Multidisciplinary Design Optimization -Assignment 3

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1 Introduction

1.1 Problem Description

The aim of this project is to perform an optimization on the shape of a 3D aircraft wing so as to minimize the aircraft fuel weight. The optimization is to be performed such that the optimized aircraft wing satisfies the given constraints. The aerodynamic analysis of the aircraft wing is performed using Q3D, XFOIL, and AVL. The structural analysis of the aicraft wing is performed using EMWET.

1.2 Tasks Summary

The tasks performed in this project are as follows

- Building a Design Structure Matrix (DSM)
- Mathematical description of the optimization problem, such as defining objective function, constraints and bounds.
- Optimization and discussion of results.

2 Design Structure Matrix

2.1 Design Structure Matrix

The aim of this optimization to reduce the fuel weight. To do so, we need to perform various analyses on the 3D wing shape to determine the corresponding weight of the fuel, that the aircraft using this particular 3D wing shape is going to require. The flow of these analyses are shown in the Design Structure Matrix in Figure 1,

NOTE : List of Symbols

- W_{to} – Aircraft take-off weight
- W_f – Aricraft fuel weight
- W_w – Aircraft wing weight
- D – Aircraft total drag
- L – Aircraft total lift
- W_{to}^t – Target or intermediate value of take-off weight
- W_f^t – Target or intermediate value of fuel weight
- W_w^t – Target or intermediate value of wing weight
- WG – Wing Geometry

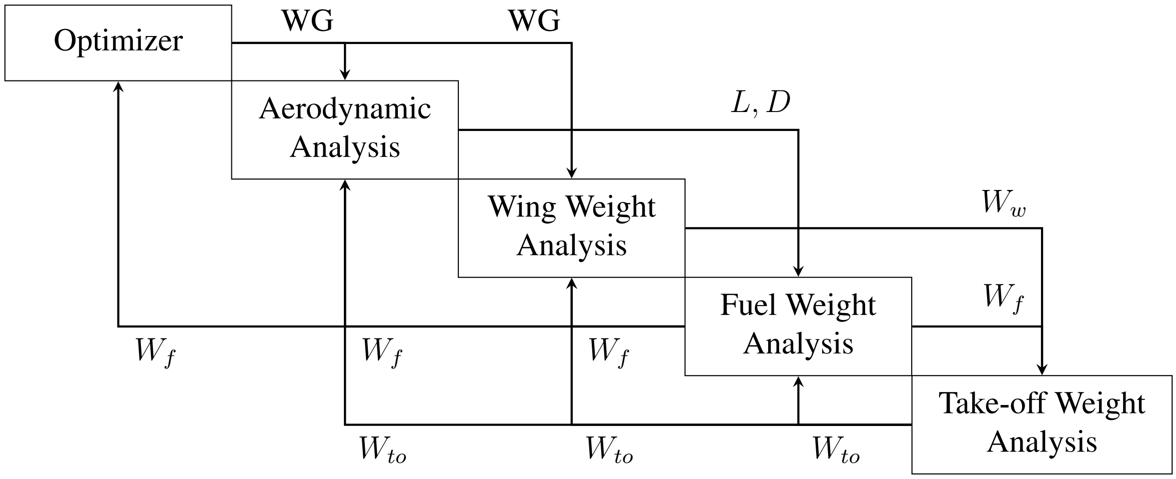

Figure 1: Design Structure Matrix

- MDA - Multidisciplinary Analysis

From Figure 1, we can see that there is an inherent coupling between the analyses as the input of one analysis is the output for another and vice-versa. Hence, to rectify this problem, **Multidisciplinary Feasible (MDF)** architecture is used in this project and **Gauss-Siedel Algorithm** is used for the MDF architecture. The flow of the entire optimization problem using the MDF architecture is shown in the Extended Design Structure Matrix in Figure 2.

2.2 Extended Design Structure Matrix

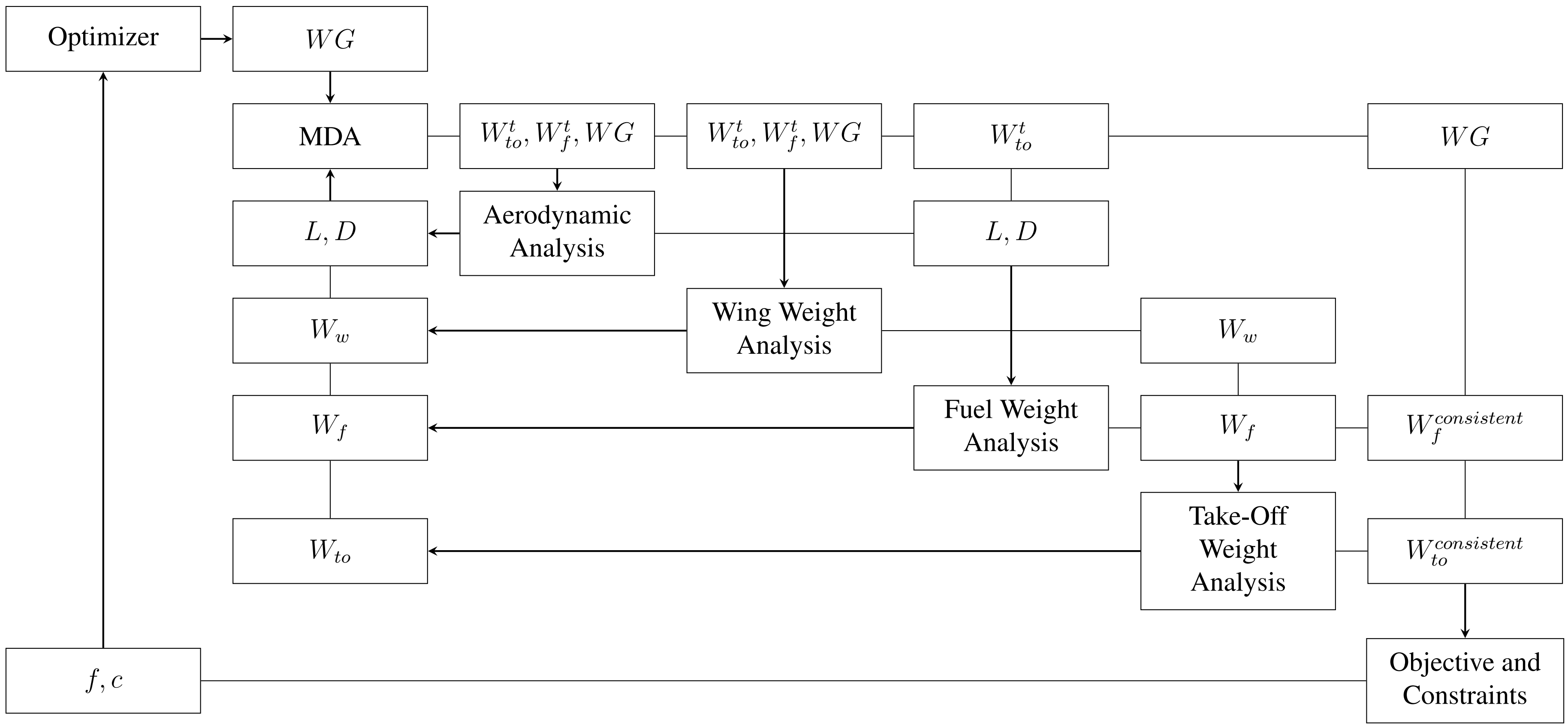

Figure 2: Extended Design Structure Matrix

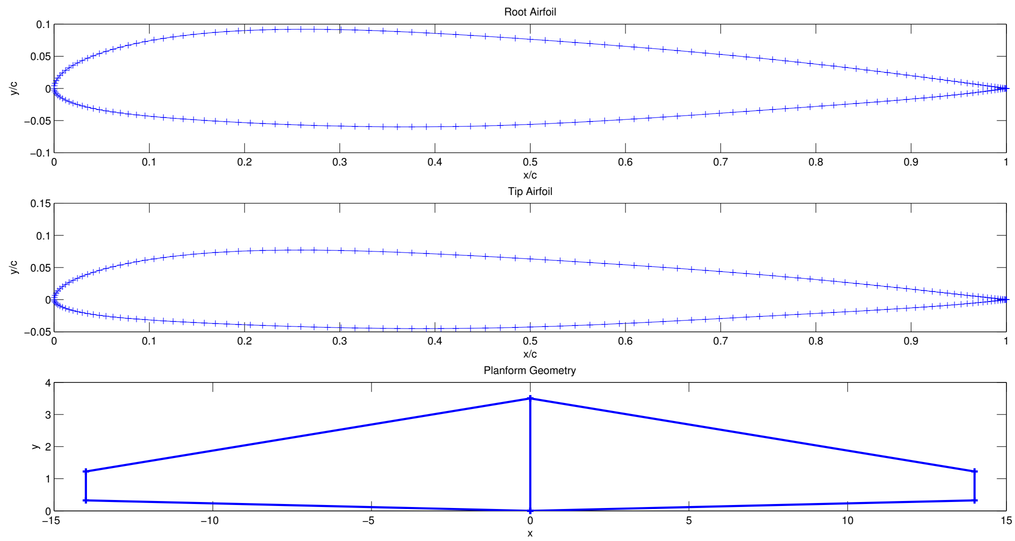

Figure 3: Initial Wing

3 Initial Wing

The initial wing is shown in the Figure 3 with the following Wing Geometry,

1. Wing Planform Geometry
   - Tip Chord = 3.5 m
   - Root Chord = 0.9 m
   - Sweep Angle = 5 deg
   - Half-Span = 14 m
2. Wing Airfoils
   - Root Airfoil = NACA 23015
   - Tip Airfoil = NACA 23012

The wing structure is made of Alluminium alloy 7075-T6 with the following properties,

$$\text{Young's Modulus, } E = 7e10 \quad N/m^2 \tag{1}$$
$$\text{Density, } \rho_{Al} = 2800 \quad kg/m^3 \tag{2}$$
$$\text{Maximum allowable tensile stress } = 4.8e8 \quad N/m^2 \tag{3}$$
$$\text{Maximum allowable compressive stress } = 4.6e10 \quad N/m^2 \tag{4}$$

Further, properties of wing structure are as follows,

$$\begin{aligned}
\text{Stringers } &= Z \text{ type} && (5)\\
\text{Rib pitch } &= 0.5 \quad m && (6)\\
\text{Position of front spar } &= 20\% \text{ of local chord} && (7)\\
\text{Position of rear spar } &= 70\% \text{ of local chord} && (8)\\
\text{Start and end of fuel tank } &= 10\% \text{ and } 70\% \text{ of semi-span respectively} && (9)\\
\text{Position of engine } &= 25\% \text{ of the semi-span} && (10)\\
\text{Engine weight } &= 1200 \quad kg && (11)
\end{aligned}$$

The flight conditions are taken as,

$$\begin{aligned}
\text{Altitude } &= 10 \quad km && (12)\\
\text{Speed of Sound, } a &= 299.46 \quad m/s && (13)\\
\text{Density, } \rho_{air} &= 0.4127 \quad kg/m^3 && (14)\\
\text{Dynamic viscosity, } \mu &= 1.46884e-5 \quad Pa.s && (15)\\
\text{Mach, } M &= 0.55 && (16)
\end{aligned}$$

The above given wing structure and flight conditions are kept **constant** throughout the optimization process. Initial weight values are given to be,

$$\begin{aligned}
\text{Wing weight, } [W_w]_{intial} &= 2107 \quad kg && (17)\\
\text{Fuel Weight, } [W_f]_{intial} &= 2220 \quad kg && (18)\\
\text{Take-Off weight, } [W_{to}]_{intial} &= 20820 \quad kg && (19)
\end{aligned}$$

4 Mathematical Formulation of the Optimization Problem

4.1 Objective Function

Objective function, or the aim of this optimization is to minimize the fuel weight of the aircraft by improving the shape of the aircraft wing. To give equal weightage to the objective function and constraints, both the objective functions and constraint values are **normalized** with the data of the initial wing. So, the objective function is,

$$f(x, y(x, y)) = \left[\frac{W_f(x, y(x, y))}{[W_f]_{initial}}\right] \tag{20}$$

$$\begin{aligned}
\text{where, } x &- \text{Design vector}\\
y &- \text{Vector of state variables}\\
W_f &- \text{Aircraft fuel weight}
\end{aligned}$$

In our case, the **design vector, x** is the **wing geometry** and the **vector of state variables, y** is **[**W_{to}, W_w, W_f, L, D**]** ie.,

$$x = \begin{bmatrix} \text{root airfoil} \\ \text{tip airfoil} \\ \text{root chord} \\ \text{tip chord} \\ \text{half span} \\ \text{sweep angle} \end{bmatrix} \tag{21}$$

$$y = \begin{bmatrix} W_{to} \\ W_w \\ W_f \\ L \\ D \end{bmatrix} \tag{22}$$

4.2 Constraints

The constraints that the optimized aircraft wing should satisfy are as follows

- The wing loading of the optimized wing should be lower than or equal to the wing loading of the initial wing
- The wing aspect ratio of the optimized wing is greater than or equal to 9.

4.2.1 Wing Loading Constraint

The wing loading constraint is given by,

$$\left[\frac{W_{to}}{S}\right](x, y(x, y)) \leq \left[\frac{W_{to}}{S}\right]_{initial} \tag{23}$$

$$\begin{aligned} \text{where, } x &- \text{Design vector} \\ y &- \text{Vector of state variables} \\ W_{to} &- \text{Aircraft take-off weight} \\ S &- \text{Wing surface area} \end{aligned}$$

The normalized version of the above equation is used as the constraint equation, which is,

$$c_1(x, y(x, y)) = \left[\frac{\left[\frac{W_{to}}{S}\right](x, y(x, y))}{\left[\frac{W_{to}}{S}\right]_{initial}}\right] - 1 \leq 0 \tag{24}$$

4.2.2 Aspect Ratio Constraint

The aspect ratio constraint is given by,

$$\left[\frac{b^2}{S}\right](x, y(x, y)) \geq 9 \tag{25}$$

$$\begin{aligned} \text{where, } x &- \text{Design vector} \\ y &- \text{Vector of state variables} \\ b &- \text{Span of wing} \\ S &- \text{Wing surface area} \end{aligned}$$

The normalized version of the above equation is used as the constraint equation, which is,

$$c_2(x, y(x, y)) = 1 - \left[\frac{\left[\frac{b^2}{S}\right](x, y(x, y))}{9}\right] \leq 0 \tag{26}$$

4.3 Design Vector

The size of design vector in this case is 24. The **first 10** elements of the design vector represents the **root airfoil CSTs'**, the **second 10** elements of the design vector represents the **tip airfoil CSTs'** (out of the 10 CST's, the **first 5** represent the **upper curve** of the airfoil and the **last 5** represent the **lower curve** of the airfoil), the 21^{st} **element** represents the **root chord**, the 22^{nd} **element** represents the **tip chord**, the 23^{rd} **element** represents the **half span**, and the 24^{th} **element** represents the **sweep angle**. That is,

$$\text{Root Airfoil} - \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{10} \end{bmatrix} \tag{27}$$

$$\text{Tip Airfoil} - \begin{bmatrix} x_{11} \\ x_{12} \\ \vdots \\ x_{20} \end{bmatrix} \tag{28}$$

$$\text{Root Chord} - x_{21} \tag{29}$$

$$\text{Tip Chord} - x_{22} \tag{30}$$

$$\text{Half Span} - x_{23} \tag{31}$$

$$\text{Sweep Angle} - x_{24} \tag{32}$$

$$\text{Design Vector, } x = \begin{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{10} \end{bmatrix} \\ \begin{bmatrix} x_{11} \\ x_{12} \\ \vdots \\ x_{20} \end{bmatrix} \\ [x_{21}] \\ [x_{22}] \\ [x_{23}] \\ [x_{24}] \end{bmatrix} \tag{33}$$

We can see that the design vector, x consists of values that are of different orders of magnitude. For example, the wing span is of the order $1e1$ where as some airfoil CST's are of the order $1e-2$. Therefore, it is important to **normalize** the design vector with the data of the initial wing and the optimizer always works with the normalized version of the design vector. This is done to make sure that the optimizer gives equal weightage to all the design variables. Hence, the initial value of the design vector with which the optimization is started will be nothing but a vector of ones' ie.,

$$x0 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix} \tag{34}$$

where, $x0$ – Initial design vector

But, it is important to make sure that the design vector is **rescaled** back to its original value before performing the analyses. As the various analyzers including Q3D, EMWET, etc., cannot work with the the normalized value of the design vector. Rescaling is performed by multiplying the design vector with the data of the initial wing.

4.4 Bounds

The optimizer tries to design a wing that has the minimum value for the objective function and satisfies the constraints. But, it is important that this wing design is a feasible design that Q3D, EMWET, XFOIL, etc., can work with. Hence, the changes that the optimizer can make on the design vector must be limited. For this particular initial wing design, the bounds are taken as follows,

$$lb(\text{planform geometry}) = 30\% \text{ of initial planform geometry} \tag{35}$$
$$ub(\text{planform geometry}) = 250\% \text{ of initial planform geometry} \tag{36}$$

$$lb(\text{airfoil CSTs'}) = 40\% \text{ of initial airfoil CSTs'} \tag{37}$$
$$ub(\text{airfoil CSTs'}) = 240\% \text{ of initial airfoil CSTs'} \tag{38}$$

$$\text{where, } lb - \text{lower bound vector}$$
$$ub - \text{upper bound vector}$$

For bounds higher than this, optimizer starts producing random infeasible wing designs that the black box software cannot work with anymore.

4.5 Optimization Problem

The entire optimization problem can be presented as follows,

$$\boxed{\begin{array}{c} \text{minimize } f(x, y(x, y)) \\ \\ \text{with respect to } x \\ \\ \text{such that} \\ \\ c_1(x, y(x, y)) \leq 0 \\ c_2(x, y(x, y)) \leq 0 \\ \\ \text{with } x \text{ being between the bounds} \\ \\ lb \leq x \leq ub \end{array}}$$

There is no equality constraints in the above optimization problem. The above presented problem is solved as a constrained gradient based optimization using **fmincon**, which is an in-built matlab optimizer.

5 Optimization Results

The formulated optimization problem is solved using fmincon. The optimizer is used with a finite difference step size of 0.05.

5.1 Optimization History

The optimization history is given in Figure 4.

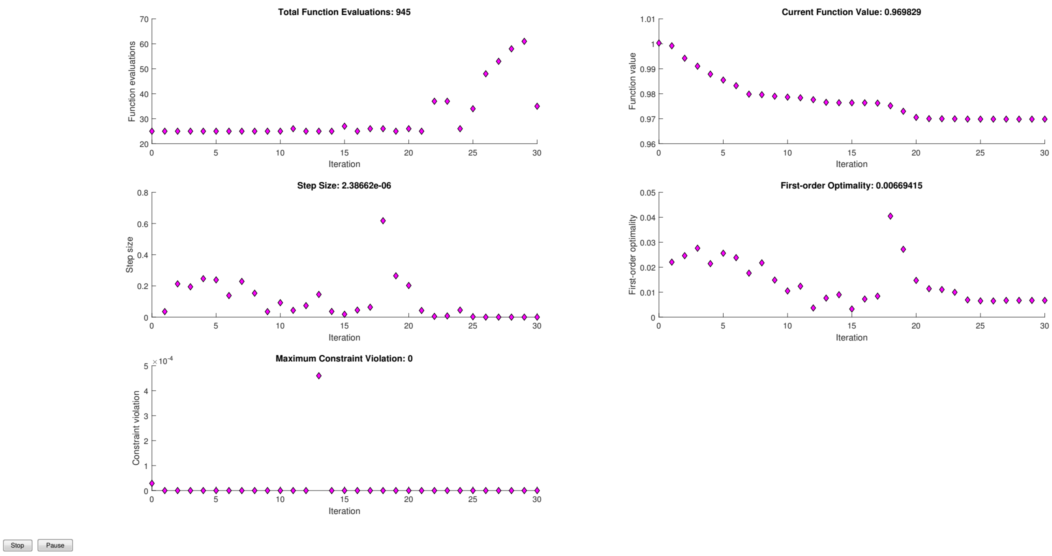

Figure 4: Optimization History

5.1.1 Stopping Criteria

Matlab stops the optimization with the report shown in Matlab 1,

```
Optimization stopped because the relative changes in all ...
    elements of x are
less than options.StepTolerance = 1.000000e-06, and the ...
    relative maximum constraint
violation, 0.000000e+00, is less than ...
    options.ConstraintTolerance = 1.000000e-06.
```

Matlab 1: Stopping Criteria

The optimizer stops because normalized step size of the design variables itself is smaller than the default value of $1e-6$, which is already a low enough value. Hence, we can conclude that the optimization can be stopped here.

5.2 Objective and Constraint Values

The initial values of objective function and constraints are as follows,

$$\boxed{\begin{aligned} f_{initial} &= 1.000321 && (39)\\ [c_1]_{initial} &= 2.8e-5 && (40)\\ [c_2]_{initial} &= 0 && (41)\end{aligned}}$$

There is a slight deviation from 1 in the initial value of objective function because initial value of the weights calculated using MDA is slightly different from the actual initial value of the weights. This is also the reason why there is a deviation from 0 in the initial constraint value. The value of objective function and constraint after optimization are as follows,

$$\boxed{\begin{aligned} f_{optimized} &= 0.9698 \quad (42)\\ [c_1]_{optimized} &= -2.1060e-4 \quad (43)\\ [c_2]_{optimized} &= -0.7273 \quad (44)\end{aligned}}$$

5.3 Design Vector Values

The size of design vector in this case is 24. The **first 10** elements of the design vector represents the **root airfoil CSTs'**, the **second 10** elements of the design vector represents the **tip airfoil CSTs'** (out of the 10 CST's, the **first 5** represent the **upper curve** of the airfoil and the **last 5** represent the **lower curve** of the airfoil), the 21^{st} **element** represents the **root chord**, the 22^{nd} **element** represents the **tip chord**, the 23^{rd} **element** represents the **half span**, and the 24^{th} **element** represents the **sweep angle**.

The **normalized initial** values of the design variables are as follows

$$x0 = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] \quad (45)$$

The **rescaled** values of the **initial** design varaibles are as follows, planform geometry,

$$[\text{Tip Chord}]_{initial} = 3.5 \quad (46)$$
$$[\text{Root Chord}]_{initial} = 0.9 \quad (47)$$
$$[\text{Sweep Angle}]_{initial} = 5 \quad (48)$$
$$[\text{Half Span}]_{initial} = 14 \quad (49)$$

airfoil CSTs',

$$\begin{aligned}[\text{Root Airfoil}]_{initial} = [&0.2577, 0.2847, 0.1495, 0.2413, 0.2003,\\ &-0.1715, -0.0909, -0.2570, -0.0583, -0.2214]\end{aligned} \quad (50)$$

$$\begin{aligned}[\text{Tip Airfoil}]_{initial} = [&0.2154, 0.2440, 0.1139, 0.2071, 0.1599,\\ &-0.1285, -0.0564, -0.2107, -0.0335, -0.1766]\end{aligned} \quad (51)$$

The **normalized optimum** values of the design variables are as follows,

$$\begin{aligned}[x]_{optimum} = [&0.5445, 1.3096, 1.9169, 1.0481, 0.4000, 0.9167, 0.8087, 1.4749, 1.4851, 1.2861,\\ &0.9274, 1.5584, 1.3881, 1.4566, 1.2016, 1.1143, 1.0047, 1.2844, 1.1024, 0.8890,\\ &1.0505, 0.3000, 1.0022, 0.7332]\end{aligned} \quad (52)$$

The **rescaled** values of the **optimum** design variables are as follows, planform geometry,

$$[\text{Tip Chord}]_{optimized} = 3.6769 \tag{53}$$

$$[\text{Root Chord}]_{optimized} = 0.2700 \tag{54}$$

$$[\text{Sweep Angle}]_{optimized} = 3.6663 \tag{55}$$

$$[\text{Half Span}]_{optimized} = 15.3393 \tag{56}$$

airfoil CSTs',

$$[\text{Root Airfoil}]_{optimized} = [0.1403, 0.3730, 0.2867, 0.2529, 0.0802, \\ -0.1573, -0.0735, -0.3792, -0.0866, -0.2848] \tag{57}$$

$$[\text{Tip Airfoil}]_{optimized} = [0.1998, 0.3803, 0.1582, 0.3018, 0.1922, \\ -0.1432, -0.0567, -0.2707, -0.0370, -0.1570] \tag{58}$$

The optimized wing in comparison with the initial wing is shown in Figure 5

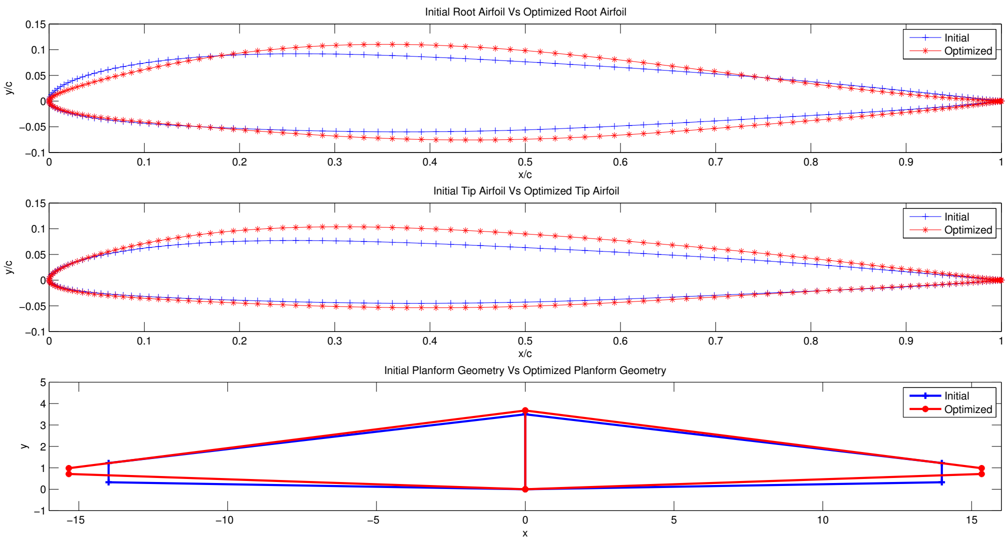

Figure 5: Optimized Wing Vs Initial Wing

5.4 Improvement in Aircraft Weight

The characteristics of the aircraft with initial wing are as follows,

$$\text{Aircraft take-off weight, } [W_{to}]_{initial} = 20820 \quad kg \tag{59}$$
$$\text{Aircraft fuel weight, } [W_f]_{initial} = 2220 \quad kg \tag{60}$$
$$\text{Aircraft wing weight, } [W_w]_{initial} = 2107 \quad kg \tag{61}$$
$$\text{Aircraft lift, } L_{initial} = 1.9554e5 \quad N \tag{62}$$
$$\text{Aircraft Drag, } D_{initial} = 1.5051e4 \quad N \tag{63}$$
$$\text{Lift over drag, } \left[\frac{L}{D}\right]_{initial} = 12.99 \tag{64}$$
$$\text{Wing area, } S_{initial} = 61.6 \quad m^2 \tag{65}$$
$$\text{Wing loading}_{initial} = 337.98 \quad kg/m^2 \tag{66}$$
$$\text{Wing aspect ratio}_{initial} = 12.7272 \tag{67}$$
$$\text{Wing taper ratio}_{initial} = 0.2571 \tag{68}$$

The characteristics of the aircraft with optimized wing are as follows,

$$\text{Aircraft take-off weight, } [W_{to}]_{optimized} = 20457 \quad kg \tag{69}$$
$$\text{Aircraft fuel weight, } [W_f]_{optimized} = 2153 \quad kg \tag{70}$$
$$\text{Aircraft wing weight, } [W_w]_{optimized} = 1811.2 \quad kg \tag{71}$$
$$\text{Aircraft lift, } L_{optimized} = 1.9224e5 \quad N \tag{72}$$
$$\text{Aircraft Drag, } D_{optimized} = 1.4369e4 \quad N \tag{73}$$
$$\text{Lift over drag, } \left[\frac{L}{D}\right]_{optimized} = 13.37 \tag{74}$$
$$\text{Wing area, } S_{optimized} = 60.54 \quad m^2 \tag{75}$$
$$\text{Wing loading}_{optimized} = 337.90 \quad kg/m^2 \tag{76}$$
$$\text{Wing aspect ratio}_{optimized} = 15.5463 \tag{77}$$
$$\text{Wing taper ratio}_{optimized} = 0.0734 \tag{78}$$

6 Conclusion

Hence, the goal of this project is reached and all the tasks mentioned in the introduction are performed. Summarizing, the following has been performed,

- Design structure matrix was built to give an outline of solving the optimization problem.
- Optimization problem was formulated to minimize aircraft fuel weight with constraints and bounds.
- Optimization results were discussed and the final optimized wing design, shown in Figure 6, has the following improvements over the initial wing design.

1.7% decrease in aircraft take-off weight
3% decrease in aircraft fuel weight
14% decrease in aircraft wing weight
3% increase in aircraft lift over drag

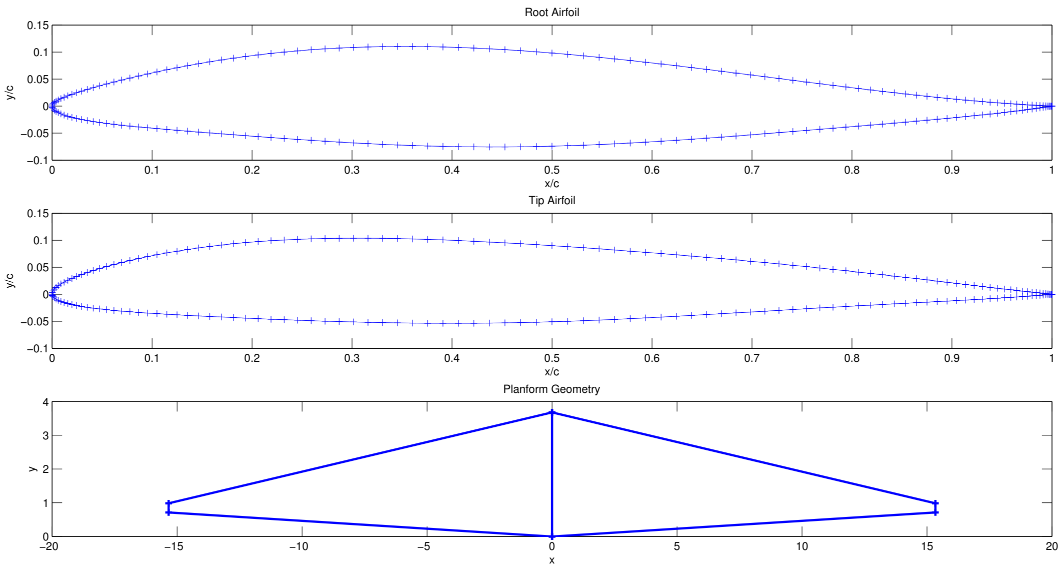

Figure 6: Optimized Wing