

Assignment 5

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Download latex-tikz codes from

https://github.com/adhvik24/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/tree/main/ASSIGNMENT_5/AI1103_Assignment5.tex

1 GATE 2019 (ST) , Q.45 (STATISTICS SECTION)

Consider the trinomial distribution with the probability mass function

$$P(X = x, Y = y) = \left(\frac{7!}{x!y!(7-x-y)!} \right) (0.6)^x (0.2)^y (0.2)^{7-x-y}$$

where $x \geq 0, y \geq 0$ and $x + y \leq 7$. Then $E[Y|X = 3]$ is equal to

2 SOLUTION

Probability mass function of a trinomial distribution is

$$\begin{aligned} P(X = x, Y = y) &= \left(\frac{7!}{x!y!(7-x-y)!} \right) (0.6)^x (0.2)^y (0.2)^{7-x-y} \\ &= \left(\frac{7!}{x!(7-x)! y!(7-x-y)!} \right) (0.6)^x (0.2)^y (0.2)^{7-x-y} \\ P(X = x, Y = y) &= {}^7C_x {}^{7-x}C_y (0.6)^x (0.2)^y (0.2)^{7-x-y} \end{aligned} \quad (2.0.1)$$

Using (2.0.1), $P(X = x)$ is

$$\begin{aligned} P(X = x) &= \sum_{y=0}^{7-x} P(X = x, Y = y) \\ &= {}^7C_x (0.6)^x \sum_{y=0}^{7-x} {}^{7-x}C_y (0.2)^y (0.2)^{7-x-y} \\ &= {}^7C_x (0.6)^x (0.4)^{7-x} \\ P(X = x) &= {}^7C_x (0.6)^x (0.4)^{7-x} \end{aligned} \quad (2.0.2)$$

We have to find $E[Y|X = 3]$,

$$E[Y|X = 3] = \sum_{y=0}^4 y P(Y = y|X = 3) \quad (2.0.3)$$

$$E[Y|X = 3] = \sum_{y=0}^4 y \left(\frac{P(X = 3, Y = y)}{P(X = 3)} \right) \quad (2.0.4)$$

By taking $X=3$ in (2.0.1) and (2.0.2) to use in (2.0.4),

$$\begin{aligned} E[Y|X = 3] &= \sum_{y=0}^4 y \left(\frac{P(X = 3, Y = y)}{P(X = 3)} \right) \\ &= \sum_{y=0}^4 y \left(\frac{{}^7C_3 {}^4C_y (0.6)^3 (0.2)^y (0.2)^{4-y}}{{}^7C_3 (0.6)^3 (0.4)^4} \right) \\ &= \sum_{y=0}^4 y \left(\frac{{}^4C_y (0.2)^4}{(0.4)^4} \right) \\ &= \sum_{y=0}^4 \frac{y({}^4C_y)}{16} \end{aligned} \quad (2.0.5)$$

$$E[Y|X = 3] = \frac{1}{16} \sum_{y=0}^4 y({}^4C_y) = 2 \quad (2.0.6)$$

Therefore the value of $E[Y|X = 3] = 2$.