Assignment 3

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Download all python codes from

```
https://github.com/adhvik24/AI1103−
PROBABILITY−AND−RANDOM−
VARIABLES/tree/main/ASSIGNMENT%203/
codes
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and latex-tikz codes from

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https://github.com/adhvik24/AI1103−
PROBABILITY−AND−RANDOM−
VARIABLES/tree/main/ASSIGNMENT%203/
AI1103_Assignment3.tex
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1 Gate XE-A-2017-QN 2

Three fair dies are rolled simultaneously. The probability of getting a sum of 5 is

(A) $\frac{1}{108}$
(B) $\frac{1}{72}$
(C) $\frac{1}{54}$
(D) $\frac{1}{36}$

2 Solution

Let $X_i \in \{1, 2, 3, 4, 5, 6\}$, i = 1,2,3, be the random variables representing the outcome for each die. As the dies are fair, the probability mass function (pmf) is expressed as

$$P_{X_i}(n) = \Pr(X_i = n) = \begin{cases} \frac{1}{6} & 1 \le n \le 6 \\ 0 & otherwise \end{cases} \tag{2.0.1}$$

Let X be a random variable denotes the desired outcome,

$$X = X_1 + X_2 + X_3 \tag{2.0.2}$$

$$\implies X \in \{3, 4, ..., 18\} \tag{2.0.3}$$

We also know that when two dies are rolled the probability mass function of a random variable Y is (Where Y denotes the sum of values appeared on each die $\implies Y = X_1 + X_2$)

$$P_Y(n) = \begin{cases} 0 & n < 1 \\ \frac{n-1}{36} & 2 \le n \le 7 \\ \frac{13-n}{36} & 7 < n \le 12 \\ 0 & n > 12 \end{cases} \tag{2.0.4}$$

$$P_Y(n) = \Pr(X_1 + X_2 = n) \tag{2.0.5}$$

We have to find $P_X(5) = \Pr(X_1 + X_2 + X_3 = 5)$

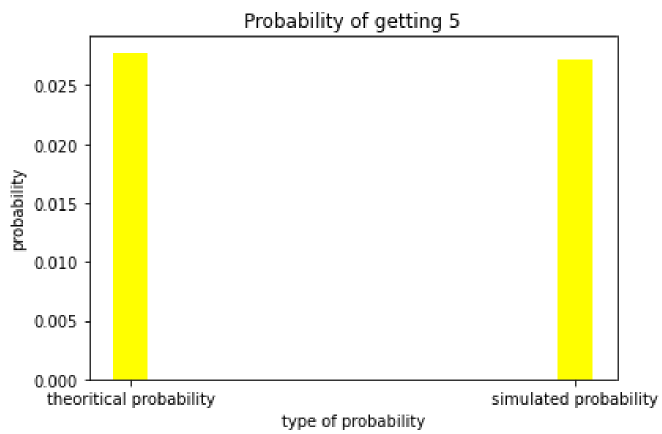

Fig. 1: Probability of getting sum of 5

$$\begin{aligned} P_X(5) &= \Pr(X_1 + X_2 + X_3 = 5) \\ &= \Pr(X_1 + X_2 = 5 - X_3) \\ &= \Sigma_{k=1}^{3} \Pr(X_1 + X_2 = 5 - k | X_3 = k) P_{X_3}(k) \end{aligned} \tag{2.0.6}$$

After unconditioning, As X_1, X_2, X_3 are independent,

$$\begin{aligned} &\Pr(X_1 + X_2 = 5 - k | X_3 = k) P_{X_3}(k) \\ &= \Pr(X_1 + X_2 = 5 - k) P_{X_3}(k) \end{aligned} \tag{2.0.7}$$

Using (2.0.7) and (2.0.1) in (2.0.6),

$$\begin{aligned} P_X(5) &= \Pr(X_1 + X_2 + X_3 = 5) \\ &= \Sigma_{k=1}^{3} \Pr(X_1 + X_2 = 5 - k | X_3 = k) P_{X_3}(k) \\ &= \frac{1}{6} \Sigma_{k=1}^{3} \Pr(X_1 + X_2 = 5 - k) \\ &= \frac{1}{6} \Sigma_{k=1}^{3} P_Y(5 - k) \end{aligned}$$

$$\therefore P_X(5) = \frac{1}{6}(P_Y(4) + P_Y(3) + P_Y(2)) \tag{2.0.8}$$

Using (2.0.4) in (2.0.8),

$$P_X(5) = \frac{1}{6}(\frac{3}{36} + \frac{2}{36} + \frac{1}{36}) \tag{2.0.9}$$

$$\therefore P_X(5) = \frac{1}{36} \tag{2.0.10}$$

Therefore the probability of getting a sum of 5 when three fair dies are rolled is $\frac{1}{36}$.