

Assignment 6

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Download all python codes from

https://github.com/adhvik24/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/tree/main/ASSIGNMENT_6/codes

and latex-tikz codes from

https://github.com/adhvik24/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/blob/main/ASSIGNMENT_6/AI1103_Assignment6.tex

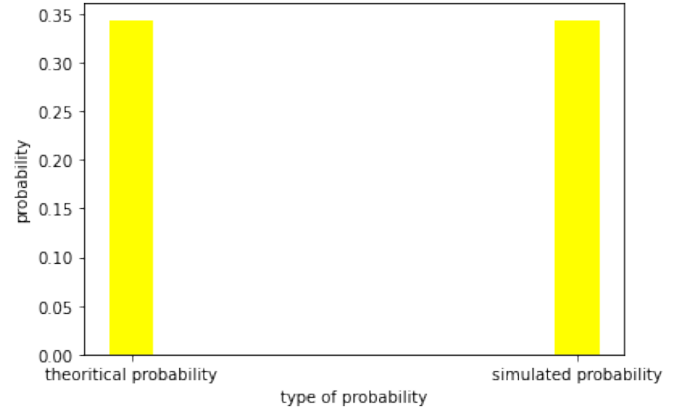


Fig. 1: probability that he/she will be caught during at least one of the trips

1 CSIR UGC NET EXAM (DEC 2015), Q.3

The probability that a ticketless traveler is caught during a trip is 0.1. If the traveler makes 4 trips, the probability that he/she will be caught during at least one of the trips is:

- (A) $1 - (0.9)^4$ (B) $(1 - 0.9)^4$
(C) $1 - (1 - 0.9)^4$ (D) $(0.9)^4$

2 SOLUTION

Let $X_i \in \{0, 1\}$ represent in the i th trip ticketless traveller is caught or not. (1 if caught and 0 if he is safe).

Given,

$$\Pr(X_i = 1) = p = 0.1 \quad (2.0.1)$$

Let,

$$X = \sum_{i=1}^n X_i \quad (2.0.2)$$

where n is the number of trips and X has a binomial distribution.

$$p_X(k) = \Pr(X = k) \quad (2.0.3)$$

$$p_X(k) = \begin{cases} {}^nC_k p^k (1-p)^{n-k}, & 0 \leq k \leq n \\ 0, & \text{otherwise} \end{cases} \quad (2.0.4)$$

Then probability of being caught in atleast one trip is,

$$\Pr(X \geq 1) = 1 - \Pr(X < 1) \quad (2.0.5)$$

$$= 1 - \Pr(X = 0) \quad (2.0.6)$$

As he/she makes 4 trips in total, Using (2.0.1) and (2.0.4),

$$\Pr(X = 0) = p_X(0) \quad (2.0.7)$$

$$= {}^4C_0 p^0 (1-p)^4 \quad (2.0.8)$$

$$\Pr(X = 0) = (0.9)^4 \quad (2.0.9)$$

Using (2.0.9) in (2.0.6),

$$\Pr(X \geq 1) = 1 - \Pr(X < 1) \quad (2.0.10)$$

$$= 1 - \Pr(X = 0) \quad (2.0.11)$$

$$= 1 - (0.9)^4 \quad (2.0.12)$$

Therefore the probability that he/she will be caught during at least one of the trips is $1 - (0.9)^4$.

ANSWER : (A)