

Assignment 2

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Download all python codes from

https://github.com/adhvik24/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/tree/main/ASSIGNMENT_2/codes

and latex-tikz codes from

https://github.com/adhvik24/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/blob/main/ASSIGNMENT_2/AI1103_Assignment2.tex

1 GATE PROBLEM No. 15

A random variable X has probability density function $f(x)$ as given below:

$$f(x) = \begin{cases} a + bx & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \quad (1.0.1)$$

If the expected value $E(X) = \frac{2}{3}$, then $\Pr(X < 0.5)$ is.....

2 SOLUTION

We know that the total probability is one,

$$\int_{-\infty}^{\infty} f(x) dx = 1 \quad (2.0.1)$$

Using (1.0.1) in (2.0.1),

$$\int_0^1 (a + bx) dx = 1 \quad (2.0.2)$$

$$\left[ax + \frac{bx^2}{2} \right]_0^1 = 1 \quad (2.0.3)$$

$$\left(a + \frac{b}{2} \right) - 0 = 1 \quad (2.0.4)$$

$$\Rightarrow a + \frac{b}{2} = 1 \quad (2.0.5)$$

We know that expectation value of X ,

$$E(X) = \int_{-\infty}^{\infty} xf(x) dx \quad (2.0.6)$$

Using $E(X) = \frac{2}{3}$ and (1.0.1) in (2.0.6), we get

$$\frac{2}{3} = \int_0^1 x(a + bx) dx \quad (2.0.7)$$

$$= \int_0^1 ax + bx^2 dx \quad (2.0.8)$$

$$= \left[\frac{ax^2}{2} + \frac{bx^3}{3} \right]_0^1 \quad (2.0.9)$$

$$= \frac{a}{2} + \frac{b}{3} - 0 \quad (2.0.10)$$

$$\Rightarrow \frac{a}{2} + \frac{b}{3} = \frac{2}{3} \quad (2.0.11)$$

By solving (2.0.5) and (2.0.11), we get

$$a = 0 \text{ and } b = 2. \quad (2.0.12)$$

Using values of a and b in (1.0.1), we get

$$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.13)$$

The graph of PDF of X is 1

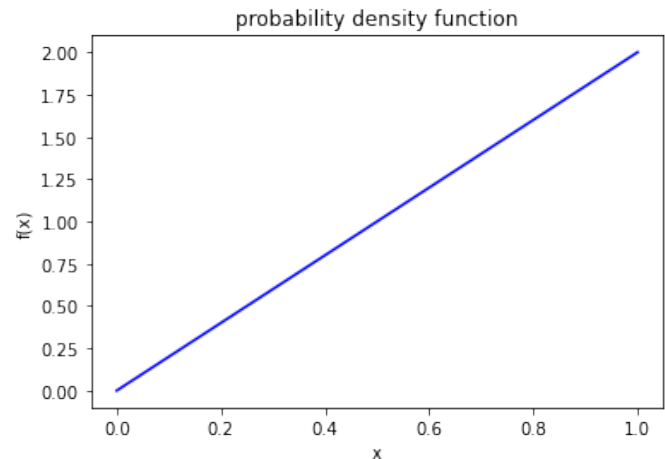


Fig. 1: Probability Density Function (PDF) of X

Let $F_X(x)$ be the cumulative distribution function of random variable X .

$$F_X(x) = \int_{-\infty}^x f(x) dx \quad (2.0.14)$$

$F_X(x)$ can be obtained from the uniform distribution of a random variable U on $(0,1)$ and let $U=X^2$.

$$0 < U < 1 \quad (2.0.15)$$

As for random variable X also,

$$0 < F_X(x) < 1 \quad (2.0.16)$$

This similarity between U and $F_X(x)$ is used to generate the random variable X from U .

$$F_X(x) = \Pr(X < x) \quad (2.0.17)$$

$$= \Pr(\sqrt{U} < x) \quad (2.0.18)$$

$$= \Pr(U < x^2) \quad (2.0.19)$$

$$= F_U(x^2) \quad (2.0.20)$$

From uniform distribution,

The graph of Probability Density Function (PDF) of U is 2

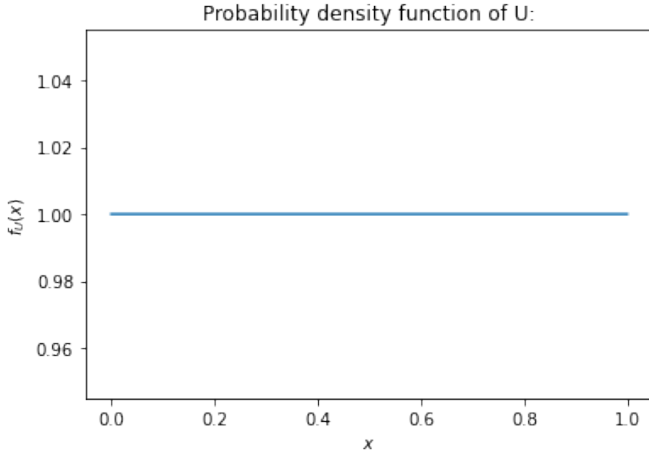


Fig. 2: Probability Density Function (PDF) of U

$$F_U(x) = \begin{cases} 0 & x \leq 0 \\ x & 0 < x < 1 \\ 1 & x \geq 1 \end{cases} \quad (2.0.21)$$

Using (2.0.21) in (2.0.20),

Cumulative distribution function (CDF) of random variable X is,

$$F_X(x) = \Pr(X < x) = \begin{cases} 0 & x \leq 0 \\ x^2 & 0 < x < 1 \\ 1 & x \geq 1 \end{cases} \quad (2.0.22)$$

The graph of Cumulative distribution function (CDF) of random variable X is 3

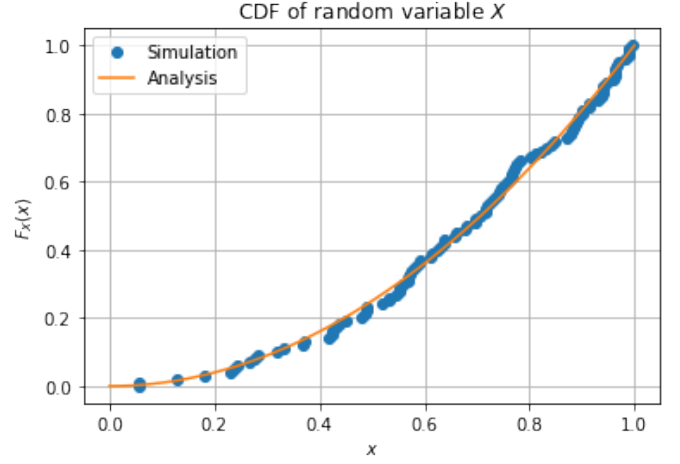


Fig. 3: Cumulative Density Function (CDF)

Now we have to find $\Pr(X < 0.5)$, Using (2.0.22),

$$\Pr(X < 0.5) = (0.5)^2 \quad (2.0.23)$$

$$\Rightarrow \Pr(X < 0.5) = 0.25 \quad (2.0.24)$$