Bit Vector Data Flow Frameworks

Uday Khedker

(www.cse.iitb.ac.in/˜uday)

Department of Computer Science and Engineering,

Indian Institute of Technology, Bombay

Jul 2018

Part 1

About These Slides

Copyright

These slides constitute the lecture notes for CS618 Program Analysis course at IIT Bombay and have been made available as teaching material accompanying the book:

- Uday Khedker, Amitabha Sanyal, and Bageshri Karkare. *Data Flow Analysis: Theory and Practice*. CRC Press (Taylor and Francis Group). 2009.

  (Indian edition published by Ane Books in 2013)

Apart from the above book, some slides are based on the material from the following books

- M. S. Hecht. *Flow Analysis of Computer Programs*. Elsevier North-Holland Inc. 1977.
- F. Nielson, H. R. Nielson, and C. Hankin. *Principles of Program Analysis*. Springer-Verlag. 1998.

Outline

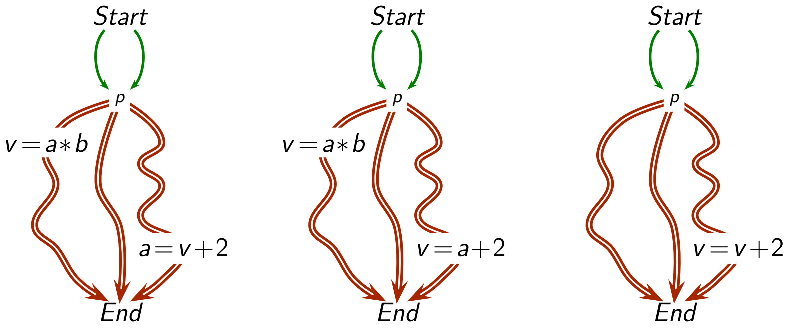
- Live Variables Analysis
- Observations about Data Flow Analysis
- Available Expressions Analysis
- Anticipable Expressions Analysis
- Reaching Definitions Analysis
- Common Features of Bit Vector Frameworks

Part 2

Live Variables Analysis

Defining Live Variables Analysis

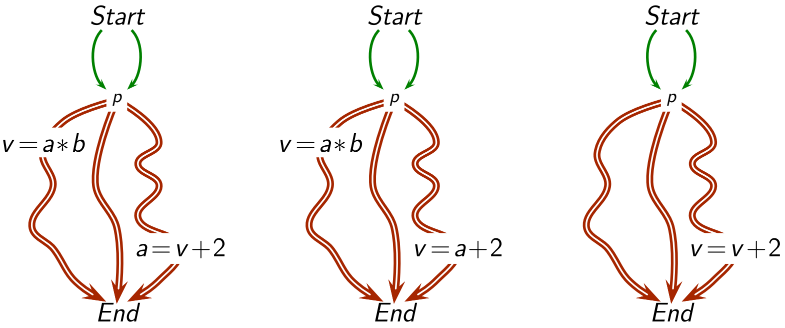
A variable v is live at a program point p, if some path from p to program exit contains an r-value occurrence of v which is not preceded by an l-value occurrence of v.



Defining Live Variables Analysis

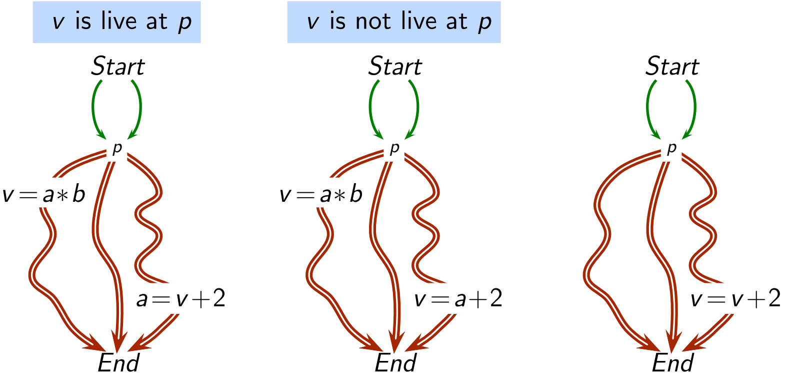
A variable v is live at a program point p, if some path from p to program exit contains an r-value occurrence of v which is not preceded by an l-value occurrence of v.

v is live at p



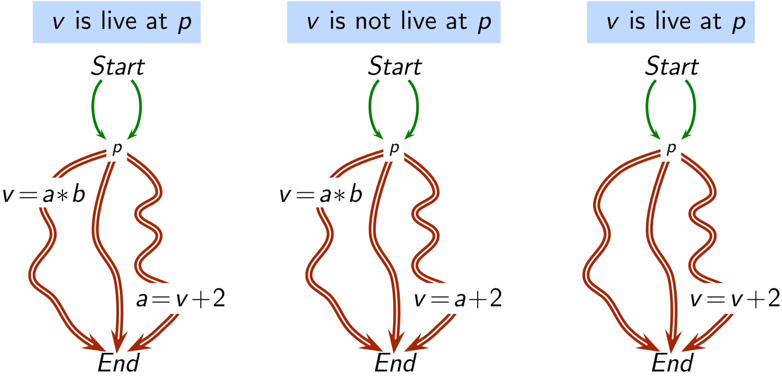
Defining Live Variables Analysis

A variable v is live at a program point p, if some path from p to program exit contains an r-value occurrence of v which is not preceded by an l-value occurrence of v.



Defining Live Variables Analysis

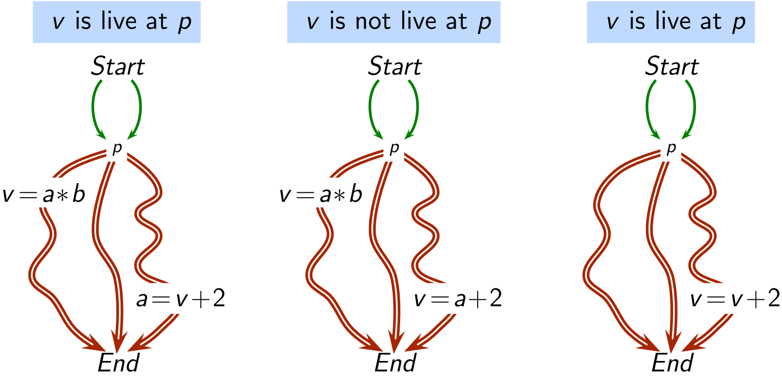
A variable v is live at a program point p, if some path from p to program exit contains an r-value occurrence of v which is not preceded by an l-value occurrence of v.



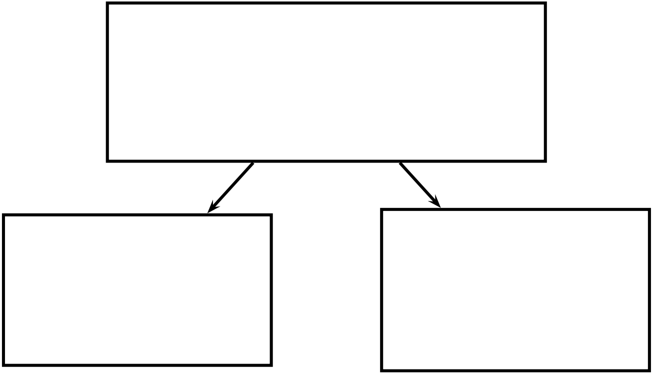
Defining Live Variables Analysis

A variable v is live at a program point p, if some path from p to program exit contains an r-value occurrence of v which is not preceded by an l-value occurrence of v.

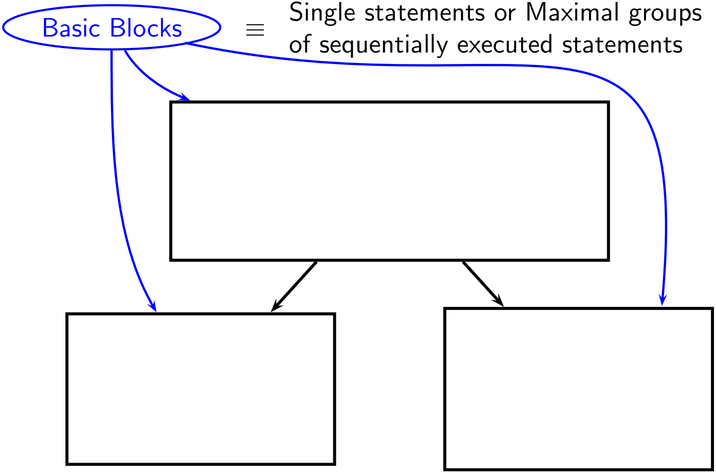
Path based specification



Defining Data Flow Analysis for Live Variables Analysis

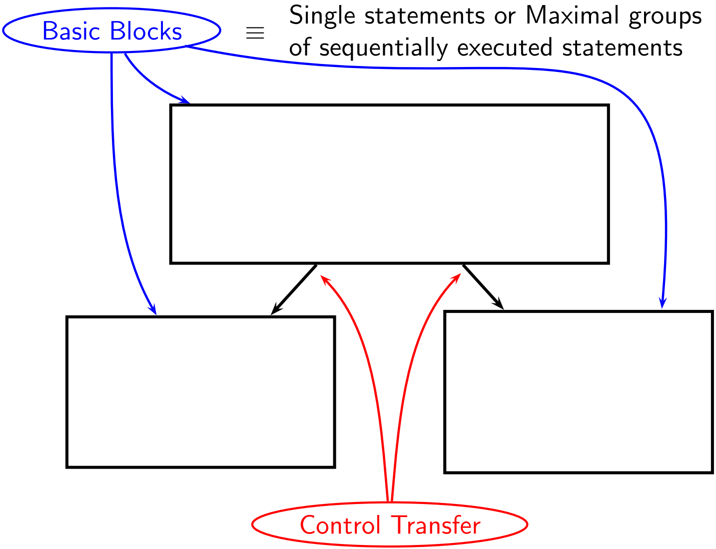


Defining Data Flow Analysis for Live Variables Analysis

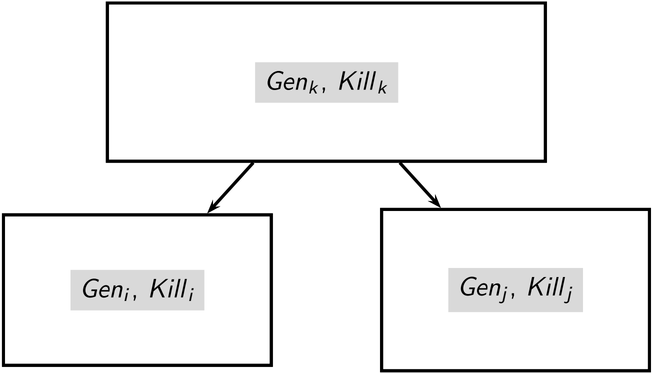


≡ Single statements or Maximal groups of sequentially executed statements

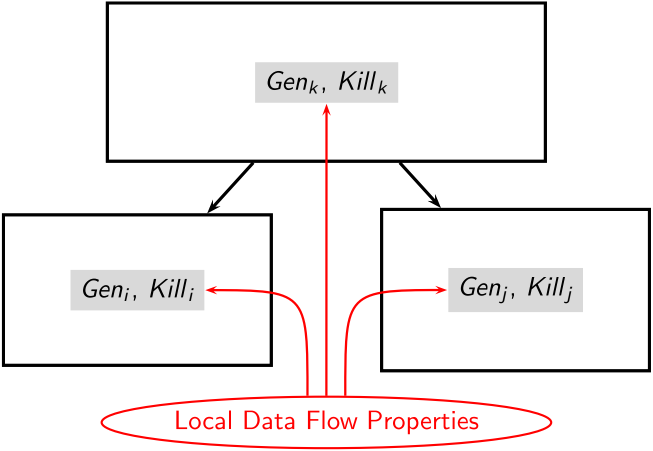
Defining Data Flow Analysis for Live Variables Analysis



Defining Data Flow Analysis for Live Variables Analysis



Defining Data Flow Analysis for Live Variables Analysis



Local Data Flow Properties for Live Variables Analysis

$$
\begin{aligned}
Gen_n &= \{\ v \mid \text{variable } v \text{ is used in basic block } n \text{ and} \\
&\qquad\quad \text{is not preceded by a definition of } v\ \} \\
Kill_n &= \{\ v \mid \text{basic block } n \text{ contains a definition of } v\ \}
\end{aligned}
$$

Local Data Flow Properties for Live Variables Analysis

r-value occurrence

Value is only read, e.g. x,y,z in

x.sum = y.data + z.data

$$Gen_n = \{ v \mid \text{variable } v \text{ is used in basic block } n \text{ and is not preceded by a definition of } v \}$$

$$Kill_n = \{ v \mid \text{basic block } n \text{ contains a definition of } v \}$$

Local Data Flow Properties for Live Variables Analysis

r-value occurrence

Value is only read, e.g. x,y,z in

x.sum = y.data + z.data

l-value occurrence

Value is modified e.g. y in

y = x.lptr

$$Gen_n = \{ v \mid \text{variable } v \text{ is used in basic block } n \text{ and is not preceded by a definition of } v \}$$

$$Kill_n = \{ v \mid \text{basic block } n \text{ contains a definition of } v \}$$

Local Data Flow Properties for Live Variables Analysis

r-value occurrence

Value is only read, e.g. x,y,z in

x.sum = y.data + z.data

l-value occurrence

Value is modified e.g. y in

y = x.lptr

$$Gen_n = \{ v \mid \text{variable } v \text{ is used in basic block } n \text{ and is not preceded by a definition of } v \}$$

$$Kill_n = \{ v \mid \text{basic block } n \text{ contains a definition of } v \}$$

within n

Local Data Flow Properties for Live Variables Analysis

r-value occurrence

Value is only read, e.g. x,y,z in

x.sum = y.data + z.data

l-value occurrence

Value is modified e.g. y in

y = x.lptr

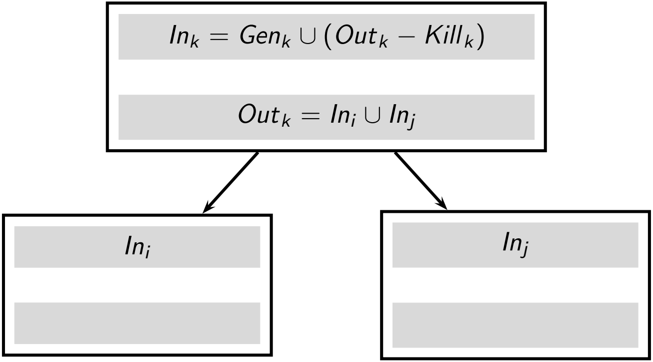
$$Gen_n = \{ v \mid \text{variable } v \text{ is used in basic block } n \text{ and is not preceded by a definition of } v \}$$

$$Kill_n = \{ v \mid \text{basic block } n \text{ contains a definition of } v \}$$

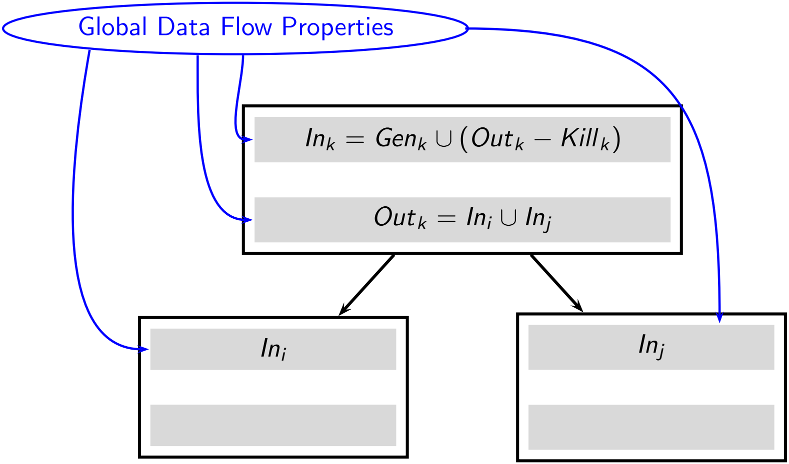
within n

anywhere in n

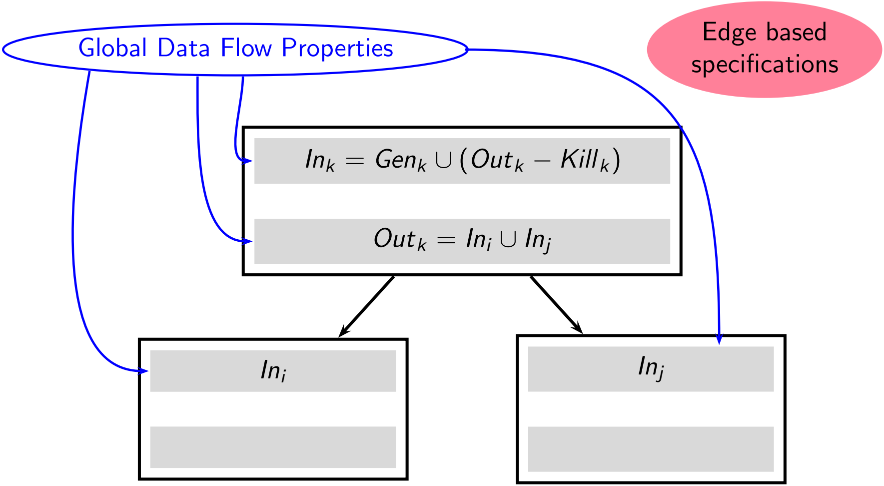
Defining Data Flow Analysis for Live Variables Analysis



Defining Data Flow Analysis for Live Variables Analysis



Defining Data Flow Analysis for Live Variables Analysis



Data Flow Equations For Live Variables Analysis

$$
\begin{aligned}
In_n &= (Out_n - Kill_n) \cup Gen_n \\
Out_n &= \begin{cases} BI & n \text{ is } End \text{ block} \\ \displaystyle\bigcup_{s \in succ(n)} In_s & \text{otherwise} \end{cases}
\end{aligned}
$$

Data Flow Equations For Live Variables Analysis

$$
\begin{aligned}
In_n &= (Out_n - Kill_n) \cup Gen_n \\
Out_n &= \begin{cases} BI & n \text{ is } End \text{ block} \\ \bigcup\limits_{s \in succ(n)} In_s & \text{otherwise} \end{cases}
\end{aligned}
$$

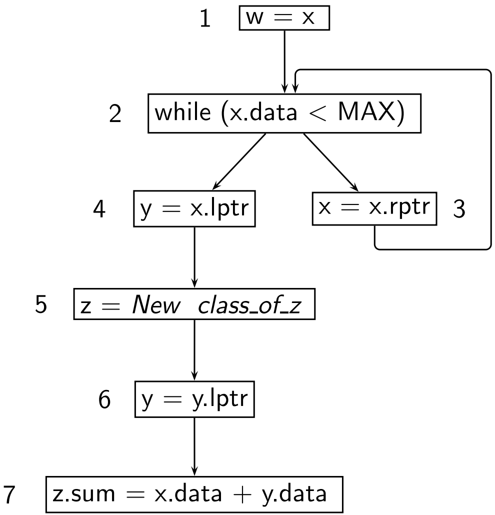
- In_n and Out_n are sets of variables

Data Flow Equations For Live Variables Analysis

$$
\begin{aligned}
In_n &= (Out_n - Kill_n) \cup Gen_n \\
Out_n &= \begin{cases} BI & n \text{ is } End \text{ block} \\ \displaystyle\bigcup_{s \in succ(n)} In_s & \text{otherwise} \end{cases}
\end{aligned}
$$

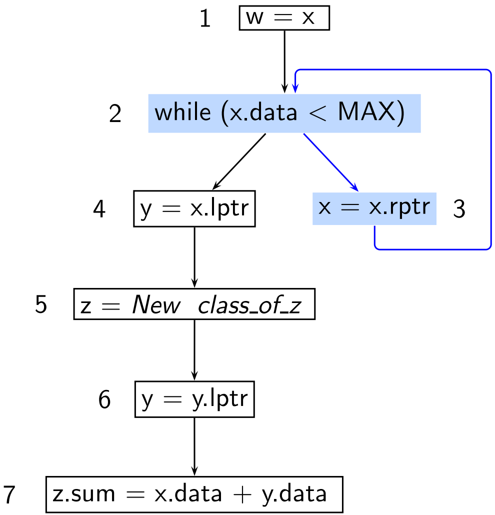
- In_n and Out_n are sets of variables
- BI is boundary information representing the effect of calling contexts
  - $\emptyset$ for local variables except for the values being returned
  - set of global variables used further in any calling context (can be safely approximated by the set of all global variables)

Data Flow Equations for Our Example



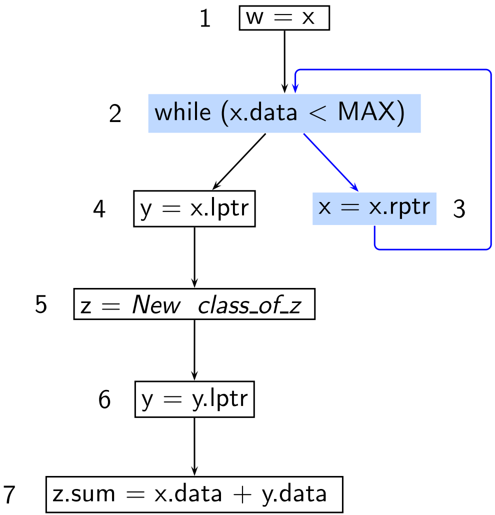
$$
\begin{aligned}
In_1 &= (Out_1 - Kill_1) \cup Gen_1 \\
Out_1 &= In_2 \\
In_2 &= (Out_2 - Kill_2) \cup Gen_2 \\
Out_2 &= In_3 \cup In_4 \\
In_3 &= (Out_3 - Kill_3) \cup Gen_3 \\
Out_3 &= In_2 \\
In_4 &= (Out_4 - Kill_4) \cup Gen_4 \\
Out_4 &= In_5 \\
In_5 &= (Out_5 - Kill_5) \cup Gen_5 \\
Out_5 &= In_6 \\
In_6 &= (Out_6 - Kill_6) \cup Gen_6 \\
Out_6 &= In_7 \\
In_7 &= (Out_7 - Kill_7) \cup Gen_7 \\
Out_7 &= \emptyset
\end{aligned}
$$

Data Flow Equations for Our Example



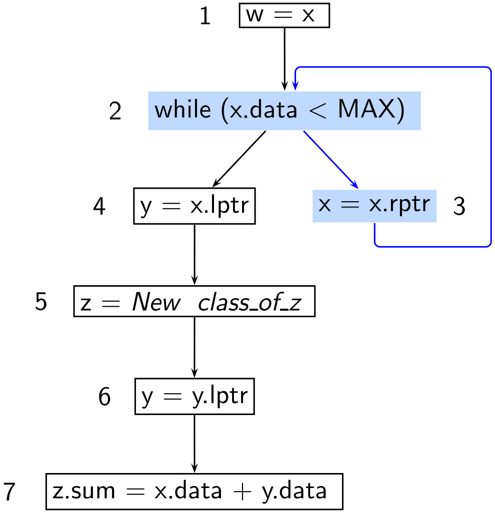
$$
\begin{aligned}
In_1 &= (Out_1 - Kill_1) \cup Gen_1\\
Out_1 &= In_2\\
In_2 &= (Out_2 - Kill_2) \cup Gen_2\\
Out_2 &= In_3 \cup In_4\\
In_3 &= (Out_3 - Kill_3) \cup Gen_3\\
Out_3 &= In_2\\
In_4 &= (Out_4 - Kill_4) \cup Gen_4\\
Out_4 &= In_5\\
In_5 &= (Out_5 - Kill_5) \cup Gen_5\\
Out_5 &= In_6\\
In_6 &= (Out_6 - Kill_6) \cup Gen_6\\
Out_6 &= In_7\\
In_7 &= (Out_7 - Kill_7) \cup Gen_7\\
Out_7 &= \emptyset
\end{aligned}
$$

Data Flow Equations for Our Example



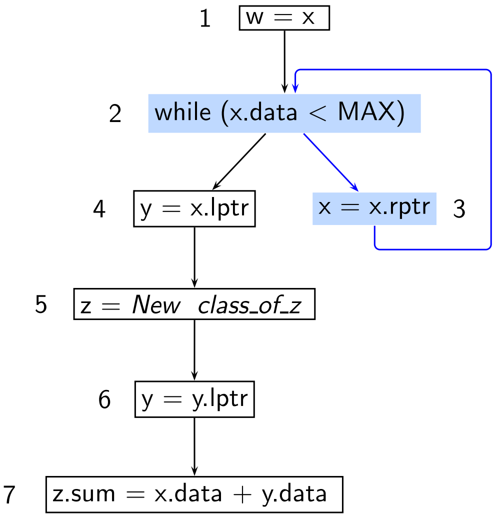
$$
\begin{aligned}
In_1 &= (Out_1 - Kill_1) \cup Gen_1\\
Out_1 &= In_2\\
In_2 &= (Out_2 - Kill_2) \cup Gen_2\\
Out_2 &= In_3 \cup In_4\\
In_3 &= (Out_3 - Kill_3) \cup Gen_3\\
Out_3 &= In_2\\
In_4 &= (Out_4 - Kill_4) \cup Gen_4\\
Out_4 &= In_5\\
In_5 &= (Out_5 - Kill_5) \cup Gen_5\\
Out_5 &= In_6\\
In_6 &= (Out_6 - Kill_6) \cup Gen_6\\
Out_6 &= In_7\\
In_7 &= (Out_7 - Kill_7) \cup Gen_7\\
Out_7 &= \emptyset
\end{aligned}
$$

Data Flow Equations for Our Example



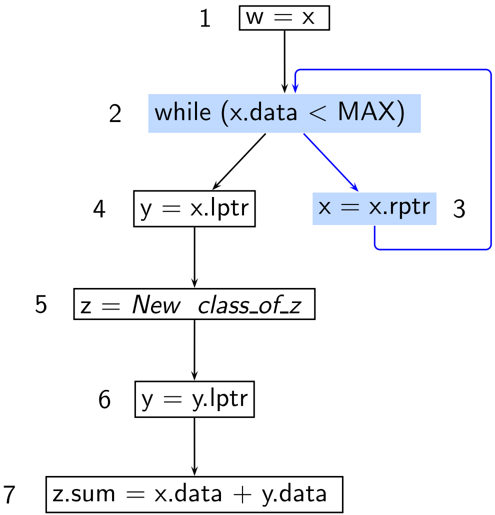
$$
\begin{aligned}
In_1 &= (Out_1 - Kill_1) \cup Gen_1 \\
Out_1 &= In_2 \\
In_2 &= (Out_2 - Kill_2) \cup Gen_2 \\
Out_2 &= In_3 \cup In_4 \\
In_3 &= (Out_3 - Kill_3) \cup Gen_3 \\
Out_3 &= In_2 \\
In_4 &= (Out_4 - Kill_4) \cup Gen_4 \\
Out_4 &= In_5 \\
In_5 &= (Out_5 - Kill_5) \cup Gen_5 \\
Out_5 &= In_6 \\
In_6 &= (Out_6 - Kill_6) \cup Gen_6 \\
Out_6 &= In_7 \\
In_7 &= (Out_7 - Kill_7) \cup Gen_7 \\
Out_7 &= \emptyset
\end{aligned}
$$

Data Flow Equations for Our Example



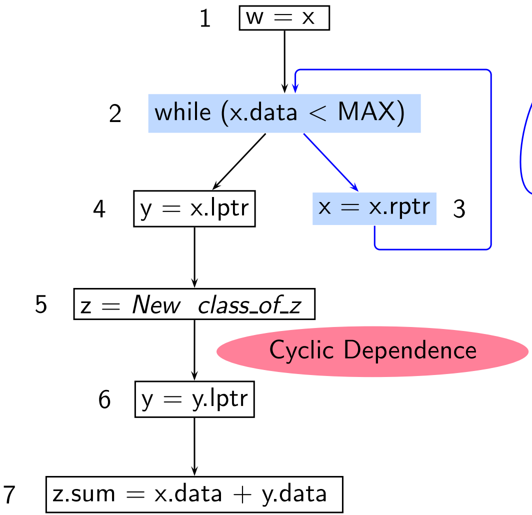
$$
\begin{aligned}
In_1 &= (Out_1 - Kill_1) \cup Gen_1 \\
Out_1 &= In_2 \\
In_2 &= (Out_2 - Kill_2) \cup Gen_2 \\
Out_2 &= In_3 \cup In_4 \\
In_3 &= (Out_3 - Kill_3) \cup Gen_3 \\
Out_3 &= In_2 \\
In_4 &= (Out_4 - Kill_4) \cup Gen_4 \\
Out_4 &= In_5 \\
In_5 &= (Out_5 - Kill_5) \cup Gen_5 \\
Out_5 &= In_6 \\
In_6 &= (Out_6 - Kill_6) \cup Gen_6 \\
Out_6 &= In_7 \\
In_7 &= (Out_7 - Kill_7) \cup Gen_7 \\
Out_7 &= \emptyset
\end{aligned}
$$

Data Flow Equations for Our Example



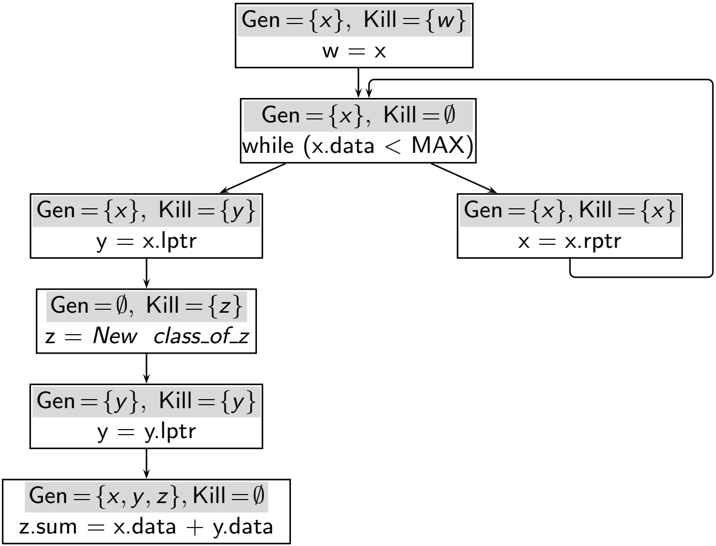
$$
\begin{aligned}
In_1 &= (Out_1 - Kill_1) \cup Gen_1 \\
Out_1 &= In_2 \\
In_2 &= (Out_2 - Kill_2) \cup Gen_2 \\
Out_2 &= In_3 \cup In_4 \\
In_3 &= (Out_3 - Kill_3) \cup Gen_3 \\
Out_3 &= In_2 \\
In_4 &= (Out_4 - Kill_4) \cup Gen_4 \\
Out_4 &= In_5 \\
In_5 &= (Out_5 - Kill_5) \cup Gen_5 \\
Out_5 &= In_6 \\
In_6 &= (Out_6 - Kill_6) \cup Gen_6 \\
Out_6 &= In_7 \\
In_7 &= (Out_7 - Kill_7) \cup Gen_7 \\
Out_7 &= \emptyset
\end{aligned}
$$

Data Flow Equations for Our Example

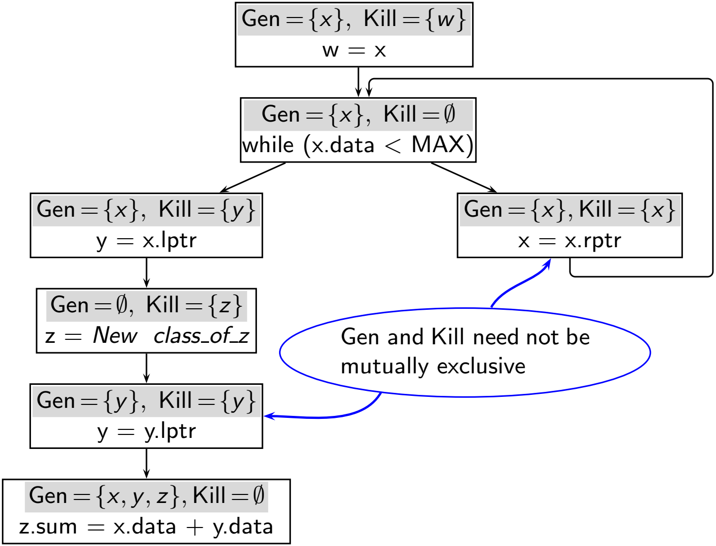


$$
\begin{aligned}
In_1 &= (Out_1 - Kill_1) \cup Gen_1 \\
Out_1 &= In_2 \\
In_2 &= (Out_2 - Kill_2) \cup Gen_2 \\
Out_2 &= In_3 \cup In_4 \\
In_3 &= (Out_3 - Kill_3) \cup Gen_3 \\
Out_3 &= In_2 \\
In_4 &= (Out_4 - Kill_4) \cup Gen_4 \\
Out_4 &= In_5 \\
In_5 &= (Out_5 - Kill_5) \cup Gen_5 \\
Out_5 &= In_6 \\
In_6 &= (Out_6 - Kill_6) \cup Gen_6 \\
Out_6 &= In_7 \\
In_7 &= (Out_7 - Kill_7) \cup Gen_7 \\
Out_7 &= \emptyset
\end{aligned}
$$

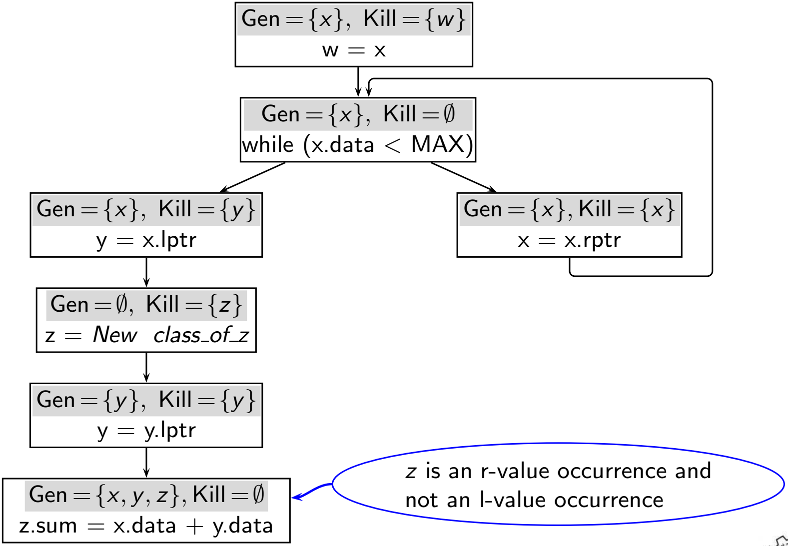
Performing Live Variables Analysis



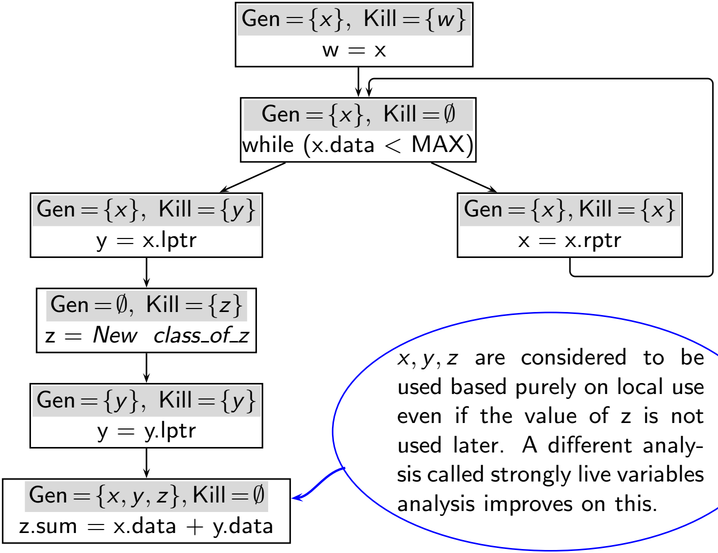
Performing Live Variables Analysis



Performing Live Variables Analysis

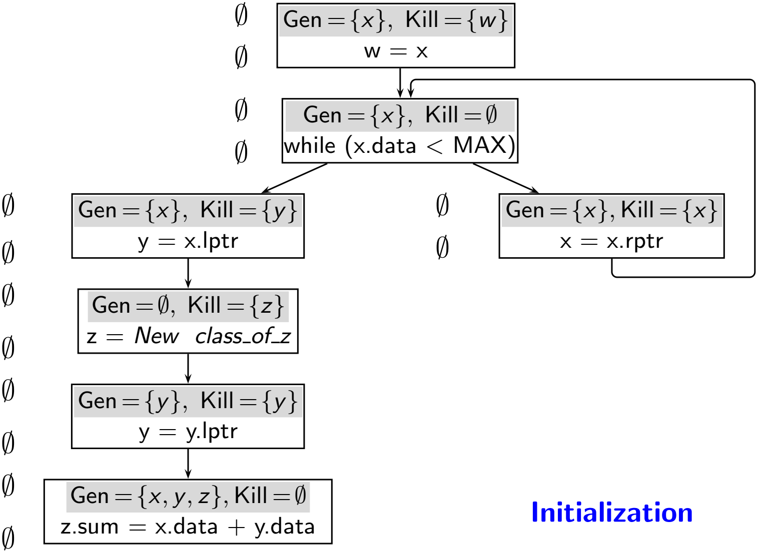


Performing Live Variables Analysis



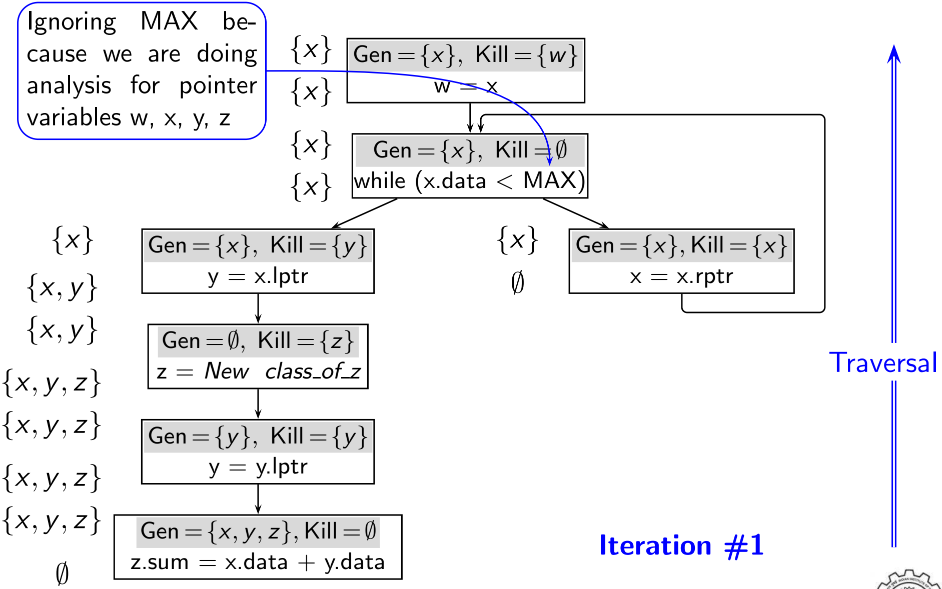
x, y, z are considered to be used based purely on local use even if the value of z is not used later. A different analysis called strongly live variables analysis improves on this.

Performing Live Variables Analysis

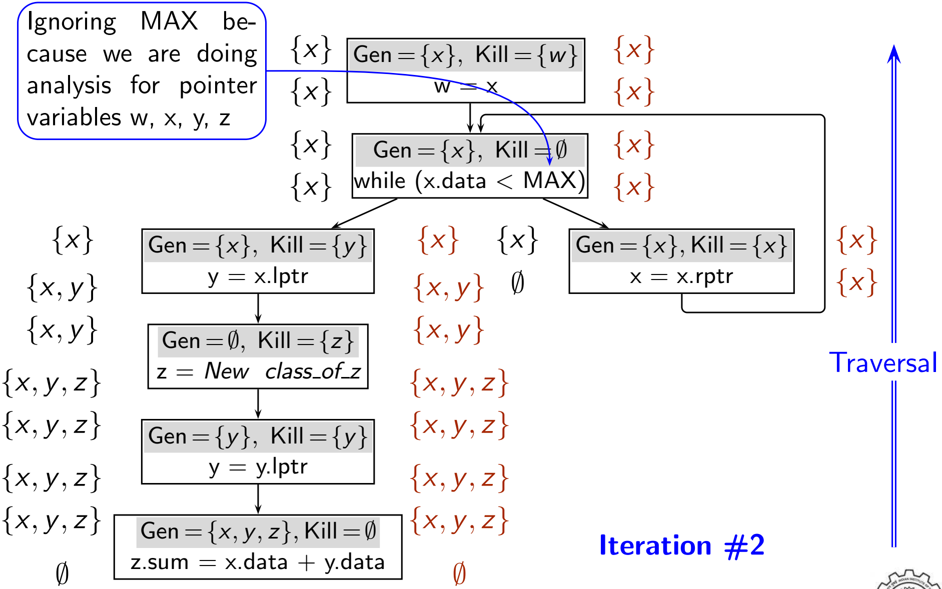


Initialization

Performing Live Variables Analysis

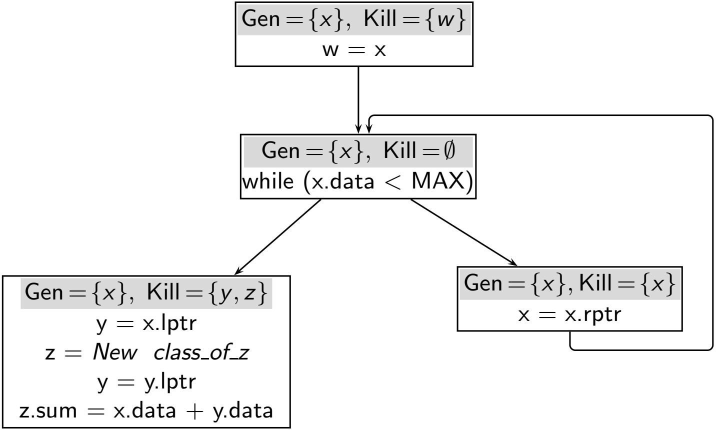


Performing Live Variables Analysis



Performing Live Variables Analysis

Local data flow properties when basic blocks contain multiple statements



Local Data Flow Properties for Live Variables Analysis

$$In_n = Gen_n \cup (Out_n - Kill_n)$$

- Gen_n : Use not preceded by definition
- $Kill_n$: Definition anywhere in a block

Local Data Flow Properties for Live Variables Analysis

$$In_n = Gen_n \cup (Out_n - Kill_n)$$

- Gen_n : Use not preceded by definition

  Upwards exposed use

- $Kill_n$: Definition anywhere in a block

  Stop the effect from being propagated across a block

Local Data Flow Properties for Live Variables Analysis

Case	Local Information		Example basic block	Explanation
1	$v \notin Gen_n$	$v \notin Kill_n$		
2	$v \in Gen_n$	$v \notin Kill_n$		
3	$v \notin Gen_n$	$v \in Kill_n$		
4	$v \in Gen_n$	$v \in Kill_n$		

Local Data Flow Properties for Live Variables Analysis

Case	Local Information		Example basic block	Explanation
1	$v \notin Gen_n$	$v \notin Kill_n$	$a = b + c$ $b = c * d$	liveness of v is unaffected by the basic block
2	$v \in Gen_n$	$v \notin Kill_n$	$a = b + c$ $b = v * d$	v becomes live before the basic block
3	$v \notin Gen_n$	$v \in Kill_n$	$a = b + c$ $v = c * d$ OR $v = a + b$ $c = v * d$	v ceases to be live before the basic block
4	$v \in Gen_n$	$v \in Kill_n$	$a = v + c$ $v = c * d$	liveness of v is killed but v becomes live before the basic block

Using Data Flow Information of Live Variables Analysis

- Used for register allocation

  If variable x is live in a basic block b, it is a potential candidate for register allocation

Using Data Flow Information of Live Variables Analysis

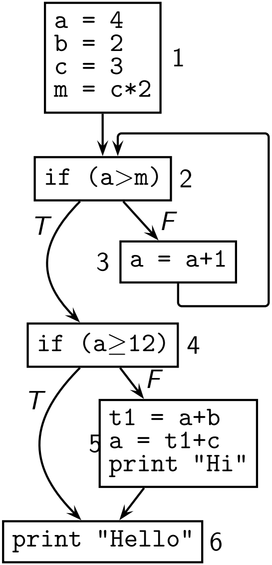
- Used for register allocation

  If variable x is live in a basic block b, it is a potential candidate for register allocation

- Used for dead code elimination

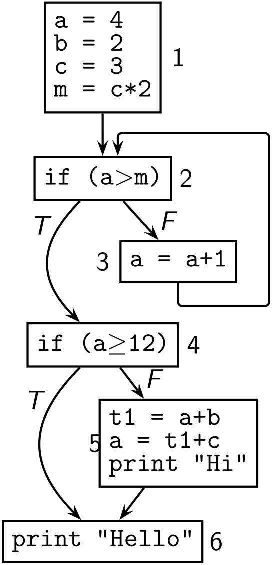
  If variable x is not live after an assignment $x = \ldots$, then the assignment is redundant and can be deleted as dead code

Tutorial Problem 1: Perform Dead Code Elimination



Local Data Flow Information		
	Gen	*Kill*
1	$\emptyset$	$\{a, b, c, m\}$
2	$\{a, m\}$	$\emptyset$
3	$\{a\}$	$\{a\}$
4	$\{a\}$	$\emptyset$
5	$\{a, b, c\}$	$\{a, t1\}$
6	$\emptyset$	$\emptyset$

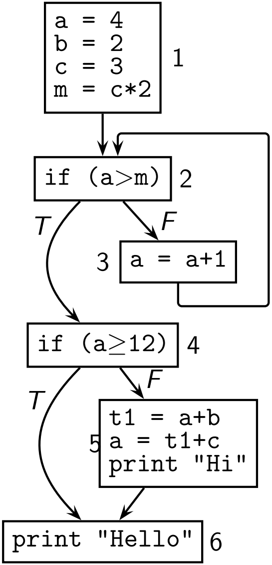
Tutorial Problem 1: Perform Dead Code Elimination



Local Data Flow Information		
	Gen	*Kill*
1	$\emptyset$	$\{a, b, c, m\}$
2	$\{a, m\}$	$\emptyset$
3	$\{a\}$	$\{a\}$
4	$\{a\}$	$\emptyset$
5	$\{a, b, c\}$	$\{a, t1\}$
6	$\emptyset$	$\emptyset$

Global Data Flow Information				
	Iteration #1		Iteration #2	
	Out	*In*	*Out*	*In*
6	$\emptyset$	$\emptyset$		
5	$\emptyset$	$\{a, b, c\}$		
4	$\{a, b, c\}$	$\{a, b, c\}$		
3	$\emptyset$	$\{a\}$		
2	$\{a, b, c\}$	$\{a, b, c, m\}$		
1	$\{a, b, c, m\}$	$\emptyset$		

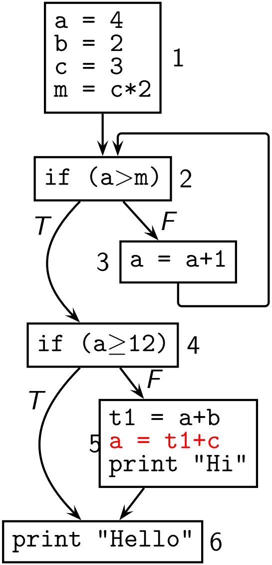
Tutorial Problem 1: Perform Dead Code Elimination



Local Data Flow Information		
	Gen	*Kill*
1	$\emptyset$	$\{a, b, c, m\}$
2	$\{a, m\}$	$\emptyset$
3	$\{a\}$	$\{a\}$
4	$\{a\}$	$\emptyset$
5	$\{a, b, c\}$	$\{a, t1\}$
6	$\emptyset$	$\emptyset$

Global Data Flow Information				
	Iteration #1		Iteration #2	
	Out	*In*	*Out*	*In*
6	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
5	$\emptyset$	$\{a, b, c\}$	$\emptyset$	$\{a, b, c\}$
4	$\{a, b, c\}$	$\{a, b, c\}$	$\{a, b, c\}$	$\{a, b, c\}$
3	$\emptyset$	$\{a\}$	$\{a, b, c, m\}$	$\{a, b, c, m\}$
2	$\{a, b, c\}$	$\{a, b, c, m\}$	$\{a, b, c, m\}$	$\{a, b, c, m\}$
1	$\{a, b, c, m\}$	$\emptyset$	$\{a, b, c, m\}$	$\emptyset$

Tutorial Problem 1: Perform Dead Code Elimination



Local Data Flow Information		
	Gen	*Kill*
1	$\emptyset$	$\{a, b, c, m\}$
2	$\{a, m\}$	$\emptyset$
3	$\{a\}$	$\{a\}$
4	$\{a\}$	$\emptyset$
5	$\{a, b, c\}$	$\{a, t1\}$
6	$\emptyset$	$\emptyset$

Global Data Flow Information				
	Iteration #1		Iteration #2	
	Out	*In*	*Out*	*In*
6	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
5	$\emptyset$	$\{a, b, c\}$	$\emptyset$	$\{a, b, c\}$
4	$\{a, b, c\}$	$\{a, b, c\}$	$\{a, b, c\}$	$\{a, b, c\}$
3	$\emptyset$	$\{a\}$	$\{a, b, c, m\}$	$\{a, b, c, m\}$
2	$\{a, b, c\}$	$\{a, b, c, m\}$	$\{a, b, c, m\}$	$\{a, b, c, m\}$
1	$\{a, b, c, m\}$	$\emptyset$	$\{a, b, c, m\}$	$\emptyset$

Tutorial Problem 1: Observeations About Round #1 of Dead Code Elimination

- We can repeat liveness analysis on the optimized code and then optimize it further

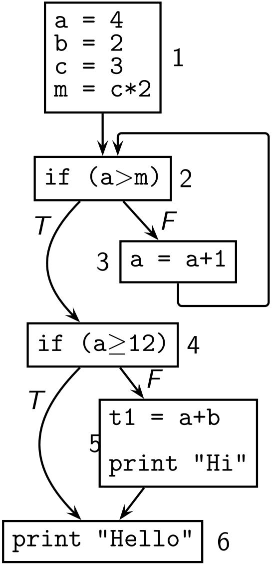
  This can continue as long code continues to change

- A better approach would be to perform strong liveness analysis

  The code needs to be optimized only once

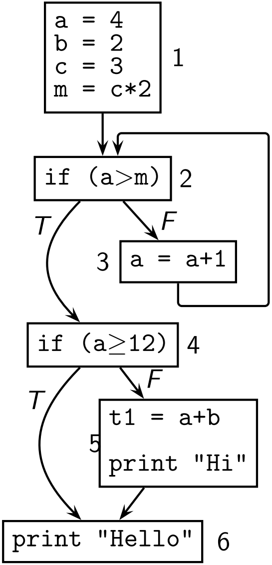
- Here we show the repeated application only to show the scope of further optimizations

Tutorial Problem 1: Round #2 of Dead Code Elimination



Local Data Flow Information		
	Gen	*Kill*
1	$\emptyset$	$\{a, b, c, m\}$
2	$\{a, m\}$	$\emptyset$
3	$\{a\}$	$\{a\}$
4	$\{a\}$	$\emptyset$
5	$\{a, b\}$	$\{t1\}$
6	$\emptyset$	$\emptyset$

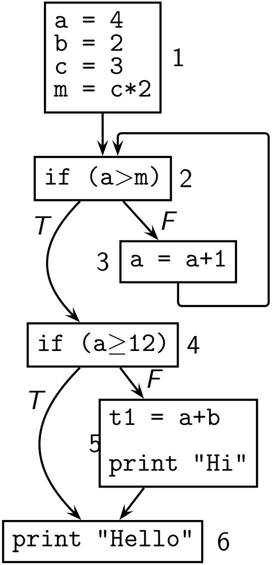
Tutorial Problem 1: Round #2 of Dead Code Elimination



Local Data Flow Information		
	Gen	*Kill*
1	$\emptyset$	$\{a, b, c, m\}$
2	$\{a, m\}$	$\emptyset$
3	$\{a\}$	$\{a\}$
4	$\{a\}$	$\emptyset$
5	$\{a, b\}$	$\{t1\}$
6	$\emptyset$	$\emptyset$

Global Data Flow Information				
	Iteration #1		Iteration #2	
	Out	*In*	*Out*	*In*
6	$\emptyset$	$\emptyset$		
5	$\emptyset$	$\{a, b\}$		
4	$\{a, b\}$	$\{a, b\}$		
3	$\emptyset$	$\{a\}$		
2	$\{a, b\}$	$\{a, b, m\}$		
1	$\{a, b, m\}$	$\emptyset$		

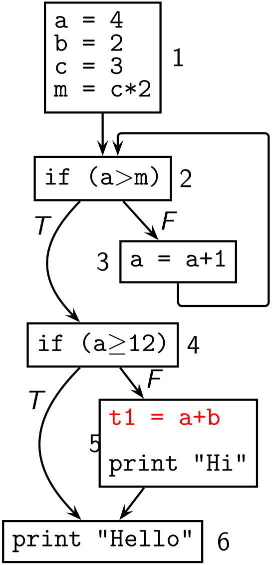
Tutorial Problem 1: Round #2 of Dead Code Elimination



Local Data Flow Information		
	Gen	*Kill*
1	$\emptyset$	$\{a, b, c, m\}$
2	$\{a, m\}$	$\emptyset$
3	$\{a\}$	$\{a\}$
4	$\{a\}$	$\emptyset$
5	$\{a, b\}$	$\{t1\}$
6	$\emptyset$	$\emptyset$

Global Data Flow Information				
	Iteration #1		Iteration #2	
	Out	*In*	*Out*	*In*
6	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
5	$\emptyset$	$\{a, b\}$	$\emptyset$	$\{a, b\}$
4	$\{a, b\}$	$\{a, b\}$	$\{a, b\}$	$\{a, b\}$
3	$\emptyset$	$\{a\}$	$\{a, b, m\}$	$\{a, b, m\}$
2	$\{a, b\}$	$\{a, b, m\}$	$\{a, b, m\}$	$\{a, b, m\}$
1	$\{a, b, m\}$	$\emptyset$	$\{a, b, m\}$	$\emptyset$

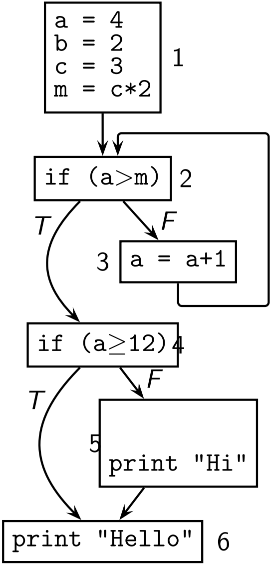
Tutorial Problem 1: Round #2 of Dead Code Elimination



Local Data Flow Information		
	Gen	*Kill*
1	$\emptyset$	$\{a, b, c, m\}$
2	$\{a, m\}$	$\emptyset$
3	$\{a\}$	$\{a\}$
4	$\{a\}$	$\emptyset$
5	$\{a, b\}$	$\{t1\}$
6	$\emptyset$	$\emptyset$

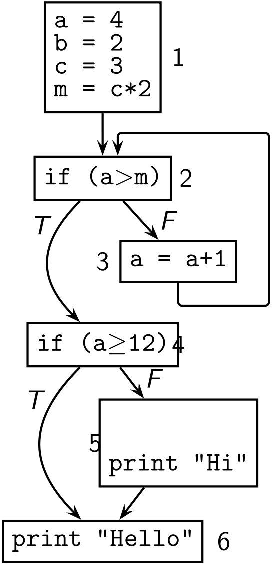
Global Data Flow Information				
	Iteration #1		Iteration #2	
	Out	*In*	*Out*	*In*
6	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
5	$\emptyset$	$\{a, b\}$	$\emptyset$	$\{a, b\}$
4	$\{a, b\}$	$\{a, b\}$	$\{a, b\}$	$\{a, b\}$
3	$\emptyset$	$\{a\}$	$\{a, b, m\}$	$\{a, b, m\}$
2	$\{a, b\}$	$\{a, b, m\}$	$\{a, b, m\}$	$\{a, b, m\}$
1	$\{a, b, m\}$	$\emptyset$	$\{a, b, m\}$	$\emptyset$

Tutorial Problem 1: Round #3 of Dead Code Elimination



Local Data Flow Information		
	Gen	*Kill*
1	$\emptyset$	$\{a, b, c, m\}$
2	$\{a, m\}$	$\emptyset$
3	$\{a\}$	$\{a\}$
4	$\{a\}$	$\emptyset$
5	$\emptyset$	$\emptyset$
6	$\emptyset$	$\emptyset$

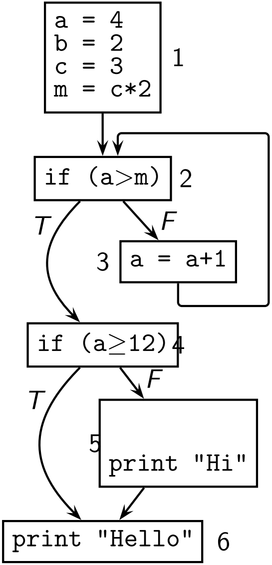
Tutorial Problem 1: Round #3 of Dead Code Elimination



Local Data Flow Information		
	Gen	*Kill*
1	$\emptyset$	$\{a, b, c, m\}$
2	$\{a, m\}$	$\emptyset$
3	$\{a\}$	$\{a\}$
4	$\{a\}$	$\emptyset$
5	$\emptyset$	$\emptyset$
6	$\emptyset$	$\emptyset$

Global Data Flow Information				
	Iteration #1		Iteration #2	
	Out	*In*	*Out*	*In*
6	$\emptyset$	$\emptyset$		
5	$\emptyset$	$\emptyset$		
4	$\emptyset$	$\{a\}$		
3	$\emptyset$	$\{a\}$		
2	$\{a\}$	$\{a, m\}$		
1	$\{a, m\}$	$\emptyset$		

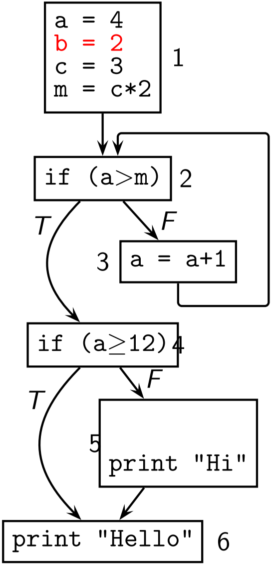
Tutorial Problem 1: Round #3 of Dead Code Elimination



Local Data Flow Information		
	Gen	*Kill*
1	$\emptyset$	$\{a, b, c, m\}$
2	$\{a, m\}$	$\emptyset$
3	$\{a\}$	$\{a\}$
4	$\{a\}$	$\emptyset$
5	$\emptyset$	$\emptyset$
6	$\emptyset$	$\emptyset$

Global Data Flow Information				
	Iteration #1		Iteration #2	
	Out	*In*	*Out*	*In*
6	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
5	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
4	$\emptyset$	$\{a\}$	$\emptyset$	$\{a\}$
3	$\emptyset$	$\{a\}$	$\{a, m\}$	$\{a, m\}$
2	$\{a\}$	$\{a, m\}$	$\{a, m\}$	$\{a, m\}$
1	$\{a, m\}$	$\emptyset$	$\{a, m\}$	$\emptyset$

Tutorial Problem 1: Round #3 of Dead Code Elimination



Local Data Flow Information		
	Gen	*Kill*
1	$\emptyset$	$\{a, b, c, m\}$
2	$\{a, m\}$	$\emptyset$
3	$\{a\}$	$\{a\}$
4	$\{a\}$	$\emptyset$
5	$\emptyset$	$\emptyset$
6	$\emptyset$	$\emptyset$

Global Data Flow Information				
	Iteration #1		Iteration #2	
	Out	*In*	*Out*	*In*
6	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
5	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
4	$\emptyset$	$\{a\}$	$\emptyset$	$\{a\}$
3	$\emptyset$	$\{a\}$	$\{a, m\}$	$\{a, m\}$
2	$\{a\}$	$\{a, m\}$	$\{a, m\}$	$\{a, m\}$
1	$\{a, m\}$	$\emptyset$	$\{a, m\}$	$\emptyset$

Part 3

Some Observations

What Does Data Flow Analysis Involve?

- Defining the analysis.
- Formulating the analysis.
- Performing the analysis.

What Does Data Flow Analysis Involve?

- Defining the analysis. Define the properties of execution paths
- Formulating the analysis.
- Performing the analysis.

What Does Data Flow Analysis Involve?

- Defining the analysis. Define the properties of execution paths
- Formulating the analysis. Define data flow equations
  - Linear simultaneous equations on sets rather than numbers
  - Later we will generalize the domain of values
- Performing the analysis.

What Does Data Flow Analysis Involve?

- Defining the analysis. Define the properties of execution paths
- Formulating the analysis. Define data flow equations
  - Linear simultaneous equations on sets rather than numbers
  - Later we will generalize the domain of values
- Performing the analysis. Solve data flow equations for the given program flow graph

What Does Data Flow Analysis Involve?

- Defining the analysis. Define the properties of execution paths
- Formulating the analysis. Define data flow equations
  - Linear simultaneous equations on sets rather than numbers
  - Later we will generalize the domain of values
- Performing the analysis. Solve data flow equations for the given program flow graph
- Many unanswered questions

  Initial value? Termination? Complexity? Properties of Solutions?

A Digression: Iterative Solution of Linear Simultaneous Equations

- Simultaneous equations represented in the form of the product of a matrix of coefficients (**A**) with the vector of unknowns (**x**)

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

- Start with approximate values
- Compute new values repeatedly from old values
- Two classical methods
  - Gauss-Seidel Method (Gauss: 1823, 1826), (Seidel: 1874)
  - Jacobi Method (Jacobi: 1845)

A Digression: An Example of Iterative Solution of Linear Simultaneous Equations

Equations	Solution
$4w = x + y + 32$ $4x = y + z + 32$ $4y = z + w + 32$ $4z = w + x + 32$	$w = x = y = z = 16$

- Rewrite the equations to define w, x, y, and z

$$
\begin{aligned}
w &= 0.25x + 0.25y + 8 \\
x &= 0.25y + 0.25z + 8 \\
y &= 0.25z + 0.25w + 8 \\
z &= 0.25w + 0.25x + 8
\end{aligned}
$$

- Assume some initial values of w_0, x_0, y_0, and z_0
- Compute w_i, x_i, y_i, and z_i within some margin of error

A Digression: Gauss-Seidel Method

Equations	Initial Values	Error Margin
$w = 0.25x + 0.25y + 8$	$w_0 = 24$	$w_{i+1} - w_i \leq 0.35$
$x = 0.25y + 0.25z + 8$	$x_0 = 24$	$x_{i+1} - x_i \leq 0.35$
$y = 0.25z + 0.25w + 8$	$y_0 = 24$	$y_{i+1} - y_i \leq 0.35$
$z = 0.25w + 0.25x + 8$	$z_0 = 24$	$z_{i+1} - z_i \leq 0.35$

Iteration 1	Iteration 2	Iteration 3

Iteration 4	Iteration 5

A Digression: Gauss-Seidel Method

Equations	Initial Values	Error Margin
$w = 0.25x + 0.25y + 8$	$w_0 = 24$	$w_{i+1} - w_i \leq 0.35$
$x = 0.25y + 0.25z + 8$	$x_0 = 24$	$x_{i+1} - x_i \leq 0.35$
$y = 0.25z + 0.25w + 8$	$y_0 = 24$	$y_{i+1} - y_i \leq 0.35$
$z = 0.25w + 0.25x + 8$	$z_0 = 24$	$z_{i+1} - z_i \leq 0.35$

Iteration 1	Iteration 2	Iteration 3
$w_1 = 6 + 6 + 8 = 20$ $x_1 = 6 + 6 + 8 = 20$ $y_1 = 6 + 6 + 8 = 20$ $z_1 = 6 + 6 + 8 = 20$		

Iteration 4	Iteration 5

A Digression: Gauss-Seidel Method

Equations	Initial Values	Error Margin
$w = 0.25x + 0.25y + 8$	$w_0 = 24$	$w_{i+1} - w_i \leq 0.35$
$x = 0.25y + 0.25z + 8$	$x_0 = 24$	$x_{i+1} - x_i \leq 0.35$
$y = 0.25z + 0.25w + 8$	$y_0 = 24$	$y_{i+1} - y_i \leq 0.35$
$z = 0.25w + 0.25x + 8$	$z_0 = 24$	$z_{i+1} - z_i \leq 0.35$

Iteration 1	Iteration 2	Iteration 3
$w_1 = 6 + 6 + 8 = 20$	$w_2 = 5 + 5 + 8 = 18$	
$x_1 = 6 + 6 + 8 = 20$	$x_2 = 5 + 5 + 8 = 18$	
$y_1 = 6 + 6 + 8 = 20$	$y_2 = 5 + 5 + 8 = 18$	
$z_1 = 6 + 6 + 8 = 20$	$z_2 = 5 + 5 + 8 = 18$	

Iteration 4	Iteration 5

A Digression: Gauss-Seidel Method

Equations	Initial Values	Error Margin
$w = 0.25x + 0.25y + 8$	$w_0 = 24$	$w_{i+1} - w_i \leq 0.35$
$x = 0.25y + 0.25z + 8$	$x_0 = 24$	$x_{i+1} - x_i \leq 0.35$
$y = 0.25z + 0.25w + 8$	$y_0 = 24$	$y_{i+1} - y_i \leq 0.35$
$z = 0.25w + 0.25x + 8$	$z_0 = 24$	$z_{i+1} - z_i \leq 0.35$

Iteration 1	Iteration 2	Iteration 3
$w_1 = 6 + 6 + 8 = 20$	$w_2 = 5 + 5 + 8 = 18$	$w_3 = 4.5 + 4.5 + 8 = 17$
$x_1 = 6 + 6 + 8 = 20$	$x_2 = 5 + 5 + 8 = 18$	$x_3 = 4.5 + 4.5 + 8 = 17$
$y_1 = 6 + 6 + 8 = 20$	$y_2 = 5 + 5 + 8 = 18$	$y_3 = 4.5 + 4.5 + 8 = 17$
$z_1 = 6 + 6 + 8 = 20$	$z_2 = 5 + 5 + 8 = 18$	$z_3 = 4.5 + 4.5 + 8 = 17$

Iteration 4	Iteration 5

A Digression: Gauss-Seidel Method

Equations	Initial Values	Error Margin
$w = 0.25x + 0.25y + 8$	$w_0 = 24$	$w_{i+1} - w_i \leq 0.35$
$x = 0.25y + 0.25z + 8$	$x_0 = 24$	$x_{i+1} - x_i \leq 0.35$
$y = 0.25z + 0.25w + 8$	$y_0 = 24$	$y_{i+1} - y_i \leq 0.35$
$z = 0.25w + 0.25x + 8$	$z_0 = 24$	$z_{i+1} - z_i \leq 0.35$

Iteration 1	Iteration 2	Iteration 3
$w_1 = 6 + 6 + 8 = 20$	$w_2 = 5 + 5 + 8 = 18$	$w_3 = 4.5 + 4.5 + 8 = 17$
$x_1 = 6 + 6 + 8 = 20$	$x_2 = 5 + 5 + 8 = 18$	$x_3 = 4.5 + 4.5 + 8 = 17$
$y_1 = 6 + 6 + 8 = 20$	$y_2 = 5 + 5 + 8 = 18$	$y_3 = 4.5 + 4.5 + 8 = 17$
$z_1 = 6 + 6 + 8 = 20$	$z_2 = 5 + 5 + 8 = 18$	$z_3 = 4.5 + 4.5 + 8 = 17$

Iteration 4	Iteration 5
$w_4 = 4.25 + 4.25 + 8 = 16.5$	
$x_4 = 4.25 + 4.25 + 8 = 16.5$	
$y_4 = 4.25 + 4.25 + 8 = 16.5$	
$z_4 = 4.25 + 4.25 + 8 = 16.5$	

A Digression: Gauss-Seidel Method

Equations	Initial Values	Error Margin
$w = 0.25x + 0.25y + 8$	$w_0 = 24$	$w_{i+1} - w_i \leq 0.35$
$x = 0.25y + 0.25z + 8$	$x_0 = 24$	$x_{i+1} - x_i \leq 0.35$
$y = 0.25z + 0.25w + 8$	$y_0 = 24$	$y_{i+1} - y_i \leq 0.35$
$z = 0.25w + 0.25x + 8$	$z_0 = 24$	$z_{i+1} - z_i \leq 0.35$

Iteration 1	Iteration 2	Iteration 3
$w_1 = 6 + 6 + 8 = 20$	$w_2 = 5 + 5 + 8 = 18$	$w_3 = 4.5 + 4.5 + 8 = 17$
$x_1 = 6 + 6 + 8 = 20$	$x_2 = 5 + 5 + 8 = 18$	$x_3 = 4.5 + 4.5 + 8 = 17$
$y_1 = 6 + 6 + 8 = 20$	$y_2 = 5 + 5 + 8 = 18$	$y_3 = 4.5 + 4.5 + 8 = 17$
$z_1 = 6 + 6 + 8 = 20$	$z_2 = 5 + 5 + 8 = 18$	$z_3 = 4.5 + 4.5 + 8 = 17$

Iteration 4	Iteration 5
$w_4 = 4.25 + 4.25 + 8 = 16.5$	$w_5 = 4.125 + 4.125 + 8 = 16.25$
$x_4 = 4.25 + 4.25 + 8 = 16.5$	$x_5 = 4.125 + 4.125 + 8 = 16.25$
$y_4 = 4.25 + 4.25 + 8 = 16.5$	$y_5 = 4.125 + 4.125 + 8 = 16.25$
$z_4 = 4.25 + 4.25 + 8 = 16.5$	$z_5 = 4.125 + 4.125 + 8 = 16.25$

A Digression: Jacobi Method

Use values from the current iteration wherever possible

Equations	Initial Values	Error Margin
$w = 0.25x + 0.25y + 8$	$w_0 = 24$	$w_{i+1} - w_i \leq 0.35$
$x = 0.25y + 0.25z + 8$	$x_0 = 24$	$x_{i+1} - x_i \leq 0.35$
$y = 0.25z + 0.25w + 8$	$y_0 = 24$	$y_{i+1} - y_i \leq 0.35$
$z = 0.25w + 0.25x + 8$	$z_0 = 24$	$z_{i+1} - z_i \leq 0.35$

Iteration 1	Iteration 2

Iteration 3	Iteration 4

A Digression: Jacobi Method

Use values from the current iteration wherever possible

Equations	Initial Values	Error Margin
$w = 0.25x + 0.25y + 8$	$w_0 = 24$	$w_{i+1} - w_i \leq 0.35$
$x = 0.25y + 0.25z + 8$	$x_0 = 24$	$x_{i+1} - x_i \leq 0.35$
$y = 0.25z + 0.25w + 8$	$y_0 = 24$	$y_{i+1} - y_i \leq 0.35$
$z = 0.25w + 0.25x + 8$	$z_0 = 24$	$z_{i+1} - z_i \leq 0.35$

Iteration 1	Iteration 2
$w_1 = 6 + 6 + 8 = 20$ $x_1 = 6 + 6 + 8 = 20$ $y_1 = 6 + 5 + 8 = 19$ $z_1 = 5 + 5 + 8 = 18$	

Iteration 3	Iteration 4

A Digression: Jacobi Method

Use values from the current iteration wherever possible

Equations	Initial Values	Error Margin
$w = 0.25x + 0.25y + 8$	$w_0 = 24$	$w_{i+1} - w_i \leq 0.35$
$x = 0.25y + 0.25z + 8$	$x_0 = 24$	$x_{i+1} - x_i \leq 0.35$
$y = 0.25z + 0.25w + 8$	$y_0 = 24$	$y_{i+1} - y_i \leq 0.35$
$z = 0.25w + 0.25x + 8$	$z_0 = 24$	$z_{i+1} - z_i \leq 0.35$

Iteration 1	Iteration 2
$w_1 = 6 + 6 + 8 = 20$	$w_2 = 5 + 4.75 + 8 = 17.75$
$x_1 = 6 + 6 + 8 = 20$	$x_2 = 4.75 + 4.5 + 8 = 17.25$
$y_1 = 6 + 5 + 8 = 19$	$y_2 = 4.5 + 4.4375 + 8 = 16.935$
$z_1 = 5 + 5 + 8 = 18$	$z_2 = 4.4375 + 4.375 + 8 = 16.8125$

Iteration 3	Iteration 4

A Digression: Jacobi Method

Use values from the current iteration wherever possible

Equations	Initial Values	Error Margin
$w = 0.25x + 0.25y + 8$	$w_0 = 24$	$w_{i+1} - w_i \leq 0.35$
$x = 0.25y + 0.25z + 8$	$x_0 = 24$	$x_{i+1} - x_i \leq 0.35$
$y = 0.25z + 0.25w + 8$	$y_0 = 24$	$y_{i+1} - y_i \leq 0.35$
$z = 0.25w + 0.25x + 8$	$z_0 = 24$	$z_{i+1} - z_i \leq 0.35$

Iteration 1	Iteration 2
$w_1 = 6 + 6 + 8 = 20$	$w_2 = 5 + 4.75 + 8 = 17.75$
$x_1 = 6 + 6 + 8 = 20$	$x_2 = 4.75 + 4.5 + 8 = 17.25$
$y_1 = 6 + 5 + 8 = 19$	$y_2 = 4.5 + 4.4375 + 8 = 16.935$
$z_1 = 5 + 5 + 8 = 18$	$z_2 = 4.4375 + 4.375 + 8 = 16.8125$

Iteration 3	Iteration 4
$w_3 = 4.3125 + 4.23375 + 8 = 16.54625$	
$x_3 = 4.23375 + 4.23375 + 8 = 16.436875$	
$y_3 = 4.23375 + 4.1365625 + 8 = 16.370$	
$z_3 = 4.1365625 + 4.11 + 8 = 16.34375$	

A Digression: Jacobi Method

Use values from the current iteration wherever possible

Equations	Initial Values	Error Margin
$w = 0.25x + 0.25y + 8$	$w_0 = 24$	$w_{i+1} - w_i \leq 0.35$
$x = 0.25y + 0.25z + 8$	$x_0 = 24$	$x_{i+1} - x_i \leq 0.35$
$y = 0.25z + 0.25w + 8$	$y_0 = 24$	$y_{i+1} - y_i \leq 0.35$
$z = 0.25w + 0.25x + 8$	$z_0 = 24$	$z_{i+1} - z_i \leq 0.35$

Iteration 1	Iteration 2
$w_1 = 6 + 6 + 8 = 20$	$w_2 = 5 + 4.75 + 8 = 17.75$
$x_1 = 6 + 6 + 8 = 20$	$x_2 = 4.75 + 4.5 + 8 = 17.25$
$y_1 = 6 + 5 + 8 = 19$	$y_2 = 4.5 + 4.4375 + 8 = 16.935$
$z_1 = 5 + 5 + 8 = 18$	$z_2 = 4.4375 + 4.375 + 8 = 16.8125$

Iteration 3	Iteration 4
$w_3 = 4.3125 + 4.23375 + 8 = 16.54625$	$w_4 = 16.20172$
$x_3 = 4.23375 + 4.23375 + 8 = 16.436875$	$x_4 = 16.17844$
$y_3 = 4.23375 + 4.1365625 + 8 = 16.370$	$y_4 = 16.13637$
$z_3 = 4.1365625 + 4.11 + 8 = 16.34375$	$z_4 = 16.09504$

Our Method of Performing Data Flow Analysis

- Round robin iteration using the Jacobi method
  (use the values from the current iteration wherever possible)
- Unknowns are the data flow variables In_i and Out_i
- Domain of values is not numbers
- Computation in a fixed order
  - either forward (reverse post order) traversal, or
  - backward (post order) traversal

  over the control flow graph

Tutorial Problem 2 for Liveness Analysis

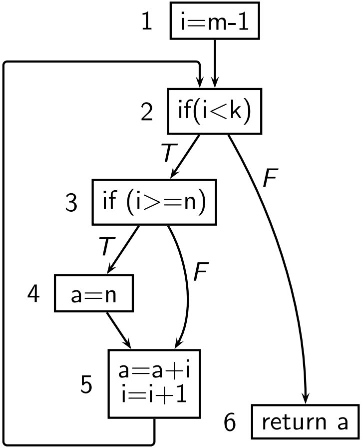
Draw the control flow graph and perform live variables analysis

```
int f(int m, int n, int k)
{
  int a,i;

  for (i=m-1; i<k; i++)
  {   if (i>=n)
          a = n;
      a = a+i;
  }
  return a;
}
```

Tutorial Problem 2 for Liveness Analysis

Draw the control flow graph and perform live variables analysis

```
int f(int m, int n, int k)
{
  int a,i;

  for (i=m-1; i<k; i++)
  {   if (i>=n)
          a = n;
      a = a+i;
  }
  return a;
}
```



The Semantics of Return Statement for Live Variables Analysis

"return a" is modelled by the statement "return_value_in_stack = a"

- If we assume that the return statement is executed *within* the block
- If we assume that the return statement is executed *outside of* the block and along the edge connecting the procedure to its caller

The Semantics of Return Statement for Live Variables Analysis

"return a" is modelled by the statement "return_value_in_stack = a"

- If we assume that the return statement is executed *within* the block
  $\Longrightarrow$ BI can be $\emptyset$

- If we assume that the return statement is executed *outside of* the block and along the edge connecting the procedure to its caller
  $\Longrightarrow$ $a \in BI$

Solution of Tutorial Problem 2

Block	Local Information		Global Information			
			Iteration # 1		Change in iteration # 2	
	Gen	*Kill*	*Out*	*In*	*Out*	*In*
6	$\{a\}$	$\emptyset$				
5	$\{a, i\}$	$\{a, i\}$				
4	$\{n\}$	$\{a\}$				
3	$\{i, n\}$	$\emptyset$				
2	$\{i, k\}$	$\emptyset$				
1	$\{m\}$	$\{i\}$				

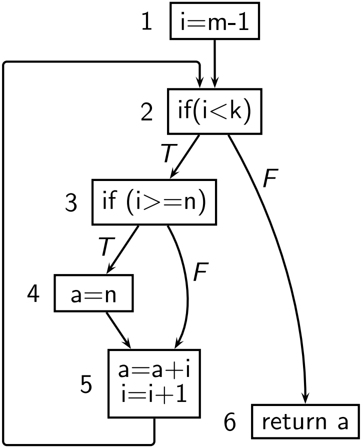
Solution of Tutorial Problem 2

Block	Local Information		Global Information			
			Iteration # 1		Change in iteration # 2	
	Gen	*Kill*	*Out*	*In*	*Out*	*In*
6	$\{a\}$	$\emptyset$	$\emptyset$	$\{a\}$		
5	$\{a, i\}$	$\{a, i\}$	$\emptyset$	$\{a, i\}$		
4	$\{n\}$	$\{a\}$	$\{a, i\}$	$\{i, n\}$		
3	$\{i, n\}$	$\emptyset$	$\{a, i, n\}$	$\{a, i, n\}$		
2	$\{i, k\}$	$\emptyset$	$\{a, i, n\}$	$\{a, i, k, n\}$		
1	$\{m\}$	$\{i\}$	$\{a, i, k, n\}$	$\{a, k, m, n\}$		

Solution of Tutorial Problem 2

Block	Local Information		Global Information			
			Iteration # 1		Change in iteration # 2	
	Gen	*Kill*	*Out*	*In*	*Out*	*In*
6	$\{a\}$	$\emptyset$	$\emptyset$	$\{a\}$		
5	$\{a, i\}$	$\{a, i\}$	$\emptyset$	$\{a, i\}$	$\{a, i, k, n\}$	$\{a, i, k, n\}$
4	$\{n\}$	$\{a\}$	$\{a, i\}$	$\{i, n\}$	$\{a, i, k, n\}$	$\{i, k, n\}$
3	$\{i, n\}$	$\emptyset$	$\{a, i, n\}$	$\{a, i, n\}$	$\{a, i, k, n\}$	$\{a, i, k, n\}$
2	$\{i, k\}$	$\emptyset$	$\{a, i, n\}$	$\{a, i, k, n\}$	$\{a, i, k, n\}$	
1	$\{m\}$	$\{i\}$	$\{a, i, k, n\}$	$\{a, k, m, n\}$		

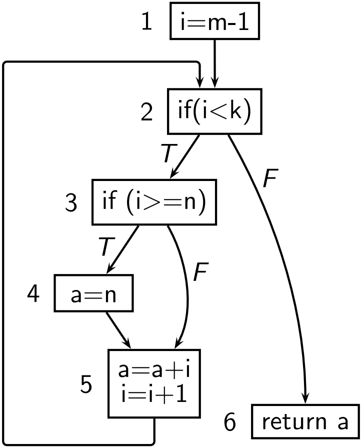
Interpreting the Result of Liveness Analysis for Tutorial Problem 2



- Is a live at the exit of node 5 at the end of iteration 1? Why?

  (We have used post order traversal)

Interpreting the Result of Liveness Analysis for Tutorial Problem 2



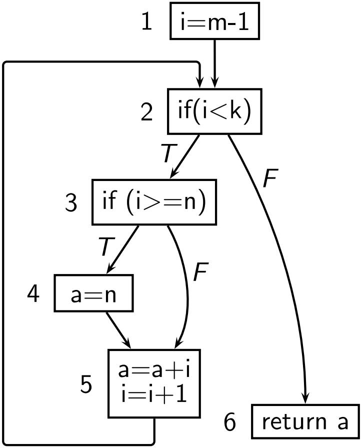
- Is a live at the exit of node 5 at the end of iteration 1? Why?

  (We have used post order traversal)

- Is a live at the exit of node 5 at the end of iteration 2? Why?

  (We have used post order traversal)

Interpreting the Result of Liveness Analysis for Tutorial Problem 2



- Is a live at the exit of node 5 at the end of iteration 1? Why?

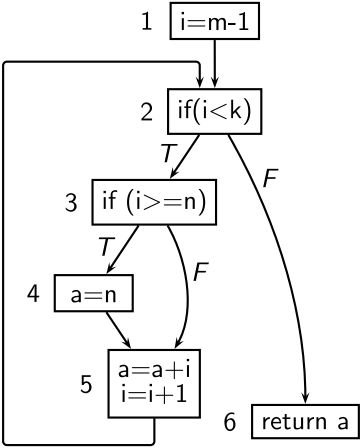
  (We have used post order traversal)

- Is a live at the exit of node 5 at the end of iteration 2? Why?

  (We have used post order traversal)

- Show an execution path along which a is live at the exit of node 5

Interpreting the Result of Liveness Analysis for Tutorial Problem 2



- Is a live at the exit of node 5 at the end of iteration 1? Why?

  (We have used post order traversal)

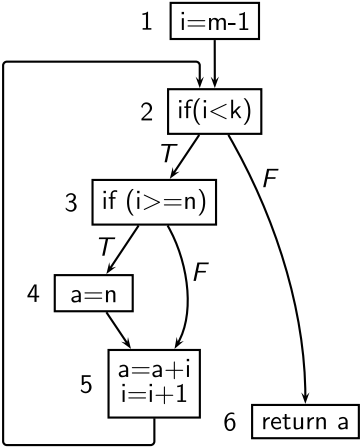
- Is a live at the exit of node 5 at the end of iteration 2? Why?

  (We have used post order traversal)

- Show an execution path along which a is live at the exit of node 5

- Show an execution path along which a is live at the exit of node 3

Interpreting the Result of Liveness Analysis for Tutorial Problem 2



- Is a live at the exit of node 5 at the end of iteration 1? Why?

  (We have used post order traversal)

- Is a live at the exit of node 5 at the end of iteration 2? Why?

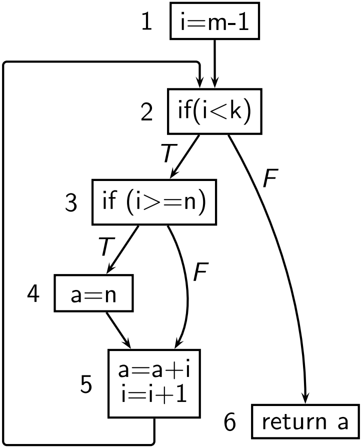
  (We have used post order traversal)

- Show an execution path along which a is live at the exit of node 5

- Show an execution path along which a is live at the exit of node 3

  $1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 2 \rightarrow \ldots$

Interpreting the Result of Liveness Analysis for Tutorial Problem 2



- Is a live at the exit of node 5 at the end of iteration 1? Why?

  (We have used post order traversal)

- Is a live at the exit of node 5 at the end of iteration 2? Why?

  (We have used post order traversal)

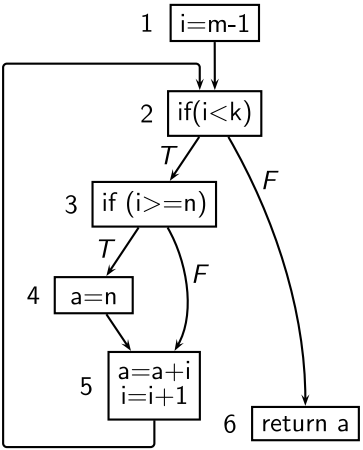
- Show an execution path along which a is live at the exit of node 5

- Show an execution path along which a is live at the exit of node 3

  $1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 2 \rightarrow \ldots$

- Show an execution path along which a is not live at the exit of node 3

Interpreting the Result of Liveness Analysis for Tutorial Problem 2



- Is a live at the exit of node 5 at the end of iteration 1? Why?

  (We have used post order traversal)

- Is a live at the exit of node 5 at the end of iteration 2? Why?

  (We have used post order traversal)

- Show an execution path along which a is live at the exit of node 5
- Show an execution path along which a is live at the exit of node 3

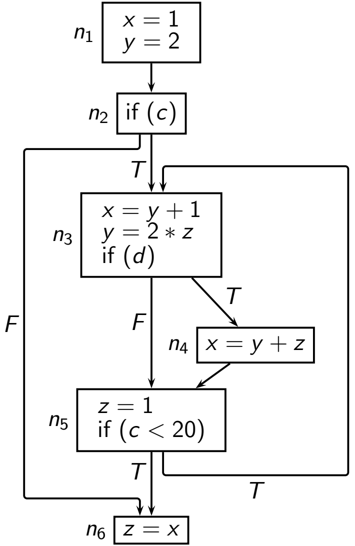
  $1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 2 \rightarrow \ldots$

- Show an execution path along which a is not live at the exit of node 3

  $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 2 \rightarrow \ldots$

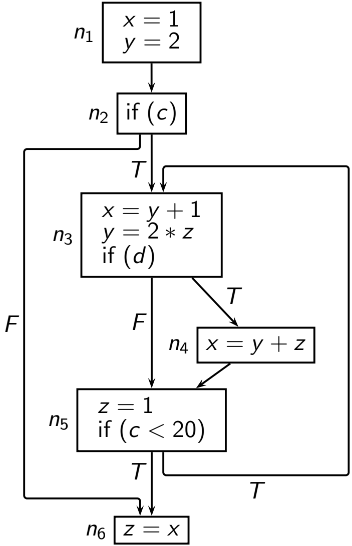
Tutorial Problem 3 for Liveness Analysis

Also write a C program for this CFG without using goto or break



Tutorial Problem 3 for Liveness Analysis

Also write a C program for this CFG without using goto or break




```
void f()
{   int x, y, z;
    int c, d;
    x = 1;
    y = 2;
    if (c)
    {   do
        {   x = y+1;
            y = 2*z;
            if (d)
                x = y+z;
            z = 1;
        } while (c < 20);
    }
    z = x;
}
```

Solution of Tutorial Problem 3

Block	Local Information		Global Information			
			Iteration # 1		Change in iteration # 2	
	Gen	*Kill*	*Out*	*In*	*Out*	*In*
n_6	$\{x\}$	$\{z\}$				
n_5	$\{c\}$	$\{z\}$				
n_4	$\{y, z\}$	$\{x\}$				
n_3	$\{y, z, d\}$	$\{x, y\}$				
n_2	$\{c\}$	$\emptyset$				
n_1	$\emptyset$	$\{x, y\}$				

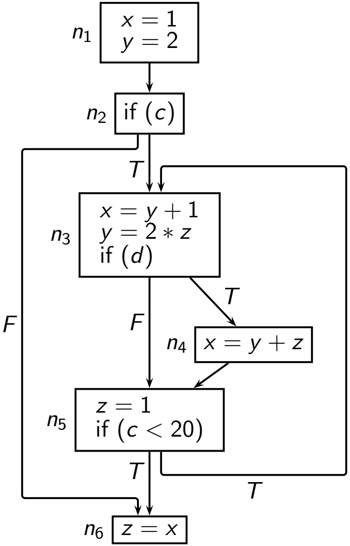
Solution of Tutorial Problem 3

Block	Local Information		Global Information			
			Iteration # 1		Change in iteration # 2	
	Gen	*Kill*	*Out*	*In*	*Out*	*In*
n_6	$\{x\}$	$\{z\}$	$\emptyset$	$\{x\}$		
n_5	$\{c\}$	$\{z\}$	$\{x\}$	$\{x, c\}$		
n_4	$\{y, z\}$	$\{x\}$	$\{x, c\}$	$\{y, z, c\}$		
n_3	$\{y, z, d\}$	$\{x, y\}$	$\{x, y, z, c\}$	$\{y, z, c, d\}$		
n_2	$\{c\}$	$\emptyset$	$\{x, y, z, c, d\}$	$\{x, y, z, c, d\}$		
n_1	$\emptyset$	$\{x, y\}$	$\{x, y, z, c, d\}$	$\{z, c, d\}$		

Solution of Tutorial Problem 3

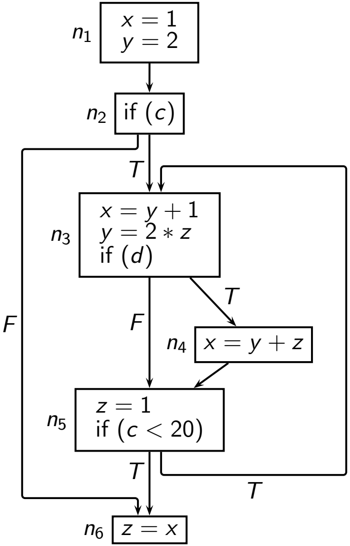
Block	Local Information		Global Information			
			Iteration # 1		Change in iteration # 2	
	Gen	*Kill*	*Out*	*In*	*Out*	*In*
n_6	$\{x\}$	$\{z\}$	$\emptyset$	$\{x\}$		
n_5	$\{c\}$	$\{z\}$	$\{x\}$	$\{x, c\}$	$\{x, y, z, c, d\}$	$\{x, y, c, d\}$
n_4	$\{y, z\}$	$\{x\}$	$\{x, c\}$	$\{y, z, c\}$	$\{x, y, c, d\}$	$\{y, z, c, d\}$
n_3	$\{y, z, d\}$	$\{x, y\}$	$\{x, y, z, c\}$	$\{y, z, c, d\}$	$\{x, y, z, c, d\}$	
n_2	$\{c\}$	$\emptyset$	$\{x, y, z, c, d\}$	$\{x, y, z, c, d\}$		
n_1	$\emptyset$	$\{x, y\}$	$\{x, y, z, c, d\}$	$\{z, c, d\}$		

Interpreting the Result of Liveness Analysis for Tutorial Problem 3



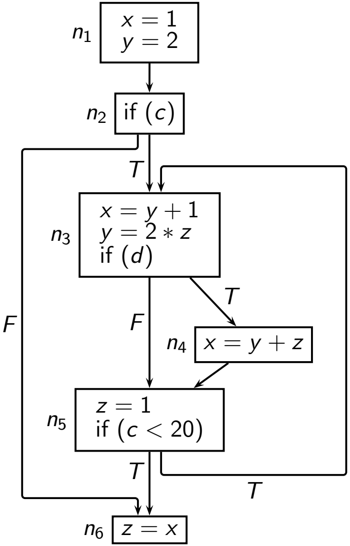
- Why is z live at the exit of n_5?

Interpreting the Result of Liveness Analysis for Tutorial Problem 3



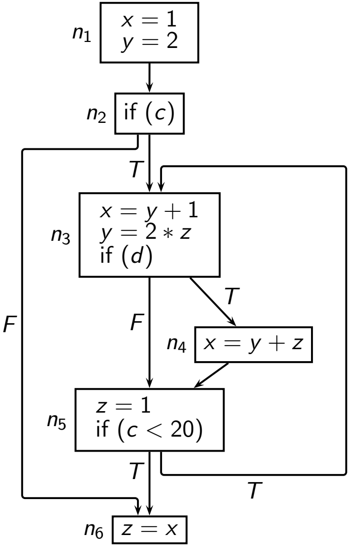
- Why is z live at the exit of n_5?
- Why is z not live at the entry of n_5?

Interpreting the Result of Liveness Analysis for Tutorial Problem 3



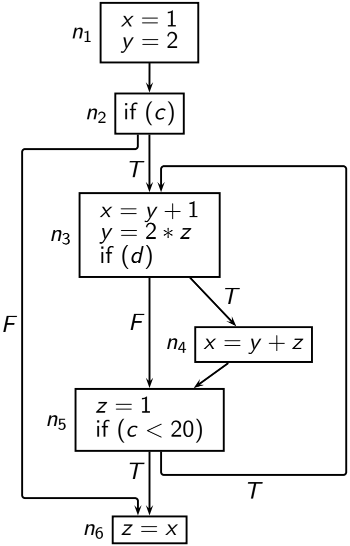
- Why is z live at the exit of n_5?
- Why is z not live at the entry of n_5?
- Why is x live at the exit of n_3 inspite of being killed in n_4?

Interpreting the Result of Liveness Analysis for Tutorial Problem 3



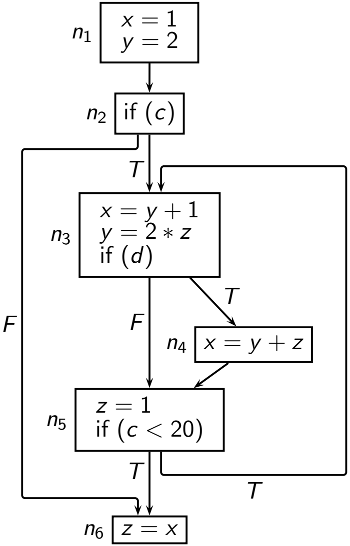
- Why is z live at the exit of n_5?
- Why is z not live at the entry of n_5?
- Why is x live at the exit of n_3 inspite of being killed in n_4?
- Identify the instance of dead code elimination

Interpreting the Result of Liveness Analysis for Tutorial Problem 3



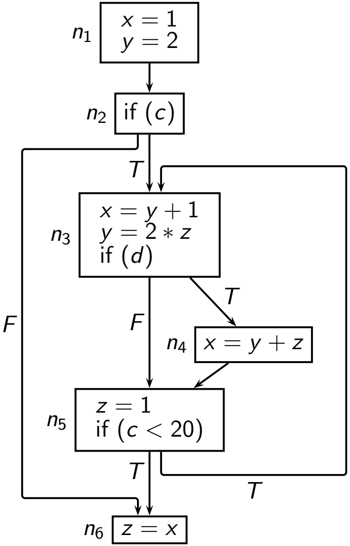
- Why is z live at the exit of n_5?
- Why is z not live at the entry of n_5?
- Why is x live at the exit of n_3 inspite of being killed in n_4?
- Identify the instance of dead code elimination $z = x$ in n_6

Interpreting the Result of Liveness Analysis for Tutorial Problem 3



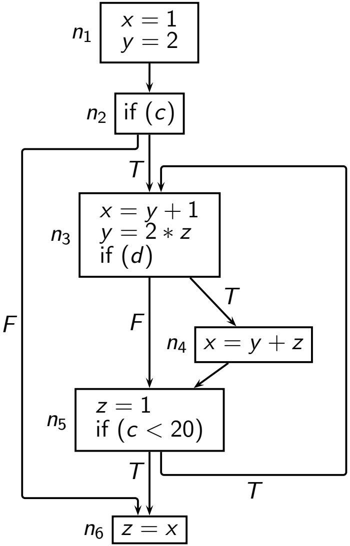
- Why is z live at the exit of n_5?
- Why is z not live at the entry of n_5?
- Why is x live at the exit of n_3 inspite of being killed in n_4?
- Identify the instance of dead code elimination $z = x$ in n_6
- Would the first round of dead code elimination cause liveness information to change?

Interpreting the Result of Liveness Analysis for Tutorial Problem 3



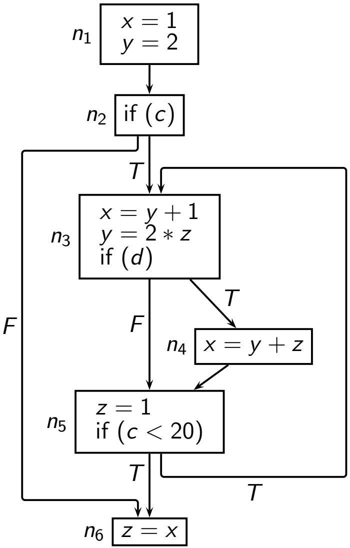
- Why is z live at the exit of n_5?
- Why is z not live at the entry of n_5?
- Why is x live at the exit of n_3 inspite of being killed in n_4?
- Identify the instance of dead code elimination $z = x$ in n_6
- Would the first round of dead code elimination cause liveness information to change? Yes

Interpreting the Result of Liveness Analysis for Tutorial Problem 3



- Why is z live at the exit of n_5?
- Why is z not live at the entry of n_5?
- Why is x live at the exit of n_3 inspite of being killed in n_4?
- Identify the instance of dead code elimination $z = x$ in n_6
- Would the first round of dead code elimination cause liveness information to change? Yes
- Would the second round of liveness analysis lead to further dead code elimination?

Interpreting the Result of Liveness Analysis for Tutorial Problem 3



- Why is z live at the exit of n_5?
- Why is z not live at the entry of n_5?
- Why is x live at the exit of n_3 inspite of being killed in n_4?
- Identify the instance of dead code elimination $z = x$ in n_6
- Would the first round of dead code elimination cause liveness information to change? Yes
- Would the second round of liveness analysis lead to further dead code elimination? Yes

Choice of Initialization

What should be the initial value of internal nodes?

The role of boundary info *BI* explained later in the context of available expressions analysis

Choice of Initialization

What should be the initial value of internal nodes?

- Confluence is $\cup$
- Identity of $\cup$ is $\emptyset$

The role of boundary info *BI* explained later in the context of available expressions analysis

Choice of Initialization

What should be the initial value of internal nodes?

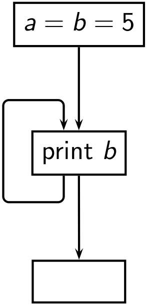
- Confluence is $\cup$
- Identity of $\cup$ is $\emptyset$
- We begin with $\emptyset$ and let the sets at each program point grow

  A revisit to a program point

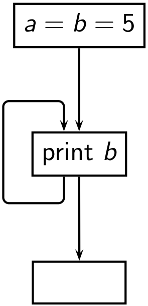
  - may consider a new execution path
  - more variables may be found to be live
  - a variable found to be live earlier does not become dead

The role of boundary info *BI* explained later in the context of available expressions analysis

How Does the Initialization Affect the Solution?



How Does the Initialization Affect the Solution?



Init.

$\emptyset$

$\emptyset$

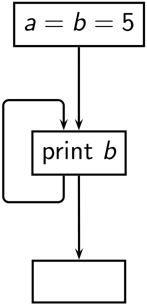
$\emptyset$

$\emptyset$

$\emptyset$

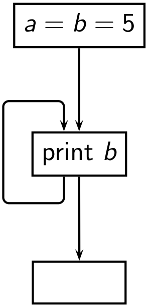
$\emptyset$

How Does the Initialization Affect the Solution?



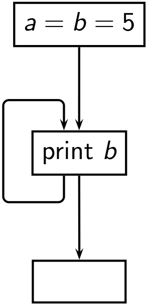
Init.	Iter. #1
$\emptyset$	
$\emptyset$	
$\emptyset$	
$\emptyset$	
$\emptyset$	
$\emptyset$	$\emptyset$

How Does the Initialization Affect the Solution?



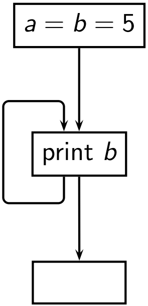
Init.	Iter. #1
$\emptyset$	
$\emptyset$	
$\emptyset$	
$\emptyset$	
$\emptyset$	$\emptyset$
$\emptyset$	$\emptyset$

How Does the Initialization Affect the Solution?



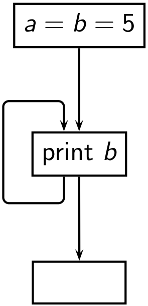
Init.	Iter. #1
$\emptyset$	
$\emptyset$	
$\emptyset$	
$\emptyset$	$\emptyset$
$\emptyset$	$\emptyset$
$\emptyset$	$\emptyset$

How Does the Initialization Affect the Solution?



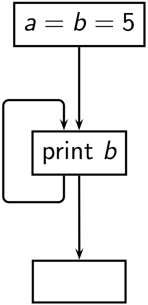
Init.	Iter. #1
$\emptyset$	
$\emptyset$	
$\emptyset$	$\{b\}$
$\emptyset$	$\emptyset$
$\emptyset$	$\emptyset$
$\emptyset$	$\emptyset$

How Does the Initialization Affect the Solution?



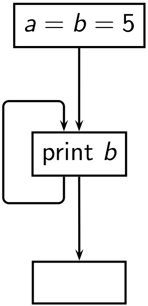
Init.	Iter. #1
$\emptyset$	
$\emptyset$	$\{b\}$
$\emptyset$	$\{b\}$
$\emptyset$	$\emptyset$
$\emptyset$	$\emptyset$
$\emptyset$	$\emptyset$

How Does the Initialization Affect the Solution?



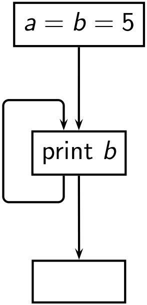
Init.	Iter. #1
$\emptyset$	$\emptyset$
$\emptyset$	$\{b\}$
$\emptyset$	$\{b\}$
$\emptyset$	$\emptyset$
$\emptyset$	$\emptyset$
$\emptyset$	$\emptyset$

How Does the Initialization Affect the Solution?

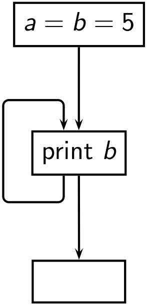


Init.	Iter. #1	Iter. #2
$\emptyset$	$\emptyset$	$\emptyset$
$\emptyset$	$\{b\}$	$\{b\}$
$\emptyset$	$\{b\}$	$\{b\}$
$\emptyset$	$\emptyset$	$\{b\}$
$\emptyset$	$\emptyset$	$\emptyset$
$\emptyset$	$\emptyset$	$\emptyset$

How Does the Initialization Affect the Solution?

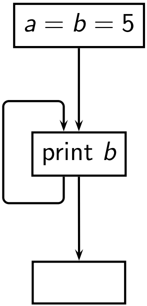


Init.	Iter. #1	Iter. #2
$\emptyset$	$\emptyset$	$\emptyset$
$\emptyset$	$\{b\}$	$\{b\}$
$\emptyset$	$\{b\}$	$\{b\}$
$\emptyset$	$\emptyset$	$\{b\}$
$\emptyset$	$\emptyset$	$\emptyset$
$\emptyset$	$\emptyset$	$\emptyset$

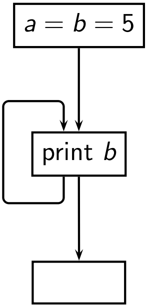


Init.
$\{a, b\}$
$\{a, b\}$
$\{a, b\}$
$\{a, b\}$
$\{a, b\}$
$\emptyset$

How Does the Initialization Affect the Solution?

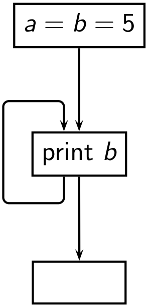


Init.	Iter. #1	Iter. #2
$\emptyset$	$\emptyset$	$\emptyset$
$\emptyset$	$\{b\}$	$\{b\}$
$\emptyset$	$\{b\}$	$\{b\}$
$\emptyset$	$\emptyset$	$\{b\}$
$\emptyset$	$\emptyset$	$\emptyset$
$\emptyset$	$\emptyset$	$\emptyset$

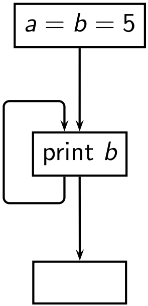


Init.	Iter. #1
$\{a, b\}$	
$\{a, b\}$	
$\{a, b\}$	
$\{a, b\}$	
$\{a, b\}$	
$\emptyset$	$\emptyset$

How Does the Initialization Affect the Solution?

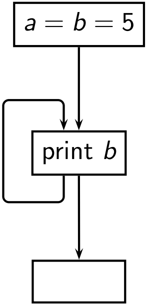


Init.	Iter. #1	Iter. #2
$\emptyset$	$\emptyset$	$\emptyset$
$\emptyset$	$\{b\}$	$\{b\}$
$\emptyset$	$\{b\}$	$\{b\}$
$\emptyset$	$\emptyset$	$\{b\}$
$\emptyset$	$\emptyset$	$\emptyset$
$\emptyset$	$\emptyset$	$\emptyset$

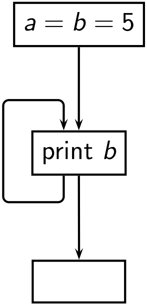


Init.	Iter. #1
$\{a, b\}$	
$\{a, b\}$	
$\{a, b\}$	
$\{a, b\}$	
$\{a, b\}$	$\emptyset$
$\emptyset$	$\emptyset$

How Does the Initialization Affect the Solution?

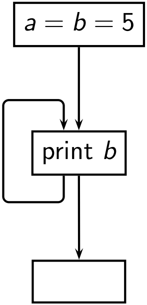


Init.	Iter. #1	Iter. #2
$\emptyset$	$\emptyset$	$\emptyset$
$\emptyset$	$\{b\}$	$\{b\}$
$\emptyset$	$\{b\}$	$\{b\}$
$\emptyset$	$\emptyset$	$\{b\}$
$\emptyset$	$\emptyset$	$\emptyset$
$\emptyset$	$\emptyset$	$\emptyset$

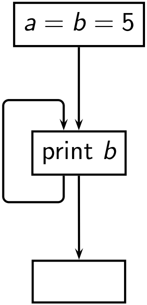


Init.	Iter. #1
$\{a, b\}$	
$\{a, b\}$	
$\{a, b\}$	
$\{a, b\}$	$\{a, b\}$
$\{a, b\}$	$\emptyset$
$\emptyset$	$\emptyset$

How Does the Initialization Affect the Solution?

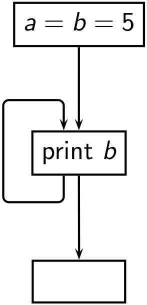


Init.	Iter. #1	Iter. #2
$\emptyset$	$\emptyset$	$\emptyset$
$\emptyset$	$\{b\}$	$\{b\}$
$\emptyset$	$\{b\}$	$\{b\}$
$\emptyset$	$\emptyset$	$\{b\}$
$\emptyset$	$\emptyset$	$\emptyset$
$\emptyset$	$\emptyset$	$\emptyset$

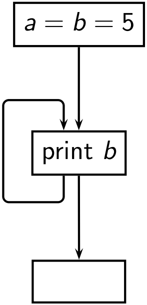


Init.	Iter. #1
$\{a, b\}$	
$\{a, b\}$	
$\{a, b\}$	$\{a, b\}$
$\{a, b\}$	$\{a, b\}$
$\{a, b\}$	$\emptyset$
$\emptyset$	$\emptyset$

How Does the Initialization Affect the Solution?

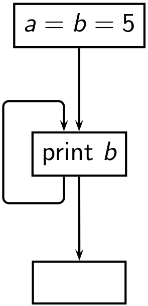


Init.	Iter. #1	Iter. #2
$\emptyset$	$\emptyset$	$\emptyset$
$\emptyset$	$\{b\}$	$\{b\}$
$\emptyset$	$\{b\}$	$\{b\}$
$\emptyset$	$\emptyset$	$\{b\}$
$\emptyset$	$\emptyset$	$\emptyset$
$\emptyset$	$\emptyset$	$\emptyset$

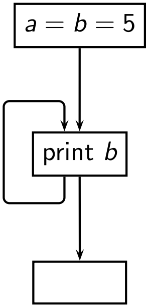


Init.	Iter. #1
$\{a, b\}$	
$\{a, b\}$	$\{a, b\}$
$\{a, b\}$	$\{a, b\}$
$\{a, b\}$	$\{a, b\}$
$\{a, b\}$	$\emptyset$
$\emptyset$	$\emptyset$

How Does the Initialization Affect the Solution?

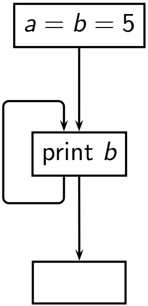


Init.	Iter. #1	Iter. #2
$\emptyset$	$\emptyset$	$\emptyset$
$\emptyset$	$\{b\}$	$\{b\}$
$\emptyset$	$\{b\}$	$\{b\}$
$\emptyset$	$\emptyset$	$\{b\}$
$\emptyset$	$\emptyset$	$\emptyset$
$\emptyset$	$\emptyset$	$\emptyset$

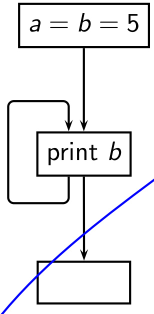


Init.	Iter. #1
$\{a, b\}$	$\emptyset$
$\{a, b\}$	$\{a, b\}$
$\{a, b\}$	$\{a, b\}$
$\{a, b\}$	$\{a, b\}$
$\{a, b\}$	$\emptyset$
$\emptyset$	$\emptyset$

How Does the Initialization Affect the Solution?



Init.	Iter. #1	Iter. #2
$\emptyset$	$\emptyset$	$\emptyset$
$\emptyset$	$\{b\}$	$\{b\}$
$\emptyset$	$\{b\}$	$\{b\}$
$\emptyset$	$\emptyset$	$\{b\}$
$\emptyset$	$\emptyset$	$\emptyset$
$\emptyset$	$\emptyset$	$\emptyset$



Init.	Iter. #1
$\{a, b\}$	$\emptyset$
$\{a, b\}$	$\{a, b\}$
$\{a, b\}$	$\{a, b\}$
$\{a, b\}$	$\{a, b\}$
$\{a, b\}$	$\emptyset$
$\emptyset$	$\emptyset$

a is spuriously marked live

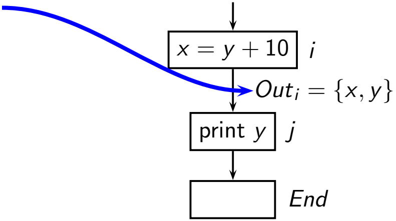
Soundness and Precision of Live Variables Analysis

Consider dead code elimination based on liveness information

Soundness and Precision of Live Variables Analysis

Consider dead code elimination based on liveness information

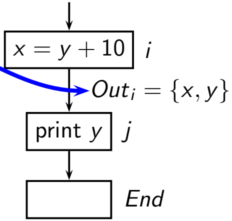
- Spurious inclusion of a non-live variable



Soundness and Precision of Live Variables Analysis

Consider dead code elimination based on liveness information

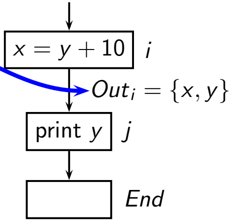
- Spurious inclusion of a non-live variable
  - A dead assignment may not be eliminated
  - Solution is sound but may be imprecise

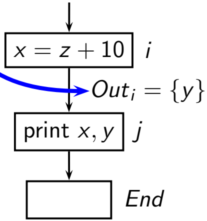


Soundness and Precision of Live Variables Analysis

Consider dead code elimination based on liveness information

- Spurious inclusion of a non-live variable
  - A dead assignment may not be eliminated
  - Solution is sound but may be imprecise
- Spurious exclusion of a live variable

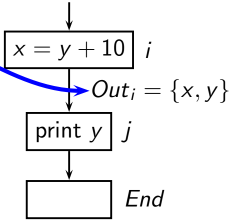




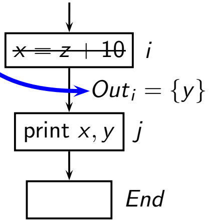
Soundness and Precision of Live Variables Analysis

Consider dead code elimination based on liveness information

- Spurious inclusion of a non-live variable
  - A dead assignment may not be eliminated
  - Solution is sound but may be imprecise



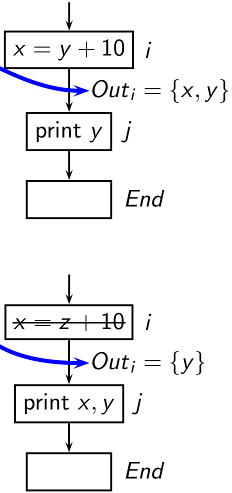
- Spurious exclusion of a live variable
  - A useful assignment may be eliminated
  - Solution is unsound



Soundness and Precision of Live Variables Analysis

Consider dead code elimination based on liveness information

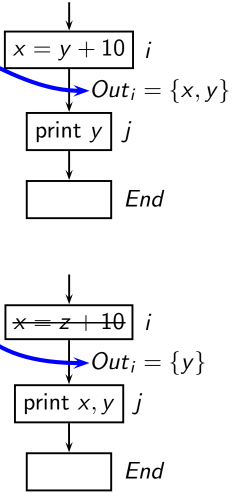
- Spurious inclusion of a non-live variable
  - A dead assignment may not be eliminated
  - Solution is sound but may be imprecise
- Spurious exclusion of a live variable
  - A useful assignment may be eliminated
  - Solution is unsound
- Given $L_2 \supseteq L_1$ representing liveness information
  - Using L_2 in place of L_1 is sound
  - Using L_1 in place of L_2 may not be sound



Soundness and Precision of Live Variables Analysis

Consider dead code elimination based on liveness information

- Spurious inclusion of a non-live variable
    - A dead assignment may not be eliminated
    - Solution is sound but may be imprecise
- Spurious exclusion of a live variable
    - A useful assignment may be eliminated
    - Solution is unsound
- Given $L_2 \supseteq L_1$ representing liveness information
    - Using L_2 in place of L_1 is sound
    - Using L_1 in place of L_2 may not be sound
- The smallest set of all live variables is most precise
    - Since liveness sets grow (confluence is $\cup$), we choose $\emptyset$ as the initial conservative value



Termination, Convergence, and Complexity

- For live variables analysis,
  - The set of all variables is finite, and
  - the confluence operation (i.e. meet) is union, hence
  - the set associated with a data flow variable can only grow

$\Longrightarrow$ Termination is guaranteed

Termination, Convergence, and Complexity

- For live variables analysis,
    - The set of all variables is finite, and
    - the confluence operation (i.e. meet) is union, hence
    - the set associated with a data flow variable can only grow

  $\Longrightarrow$ Termination is guaranteed

- Since initial value is $\emptyset$, live variables analysis converges on the smallest set

Termination, Convergence, and Complexity

- For live variables analysis,
    - The set of all variables is finite, and
    - the confluence operation (i.e. meet) is union, hence
    - the set associated with a data flow variable can only grow

  $\Longrightarrow$ Termination is guaranteed
- Since initial value is $\emptyset$, live variables analysis converges on the smallest set
- How many iterations do we need for reaching the convergence?

Termination, Convergence, and Complexity

- For live variables analysis,
  - The set of all variables is finite, and
  - the confluence operation (i.e. meet) is union, hence
  - the set associated with a data flow variable can only grow

  $\Longrightarrow$ Termination is guaranteed
- Since initial value is $\emptyset$, live variables analysis converges on the smallest set
- How many iterations do we need for reaching the convergence?
- Going beyond live variables analysis
  - Do the sets always grow for other data flow frameworks?
  - What is the complexity of round robin analysis for other analyses?

Termination, Convergence, and Complexity

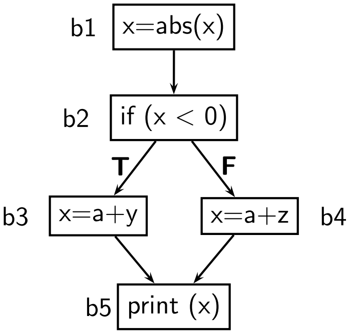
- For live variables analysis,
  - The set of all variables is finite, and
  - the confluence operation (i.e. meet) is union, hence
  - the set associated with a data flow variable can only grow

$\Longrightarrow$ Termination is guaranteed

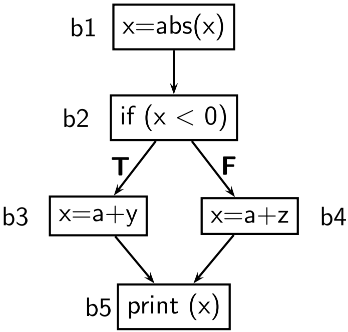
- Since initial value is $\emptyset$, live variables analysis converges on the smallest set
- How many iterations do we need for reaching the convergence?
- Going beyond live variables analysis
  - Do the sets always grow for other data flow frameworks?
  - What is the complexity of round robin analysis for other analyses?

Answered formally in module 2 (Theoretical Abstractions)

Conservative Nature of Analysis (1)

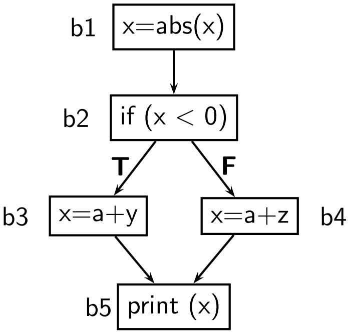


Conservative Nature of Analysis (1)



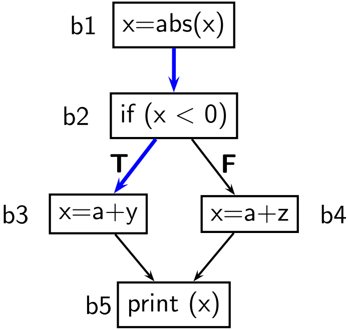
- abs(n) returns the absolute value of n

Conservative Nature of Analysis (1)



- abs(n) returns the absolute value of n
- Is y live on entry to block b2?

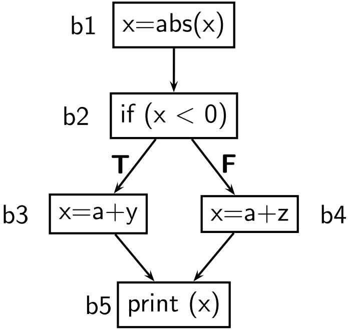
Conservative Nature of Analysis (1)



- abs(n) returns the absolute value of n
- Is y live on entry to block b2?
- By execution semantics, NO

  Path b1→b2→b3 is an infeasible execution path

Conservative Nature of Analysis (1)



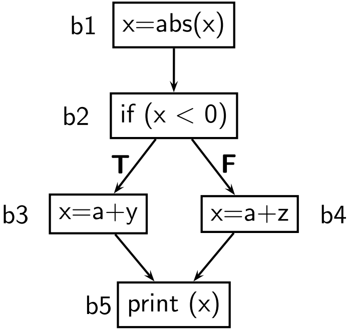
- abs(n) returns the absolute value of n
- Is y live on entry to block b2?
- By execution semantics, NO

  Path b1→b2→b3 is an infeasible execution path
- A compiler makes conservative assumptions:

  All branch outcomes are possible

  ⇒ Consider every path in CFG as a potential execution path

Conservative Nature of Analysis (1)



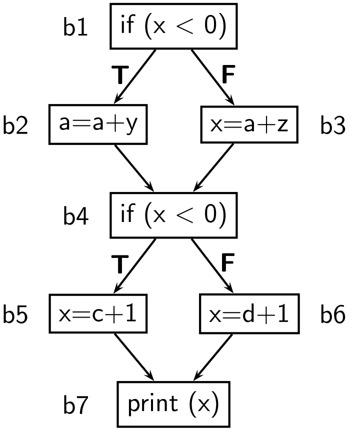
- abs(n) returns the absolute value of n
- Is y live on entry to block b2?
- By execution semantics, NO

  Path b1→b2→b3 is an infeasible execution path
- A compiler makes conservative assumptions:

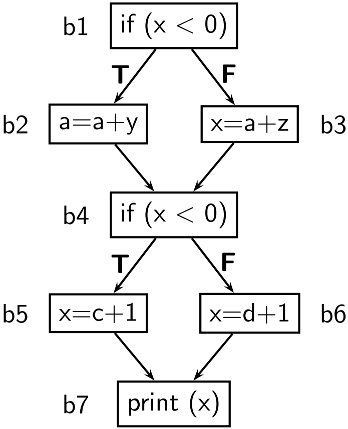
  All branch outcomes are possible

  ⇒ Consider every path in CFG as a potential execution path
- Our analysis concludes that y is live on entry to block b2

Conservative Nature of Analysis (2)

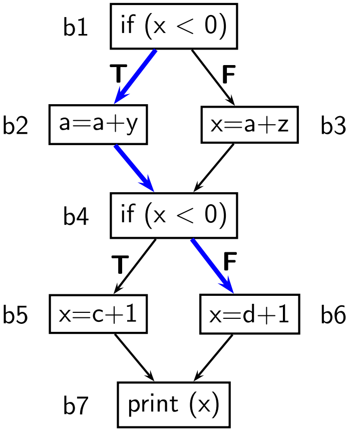


Conservative Nature of Analysis (2)



- Is d live on entry to block b2?

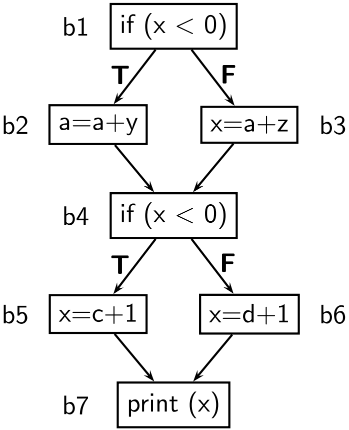
Conservative Nature of Analysis (2)



- Is d live on entry to block b2?
- By execution semantics, NO

  Path b1→b2→b4→b6 is an infeasible execution path

Conservative Nature of Analysis (2)

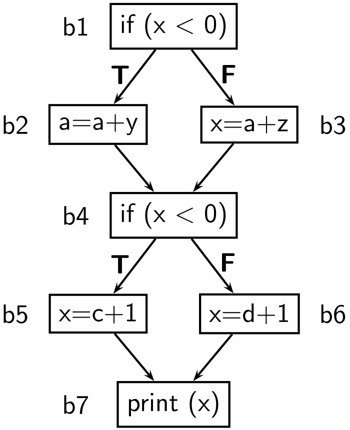


- Is d live on entry to block b2?
- By execution semantics, NO

  Path b1→b2→b4→b6 is an infeasible execution path
- Is c live on entry to block b3?

  Path b1→b3→b4→b6 is a feasible execution path

Conservative Nature of Analysis (2)



- Is d live on entry to block b2?
- By execution semantics, NO

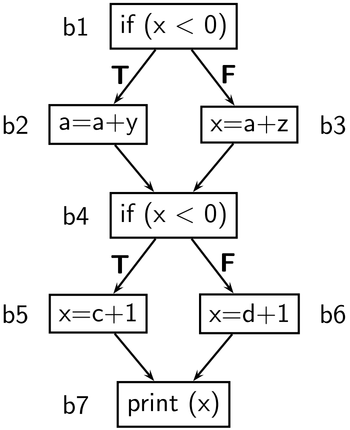
  Path b1→b2→b4→b6 is an infeasible execution path
- Is c live on entry to block b3?

  Path b1→b3→b4→b6 is a feasible execution path
- A compiler make conservative assumptions

  ⇒ our analysis is *path insensitive*

  Note: It is *flow sensitive* (i.e. information is computed for every control flow points)

Conservative Nature of Analysis (2)



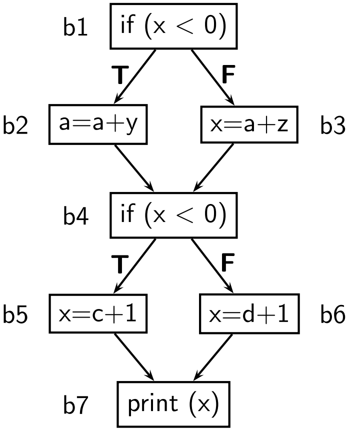
- Is d live on entry to block b2?
- By execution semantics, NO

  Path b1→b2→b4→b6 is an infeasible execution path
- Is c live on entry to block b3?

  Path b1→b3→b4→b6 is a feasible execution path
- A compiler make conservative assumptions ⇒ our analysis is *path insensitive*

  Note: It is *flow sensitive* (i.e. information is computed for every control flow points)
- Our analysis concludes that d is live at the entry of b2

Conservative Nature of Analysis (2)



- Is d live on entry to block b2?
- By execution semantics, NO

  Path b1→b2→b4→b6 is an infeasible execution path
- Is c live on entry to block b3?

  Path b1→b3→b4→b6 is a feasible execution path
- A compiler make conservative assumptions ⇒ our analysis is *path insensitive*

  Note: It is *flow sensitive* (i.e. information is computed for every control flow points)
- Our analysis concludes that d is live at the entry of b2
- Is c live at the entry of b3?

Conservative Nature of Analysis at Intraprocedural Level

- We assume that all paths are potentially executable
- Our analysis is path insensitive
  - The data flow information at a program point p is path insensitive
    - information at p is merged along all paths reaching p
  - The data flow information reaching p is computed path insensitively
    - information is merged at all shared points in paths reaching p
    - may generate spurious information due to non-distributive flow functions

More about it in module 2

Conservative Nature of Analysis at Interprocedural Level

- Context insensitivity
  - Merges of information across all calling contexts
- Flow insensitivity
  - Disregards the control flow

More about it in module 4

What About Soundness of Analysis Results?

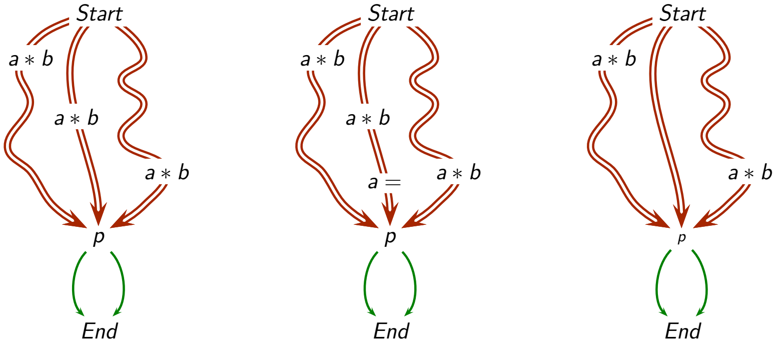
- No compromises
- We will study it in module 2

Part 4

Available Expressions Analysis

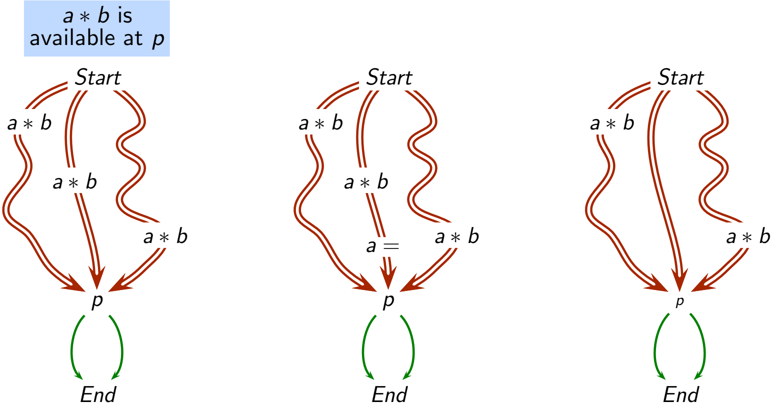
Defining Available Expressions Analysis

An expression e is available at a program point p, if
every path from program entry to p contains an evaluation of e
which is not followed by a definition of any operand of e.



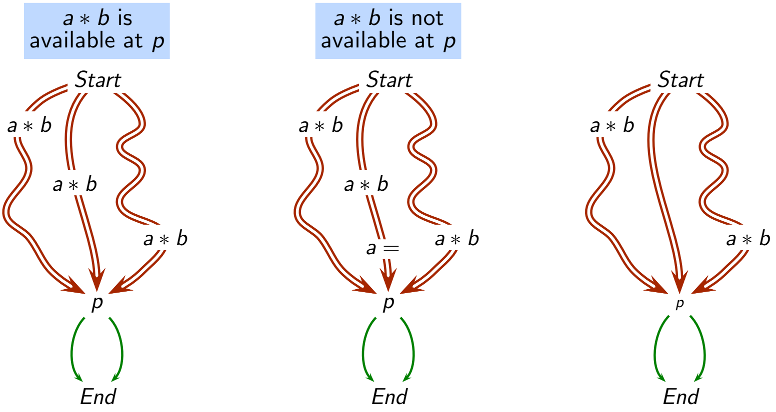
Defining Available Expressions Analysis

An expression e is available at a program point p, if every path from program entry to p contains an evaluation of e which is not followed by a definition of any operand of e.



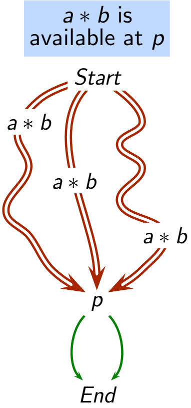
Defining Available Expressions Analysis

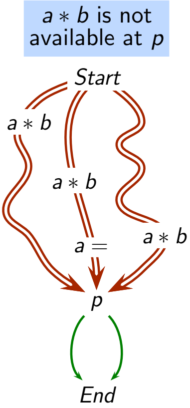
An expression e is available at a program point p, if every path from program entry to p contains an evaluation of e which is not followed by a definition of any operand of e.

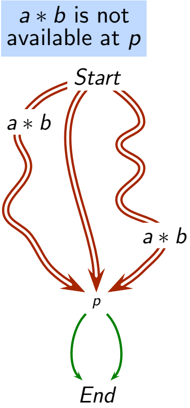


Defining Available Expressions Analysis

An expression e is available at a program point p, if
every path from program entry to p contains an evaluation of e
which is not followed by a definition of any operand of e.







Local Data Flow Properties for Available Expressions Analysis

$$Gen_n = \{ e \mid \text{expression } e \text{ is evaluated in basic block } n \text{ and this evaluation is not followed by a definition of any operand of } e\}$$

$$Kill_n = \{ e \mid \text{basic block } n \text{ contains a definition of an operand of } e\}$$

	Entity	Manipulation	Exposition
Gen_n	Expression	Use	Downwards
$Kill_n$	Expression	Modification	Anywhere

Data Flow Equations For Available Expressions Analysis

$$In_n = \begin{cases} BI & n \text{ is } \mathit{Start} \text{ block} \\ \bigcap\limits_{p \in pred(n)} Out_p & \text{otherwise} \end{cases}$$

$$Out_n = Gen_n \cup (In_n - Kill_n)$$

Data Flow Equations For Available Expressions Analysis

$$In_n = \begin{cases} BI & n \text{ is } \mathit{Start} \text{ block} \\ \bigcap\limits_{p \in pred(n)} Out_p & \text{otherwise} \end{cases}$$

$$Out_n = Gen_n \cup (In_n - Kill_n)$$

Alternatively,

$$Out_n = f_n(In_n), \qquad \text{where}$$

$$f_n(X) = Gen_n \cup (X - Kill_n)$$

Data Flow Equations For Available Expressions Analysis

$$
\begin{aligned}
In_n &= \begin{cases} BI & n \text{ is } \mathit{Start} \text{ block} \\ \displaystyle\bigcap_{p \in pred(n)} Out_p & \text{otherwise} \end{cases} \\
Out_n &= Gen_n \cup (In_n - Kill_n)
\end{aligned}
$$

Alternatively,

$$
\begin{aligned}
Out_n &= f_n(In_n), \qquad \text{where} \\
f_n(X) &= Gen_n \cup (X - Kill_n)
\end{aligned}
$$

- In_n and Out_n are sets of expressions

Data Flow Equations For Available Expressions Analysis

$$
In_n = \begin{cases} BI & n \text{ is } \mathit{Start} \text{ block} \\ \displaystyle\bigcap_{p \in pred(n)} Out_p & \text{otherwise} \end{cases}
$$

$$
Out_n = Gen_n \cup (In_n - Kill_n)
$$

Alternatively,

$$
Out_n = f_n(In_n), \qquad \text{where}
$$

$$
f_n(X) = Gen_n \cup (X - Kill_n)
$$

- In_n and Out_n are sets of expressions
- BI is $\emptyset$ for expressions involving a local variable

Using Data Flow Information of Available Expressions Analysis

- Common subexpression elimination

Using Data Flow Information of Available Expressions Analysis

- Common subexpression elimination
    - If an expression is available at the entry of a block n (In_n) **and**

Using Data Flow Information of Available Expressions Analysis

- Common subexpression elimination
  - If an expression is available at the entry of a block n (In_n) **and**
  - a computation of the expression exists in n **such that**

Using Data Flow Information of Available Expressions Analysis

- Common subexpression elimination
  - If an expression is available at the entry of a block n (In_n) **and**
  - a computation of the expression exists in n **such that**
  - it is not preceded by a definition of any of its operands ($AntGen_n$)

Using Data Flow Information of Available Expressions Analysis

- Common subexpression elimination
  - If an expression is available at the entry of a block n (In_n) **and**
  - a computation of the expression exists in n **such that**
  - it is not preceded by a definition of any of its operands ($AntGen_n$)

  Then the expression is redundant

$$Redundant_n = In_n \cap AntGen_n$$

Using Data Flow Information of Available Expressions Analysis

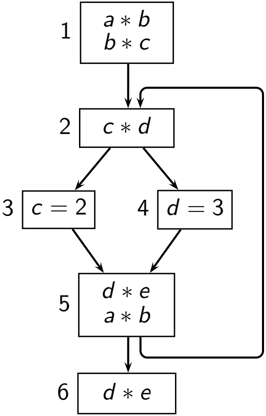
- Common subexpression elimination
  - If an expression is available at the entry of a block n (In_n) **and**
  - a computation of the expression exists in n **such that**
  - it is not preceded by a definition of any of its operands ($AntGen_n$)

  Then the expression is redundant

$$Redundant_n = In_n \cap AntGen_n$$

- A redundant expression is upwards exposed whereas the expressions in Gen_n are downwards exposed

An Example of Available Expressions Analysis

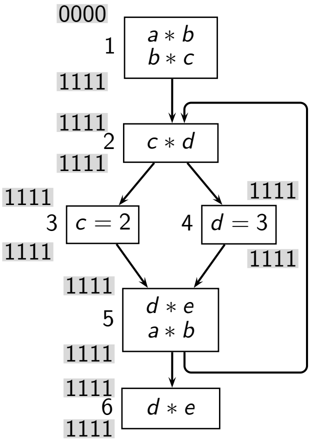


Let $e_1 \equiv a * b$, $e_2 \equiv b * c$, $e_3 \equiv c * d$, $e_4 \equiv d * e$

Node	*Gen*		*Kill*		Available		Redund.	
1	$\{e_1, e_2\}$	1100	$\emptyset$	0000	$\emptyset$	0000	$\emptyset$	0000
2	$\{e_3\}$	0010	$\emptyset$	0000	$\{e_1\}$	1000	$\emptyset$	0000
3	$\emptyset$	0000	$\{e_2, e_3\}$	0110	$\{e_1, e_3\}$	1010	$\emptyset$	0000
4	$\emptyset$	0000	$\{e_3, e_4\}$	0011	$\{e_1, e_3\}$	1010	$\emptyset$	0000
5	$\{e_1, e_4\}$	1001	$\emptyset$	0000	$\{e_1\}$	1000	$\{e_1\}$	1000
6	$\{e_4\}$	0001	$\emptyset$	0000	$\{e_1, e_4\}$	1001	$\{e_4\}$	0001

An Example of Available Expressions Analysis

Initialisation

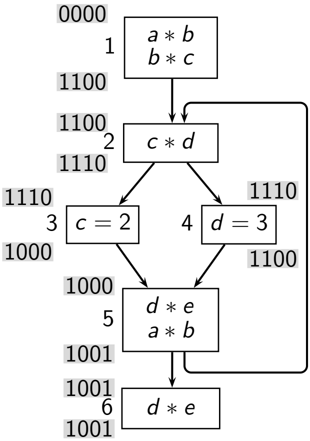


Let $e_1 \equiv a * b$, $e_2 \equiv b * c$, $e_3 \equiv c * d$, $e_4 \equiv d * e$

Node	*Gen*		*Kill*		Available		Redund.	
1	$\{e_1, e_2\}$	1100	$\emptyset$	0000	$\emptyset$	0000	$\emptyset$	0000
2	$\{e_3\}$	0010	$\emptyset$	0000	$\{e_1\}$	1000	$\emptyset$	0000
3	$\emptyset$	0000	$\{e_2, e_3\}$	0110	$\{e_1, e_3\}$	1010	$\emptyset$	0000
4	$\emptyset$	0000	$\{e_3, e_4\}$	0011	$\{e_1, e_3\}$	1010	$\emptyset$	0000
5	$\{e_1, e_4\}$	1001	$\emptyset$	0000	$\{e_1\}$	1000	$\{e_1\}$	1000
6	$\{e_4\}$	0001	$\emptyset$	0000	$\{e_1, e_4\}$	1001	$\{e_4\}$	0001

An Example of Available Expressions Analysis

Iteration #1

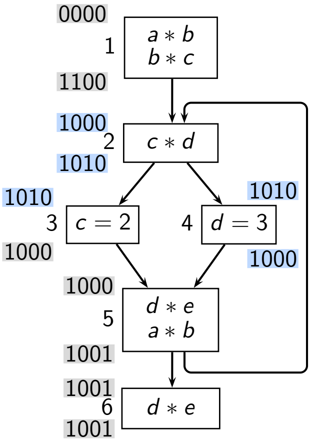


Let $e_1 \equiv a * b$, $e_2 \equiv b * c$, $e_3 \equiv c * d$, $e_4 \equiv d * e$

Node	*Gen*		*Kill*		Available		Redund.	
1	$\{e_1, e_2\}$	1100	$\emptyset$	0000	$\emptyset$	0000	$\emptyset$	0000
2	$\{e_3\}$	0010	$\emptyset$	0000	$\{e_1\}$	1000	$\emptyset$	0000
3	$\emptyset$	0000	$\{e_2, e_3\}$	0110	$\{e_1, e_3\}$	1010	$\emptyset$	0000
4	$\emptyset$	0000	$\{e_3, e_4\}$	0011	$\{e_1, e_3\}$	1010	$\emptyset$	0000
5	$\{e_1, e_4\}$	1001	$\emptyset$	0000	$\{e_1\}$	1000	$\{e_1\}$	1000
6	$\{e_4\}$	0001	$\emptyset$	0000	$\{e_1, e_4\}$	1001	$\{e_4\}$	0001

An Example of Available Expressions Analysis

Iteration #2

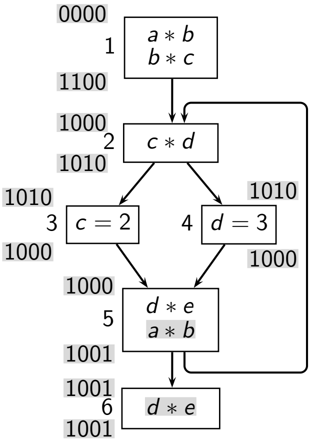


Let $e_1 \equiv a * b$, $e_2 \equiv b * c$, $e_3 \equiv c * d$, $e_4 \equiv d * e$

Node	*Gen*		*Kill*		Available		Redund.	
1	$\{e_1, e_2\}$	1100	$\emptyset$	0000	$\emptyset$	0000	$\emptyset$	0000
2	$\{e_3\}$	0010	$\emptyset$	0000	$\{e_1\}$	1000	$\emptyset$	0000
3	$\emptyset$	0000	$\{e_2, e_3\}$	0110	$\{e_1, e_3\}$	1010	$\emptyset$	0000
4	$\emptyset$	0000	$\{e_3, e_4\}$	0011	$\{e_1, e_3\}$	1010	$\emptyset$	0000
5	$\{e_1, e_4\}$	1001	$\emptyset$	0000	$\{e_1\}$	1000	$\{e_1\}$	1000
6	$\{e_4\}$	0001	$\emptyset$	0000	$\{e_1, e_4\}$	1001	$\{e_4\}$	0001

An Example of Available Expressions Analysis

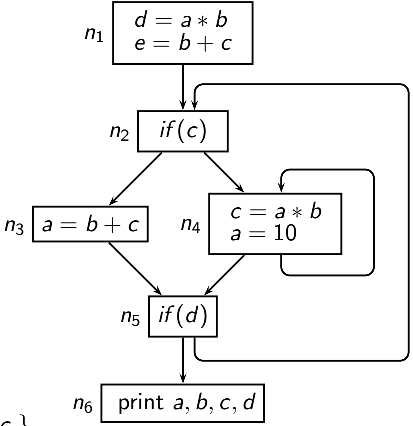
Final Result



Let $e_1 \equiv a * b$, $e_2 \equiv b * c$, $e_3 \equiv c * d$, $e_4 \equiv d * e$

Node	*Gen*		*Kill*		Available		Redund.	
1	$\{e_1, e_2\}$	1100	$\emptyset$	0000	$\emptyset$	0000	$\emptyset$	0000
2	$\{e_3\}$	0010	$\emptyset$	0000	$\{e_1\}$	1000	$\emptyset$	0000
3	$\emptyset$	0000	$\{e_2, e_3\}$	0110	$\{e_1, e_3\}$	1010	$\emptyset$	0000
4	$\emptyset$	0000	$\{e_3, e_4\}$	0011	$\{e_1, e_3\}$	1010	$\emptyset$	0000
5	$\{e_1, e_4\}$	1001	$\emptyset$	0000	$\{e_1\}$	1000	$\{e_1\}$	1000
6	$\{e_4\}$	0001	$\emptyset$	0000	$\{e_1, e_4\}$	1001	$\{e_4\}$	0001

Tutorial Problem 2 for Available Expressions Analysis



$\mathbb{E}\text{xpr} = \{\ a * b,\ b + c\ \}$

Solution of the Tutorial Problem 2

Bit vector | $a * b$ | $b + c$ |

Node	Local Information			Global Information				
				Iteration # 1		Changes in iteration # 2		$Redundant_n$
	Gen_n	$Kill_n$	$AntGen_n$	In_n	Out_n	In_n	Out_n	
n_1	11	00	11					
n_2	00	00	00					
n_3	01	10	01					
n_4	00	11	10					
n_5	00	00	00					
n_6	00	00	00					

Solution of the Tutorial Problem 2

Bit vector $\boxed{a * b \mid b + c}$

Node	Local Information			Global Information				
				Iteration # 1		Changes in iteration # 2		$Redundant_n$
	Gen_n	$Kill_n$	$AntGen_n$	In_n	Out_n	In_n	Out_n	
n_1	11	00	11	00	11			
n_2	00	00	00	11	11			
n_3	01	10	01	11	01			
n_4	00	11	10	11	00			
n_5	00	00	00	00	00			
n_6	00	00	00	00	00			

Solution of the Tutorial Problem 2

Bit vector $\boxed{a * b \mid b + c}$

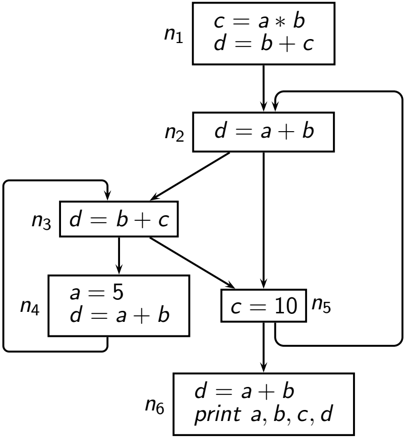
Node	Local Information			Global Information				
				Iteration # 1		Changes in iteration # 2		$Redundant_n$
	Gen_n	$Kill_n$	$AntGen_n$	In_n	Out_n	In_n	Out_n	
n_1	11	00	11	00	11			
n_2	00	00	00	11	11	00	00	
n_3	01	10	01	11	01	00		
n_4	00	11	10	11	00	00		
n_5	00	00	00	00	00			
n_6	00	00	00	00	00			

Solution of the Tutorial Problem 2

Bit vector | $a * b$ | $b + c$ |

Node	Local Information			Global Information				
				Iteration # 1		Changes in iteration # 2		$Redundant_n$
	Gen_n	$Kill_n$	$AntGen_n$	In_n	Out_n	In_n	Out_n	
n_1	11	00	11	00	11			00
n_2	00	00	00	11	11	00	00	00
n_3	01	10	01	11	01	00		00
n_4	00	11	10	11	00	00		00
n_5	00	00	00	00	00			00
n_6	00	00	00	00	00			00

Tutorial Problem 3 for Available Expressions Analysis



$\mathbb{E}\text{xpr} = \{\ a * b,\ b + c,\ a + b\ \}$

Solution of the Tutorial Problem 3

Bit vector	$a * b$	$b + c$	$a + b$

Node	Local Information			Global Information						
				Iteration # 1		Changes in Iteration # 2		Changes in Iteration # 3		$Redundant_n$
	Gen_n	$Kill_n$	$AntGen_n$	In_n	Out_n	In_n	Out_n	In_n	Out_n	
n_1	110	010	100							
n_2	001	000	001							
n_3	010	000	010							
n_4	001	101	000							
n_5	000	010	000							
n_6	001	000	001							

Solution of the Tutorial Problem 3

Bit vector	$a * b$	$b + c$	$a + b$

Node	Local Information			Global Information						
				Iteration # 1		Changes in Iteration # 2		Changes in Iteration # 3		$Redundant_n$
	Gen_n	$Kill_n$	$AntGen_n$	In_n	Out_n	In_n	Out_n	In_n	Out_n	
n_1	110	010	100	000	110					
n_2	001	000	001	110	111					
n_3	010	000	010	111	111					
n_4	001	101	000	111	011					
n_5	000	010	000	111	101					
n_6	001	000	001	101	101					

Solution of the Tutorial Problem 3

Bit vector	$a * b$	$b + c$	$a + b$

Node	Local Information			Global Information						
				Iteration # 1		Changes in Iteration # 2		Changes in Iteration # 3		$Redundant_n$
	Gen_n	$Kill_n$	$AntGen_n$	In_n	Out_n	In_n	Out_n	In_n	Out_n	
n_1	110	010	100	000	110					
n_2	001	000	001	110	111	100	101			
n_3	010	000	010	111	111	001	011			
n_4	001	101	000	111	011	011				
n_5	000	010	000	111	101	001	001			
n_6	001	000	001	101	101	001	001			

Solution of the Tutorial Problem 3

Bit vector	$a * b$	$b + c$	$a + b$

Node	Local Information			Global Information						
				Iteration # 1		Changes in Iteration # 2		Changes in Iteration # 3		$Redundant_n$
	Gen_n	$Kill_n$	$AntGen_n$	In_n	Out_n	In_n	Out_n	In_n	Out_n	
n_1	110	010	100	000	110					
n_2	001	000	001	110	111	100	101	000	001	
n_3	010	000	010	111	111	001	011			
n_4	001	101	000	111	011	011				
n_5	000	010	000	111	101	001	001			
n_6	001	000	001	101	101	001	001			

Solution of the Tutorial Problem 3

Bit vector	$a * b$	$b + c$	$a + b$

Node	Local Information			Global Information						
				Iteration # 1		Changes in Iteration # 2		Changes in Iteration # 3		$Redundant_n$
	Gen_n	$Kill_n$	$AntGen_n$	In_n	Out_n	In_n	Out_n	In_n	Out_n	
n_1	110	010	100	000	110					000
n_2	001	000	001	110	111	100	101	000	001	000
n_3	010	000	010	111	111	001	011			000
n_4	001	101	000	111	011	011				000
n_5	000	010	000	111	101	001	001			000
n_6	001	000	001	101	101	001	001			001

Solution of the Tutorial Problem 3

Bit vector

$a * b$	$b + c$	$a + b$

Node	Local Information			Global Information						
				Iteration # 1		Changes in Iteration # 2		Changes in Iteration # 3		$Redundant_n$
	Gen_n	$Kill_n$	$AntGen_n$	In_n	Out_n	In_n	Out_n	In_n	Out_n	
n_1	110	010	100	000	110					000
n_2	001	000	001	110	111	100	101	000	001	000
n_3	010	000	010	111	111	001	011			000
n_4	001	101	000	111	011	011				000
n_5	000	010	000	111	101	001	001			000
n_6	001	000	001	101	101	001	001			001

Why do we need 3 iterations as against 2 for previous problems?

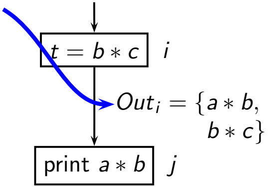
Soundness and Precision of Available Expressions Analysis

Consider common subexpression elimination based on availability information

Soundness and Precision of Available Expressions Analysis

Consider common subexpression elimination based on availability information

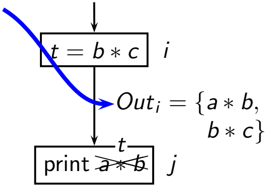
- Spurious inclusion of a non-available expression $a * b$



Soundness and Precision of Available Expressions Analysis

Consider common subexpression elimination based on availability information

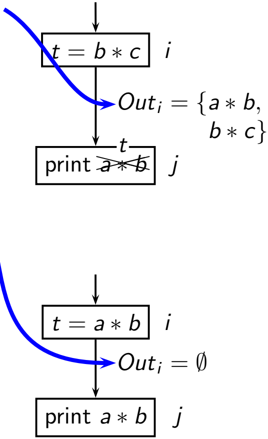
- Spurious inclusion of a non-available expression $a * b$
  - An occurrence of $a * b$ may be eliminated
  - Solution is unsound



Soundness and Precision of Available Expressions Analysis

Consider common subexpression elimination based on availability information

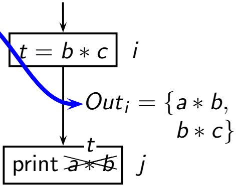
- Spurious inclusion of a non-available expression $a * b$
  - An occurrence of $a * b$ may be eliminated
  - Solution is unsound
- Spurious exclusion of an available variable

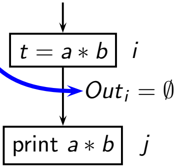


Soundness and Precision of Available Expressions Analysis

Consider common subexpression elimination based on availability information

- Spurious inclusion of a non-available expression $a * b$
  - An occurrence of $a * b$ may be eliminated
  - Solution is unsound
- Spurious exclusion of an available variable
  - An occurrence of $a * b$ may not be eliminated
  - Solution is sound but may be imprecise

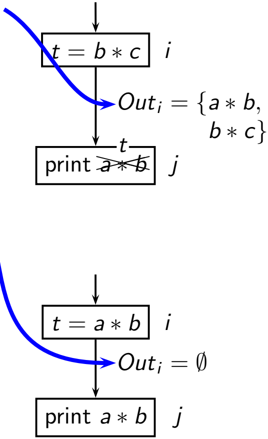




Soundness and Precision of Available Expressions Analysis

Consider common subexpression elimination based on availability information

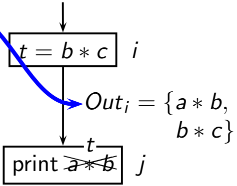
- Spurious inclusion of a non-available expression $a * b$
  - An occurrence of $a * b$ may be eliminated
  - Solution is unsound
- Spurious exclusion of an available variable
  - An occurrence of $a * b$ may not be eliminated
  - Solution is sound but may be imprecise
- Given $A_2 \supseteq A_1$ representing availability information
  - Using A_1 in place of A_2 is sound
  - Using A_2 in place of A_1 may not be sound



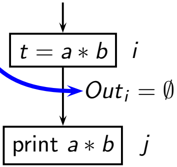
Soundness and Precision of Available Expressions Analysis

Consider common subexpression elimination based on availability information

- Spurious inclusion of a non-available expression $a * b$
  - An occurrence of $a * b$ may be eliminated
  - Solution is unsound

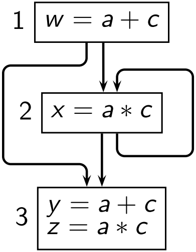


- Spurious exclusion of an available variable
  - An occurrence of $a * b$ may not be eliminated
  - Solution is sound but may be imprecise



- Given $A_2 \supseteq A_1$ representing availability information
  - Using A_1 in place of A_2 is sound
  - Using A_2 in place of A_1 may not be sound
- The largest set of available expressions is most precise
  - Since availability sets shrink (confluence is $\cap$), we choose $\mathbb{U}$ as the initial conservative value

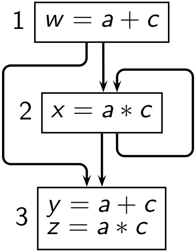
The Effect of BI and Initialization on a Solution



The Effect of BI and Initialization on a Solution

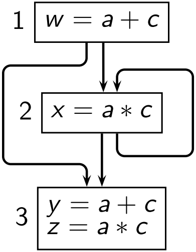
Bit Vector

$a + c$	$a * c$



BI	Node	Initialization $\mathbb{U}$		Initialization $\emptyset$	
		In_n	Out_n	In_n	Out_n
$\emptyset$	1				
	2				
	3				
$\mathbb{U}$	1				
	2				
	3				

The Effect of BI and Initialization on a Solution

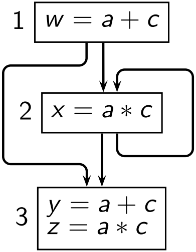


Bit Vector

$a + c$	$a * c$

BI	Node	Initialization $\mathbb{U}$		Initialization $\emptyset$	
		In_n	Out_n	In_n	Out_n
$\emptyset$	1	00	10		
	2	10	11		
	3	10	11		
$\mathbb{U}$	1				
	2				
	3				

The Effect of BI and Initialization on a Solution

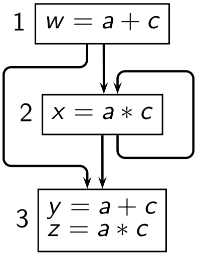


Bit Vector

$a + c$	$a * c$

BI	Node	Initialization $\mathbb{U}$		Initialization $\emptyset$	
		In_n	Out_n	In_n	Out_n
$\emptyset$	1	00	10	00	10
	2	10	11	00	01
	3	10	11	00	11
$\mathbb{U}$	1				
	2				
	3				

The Effect of BI and Initialization on a Solution



Bit Vector

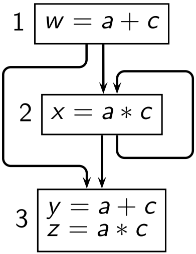
$a + c$	$a * c$

BI	Node	Initialization $\mathbb{U}$		Initialization $\emptyset$	
		In_n	Out_n	In_n	Out_n
$\emptyset$	1	00	10	00	10
	2	10	11	00	01
	3	10	11	00	11
$\mathbb{U}$	1	11	11		
	2	11	11		
	3	11	11		

The Effect of *BI* and Initialization on a Solution

Bit Vector

$a + c$	$a * c$

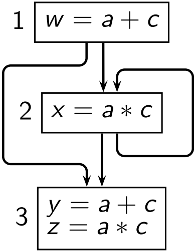


BI	Node	Initialization $\mathbb{U}$		Initialization $\emptyset$	
		In_n	Out_n	In_n	Out_n
$\emptyset$	1	00	10	00	10
	2	10	11	00	01
	3	10	11	00	11
$\mathbb{U}$	1	11	11	11	11
	2	11	11	00	01
	3	11	11	01	11

The Effect of *BI* and Initialization on a Solution

Bit Vector

$a + c$	$a * c$



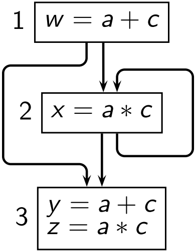
BI	Node	Initialization $\mathbb{U}$		Initialization $\emptyset$	
		In_n	Out_n	In_n	Out_n
$\emptyset$	1	00	10	00	10
	2	10	11	00	01
	3	10	11	00	11
$\mathbb{U}$	1	11	11	11	11
	2	11	11	00	01
	3	11	11	01	11

This represents the expected availability information leading to elimination of $a + c$ in node 3 ($a * c$ is not redundant in node 3)

The Effect of *BI* and Initialization on a Solution

Bit Vector

$a + c$	$a * c$



BI	Node	Initialization $\mathbb{U}$		Initialization $\emptyset$	
		In_n	Out_n	In_n	Out_n
$\emptyset$	1	00	10	00	10
	2	10	11	00	01
	3	10	11	00	11
$\mathbb{U}$	1	11	11	11	11
	2	11	11	00	01
	3	11	11	01	11

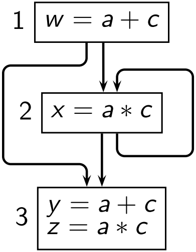
This misses the availability of $a + c$ in node 3

The Effect of *BI* and Initialization on a Solution

This makes $a * c$ available in node 3 although its computation in node 3 is not redundant

Bit Vector

$a + c$	$a * c$



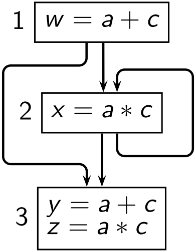
BI	Node	Initialization $\mathbb{U}$		Initialization $\emptyset$	
		In_n	Out_n	In_n	Out_n
$\emptyset$	1	00	10	00	10
	2	10	11	00	01
	3	10	11	00	11
$\mathbb{U}$	1	11	11	11	11
	2	11	11	00	01
	3	11	11	01	11

The Effect of BI and Initialization on a Solution

This make $a * c$ available in node 3 and but misses the availability of $a + c$ in node 3

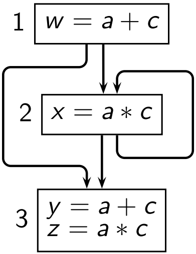
Bit Vector

$a + c$	$a * c$



BI	Node	Initialization $\mathbb{U}$		Initialization $\emptyset$	
		In_n	Out_n	In_n	Out_n
$\emptyset$	1	00	10	00	10
	2	10	11	00	01
	3	10	11	00	11
$\mathbb{U}$	1	11	11	11	11
	2	11	11	00	01
	3	11	11	01	11

The Effect of *BI* and Initialization on a Solution



Bit Vector

$a + c$	$a * c$

BI	Node	Initialization $\mathbb{U}$		Initialization $\emptyset$	
		In_n	Out_n	In_n	Out_n
$\emptyset$	1	Sound & Precise		Sound & Imprecise	
	2				
	3				
$\mathbb{U}$	1	Unsound		Unsound	
	2				
	3				

Some Observations

- Data flow equations do not require a particular order of computation
  - Specification. Data flow equations define what needs to be computed and not how it is to be computed
  - Implementation. Round robin iterations perform the actual computation
  - Specification and implementation are distinct
- Initialization governs the quality of solution found
  - Only precision is affected, soundness is guaranteed
  - Associated with "internal" nodes
- *BI* depends on the semantics of the calling context
  - May cause unsoundness
  - Associated with "boundary" node (specified by data flow equations)
    Does not vary with the method or order of traversal

Still More Tutorial Problems ☺

A New Data Flow Framework: Partially available expressions analysis

- Expressions that are computed and remain unmodified along some path reaching p
- The data flow equations are same as that of available expressions analysis except that the confluence is changed to $\cup$

Perform partially available expressions analysis for the example program used for available expressions analysis

Solution of the Tutorial Problem 2 for Partial Availability Analysis

Bit vector | $a * b$ | $b + c$ |

Node	Local Information			Global Information		
				Iteration # 1		$ParRedund_n$
	Gen_n	$Kill_n$	$AntGen_n$	$PavIn_n$	$PavOut_n$	
n_1	11	00	11			
n_2	00	00	00			
n_3	01	10	01			
n_4	00	11	10			
n_5	00	00	00			
n_6	00	00	00			

Solution of the Tutorial Problem 2 for Partial Availability Analysis

Bit vector | $a * b$ | $b + c$ |

Node	Local Information			Global Information		
				Iteration # 1		$ParRedund_n$
	Gen_n	$Kill_n$	$AntGen_n$	$PavIn_n$	$PavOut_n$	
n_1	11	00	11	00	11	
n_2	00	00	00	11	11	
n_3	01	10	01	11	01	
n_4	00	11	10	11	00	
n_5	00	00	00	01	01	
n_6	00	00	00	01	01	

Solution of the Tutorial Problem 2 for Partial Availability Analysis

Bit vector | $a * b$ | $b + c$ |

Node	Local Information			Global Information		
				Iteration # 1		$ParRedund_n$
	Gen_n	$Kill_n$	$AntGen_n$	$PavIn_n$	$PavOut_n$	
n_1	11	00	11	00	11	00
n_2	00	00	00	11	11	00
n_3	01	10	01	11	01	01
n_4	00	11	10	11	00	10
n_5	00	00	00	01	01	00
n_6	00	00	00	01	01	00

Solution of the Tutorial Problem 3 for Partial Availability Analysis

Bit vector

$a * b$	$b + c$	$a + b$

Node	Local Information			Global Information				
				Iteration # 1		Changes in iteration # 2		$ParRedund_n$
	Gen_n	$Kill_n$	$AntGen_n$	$PavIn_n$	$PavOut_n$	In_n	Out_n	
n_1	110	010	100					
n_2	001	000	001					
n_3	010	000	010					
n_4	001	101	000					
n_5	000	010	000					
n_6	001	000	001					

Solution of the Tutorial Problem 3 for Partial Availability Analysis

Bit vector	$a*b$	$b+c$	$a+b$

Node	Local Information			Global Information				
				Iteration # 1		Changes in iteration # 2		$ParRedund_n$
	Gen_n	$Kill_n$	$AntGen_n$	$PavIn_n$	$PavOut_n$	In_n	Out_n	
n_1	110	010	100	000	110			
n_2	001	000	001	110	111			
n_3	010	000	010	111	111			
n_4	001	101	000	111	011			
n_5	000	010	000	111	101			
n_6	001	000	001	101	101			

Solution of the Tutorial Problem 3 for Partial Availability Analysis

Bit vector	$a * b$	$b + c$	$a + b$

Node	Local Information			Global Information				
				Iteration # 1		Changes in iteration # 2		$ParRedund_n$
	Gen_n	$Kill_n$	$AntGen_n$	$PavIn_n$	$PavOut_n$	In_n	Out_n	
n_1	110	010	100	000	110			
n_2	001	000	001	110	111	111		
n_3	010	000	010	111	111			
n_4	001	101	000	111	011			
n_5	000	010	000	111	101			
n_6	001	000	001	101	101			

Solution of the Tutorial Problem 3 for Partial Availability Analysis

Bit vector

$a * b$	$b + c$	$a + b$

Node	Local Information			Global Information				
				Iteration # 1		Changes in iteration # 2		$ParRedund_n$
	Gen_n	$Kill_n$	$AntGen_n$	$PavIn_n$	$PavOut_n$	In_n	Out_n	
n_1	110	010	100	000	110			000
n_2	001	000	001	110	111	111		001
n_3	010	000	010	111	111			010
n_4	001	101	000	111	011			000
n_5	000	010	000	111	101			000
n_6	001	000	001	101	101			001

Part 5

Reaching Definitions Analysis

Defining Reaching Definitions Analysis

- A definition $d_x : x = e$ reaches a program point p if it appears (without a redefinition of x) on some path from program entry to p

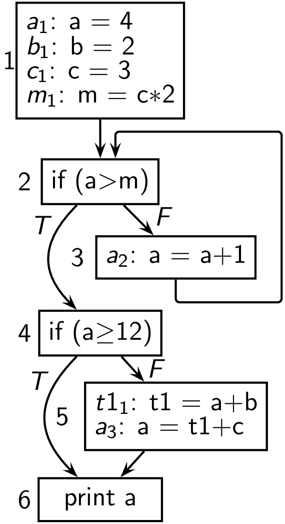
  (x is a variable and e is an expression)

- Application : Copy Propagation

  A use of a variable x at a program point p can be replaced by y if $d_x : x = y$ is the only definition which reaches p and y is not modified between the point of d_x and p.

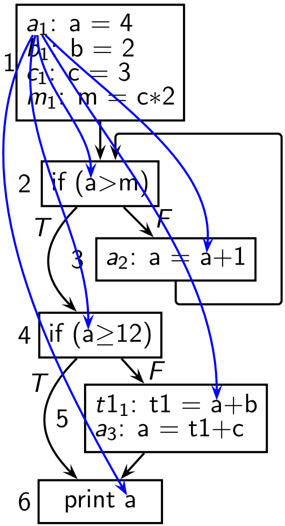
Using Reaching Definitions for Def-Use and Use-Def Chains

Def-Use Chains



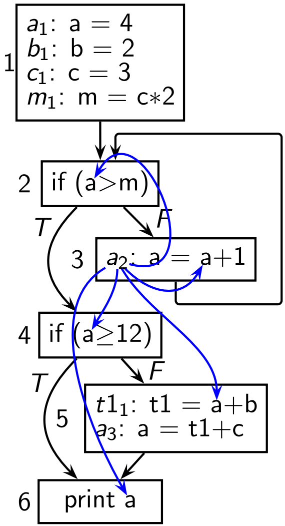
Using Reaching Definitions for Def-Use and Use-Def Chains

Def-Use Chains



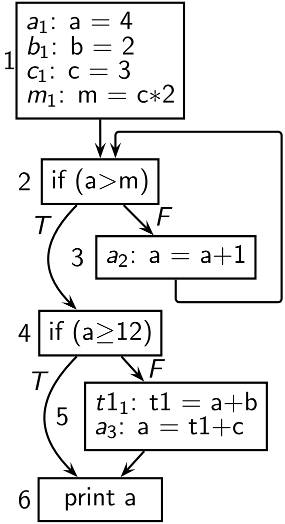
Using Reaching Definitions for Def-Use and Use-Def Chains

Def-Use Chains

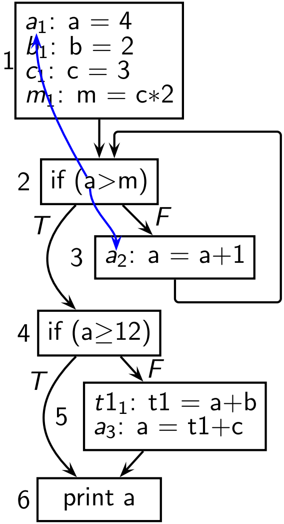


Using Reaching Definitions for Def-Use and Use-Def Chains

Def-Use Chains

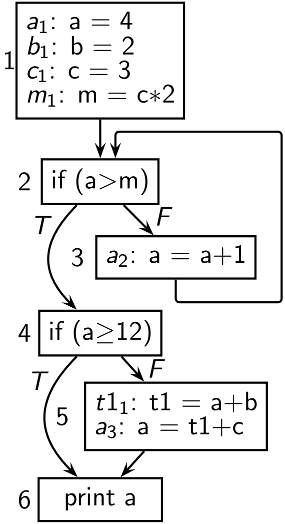


Use-Def Chains

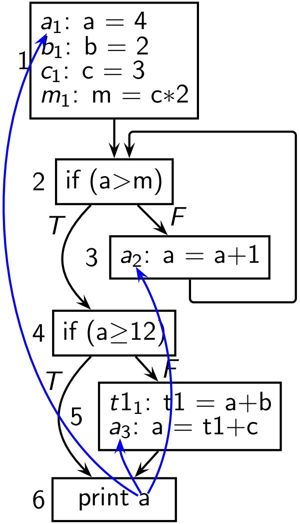


Using Reaching Definitions for Def-Use and Use-Def Chains

Def-Use Chains

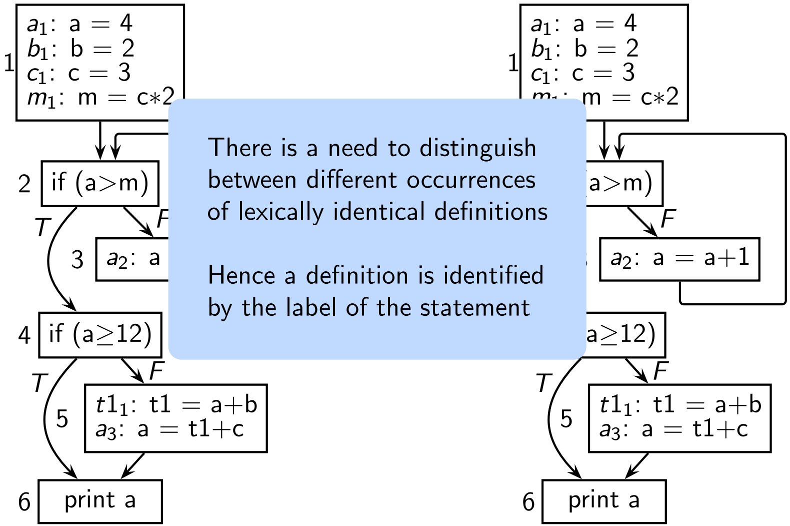


Use-Def Chains



Using Reaching Definitions for Def-Use and Use-Def Chains

Def-Use Chains *Use-Def Chains*



There is a need to distinguish between different occurrences of lexically identical definitions

Hence a definition is identified by the label of the statement

Defining Data Flow Analysis for Reaching Definitions Analysis

Let d_v be a definition of variable v

$$Gen_n = \{ d_v \mid \text{variable } v \text{ is defined in basic block } n \text{ and this definition is not followed (within } n\text{) by a definition of } v\}$$

$$Kill_n = \{ d_v \mid \text{basic block } n \text{ contains a definition of } v\}$$

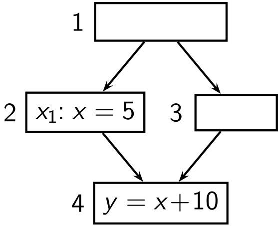
	Entity	Manipulation	Exposition
Gen_n	Definition	Occurrence	Downwards
$Kill_n$	Definition	Occurrence	Anywhere

Data Flow Equations for Reaching Definitions Analysis

$$
\begin{aligned}
In_n &= \begin{cases} BI & n \text{ is } \mathit{Start} \text{ block} \\ \displaystyle\bigcup_{p \in pred(n)} Out_p & \text{otherwise} \end{cases} \\
Out_n &= Gen_n \cup (In_n - Kill_n) \\
BI &= \{d_x : x = undef \mid x \in \mathbb{V}\text{ar}\}
\end{aligned}
$$

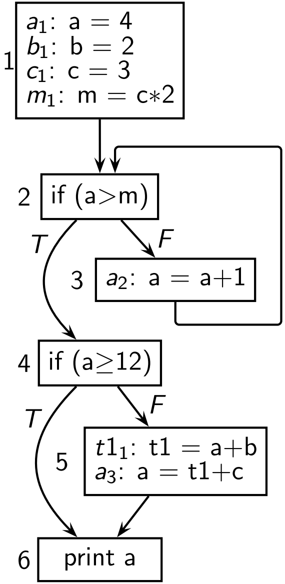
$\boxed{In_n \text{ and } Out_n \text{ are sets of definitions}}$

The Role of Boundary Information for Reaching Definitions Analysis

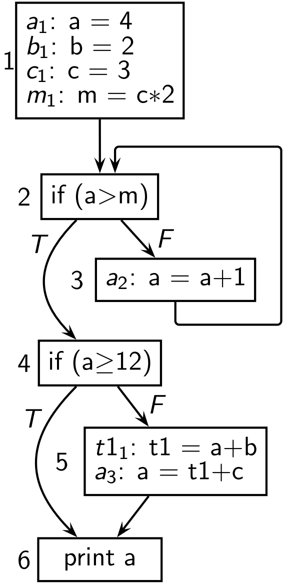


- Only one definition of x (x_1) reaches node 4
- Can we perform copy propagation in node 4 by replacing x by 5?
- Boundary information x_0: $x = \mathit{undef}$ prohibits it for soundness

Tutorial Problem for Copy Propagation

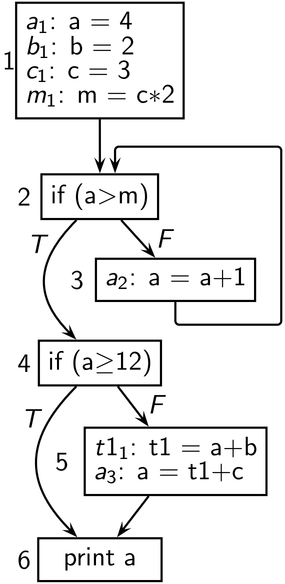


Tutorial Problem for Copy Propagation

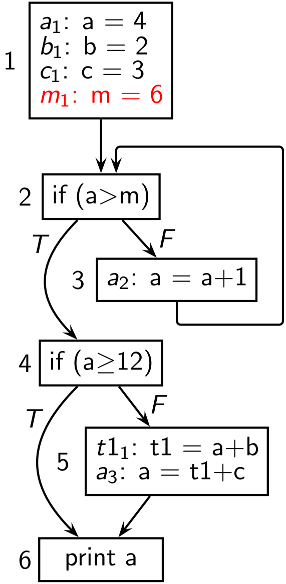


Local copy propagation and constant folding

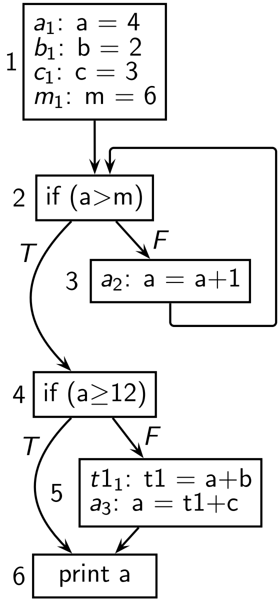
Tutorial Problem for Copy Propagation



Local copy propagation and constant folding

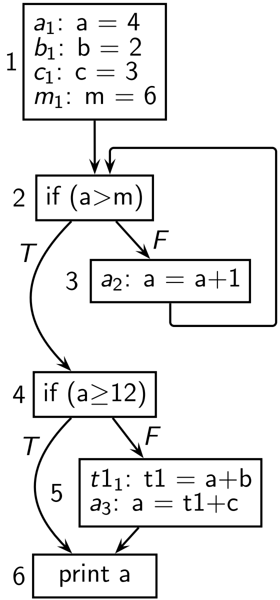


Tutorial Problem for Copy Propagation



	Gen	*Kill*
1	$\{a_1, b_1, c_1, m_1\}$	$\{a_0, a_1, a_2, a_3, b_0, b_1, c_0, c_1, m_0, m_1\}$
2	$\emptyset$	$\emptyset$
3	$\{a_2\}$	$\{a_0, a_1, a_2, a_3\}$
4	$\emptyset$	$\emptyset$
5	$\{a_3\}$	$\{a_0, a_1, a_2, a_3\}$
6	$\emptyset$	$\emptyset$

Tutorial Problem for Copy Propagation

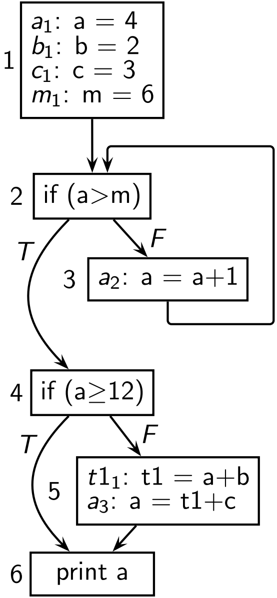


	Gen	*Kill*
1	$\{a_1, b_1, c_1, m_1\}$	$\{a_0, a_1, a_2, a_3, b_0, b_1, c_0, c_1, m_0, m_1\}$
2	$\emptyset$	$\emptyset$
3	$\{a_2\}$	$\{a_0, a_1, a_2, a_3\}$
4	$\emptyset$	$\emptyset$
5	$\{a_3\}$	$\{a_0, a_1, a_2, a_3\}$
6	$\emptyset$	$\emptyset$

- Temporary variable t1 is ignored
- For variable v, v_0 denotes the definition $v = undef$

  This is used for defining *BI*

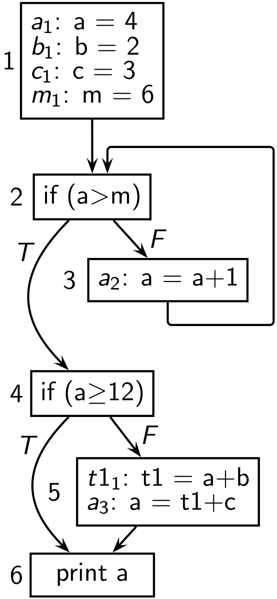
Tutorial Problem for Copy Propagation



	Gen	*Kill*
1	$\{a_1, b_1, c_1, m_1\}$	$\{a_0, a_1, a_2, a_3, b_0, b_1, c_0, c_1, m_0, m_1\}$
2	$\emptyset$	$\emptyset$
3	$\{a_2\}$	$\{a_0, a_1, a_2, a_3\}$
4	$\emptyset$	$\emptyset$
5	$\{a_3\}$	$\{a_0, a_1, a_2, a_3\}$
6	$\emptyset$	$\emptyset$

	Iteration #1	
	In	*Out*
1	$\{a_0, b_0, c_0, m_0\}$	$\{a_1, b_1, c_1, m_1\}$
2	$\{a_1, b_1, c_1, m_1\}$	$\{a_1, b_1, c_1, m_1\}$
3	$\{a_1, b_1, c_1, m_1\}$	$\{a_2, b_1, c_1, m_1\}$
4	$\{a_1, b_1, c_1, m_1\}$	$\{a_1, b_1, c_1, m_1\}$
5	$\{a_1, b_1, c_1, m_1\}$	$\{a_3, b_1, c_1, m_1\}$
6	$\{a_1, a_3, b_1, c_1, m_1\}$	$\{a_1, a_3, b_1, c_1, m_1\}$

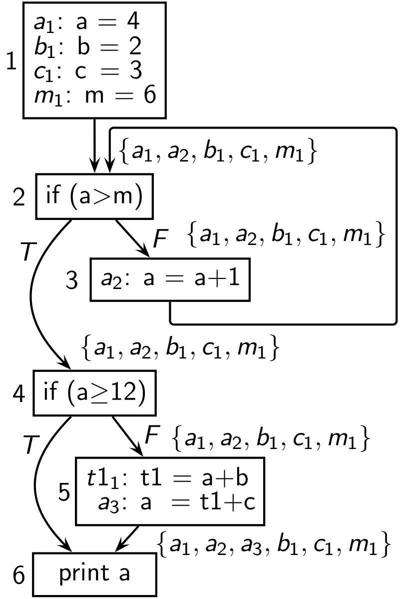
Tutorial Problem for Copy Propagation



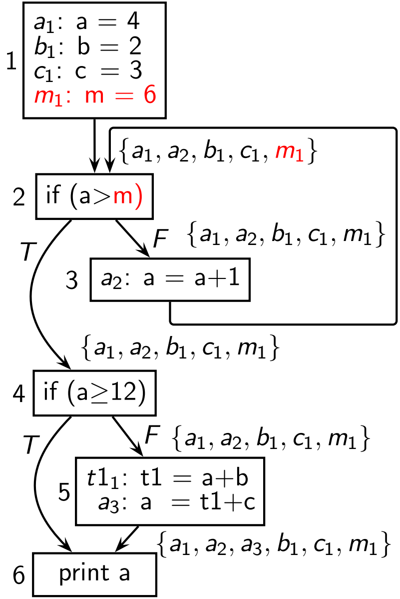
	Gen	*Kill*
1	$\{a_1, b_1, c_1, m_1\}$	$\{a_0, a_1, a_2, a_3, b_0, b_1, c_0, c_1, m_0, m_1\}$
2	$\emptyset$	$\emptyset$
3	$\{a_2\}$	$\{a_0, a_1, a_2, a_3\}$
4	$\emptyset$	$\emptyset$
5	$\{a_3\}$	$\{a_0, a_1, a_2, a_3\}$
6	$\emptyset$	$\emptyset$

	Iteration #2	
	In	*Out*
1	$\{a_0, b_0, c_0, m_0\}$	$\{a_1, b_1, c_1, m_1\}$
2	$\{a_1, a_2, b_1, c_1, m_1\}$	$\{a_1, a_2, b_1, c_1, m_1\}$
3	$\{a_1, a_2, b_1, c_1, m_1\}$	$\{a_2, b_1, c_1, m_1\}$
4	$\{a_1, a_2, b_1, c_1, m_1\}$	$\{a_1, a_2, b_1, c_1, m_1\}$
5	$\{a_1, a_2, b_1, c_1, m_1\}$	$\{a_3, b_1, c_1, m_1\}$
6	$\{a_1, a_2, a_3, b_1, c_1, m_1\}$	$\{a_1, a_2, a_3, b_1, c_1, m_1\}$

Tutorial Problem for Copy Propagation

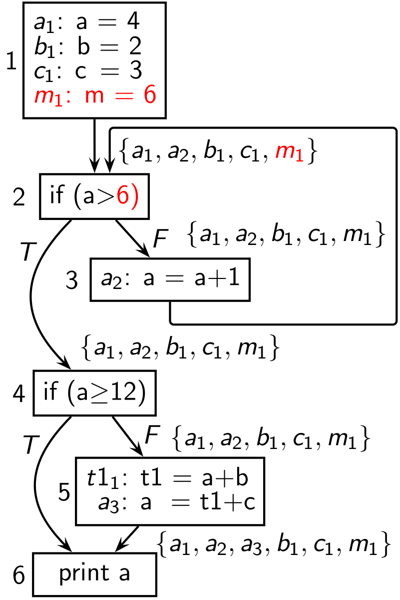


Tutorial Problem for Copy Propagation



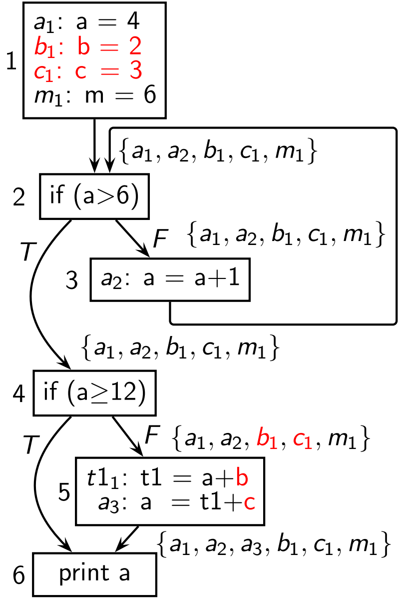
- RHS of m_1 is constant and hence cannot change

Tutorial Problem for Copy Propagation



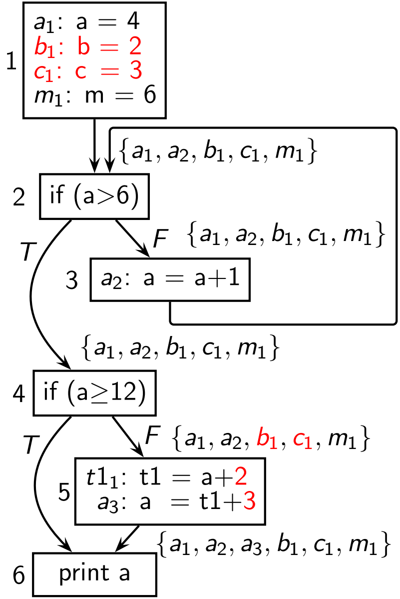
- RHS of m_1 is constant and hence cannot change
- In block 2, m can be replaced by 6

Tutorial Problem for Copy Propagation



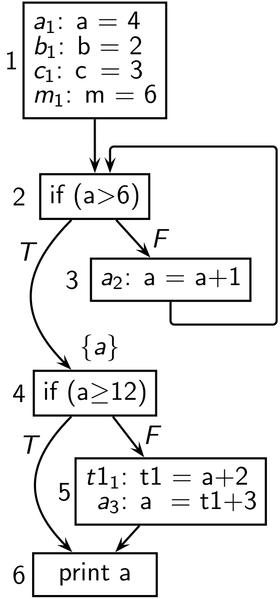
- RHS of m_1 is constant and hence cannot change
- In block 2, m can be replaced by 6
- RHS of b_1 and c_1 are constant and hence cannot change

Tutorial Problem for Copy Propagation

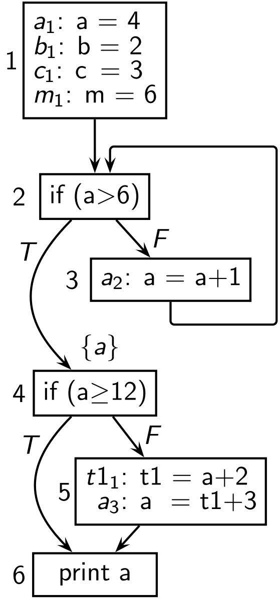


- RHS of m_1 is constant and hence cannot change
- In block 2, m can be replaced by 6
- RHS of b_1 and c_1 are constant and hence cannot change
- In block 5, b can be replaced by 2 and c can be replaced by 3

Tutorial Problem for Copy Propagation

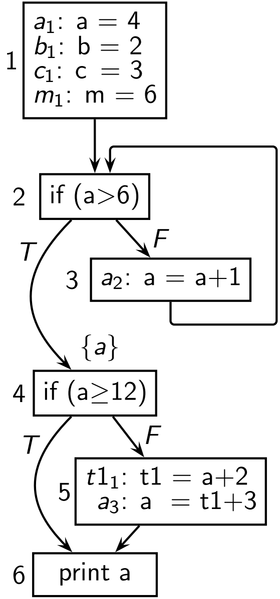


Tutorial Problem for Copy Propagation



So what is the advantage?

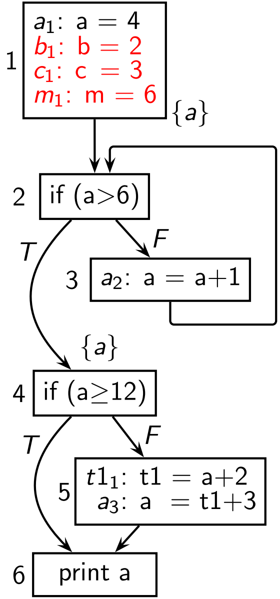
Tutorial Problem for Copy Propagation



So what is the advantage?

Dead Code Elimination

Tutorial Problem for Copy Propagation

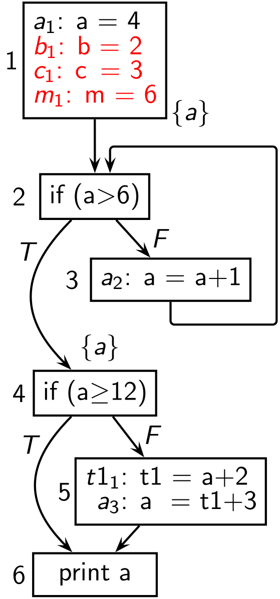


So what is the advantage?

Dead Code Elimination

- Only a is live at the exit of 1

Tutorial Problem for Copy Propagation

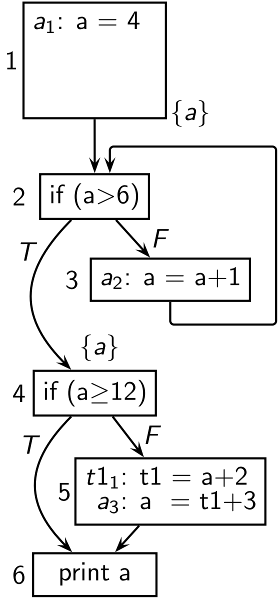


So what is the advantage?

Dead Code Elimination

- Only a is live at the exit of 1
- Assignments of b, c, and m are dead code

Tutorial Problem for Copy Propagation



So what is the advantage?

Dead Code Elimination

- Only a is live at the exit of 1
- Assignments of b, c, and m are dead code
- Can be deleted

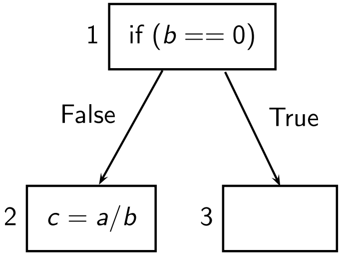
Part 6

Anticipable Expressions Analysis

Defining Anticipable Expressions Analysis

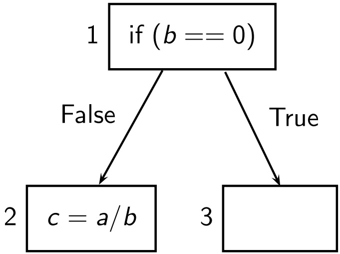
- An expression e is anticipable at a program point p, if every path from p to the program exit contains an evaluation of e which is not preceded by a redefinition of any operand of e.
- Application : Safety of Code Placement

Safety of Code Placement

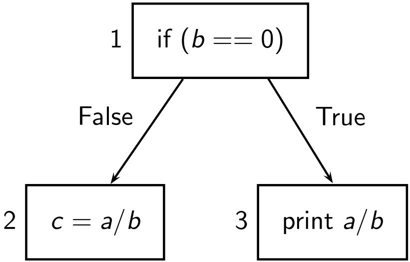


Placing a/b at the exit of 1 is unsafe
($\equiv$ can change the behaviour of the optimized program)

Safety of Code Placement

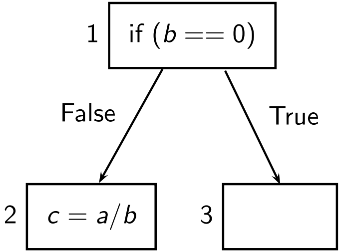


Placing a/b at the exit of 1 is unsafe ($\equiv$ can change the behaviour of the optimized program)

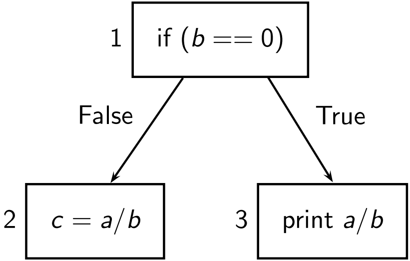


??

Safety of Code Placement



Placing a/b at the exit of 1 is unsafe
($\equiv$ can change the behaviour of the optimized program)



??

A guarded computation of an expression should not be converted to an unguarded computation

Defining Data Flow Analysis for Anticipable Expressions Analysis

$$Gen_n = \{\, e \mid \text{expression } e \text{ is evaluated in basic block } n \text{ and this evaluation is not preceded (within } n\text{) by a definition of any operand of } e \,\}$$

$$Kill_n = \{\, e \mid \text{basic block } n \text{ contains a definition of an operand of } e \,\}$$

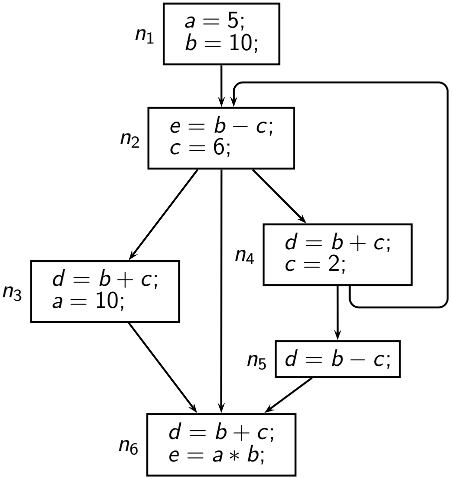
	Entity	Manipulation	Exposition
Gen_n	Expression	Use	Upwards
$Kill_n$	Expression	Modification	Anywhere

Data Flow Equations for Anticipable Expressions Analysis

$$
\begin{aligned}
In_n &= Gen_n \cup (Out_n - Kill_n) \\
Out_n &= \begin{cases} BI & n \text{ is } End \text{ block} \\ \bigcap\limits_{s \in succ(n)} In_s & \text{otherwise} \end{cases}
\end{aligned}
$$

In_n and Out_n are sets of expressions

Tutorial Problem 1 for Anticipable Expressions Analysis



$\mathbb{E}\text{xpr} = \{\ a * b,\ b + c, b - c\ \}$

Solution of Tutorial Problem 1

Block	Local Information		Global Information			
			Iteration # 1		Change in iteration # 2	
	Gen_n	$Kill_n$	Out_n	In_n	Out_n	In_n
n_6	110	000				
n_5	001	000				
n_4	010	011				
n_3	010	100				
n_2	001	011				
n_1	000	111				

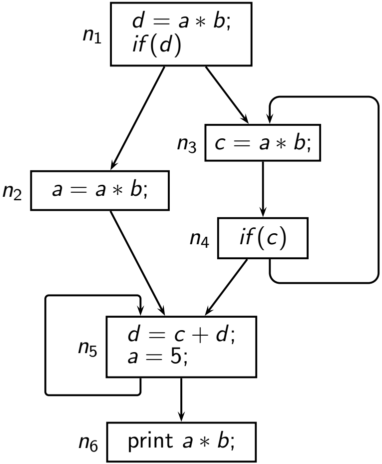
Solution of Tutorial Problem 1

Block	Local Information		Global Information			
			Iteration # 1		Change in iteration # 2	
	Gen_n	$Kill_n$	Out_n	In_n	Out_n	In_n
n_6	110	000	000	110		
n_5	001	000	110	111		
n_4	010	011	111	110		
n_3	010	100	110	010		
n_2	001	011	010	001		
n_1	000	111	001	000		

Solution of Tutorial Problem 1

Block	Local Information		Global Information			
			Iteration # 1		Change in iteration # 2	
	Gen_n	$Kill_n$	Out_n	In_n	Out_n	In_n
n_6	110	000	000	110		
n_5	001	000	110	111		
n_4	010	011	111	110	001	010
n_3	010	100	110	010		
n_2	001	011	010	001		
n_1	000	111	001	000		

Tutorial Problem 2 for Anticipable Expressions Analysis



$\mathbb{E}\text{xpr} = \{\ a * b,\ c + d\ \}$

Solution of Tutorial Problem 2

Block	Local Information		Global Information			
			Iteration # 1		Change in iteration # 2	
	Gen_n	$Kill_n$	Out_n	In_n	Out_n	In_n
n_6	10	00				
n_5	01	11				
n_4	00	00				
n_3	10	01				
n_2	10	10				
n_1	10	01				

Solution of Tutorial Problem 2

Block	Local Information		Global Information			
			Iteration # 1		Change in iteration # 2	
	Gen_n	$Kill_n$	Out_n	In_n	Out_n	In_n
n_6	10	00	00	10		
n_5	01	11	10	01		
n_4	00	00	01	01		
n_3	10	01	01	10		
n_2	10	10	01	11		
n_1	10	01	10	10		

Solution of Tutorial Problem 2

Block	Local Information		Global Information			
			Iteration # 1		Change in iteration # 2	
	Gen_n	$Kill_n$	Out_n	In_n	Out_n	In_n
n_6	10	00	00	10		
n_5	01	11	10	01	00	
n_4	00	00	01	01	00	00
n_3	10	01	01	10	00	
n_2	10	10	01	11		
n_1	10	01	10	10		

Part 7

Common Features of Bit Vector Data Flow Frameworks

Defining Local Data Flow Properties

- Live variables analysis

	Entity	Manipulation	Exposition
Gen_n	Variable	Use	Upwards
$Kill_n$	Variable	Modification	Anywhere

- Analysis of expressions

	Entity	Manipulation	Exposition	
			Availability	Anticipability
Gen_n	Expression	Use	Downwards	Upwards
$Kill_n$	Expression	Modification	Anywhere	Anywhere

Common Form of Data Flow Equations

$$
\begin{aligned}
X_i &= f(Y_i) \\
Y_i &= \bigsqcap X_j
\end{aligned}
$$

Common Form of Data Flow Equations

Data Flow Information

So far we have seen sets (or bit vectors).
Could be entities other than sets.

$$
\begin{aligned}
X_i &= f(Y_i) \\
Y_i &= \bigsqcap X_j
\end{aligned}
$$

Common Form of Data Flow Equations

Data Flow Information

So far we have seen sets (or bit vectors). Could be entities other than sets.

Flow Function

$$X_i = f(Y_i)$$
$$Y_i = \bigsqcap X_j$$

So far we have seen constant *Gen* and *Kill*. Could be dependent *Gen* and *Kill*.

Common Form of Data Flow Equations

Data Flow Information

So far we have seen sets (or bit vectors).
Could be entities other than sets.

Flow Function

$$X_i = f(Y_i)$$
$$Y_i = \bigsqcap X_j$$

So far we have seen constant *Gen* and *Kill*. Could be dependent *Gen* and *Kill*.

Confluence

So far we have seen $\cup$ and $\cap$.
Could be other operations.

A Taxonomy of Bit Vector Data Flow Frameworks

	Confluence	
	Union	Intersection
Forward	Reaching Definitions	Available Expressions
Backward	Live Variables	Anticipable Expressions
Bidirectional (limited)		Partial Redundancy Elimination (Original M-R Formulation)

A Taxonomy of Bit Vector Data Flow Frameworks

Any Path

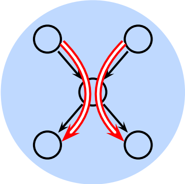
	Confluence	
	Union	Intersection
Forward	Reaching Definitions	Available Expressions
Backward	Live Variables	Anticipable Expressions
Bidirectional (limited)		Partial Redundancy Elimination (Original M-R Formulation)

A Taxonomy of Bit Vector Data Flow Frameworks

Any Path → Union; All Paths → Intersection

	Confluence	
	Union	Intersection
Forward	Reaching Definitions	Available Expressions
Backward	Live Variables	Anticipable Expressions
Bidirectional (limited)		Partial Redundancy Elimination (Original M-R Formulation)

A Taxonomy of Bit Vector Data Flow Frameworks

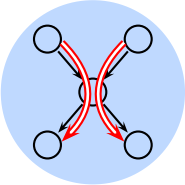


Any Path

All Paths

	Confluence	
	Union	Intersection
Forward	Reaching Definitions	Available Expressions
Backward	Live Variables	Anticipable Expressions
Bidirectional (limited)		Partial Redundancy Elimination (Original M-R Formulation)

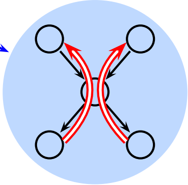
A Taxonomy of Bit Vector Data Flow Frameworks



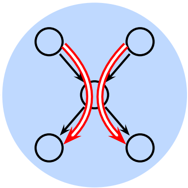
Any Path

All Paths

	Confluence	
	Union	Intersection
Forward	Reaching Definitions	Available Expressions
Backward	Live Variables	Anticipable Expressions
Bidirectional (limited)		Partial Redundancy Elimination (Original M-R Formulation)



A Taxonomy of Bit Vector Data Flow Frameworks



Any Path

All Paths

	Confluence	
	Union	Intersection
Forward	Reaching Definitions	Available Expressions
Backward	Live Variables	Anticipable Expressions
Bidirectional (limited)		Partial Redundancy Elimination (Original M-R Formulation)