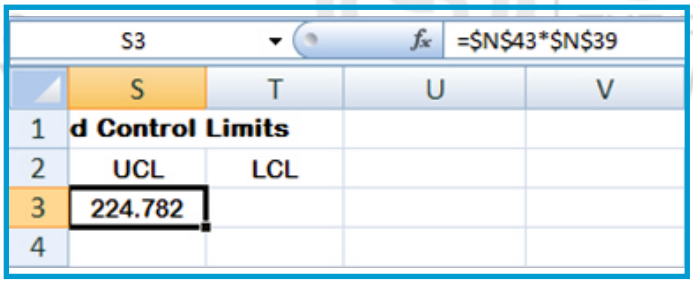
ii) From equation (9), the upper control limit is $UCL = B_4\bar{S}_{new}$. The values of B_4 and $\bar{S}_{new}$ are given in Cells N43 and N39, respectively (see Figs. 5.10 and 5.11). So we type "=$N$43*$N$39" in Cell S3 as shown in Fig. 5.12b.

The formula with dollar sign ($) is used for an absolute reference.

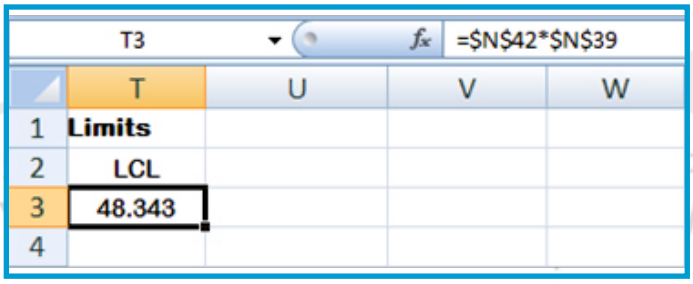
iii) Similarly, we calculate the lower control limit $LCL = B_3\bar{S}_{new}$ (see equation 10) by typing "=$N$42*$N$39" in Cell T3 as shown in Fig. 5.12c.



(a)



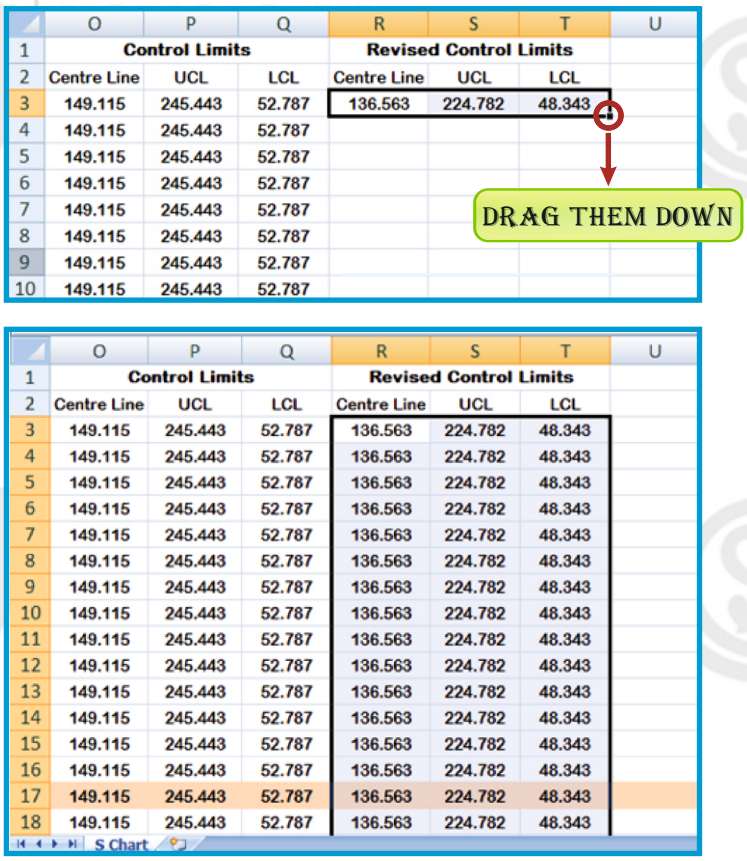
(b)



(c)

Fig. 5.12

Step 5: For plotting control limits on chart using Excel 2007, we select Cells R3:T3 and drag them down up to Row 37.



	O	P	Q	R	S	T	U
1	Control Limits			Revised Control Limits			
2	Centre Line	UCL	LCL	Centre Line	UCL	LCL	
3	149.115	245.443	52.787	136.563	224.782	48.343	
4	149.115	245.443	52.787				
5	149.115	245.443	52.787				
6	149.115	245.443	52.787				
7	149.115	245.443	52.787				
8	149.115	245.443	52.787				
9	149.115	245.443	52.787				
10	149.115	245.443	52.787				

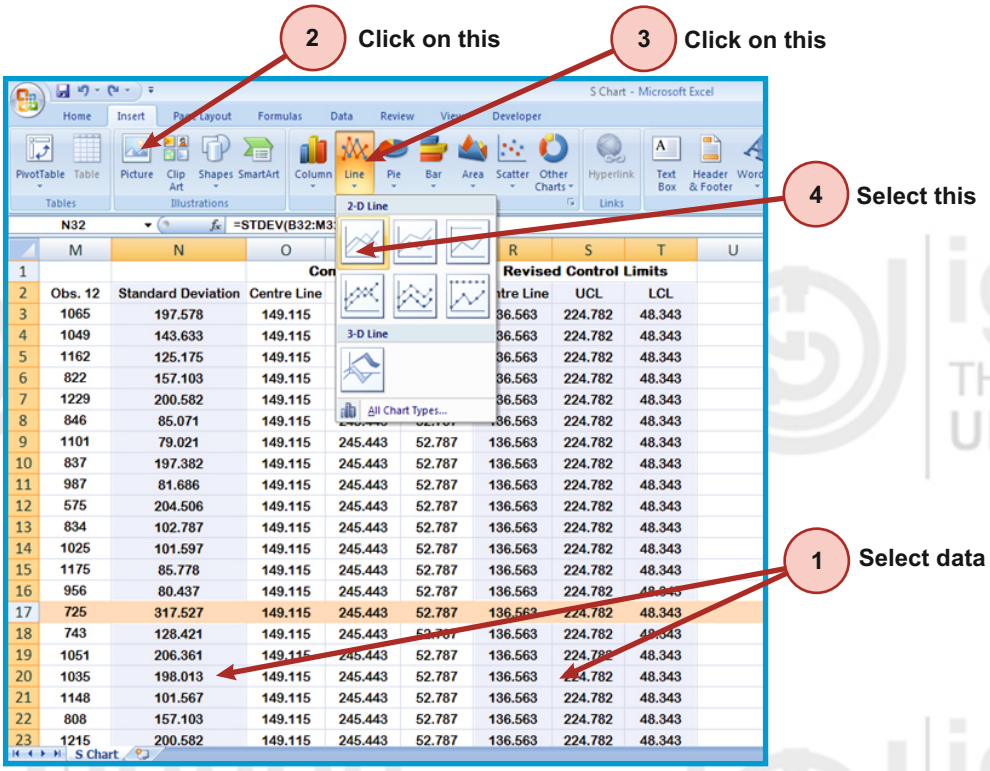
	O	P	Q	R	S	T	U
1	Control Limits			Revised Control Limits			
2	Centre Line	UCL	LCL	Centre Line	UCL	LCL	
3	149.115	245.443	52.787	136.563	224.782	48.343	
4	149.115	245.443	52.787	136.563	224.782	48.343	
5	149.115	245.443	52.787	136.563	224.782	48.343	
6	149.115	245.443	52.787	136.563	224.782	48.343	
7	149.115	245.443	52.787	136.563	224.782	48.343	
8	149.115	245.443	52.787	136.563	224.782	48.343	
9	149.115	245.443	52.787	136.563	224.782	48.343	
10	149.115	245.443	52.787	136.563	224.782	48.343	
11	149.115	245.443	52.787	136.563	224.782	48.343	
12	149.115	245.443	52.787	136.563	224.782	48.343	
13	149.115	245.443	52.787	136.563	224.782	48.343	
14	149.115	245.443	52.787	136.563	224.782	48.343	
15	149.115	245.443	52.787	136.563	224.782	48.343	
16	149.115	245.443	52.787	136.563	224.782	48.343	
17	149.115	245.443	52.787	136.563	224.782	48.343	
18	149.115	245.443	52.787	136.563	224.782	48.343	

S Chart

Fig. 5.13

Step 6: To obtain the S-chart, we refer to Fig. 5.14. It means that we

1. select Cells N2:N16, N18:N32, N34:N37, R2:T11, R18:T32 and R34:T37 by holding *Ctrl* key,
2. click on the *Insert* tab,
3. select the *Line* option, and
4. choose the chart subtype



	M	N	O	R	S	T	U
1			Con	Revised Control Limits			
2	Obs. 12	Standard Deviation	Centre Line	tre Line	UCL	LCL	
3	1065	197.578	149.115	36.563	224.782	48.343	
4	1049	143.633	149.115	36.563	224.782	48.343	
5	1162	125.175	149.115	36.563	224.782	48.343	
6	822	157.103	149.115	36.563	224.782	48.343	
7	1229	200.582	149.115	36.563	224.782	48.343	
8	846	85.071	149.115	36.563	224.782	48.343	
9	1101	79.021	149.115	245.443	52.787	136.563	224.782	48.343
10	837	197.382	149.115	245.443	52.787	136.563	224.782	48.343
11	987	81.686	149.115	245.443	52.787	136.563	224.782	48.343
12	575	204.506	149.115	245.443	52.787	136.563	224.782	48.343
13	834	102.787	149.115	245.443	52.787	136.563	224.782	48.343
14	1025	101.597	149.115	245.443	52.787	136.563	224.782	48.343
15	1175	85.778	149.115	245.443	52.787	136.563	224.782	48.343
16	956	80.437	149.115	245.443	52.787	136.563	224.782	48.343
17	725	317.527	149.115	245.443	52.787	136.563	224.782	48.343
18	743	128.421	149.115	245.443	52.787	136.563	224.782	48.343
19	1051	206.361	149.115	245.443	52.787	136.563	224.782	48.343
20	1035	198.013	149.115	245.443	52.787	136.563	224.782	48.343
21	1148	101.567	149.115	245.443	52.787	136.563	224.782	48.343
22	808	157.103	149.115	245.443	52.787	136.563	224.782	48.343
23	1215	200.582	149.115	245.443	52.787	136.563	224.782	48.343

Fig. 5.14

Step 7: We format the chart and change the horizontal axis as discussed in Steps 9 and 10 of Sec. 2.5 of Lab Session 2. The resulting S-chart is shown in Fig. 5.15.

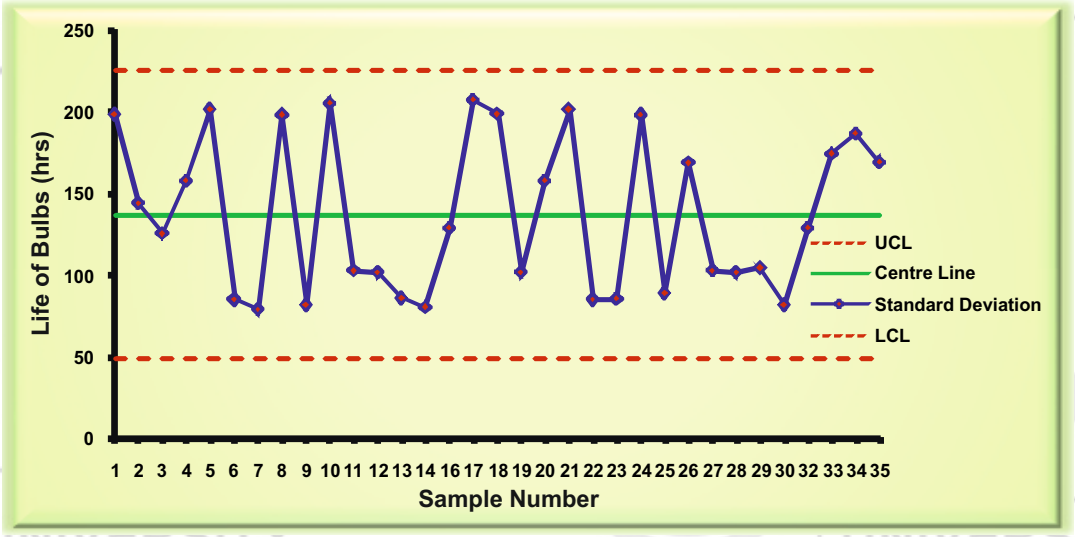


Fig. 5.15

If one or more points lie outside the revised control limits, we calculate the revised control limits of S-chart again. This process is continued until the process is brought under control.

Interpretation

This S-chart indicates that all points lie within the control limits. So the process variability is under statistical control.

After controlling the process variability, we now study the process mean. For this, we calculate the control limits of $\bar{X}$-chart using the remaining samples.

Before studying the next section, you should perform Activity 1.

Activity 1

You can also determine the revised control limits using another approach. For this purpose, follow the steps given below:

- ✓ Select Cells A2:M16, A18:M32, A34:M37.
- ✓ Choose Cell A48 or any other cell and paste the values. You can also use a separate Excel sheet where you can paste these values.
- ✓ Repeat all the steps given in Sec. 5.4.

It will give you the same results as you have obtained in Sec. 5.5.

5.6 REVISED $\bar{X}$-CHART

Recall from Unit 2 of MSTE-001 that we control the process mean and process variability simultaneously. So we use both $\bar{X}$ and S-charts together. We have already revised the S-chart in Sec. 5.5 to control the process variability. Now, we use $\bar{S}_{new}$ to estimate the process variability and calculate $\bar{\bar{X}}_{new}$ by deleting the out-of-control samples. We calculate the revised control

limits for the $\bar{X}$-chart in the same manner as discussed in Sec. 3.5 of the Lab Session 3.

We compute the revised centre line and control limits for the $\bar{X}$-chart in Excel 2007 for the remaining samples as follows:

Step 1: We determine the sample means for all samples in Cells U3:U37 as explained in Steps 1-5 of Sec. 1.4 of Lab Session 1. The output is shown in Fig. 5.16.

U3 =AVERAGE(B3:M3)

	M	N	O	P	Q	R	S	T	U	V
1			Control Limits			Revised Control Limits				
2	Obs. 12	Standard Deviation	Centre Line	UCL	LCL	Centre Line	UCL	LCL	Sample Mean	
3	1065	197.578	149.115	245.443	52.787	136.563	224.782	48.343	930.667	
4	1049	143.633	149.115	245.443	52.787	136.563	224.782	48.343	901.250	
5	1162	125.175	149.115	245.443	52.787	136.563	224.782	48.343	1074.083	
6	822	157.103	149.115	245.443	52.787	136.563	224.782	48.343	977.833	
7	1229	200.582	149.115	245.443	52.787	136.563	224.782	48.343	1050.000	
8	846	85.071	149.115	245.443	52.787	136.563	224.782	48.343	877.417	
9	1101	79.021	149.115	245.443	52.787	136.563	224.782	48.343	1080.250	
10	837	197.382	149.115	245.443	52.787	136.563	224.782	48.343	872.417	
11	987	81.686	149.115	245.443	52.787	136.563	224.782	48.343	1078.917	
12	575	204.506	149.115	245.443	52.787	136.563	224.782	48.343	934.250	
13	834	102.787	149.115	245.443	52.787	136.563	224.782	48.343	922.417	
14	1025	101.597	149.115	245.443	52.787	136.563	224.782	48.343	1011.083	
15	1175	85.778	149.115	245.443	52.787	136.563	224.782	48.343	1082.917	
16	956	80.437	149.115	245.443	52.787	136.563	224.782	48.343	941.333	
17	725	317.527	149.115	245.443	52.787	136.563	224.782	48.343	1110.333	
18	743	128.421	149.115	245.443	52.787	136.563	224.782	48.343	910.333	
19	1051	206.361	149.115	245.443	52.787	136.563	224.782	48.343	911.583	
20	1035	198.013	149.115	245.443	52.787	136.563	224.782	48.343	873.667	
21	1148	101.567	149.115	245.443	52.787	136.563	224.782	48.343	1049.417	
22	808	157.103	149.115	245.443	52.787	136.563	224.782	48.343	963.833	
23	1215	200.582	149.115	245.443	52.787	136.563	224.782	48.343	1036.000	

S Chart

Fig. 5.16

Step 2: We calculate $\bar{\bar{X}}_{new}$ by typing "=(Sum(U3:U37)-U12-U33)/(U40-U44)" in Cell U38 as shown in Fig. 5.17.

U38 =(SUM(U3:U37)-U17-U33)/(N40-N44)

	S	T	U	V	W	X
38		Revised Average	969.235			
39						

Fig. 5.17

Step 3: We already have d = 2 in Cell N44 (see Fig. 5.10). We now type the value of A_3 for n=12 in Cell N45 as shown in Fig. 5.18.

	K	L	M	N	O
44			d	2	
45			A_3 Value from Table	0.886	
46					

Fig. 5.18

Step 4: We compute the centre line, UCL and LCL shown in Fig. 5.19 in Columns V, W and X, respectively, as explained in Sec. 3.5 of Lab Session 3.

	U	V	W	X
1		Control Limits		
2	Sample Mean	Centre Line	UCL	LCL
3	930.667	969.235	1090.229	848.240
4	901.250	969.235	1090.229	848.240
5	1074.083	969.235	1090.229	848.240
6	977.833	969.235	1090.229	848.240
7	1050.000	969.235	1090.229	848.240
8	877.417	969.235	1090.229	848.240
9	1080.250	969.235	1090.229	848.240
10	872.417	969.235	1090.229	848.240
11	1078.917	969.235	1090.229	848.240
12	934.250	969.235	1090.229	848.240
13	922.417	969.235	1090.229	848.240
14	1011.083	969.235	1090.229	848.240
15	1082.917	969.235	1090.229	848.240
16	941.333	969.235	1090.229	848.240
17	1110.333	969.235	1090.229	848.240
18	910.333	969.235	1090.229	848.240
19	911.583	969.235	1090.229	848.240
20	873.667	969.235	1090.229	848.240
21	1049.417	969.235	1090.229	848.240
22	963.833	969.235	1090.229	848.240
23	1036.000	969.235	1090.229	848.240

S Chart

Fig. 5.19

Step 5: To plot the $\bar{X}$-chart, we

1. select Cells U2:X16, U18:X32 and U34:X37 by holding *Ctrl* key,
2. click on the *Insert* tab,
3. select the *Line* option, and
4. choose the chart subtype.

We format the chart as explained in Sec. 1.4 of Lab Session 1 and obtain the revised $\bar{X}$-chart shown in Fig. 5.20.

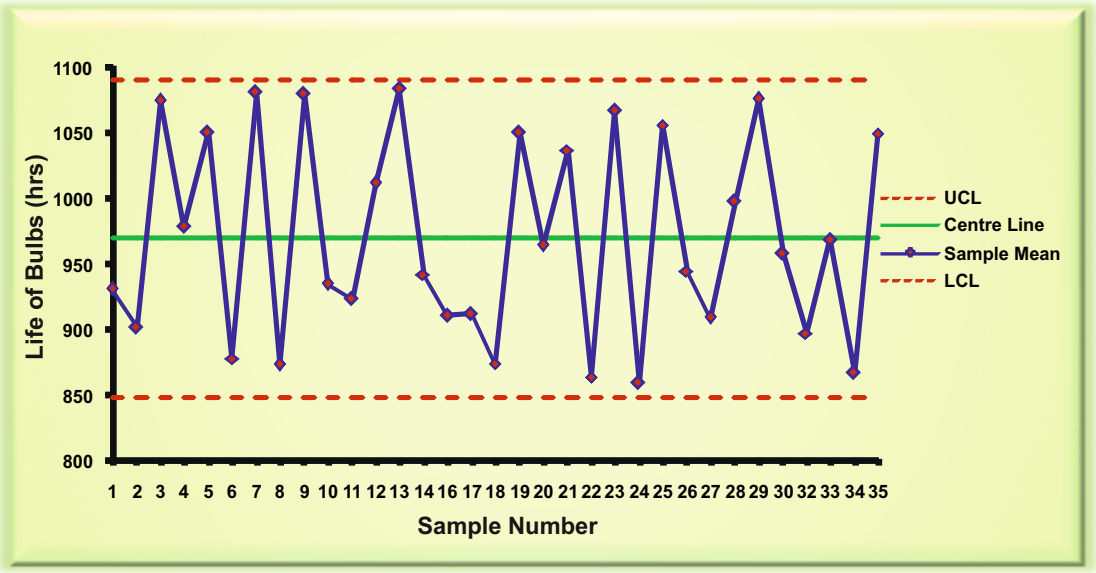


Fig. 5.20

Interpretation

Fig. 5.20 reveals that no point lies outside the control limits of the revised $\bar{X}$-chart and the process mean is under control. Hence, we may conclude that the process is under statistical control with respect to both process variability and process mean.

Now, you can check your understanding of constructing the S and $\bar{X}$-chart by solving the following exercises.

Activity 2

Construct a control charts for standard deviation with the help of MS Excel 2007 and interpret the results for

A1) Example 6 given in Unit 2 of MSTE-001.

A2) Exercise E12 given in Unit 2 of MSTE-001.

Match the results with the manual calculation done in Unit 2 of MSTE-001.

Continuous Assessment 5

Consider a problem related to a process which maintains the quality of bottling procedure. Suppose a fruit juice manufacturing company uses automatic machines to fill 500 ml juice bottles. A quality control inspector at this juice manufacturing company collects 25 samples of 10 observations of the juice bottles at different times and measures the volume of each filled bottle. The data is shown in Table 2.

Table 2: Volume of filled fruit juice in the bottles

Sample Number	Volume of Juice per bottle (in ml)									
	Obs. 1	Obs. 2	Obs. 3	Obs. 4	Obs. 5	Obs. 6	Obs. 7	Obs. 8	Obs. 9	Obs.10
1	497.32	500.62	498.68	497.82	497.32	498.68	497.32	500.62	498.68	497.82
2	504.76	500.00	498.32	500.32	504.76	498.32	504.76	500.00	498.32	500.32
3	499.24	497.18	498.12	498.68	499.24	498.12	499.24	497.18	498.12	499.68
4	499.26	496.32	498.88	497.82	499.26	498.88	499.26	499.32	498.88	498.82
5	498.32	500.62	499.56	500.12	498.32	499.56	498.32	500.62	499.56	500.12
6	499.12	500.32	499.38	500.94	499.12	499.38	499.12	500.32	499.38	500.94
7	499.34	498.32	497.32	497.62	499.34	497.32	499.34	498.32	499.32	498.62
8	499.38	498.12	500.62	498.12	499.38	500.62	499.38	498.12	500.62	498.12
9	499.26	498.38	500.68	500.38	499.26	500.68	499.26	498.38	500.68	500.38
10	498.60	497.62	499.25	498.56	498.60	499.25	498.60	497.62	499.25	498.56
11	499.44	500.00	501.32	499.38	499.44	501.32	499.44	500.00	499.32	499.38
12	498.26	500.32	500.76	499.68	498.26	500.76	498.26	500.32	500.76	499.68
13	497.32	498.50	497.18	499.38	499.32	498.18	498.32	498.50	499.18	499.38

Sample Number	Volume of Juice per bottle (in ml)									
	Obs. 1	Obs. 2	Obs. 3	Obs. 4	Obs. 5	Obs. 6	Obs. 7	Obs. 8	Obs. 9	Obs.10
14	499.56	498.00	498.76	501.12	499.56	498.76	499.56	498.00	498.76	501.12
15	500.24	500.32	499.12	499.25	500.24	499.12	500.24	500.32	499.12	499.25
16	500.76	500.50	499.68	498.12	500.76	499.68	500.76	500.50	499.68	498.12
17	500.65	497.82	494.06	496.25	500.65	494.06	500.65	497.82	494.06	496.25
18	499.12	500.26	500.44	498.76	499.12	500.44	499.12	500.26	500.44	498.76
19	499.50	500.50	499.56	500.76	499.50	499.56	499.50	500.50	499.56	500.76
20	497.50	498.82	499.76	497.82	497.50	499.76	497.50	498.82	499.76	497.82
21	499.44	500.62	500.00	501.26	499.44	500.00	499.44	499.62	500.00	500.26
22	499.38	498.38	497.56	498.56	499.38	497.56	499.38	498.38	497.56	498.56
23	501.56	499.56	498.00	499.82	501.56	498.00	501.56	499.56	498.00	499.82
24	498.32	497.32	499.56	498.62	498.32	499.56	498.32	497.32	499.56	498.62
25	499.50	500.12	498.50	500.38	499.50	498.50	499.50	500.12	498.50	500.38

Develop control limits for S-chart as well as $\bar{X}$-chart to check whether the process of bottling is under control or out-of-control. Also plot the revised control chart, if necessary.

Home Work: Do It Yourself

1) Follow the steps explained in Secs. 5.4, 5.5 and 5.6 to construct the control charts for the data of Table 1. Use a different format for the control charts. Take their screenshots and keep them in your record book.

2) Develop the spreadsheet for the exercise "Continuous Assessment 5" as explained in this lab session. Take screenshots of the final spreadsheet and the charts.

3) **Do not forget** to keep the screenshots in your record book as these will contribute in your continuous assessment in the Laboratory.