UiO : **Department of Mathematics**
University of Oslo

Clustered Conformal Prediction for the Housing Market

The 13th Symposium on Conformal and Probabilistic Prediction with Applications, 2024

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Collaborators

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Motivation

- Estimating the current value of a home is essential for homeowners, banks, real estate agents, insurance companies, investors, government, etc.
- Increasing use of automated valuation models (AVMs) instead of manual appraisal
- Extremely noisy prediction problem $\implies$ need to quantify prediction uncertainty
- State-of-the-art: Tree-based models combined with temporal and spatial smoothing

AI in Property Valuation: The Most Consequential Algorithms You've Never Heard Of

ALEX ENGLER, SYLVIA BROWN / OCT 9, 2023

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If we told you about an AI built on the latest foundation models that shapes multi-trillion-dollar markets and 'walks' through every home in the United States, would you say it was science fiction?

Well, let us introduce you to Automated Valuation Models, or AVMs, invented a century ago.

Article by researchers at at Brookings Institution and Georgetown University, published in *Tech Policy* on 9th of October 2023.

Wall Street Journal article from 17th of November 2021.

Quantifying Uncertainty in AVMs

- CP applied to the housing market previously:
  - Bellotti 2017: Adjust for temporal drift (London, UK)
  - Lim and Bellotti 2021: Design novel non-conformity scores for AVMs (Ames, US)
  - Hjort et al. 2024, preprint: Spatially-weighted CP (Oslo, Norway)
  - Bastos and Paquette 2024, preprint: Conformalized QR outperforms QR (San Francisco, US)

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- Our target: Approximately conditional coverage across municipalities
- We study $N = 84\,975$ transactions from $K = 286$ different municipalities in Norway

Conformal prediction

Indcutive conformal prediction approach:

- Split data set at random into training, calibration, test set
- Train a regression model $\hat{f} : \mathcal{X} \mapsto \mathcal{Y}$ on training set
- Calculate scores $s_i = \Psi(X_i, Y_i; \hat{f})$ on calibration set
- On test set:

$$C_{1-\alpha}(X_{N+1}) = \{y \in \mathcal{Y} : \quad \Psi(X_{N+1}, y; \hat{f}) \leq \hat{q}_{1-\alpha}\}$$

where $\hat{q}_{1-\alpha}$ is an empirical quantile of $s_1, ..., s_{N_{\text{calib}}}$.

Challenges

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  ✓ Theoretical guarantees on coverage per region
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  ✓ Some theoretical guarantees (Mao et al. 2023), and empirical success (Hjort et al. 2024, preprint)
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  ✗ Fails if data is sparse
- Clustered CP: Cluster together similar regions, calculate $\hat{q}_{1-\alpha}$ per cluster
  ✓ Works well in classification (Ding et al. 2023)
  ✗ Small bias in coverage guarantees if clustering is poor

Clustered CP

Algorithm:

- Use fraction $\gamma \in (0, 1)$ of calibration data for clustering
- Cluster the ECDFs $\hat{F}_1, ..., \hat{F}_K$ into $M < K$ clusters, minimizing within-cluster variance
- Let $\hat{q}_{1-\alpha}^{(m)}$ be the $(1-\alpha)$th quantile of scores in cluster m
- Calibrate cluster-wise: for every observation in any class k in cluster m we use $\hat{q}_{1-\alpha}^{(m)}$ to create the prediction interval

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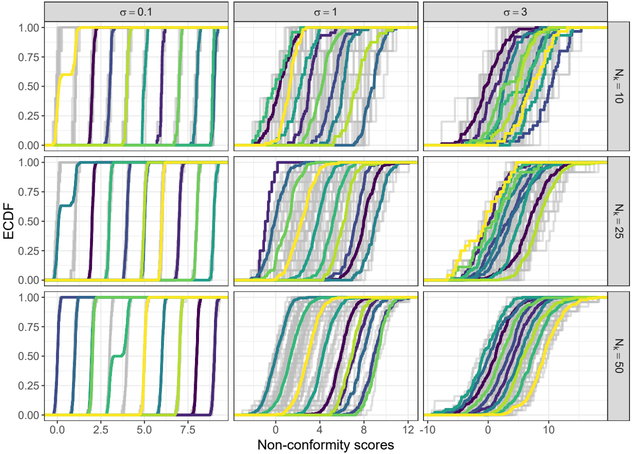
Theoretical properties: Let ε_m be the maximum Kolmogorov-Smirnov distance between two classes in cluster m. Then,

$$P\Big(Y_{N+1} \in C(X_{N+1})|\text{class } \ k\Big) \geq 1 - \alpha - \varepsilon_m, \quad \forall k \in m.$$

Clustered CP: Synthetic data



ECDFs of $K = 100$ classes (in grey) and $M = 10$ clusters (in colors). Non-conformity scores in class k is drawn from from $\mathcal{N}(\mu_k, \sigma^2)$, with $\mu_k \sim U(0, 1, ..., 10)$.

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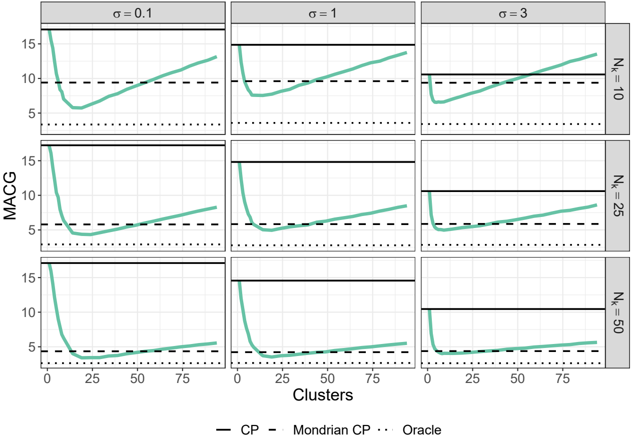


Mean Absolute Coverage Gap (MACG) as a function of the number of clusters.

The data set

We study $N = 84\,975$ from the Norwegian housing market in 2015. Transactions come from $K = 286$ different municipalities; $N_k < 100$ for more than 167 municipalities and $N_k > 1\,000$ for 16 municipalities.

Variable	Unit	Mean	St. Dev.	Min	Max	Type
Sale Price	NOK (mill.)	3.07	1.72	0.02	28.7	Numerical
Size	m^2	100	54	0	819	Numerical
Gross Size	m^2	112.42	67.48	0	1131	Numerical
Longitude	degrees	9.82	2.90	4.79	30.47	Numerical
Latitude	degrees	60.71	2.37	57.99	70.72	Numerical
Altitude	m	101.69	136.49	0	1151	Numerical
Bedrooms	-	2.56	1.20	0	15	Numerical
Municipality	-	-	-	-	-	Categorical

Experimental setup

- Random split into training (25%), calibration (50%) and test (25%)
- Three non-conformity scores:

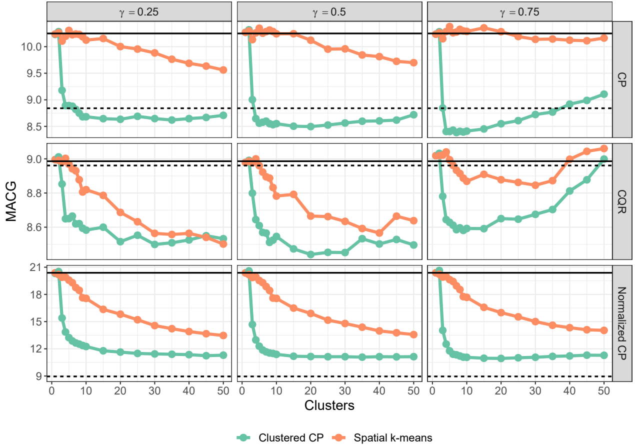
$$\Psi_{CP}(X_i, Y_i) = |Y_i - \hat{f}(X_i)| \tag{CP}$$

$$\Psi_{Norm.CP}(X_i, Y_i) = |Y_i - \hat{f}(X_i)|/\hat{f}(X_i) \tag{Normalized CP}$$

$$\Psi_{CQR}(X_i, Y_i) = \max\{\hat{Q}_{\alpha/2}(X_i) - Y_i, Y_i - \hat{Q}_{1-\alpha/2}(X_i)\} \tag{CQR}$$

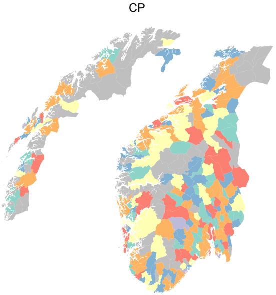
- We use a random forest to train $\hat{f}$, and quantile regression forest (Meinshausen 2006) for CQR
- Clustering:
  - Experiment with cluster fractions $\gamma \in (0.25, 0.5, 0.75)$.
  - Discretize each ECDF, i.e., $\hat{F}_k \approx [q^k_{10}, q^k_{20}, \ldots, q^k_{90}]$. Solve by M-means clustering in $\mathbb{R}^9$.
  - If $N_k < 10$: Assign to NULL cluster, calibrate globally.

Results



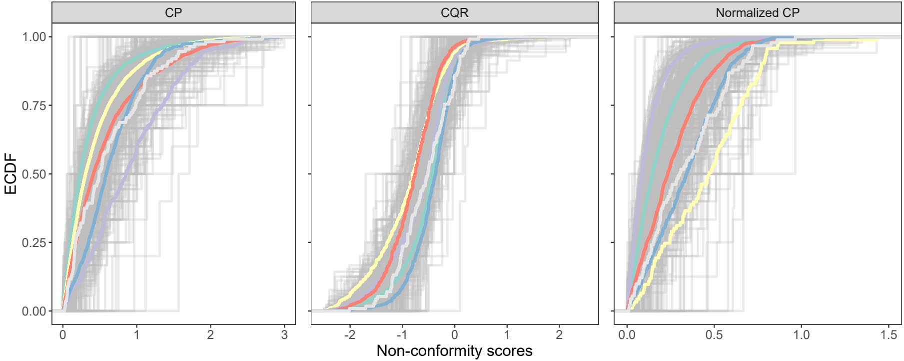
Straight line: Global calibration with $\gamma = 0$ (CP). Dotted: Mondrian CP with $\gamma = 0$. Note that the range of MACG is different for the different non-conformity scores.

Results



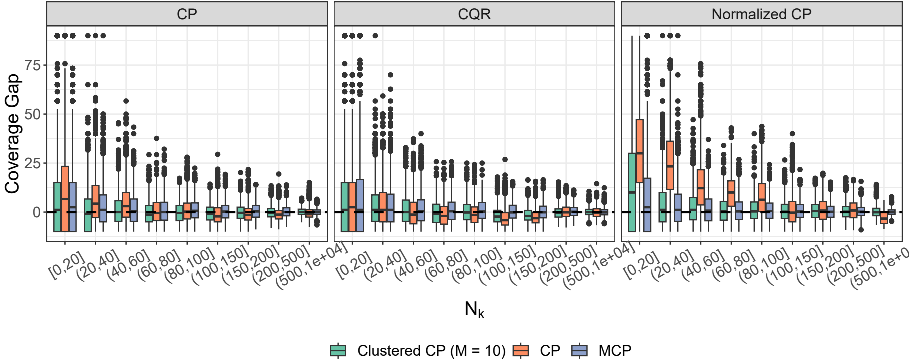
An example of the identified clusters with the Clustered CP methodology for $M = 6$ clusters. The grey municipalities either have no observations or are part of the NULL cluster.

Results



The ECDF of the identified clusters with Clustered CP for $M = 6$, overlaying the individual ECDFs for each municipality.

Results



Coverage gap for different bins of N_k for MCP, CP, and Clustered CP with $M = 10$. The results are for confidence level $\alpha = 0.1$ with a fraction $\gamma = 0.5$ set aside for clustering in Clustered CP.

Discussion

- Clustered CP is a pragmatic version of Mondrian CP where similar classes are pooled together
- Induces a small coverage gap ε_m in theory which is reduced if the clustering is good
- Clustering based on ECDFs outperforms clustering based on spatial distance
- Open questions:
  - How to decide the optimal number of clusters a priori?
  - How to handle the imbalanced classes?
  - Adjusting the CP intervals for temporal drift in the housing market

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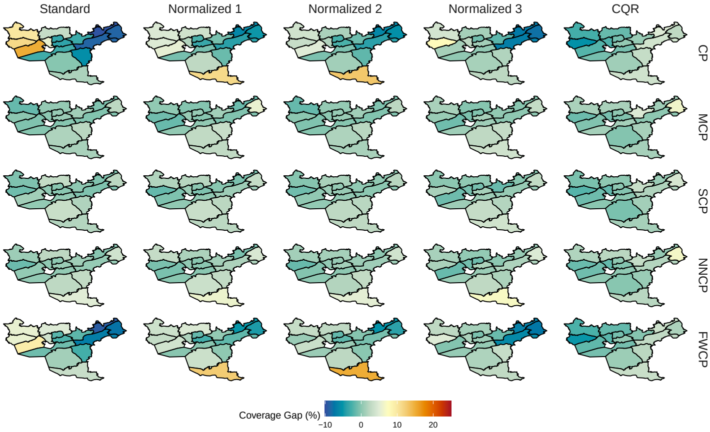
Appendix: Synthetic data, details

Draw data from $K = 100$ different classes. Each class is drawn from a normal $\mathcal{N}(\mu_k, \sigma^2)$. Importantly: Some of the groups are drawn with similar μ_k!

$$
\begin{aligned}
G &\sim U(1, ..., K) \\
\mu_k &\sim U(1, 2, ..., \sqrt{K}) \\
S|G = k &\sim \mathcal{N}(\mu_k, \sigma^2).
\end{aligned}
$$

Appendix: Results from Hjort et al. 2024



The map shows the performance for different non-conformity measures (horizontally) and weighting methods (vertically) on a data set of $N = 26\,362$ observations from Oslo (2016-2017).

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