

HW3: Induction

Golam Md Muktadir

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$$\frac{\langle x, \sigma \rangle \Downarrow \sigma(x) \quad \frac{\langle y, \sigma \rangle \Downarrow \sigma(y) \quad \langle z, \sigma \rangle \Downarrow \sigma(z)}{\langle y + z, \sigma \rangle \Downarrow \sigma(y) + \sigma(z)}}{\langle x * (y + z), \sigma \rangle \Downarrow \sigma(x) * (\sigma(y) + \sigma(z))}$$

1 Problem 1

1.1 Question

In the WHILE language, prove or disprove the equivalence of the two commands:

$$t := x; x := y; y := t$$

and

$$t := y; y := x; x := t$$

(where x; y, and t are distinct locations).

1.2 Answer

We can disprove it with a counter example with a state s in which two sequences yield different results.

Let

$$s = [t \rightarrow 0, x \rightarrow 5, y \rightarrow 100]$$

Then,

$$\begin{aligned} \langle t := x; x := y; y := t, s \rangle &\Downarrow s[t \rightarrow 5, x \rightarrow 100, y \rightarrow 5] \\ \langle t := y; y := x; x := t, s \rangle &\Downarrow s[t \rightarrow 100, x \rightarrow 100, y \rightarrow 5] \end{aligned}$$

So, location, t , has different values after execution of the sequences under this s .

2 Problem 2

2.1 Question

In the WHILE language, prove that if

$$\langle \text{while } b \text{ do } y := y - x, s \rangle \Downarrow s'$$

then there exists an integer k such that

$$s(y) = s'(y) + k * s(x)$$

Please make it explicit if/when you reason by induction on derivations, stating your induction hypothesis.

2.2 Answer

3 Problem 3

3.1 Question

In the WHILE language, prove:

$$\forall c1, c2, c3 : c1; (c2; c3) \equiv (c1; c2); c3$$

3.2 Answer