Assignment 4

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Download all python codes from

https://github.com/adhvik24/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/tree/main/ASSIGNMENT_4/codes

and latex-tikz codes from

https://github.com/adhvik24/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/tree/main/ASSIGNMENT_4/AI1103_Assignment4.tex

1 GATE 2016 (XE-A),Q.8 (Engg. Maths section)

A diagnostic test for a certain disease is 90% accurate. That is, the probability of a person having (respectively, not having) the disease tested positive (respectively, negative) is 0.9. Fifty percent of the population has the disease. What is the probability that a randomly chosen person has the disease given that the person tested negative?

2 SOLUTION

Let X and Y be two Bernoulli random variables such that X,Y$\in\{0,1\}$ and as given fifty percent of the population has the disease, the probability mass function of X is

$$p_X(n) = \Pr(X=n) = \begin{cases} 0.5 & n=1 \\ 0.5 & n=0 \\ 0 & otherwise \end{cases} \tag{2.0.1}$$

where X denotes the health status of a person(X=1 if person is healthy and X=0 if person is diseased) and Y denotes the diagnostic test result (Y=1 if it is positive and Y=0 if it is negative).
Given the probabilities of,

$$\Pr(Y=1|X=0) = 0.9 \tag{2.0.2}$$

$$\Pr(Y=0|X=1) = 0.9 \tag{2.0.3}$$

we need to find $\Pr(X=0|Y=0)$,

$$\Pr(X=0|Y=0) = \frac{\Pr(X=0 \cap Y=0)}{\Pr(Y=0)} \tag{2.0.4}$$

$$\Pr(X=0|Y=0) = \frac{\Pr(Y=0|X=0)\Pr(X=0)}{\Pr(Y=0)} \tag{2.0.5}$$

$$\Pr(Y=0) = \Pr(Y=0|X=1)\Pr(X=1) + \Pr(Y=0|X=0)\Pr(X=0) \tag{2.0.6}$$

Using (2.0.1),(2.0.2) and (2.0.3) in (2.0.6),

$$\Pr(Y=0) = 0.9(0.5) + (1-0.9)0.5$$
$$\Pr(Y=0) = 0.5 \tag{2.0.7}$$

Using (2.0.1),(2.0.2) and (2.0.7) in (2.0.5)

$$\Pr(X=0|Y=0) = \frac{(1-0.9)0.5}{0.5} \tag{2.0.8}$$

$$\Pr(X=0|Y=0) = 0.1 \tag{2.0.9}$$

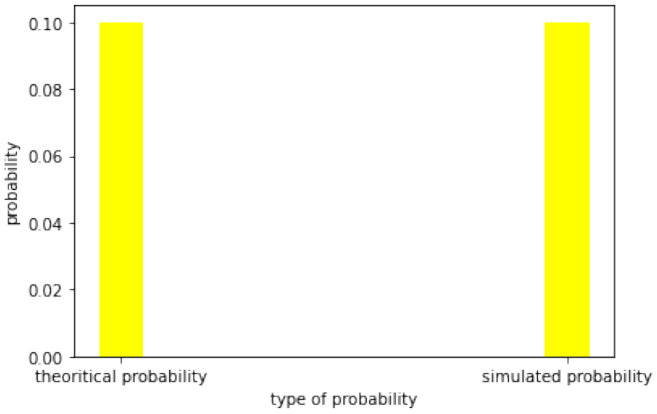

Fig. 1: probability that a randomly chosen person has the disease given that the person tested negative

Therefore the probability that a randomly chosen person has the disease given that the person tested negative is 0.1.