Assignment 5

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Download latex-tikz codes from

https://github.com/adhvik24/AI1103−PROBABILITY−AND−RANDOM−VARIABLES/tree/main/ASSIGNMENT_5/AI1103_Assignment5.tex

1 GATE 2019 (ST) , Q.45 (statistics section)

Consider the trinomial distribution with the probability mass function

$$P(X=x,Y=y) = \left(\frac{7!}{x!y!(7-x-y)!}\right)(0.6)^x(0.2)^y(0.2)^{7-x-y}$$

where $x \geq 0, y \geq 0$ *and* $x+y \leq 7$. Then $E[Y|X=3]$ is equal to

2 SOLUTION

Probability mass function of a trinomial distribution is

$$P(X=x,Y=y) = \left(\frac{7!}{x!y!(7-x-y)!}\right)(0.6)^x(0.2)^y(0.2)^{7-x-y}$$

$$= \left(\frac{7!}{x!(7-x)!}\frac{(7-x)!}{y!(7-x-y)!}\right)(0.6)^x(0.2)^y(0.2)^{7-x-y}$$

$$P(X=x,Y=y) = {}^7C_x{}^{7-x}C_y(0.6)^x(0.2)^y(0.2)^{7-x-y} \tag{2.0.1}$$

Using (2.0.1), $P(X=x)$ is

$$P(X=x) = {}^7C_x(0.6)^x\,\Sigma_{y=0}^{7-x}\,{}^{7-x}C_y(0.2)^y(0.2)^{7-x-y}$$

$$= {}^7C_x(0.6)^x\,(0.4)^{7-x}$$

$$P(X=x) = {}^7C_x(0.6)^x\,(0.4)^{7-x} \tag{2.0.2}$$

We have to find $E[Y|X=3]$,

$$E[Y|X=3] = \Sigma_{y=0}^{4}\,yP(Y=y|X=3) \tag{2.0.3}$$

$$= \Sigma_{y=0}^{4}y\frac{P(Y=y\cap X=3)}{P(X=3)} \tag{2.0.4}$$

$$E[Y|X=3] = \Sigma_{y=0}^{4}y\frac{P(X=3,Y=y)}{P(X=3)} \tag{2.0.5}$$

Using (2.0.1) and (2.0.2) in (2.0.5),

$$E[Y|X=3] = \Sigma_{y=0}^{4}y\frac{P(X=3,Y=y)}{P(X=3)}$$

$$= \Sigma_{y=0}^{4}y\left(\frac{{}^7C_3{}^4C_y(0.6)^3(0.2)^y(0.2)^{4-y}}{{}^7C_3(0.6)^3\,(0.4)^4}\right)$$

$$= \Sigma_{y=0}^{4}y\left(\frac{{}^4C_y(0.2)^4}{(0.4)^4}\right)$$

$$= \Sigma_{y=0}^{4}\frac{y({}^4C_y)}{16} \tag{2.0.6}$$

$$E[Y|X=3] = \frac{1}{16}\Sigma_{y=0}^{4}\,y({}^4C_y) = 2 \tag{2.0.7}$$

Therefore the value of $E[Y|X=3]=2$.