Assignment 5

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Download latex-tikz codes from

https://github.com/adhvik24/AI1103−PROBABILITY−AND−RANDOM−VARIABLES/tree/main/ASSIGNMENT_5/AI1103_Assignment5.tex

1 GATE 2019 (ST) , Q.45 (Statistics section)

Consider the trinomial distribution with the probability mass function

$$P(X = x, Y = y) = \left(\frac{7!}{x!y!(7-x-y)!}\right)(0.6)^x(0.2)^y(0.2)^{7-x-y}$$

where $x \geq 0, y \geq 0$ *and* $x + y \leq 7$. Then $E[Y|X = 3]$ is equal to

2 SOLUTION

Probability mass function of a trinomial distribution is :

$$\begin{aligned}P(X = x, Y = y) &= \left(\frac{7!}{x!y!(7-x-y)!}\right)(0.6)^x(0.2)^y(0.2)^{7-x-y} \\ &= \left(\frac{7!}{x!(7-x)!}\frac{(7-x)!}{y!(7-x-y)!}\right)(0.6)^x(0.2)^y(0.2)^{7-x-y}\end{aligned}$$

$$P(X = x, Y = y) = {}^7C_x{}^{7-x}C_y(0.6)^x(0.2)^y(0.2)^{7-x-y} \tag{2.0.1}$$

Using (2.0.1), $P(X = x)$ is

$$\begin{aligned}P(X = x) &= \sum_{y=0}^{7-x} P(X = x, Y = y) \\ &= {}^7C_x(0.6)^x \sum_{y=0}^{7-x} {}^{7-x}C_y(0.2)^y(0.2)^{7-x-y} \\ &= {}^7C_x(0.6)^x\,(0.4)^{7-x}\end{aligned}$$

$$P(X = x) = {}^7C_x(0.6)^x\,(0.4)^{7-x} \tag{2.0.2}$$

We have to find $E[Y|X = 3]$,

$$E[Y|X = 3] = \sum_{y=0}^{4} yP(Y = y|X = 3) \tag{2.0.3}$$

$$E[Y|X = 3] = \sum_{y=0}^{4} y\left(\frac{P(X = 3, Y = y)}{P(X = 3)}\right) \tag{2.0.4}$$

By taking X=3 in (2.0.1) and (2.0.2) to use in (2.0.4),

$$\begin{aligned}E[Y|X = 3] &= \sum_{y=0}^{4} y\left(\frac{P(X = 3, Y = y)}{P(X = 3)}\right) \\ &= \sum_{y=0}^{4} y\left(\frac{{}^7C_3{}^4C_y(0.6)^3(0.2)^y(0.2)^{4-y}}{{}^7C_3(0.6)^3\,(0.4)^4}\right) \\ &= \sum_{y=0}^{4} y\left(\frac{{}^4C_y(0.2)^4}{(0.4)^4}\right)\end{aligned}$$

$$= \sum_{y=0}^{4} \frac{y({}^4C_y)}{16} \tag{2.0.5}$$

$$E[Y|X = 3] = \frac{1}{16}\sum_{y=0}^{4} y({}^4C_y) \tag{2.0.6}$$

$$= \frac{1}{16}\sum_{y=1}^{4} y\left(\frac{4}{y}\right)({}^3C_{y-1}) \tag{2.0.7}$$

$$= \frac{1}{4}\sum_{k=0}^{3}({}^3C_k) \tag{2.0.8}$$

$$= \frac{1}{4}(1+1)^3 = \frac{1}{4}(8) \tag{2.0.9}$$

$$E[Y|X = 3] = 2 \tag{2.0.10}$$

Therefore the value of $E[Y|X = 3] = 2$.