Assignment 2

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Download all python codes from

```
https://github.com/adhvik24/AI1103−
    PROBABILITY−AND−RANDOM−
    VARIABLES/tree/main/ASSIGNMENT%202/
    codes/assign2.py
```

and latex-tikz codes from

```
https://github.com/adhvik24/AI1103−
    PROBABILITY−AND−RANDOM−
    VARIABLES/tree/main/ASSIGNMENT%202/
    AI1103_Assignment2.tex
```

1 Gate Problem No. 15

A random variable X has probability density function $f(x)$ as given below:

$$f(x) = \begin{cases} a + bx & 0 < x < 1 \\ 0 & otherwise \end{cases} \tag{1.0.1}$$

If the expected value $E(X) = \frac{2}{3}$, then $\Pr(X < 0.5)$ is..........

2 Solution

We know that the total probability is one,

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \tag{2.0.1}$$

Using (1.0.1) in (2.0.1),

$$\int_0^1 (a + bx)\,dx = 1 \tag{2.0.2}$$

$$\left[ax + \frac{bx^2}{2}\right]_0^1 = 1 \tag{2.0.3}$$

$$\left(a + \frac{b}{2}\right) - 0 = 1 \tag{2.0.4}$$

$$\implies a + \frac{b}{2} = 1 \tag{2.0.5}$$

We know that expectation value of X,

$$E(X) = \int_{-\infty}^{\infty} xf(x)\,dx \tag{2.0.6}$$

Using $E(X) = \frac{2}{3}$ and (1.0.1) in (2.0.6), we get

$$\frac{2}{3} = \int_0^1 x(a + bx)\,dx \tag{2.0.7}$$

$$= \int_0^1 ax + bx^2\,dx \tag{2.0.8}$$

$$= \left[\frac{ax^2}{2} + \frac{bx^3}{3}\right]_0^1 \tag{2.0.9}$$

$$= \frac{a}{2} + \frac{b}{3} - 0 \tag{2.0.10}$$

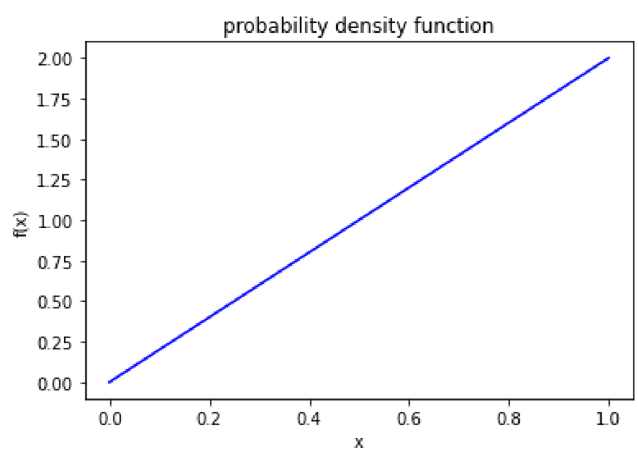
$$\implies \frac{a}{2} + \frac{b}{3} = \frac{2}{3} \tag{2.0.11}$$

By solving (2.0.5) and (2.0.11), we get

$$a = 0 \text{ and } b = 2.$$

Using values of a and b in (1.0.1), we get

$$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & otherwise \end{cases} \tag{2.0.12}$$



Let $F_X(x)$ be the cumulative distribution function of random variable X.

$$F_X(x) = \int_{-\infty}^{x} f(x)\,dx \tag{2.0.13}$$

$F_X(x)$ can be obtained from the uniform distribution of a random variable U,
Such that $0 < U < 1$ and U=X^2.
As for random variable X also $\implies 0 < F_X(x) < 1$ for $0 < x < 1$.
This similarity between U and $F_X(x)$ is used to generate the random variable X from U.

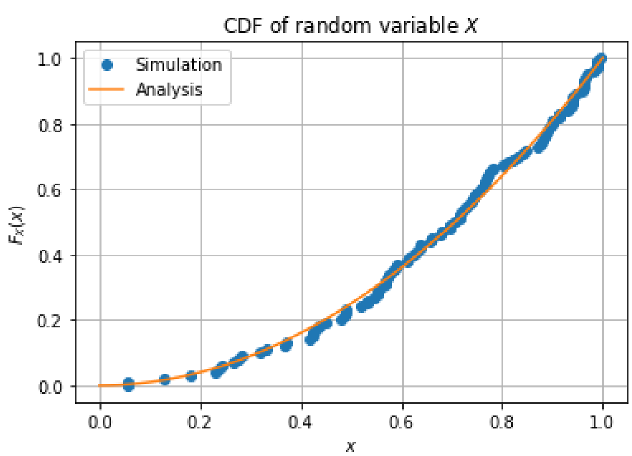
$$F_X(x) = \Pr(X < x) \tag{2.0.14}$$
$$= \Pr\left(\sqrt{U} < x\right) \tag{2.0.15}$$
$$= \Pr\left(U < x^2\right) \tag{2.0.16}$$
$$= F_U(x^2) \tag{2.0.17}$$

From uniform distribution,

$$F_U(x) = \begin{cases} 0 & x \leq 0 \\ x & 0 < x < 1 \\ 1 & x \geq 1 \end{cases} \tag{2.0.18}$$

Using (2.0.18) in (2.0.17),
Cumulative distribution function (CDF) of random variable X is,

$$F_X(x) = \Pr(X < x) = \begin{cases} 0 & x \leq 0 \\ x^2 & 0 < x < 1 \\ 1 & x \geq 1 \end{cases} \tag{2.0.19}$$



Now we have to find $\Pr(X < 0.5)$,
Using (2.0.19),

$$\Pr(X < 0.5) = (0.5)^2 \tag{2.0.20}$$
$$\implies \Pr(X < 0.5) = 0.25 \tag{2.0.21}$$