## Assignment 7

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Download latex-tikz codes from

https://github.com/adhvik24/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/blob/main/ASSIGNMENT\_7/ AI1103\_Assignment7.tex

## 1 GATE 2021 (ME-SET1), Q.20 (ME section)

Robot Ltd. wishes to maintain enough safety stock during the lead time period between starting a new production run and its completion such that the probability of satisfying the customer demand during the lead time period is 95%. The lead time periods is 5 days and daily customer demand can be assumed to follow the Gaussian (normal) distribution with mean 50 units and a standard deviation of 10 units. Using  $\phi^{-1}(0.95) = 1.64$ , where  $\phi$  represents the cumulative distribution function of the standard normal random variable, the amount of safety stock that must be maintained by Robot Ltd. to achieve this demand fulfillment probability for the lead time period is \_\_\_\_\_ units (round off to two decimal places).

## 2 SOLUTION

Let X be the normal R.V denoting the required amount of stock of customer demand over the lead time.

Let  $X_1$  be the normal R.V denoting daily customer demand with mean and standard deviation as follows,

parameter	value(in units)
Mean of $X_1(\mu)$	50
Standard deviation of $X_1$ ( $\sigma$ )	10

TABLE 0: mean and standard deviation of  $X_1$ 

Probability of satisfying customer demand is 0.95. Let Z be a standard normal R.V such that,

$$Z = \frac{X_1 - \mu}{\sigma} \tag{2.0.1}$$

Referring table(0) to use in (2.0.1),

$$Z = \frac{X_1 - 50}{10} \tag{2.0.2}$$

Given that,

$$\phi^{-1}(0.95) = 1.64 \tag{2.0.3}$$

$$\implies \phi(1.64) = 0.95$$
 (2.0.4)

$$\phi(1.64) = \Pr(Z \le 1.64) = 0.95$$
 (2.0.5)

$$\implies Z \le 1.64 \iff \frac{X_1 - 50}{10} \le 1.64$$
 (2.0.6)

$$\implies X_1 - 50 \le 1.64(10) \tag{2.0.7}$$

$$X_1 \le 66.4$$
 (2.0.8)

The demand in one day is independent of demand in the other day and the lead time is 5 days.

$$\implies X = 5(X_1) = 5(66.4) = 332$$
 (2.0.9)

Therefore the amount of safety stock that must be maintained by Robot Ltd. to achieve this demand fulfillment probability for the lead time period is 332 units.