Assignment 7

Adhvik Mani Sai Murarisetty - AI20BTECH11015

Download latex-tikz codes from

https://github.com/adhvik24/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/blob/main/ASSIGNMENT_7/AI1103_Assignment7.tex

1 GATE 2021 (ME-SET1), Q.20 (ME section)

Robot Ltd. wishes to maintain enough safety stock during the lead time period between starting a new production run and its completion such that the probability of satisfying the customer demand during the lead time period is 95%. The lead time periods is 5 days and daily customer demand can be assumed to follow the Gaussian (normal) distribution with mean 50 units and a standard deviation of 10 units. Using $\phi^{-1}(0.95)$ =1.64 , where ϕ represents the cumulative distribution function of the standard normal random variable, the amount of safety stock that must be maintained by Robot Ltd. to achieve this demand fulfillment probability for the lead time period is _____ units (round off to two decimal places).

2 SOLUTION

Let X be the normal R.V denoting the required amount of stock of customer demand over the lead time.

Let X_1 be the normal R.V denoting daily customer demand with mean and standard deviation as follows,

parameter	value(in units)
Mean of X_1 (μ)	50
Standard deviation of X_1 (σ)	10

TABLE 0: mean and standard deviation of X_1

Probability of satisfying customer is 0.95.

Let Z be a standard normal R.V such that,

$$Z = \frac{X_1 - \mu}{\sigma} \tag{2.0.1}$$

Referring table(0) to use in (2.0.1),

$$Z = \frac{X_1 - 50}{10} \tag{2.0.2}$$

Given that,

$$\phi^{-1}(0.95) = 1.64 \tag{2.0.3}$$

$$\implies \phi(1.64) = 0.95 \tag{2.0.4}$$

$$\phi(1.64) = \Pr(Z \leq 1.64) = 0.95 \tag{2.0.5}$$

$$\implies Z \leq 1.64 \iff \frac{X_1 - 50}{10} \leq 1.64 \tag{2.0.6}$$

$$\implies X_1 - 50 \leq 1.64(10) \tag{2.0.7}$$

$$\therefore X_1 \leq 66.4 \tag{2.0.8}$$

The demand in one day is independent of demand in the other day and the lead time is 5 days.

$$\implies X = 5(X_1) = 5(66.4) = 332 \tag{2.0.9}$$

Therefore the amount of safety stock that must be maintained by Robot Ltd. to achieve this demand fulfillment probability for the lead time period is 332.