

Assignment 2

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Download all python codes from

<https://github.com/adhvik24/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/tree/main/ASSIGNMENT%202/codes/assign2.py>

and latex-tikz codes from

https://github.com/adhvik24/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/tree/main/ASSIGNMENT%202/AI1103_Assignment2.tex

1 GATE PROBLEM No. 15

A random variable X has probability density function $f(x)$ as given below:

$$f(x) = \begin{cases} a + bx & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \quad (1.0.1)$$

If the expected value $E(X) = \frac{2}{3}$, then $\Pr(X < 0.5)$ is.....

2 SOLUTION

We know that the total probability is one,

$$\int_{-\infty}^{\infty} f(x) dx = 1 \quad (2.0.1)$$

Using (1.0.1) in (2.0.1),

$$\int_0^1 (a + bx) dx = 1 \quad (2.0.2)$$

$$\left[ax + \frac{bx^2}{2} \right]_0^1 = 1 \quad (2.0.3)$$

$$\left(a + \frac{b}{2} \right) - 0 = 1 \quad (2.0.4)$$

$$\Rightarrow a + \frac{b}{2} = 1 \quad (2.0.5)$$

We know that expectation value of X ,

$$E(X) = \int_{-\infty}^{\infty} xf(x) dx \quad (2.0.6)$$

Using $E(X) = \frac{2}{3}$ and (1.0.1) in (2.0.6), we get

$$\frac{2}{3} = \int_0^1 x(a + bx) dx \quad (2.0.7)$$

$$= \int_0^1 ax + bx^2 dx \quad (2.0.8)$$

$$= \left[\frac{ax^2}{2} + \frac{bx^3}{3} \right]_0^1 \quad (2.0.9)$$

$$= \frac{a}{2} + \frac{b}{3} - 0 \quad (2.0.10)$$

$$\Rightarrow \frac{a}{2} + \frac{b}{3} = \frac{2}{3} \quad (2.0.11)$$

By solving (2.0.5) and (2.0.11), we get

$$a = 0 \text{ and } b = 2.$$

Using values of a and b in (1.0.1), we get

$$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.12)$$

$$(2.0.13)$$

Now we have to find $\Pr(X < 0.5)$,

$$\Pr(X < 0.5) = \int_{-\infty}^{0.5} f(x) dx \quad (2.0.14)$$

Using (2.0.12) in (2.0.14)

$$\Pr(X < 0.5) = \int_0^{0.5} 2x dx \quad (2.0.15)$$

$$= \left[x^2 \right]_0^{0.5} \quad (2.0.16)$$

$$= (0.5)^2 - 0 \quad (2.0.17)$$

$$\Rightarrow \Pr(X < 0.5) = 0.25 \quad (2.0.18)$$