# Challenging question 5

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Download latex-tikz codes from

https://github.com/adhvik24/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/blob/main/ch_pr_5/ch_pr_5.tex

## 1 Challenging question 5

Suppose $X_1, X_2, X_3$ and $X_4$ are independent and identically distributed random variables, having density function f. Then,

1) $\Pr(X_4 > Max(X_1, X_2) > X_3) = \frac{1}{6}$
2) $\Pr(X_4 > Max(X_1, X_2) > X_3) = \frac{1}{8}$
3) $\Pr(X_4 > X_3 > Max(X_1, X_2)) = \frac{1}{12}$
4) $\Pr(X_4 > X_3 > Max(X_1, X_2)) = \frac{1}{6}$

## 2 SOLUTION

Total number of possible arrangements of these four R.V's in an order = 4!=24 ways.
As given $X_1, X_2, X_3$ and $X_4$ are independent and identically distributed random variables they all are equally likely for a position.
Required orders are:

1) $X_4 > Max(X_1, X_2) > X_3$, for this possible orders are:
   a) $X_4 > X_1 > X_2 > X_3$
   b) $X_4 > X_1 > X_3 > X_2$
   c) $X_4 > X_2 > X_1 > X_3$
   d) $X_4 > X_2 > X_3 > X_1$

   Therefore four possibilities. So,

$$\Pr(X_4 > Max(X_1, X_2) > X_3) = \frac{4}{24} \quad (2.0.1)$$
$$= \frac{1}{6} \quad (2.0.2)$$

2) $X_4 > X_3 > Max(X_1, X_2)$, for this possible orders are:
   a) $X_4 > X_3 > X_1 > X_2$
   b) $X_4 > X_3 > X_2 > X_1$

   Therefore two possibilities. So,

$$\Pr(X_4 > X_3 > Max(X_1, X_2)) = \frac{2}{24} \quad (2.0.3)$$
$$= \frac{1}{12} \quad (2.0.4)$$

Therefore correct answers are **(1) and (3)**.