Assignment 8

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Download latex-tikz codes from

https://github.com/adhvik24/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/tree/main/ASSIGNMENT_8/AI1103_Assignment8.tex

1 CSIR UGC NET EXAM (June 2013), Q.71

Let X be a random variable with probability density function,

$$f(x) = \alpha(x-\mu)^{\alpha-1}e^{-(x-\mu)^{\alpha}} \tag{1.0.1}$$

such that $-\infty < \mu < \infty$; $\alpha > 0$; $x > \mu$, The hazard function is:

1) constant for all α
2) an increasing function for some α
3) independent of α
4) independent of μ when $\alpha = 1$

2 SOLUTION

Given PDF of X,

$$f(x) = \alpha(x-\mu)^{\alpha-1}e^{-(x-\mu)^{\alpha}} \tag{2.0.1}$$

Important property(using in (2.0.7) as $x > \mu$): Given $x - y > 0$ and $-\infty < y < \infty$, then

$$\lim_{x\to-\infty} x - y = 0 \tag{2.0.2}$$

CDF of X,

$$F(x) = \int_{-\infty}^{x} f(x)\,dx \tag{2.0.3}$$
$$= \int_{-\infty}^{x} \alpha(x-\mu)^{\alpha-1}e^{-(x-\mu)^{\alpha}}dx \tag{2.0.4}$$
$$= \int_{-\infty}^{x} e^{-(x-\mu)^{\alpha}}d(x-\mu)^{\alpha} \tag{2.0.5}$$
$$= \left[\frac{e^{-(x-\mu)^{\alpha}}}{-1}\right]_{-\infty}^{x} \tag{2.0.6}$$
$$= -e^{-(x-\mu)^{\alpha}} - \lim_{x\to-\infty}\frac{e^{-(x-\mu)^{\alpha}}}{-1} \tag{2.0.7}$$
$$= -e^{-(x-\mu)^{\alpha}} + e^{-(0)^{\alpha}} \tag{2.0.8}$$
$$F(x) = 1 - e^{-(x-\mu)^{\alpha}} \tag{2.0.9}$$

Hazard function $\beta(x)$,(using (2.0.1) and (2.0.9))

$$\beta(x) = \frac{f(x)}{1-F(x)} \tag{2.0.10}$$
$$= \frac{\alpha(x-\mu)^{\alpha-1}e^{-(x-\mu)^{\alpha}}}{1-(1-e^{-(x-\mu)^{\alpha}})} \tag{2.0.11}$$
$$= \frac{\alpha(x-\mu)^{\alpha-1}e^{-(x-\mu)^{\alpha}}}{e^{-(x-\mu)^{\alpha}}} \tag{2.0.12}$$
$$\beta(x) = \alpha(x-\mu)^{\alpha-1} \tag{2.0.13}$$

1) $\beta(x)$ is not constant for all α
2) $\beta(x) = \alpha(x-\mu)^{\alpha-1}$ is an increasing function for $\alpha < 0 \; or \; \alpha > 1$ as given $x - \mu > 0$ for all x.
   Proof: Using first derivative test, A function is increasing iff its first derivative is positive for all x.
   $$\frac{d}{dx}\beta(x) = \frac{d}{dx}\alpha(x-\mu)^{\alpha-1} \tag{2.0.14}$$
   $$= \alpha(\alpha-1)(x-\mu)^{\alpha-2} \tag{2.0.15}$$
   For (2.0.15) to be positive, (As given $x-\mu > 0$)
   $$\alpha(\alpha-1)(x-\mu)^{\alpha-2} > 0 \tag{2.0.16}$$
   $$\alpha(\alpha-1) > 0 \tag{2.0.17}$$
   $$\implies \alpha \in (-\infty, 0) \cup (1, \infty) \tag{2.0.18}$$
   $\therefore \beta(x)$ an increasing function for some α
3) $\beta(x)$ is dependent of α
4) when $\alpha = 1$,
   $$\beta(x) = \alpha(x-\mu)^{0} = \alpha \tag{2.0.19}$$
   Therefore the hazard function is independent of μ when $\alpha = 1$.

ANSWER: (2) and (4)