```
{ true }
z = 0;
sum = 0;
while i != x
  i := i + 1;
  sum := sum + y;
{ sum = x * y }
```

Let w be the while loop.

1 Question 1

Using Hoare's rules, prove:

$$\{x = y\}x := x + 1; y := y + 1\{x = y\}$$

1.1 Answer

Here all the rules belong to Hoare's rules for While. So, ignored the $\vdash$ in the triples.

$$\dfrac{\dfrac{x = y \Rightarrow true \qquad \dfrac{\{x+1 = y+1\}x := x+1\{x = y+1\}}{\{true\}x := x+1\{x = y+1\}}\,(ass_t \text{ rule})}{\{x = y\}x := x+1\{x = y+1\}}\,(cons_t) \qquad \dfrac{\{[y+1/y](x = y)\}y := y+1\{x = y\}}{\{x = y+1\}y := y+1\{x = y\}}\,(ass_t)}{\{x = y\}x := x+1; y := y+1\{x = y\}}\,(\ comp_t\ )$$

2 Question 2

Using Hoare's rules, prove

$$\{y = z\} \text{ while b do } y := y - x\{\exists k.z = (y + k * x)\}$$

for an arbitrary boolean expression b.

2.1 Answer

Let's define W = $\text{ while b do } y := y - x$
First we need to find the lopp invariant I so that we can apply the while rule. In this case the invariant is $x = n$
Loop invariant: $\exists j \exists k.y = z - j * x \wedge k = j + 1$

$$\dfrac{y = z \Rightarrow \exists j \exists k. y = z - j * x \wedge k = j + 1 \qquad \dfrac{D1 : \{\exists j \exists k. y = z - j * x \wedge k = j + 1 \wedge b\} y := y - x \{\exists j \exists k. y = z - (j+1) * x \wedge k = j + 1\}}{\{\exists j \exists k. y = z - j * x \wedge k = j + 1\} W \{\exists j \exists k. y = z - (j+1) * x \wedge k = j + 1 \wedge \neg b\}}}{\{y = z\} \text{ while b do } y := y - x \{\exists k. z = (y + k * x)\}}$$

D1: $$\dfrac{\exists j \exists k. y = z - j * x \wedge k = j + 1 \wedge b \Rightarrow true \qquad \dfrac{\{[y/y - x](\exists j \exists k. y = z - (j+1) * x \wedge k = j + 1)\} y := y - x \{\exists j \exists k. y = z - (j+1) * x \wedge k = j + 1}{\{true\} y := y - x \{\exists j \exists k. y = z - (j+1) * x \wedge k = j + 1\}}}{\{\exists j \exists k. y = z - j * x \wedge k = j + 1 \wedge b\} y := y - x \{\exists j \exists k. y = z - (j+1) * x \wedge k = j + 1\}}$$