

Assignment 3

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Download all python codes from

<https://github.com/adhvik24/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/tree/main/ASSIGNMENT%203/codes>

and latex-tikz codes from

https://github.com/adhvik24/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/tree/main/ASSIGNMENT%203/AI1103_Assignment3.tex

1 GATE XE-A-2017-QN 2

Three fair dies are rolled simultaneously. The probability of getting a sum of 5 is

- (A) $\frac{1}{108}$
- (B) $\frac{1}{72}$
- (C) $\frac{1}{54}$
- (D) $\frac{1}{36}$

2 SOLUTION

Let $X_i \in \{1, 2, 3, 4, 5, 6\}$, $i = 1, 2, 3$, be the random variables representing the outcome for each die. As the dies are fair, the probability mass function (pmf) is expressed as

$$P_{X_i}(n) = \Pr(X_i = n) = \begin{cases} \frac{1}{6} & 1 \leq n \leq 6 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.1)$$

Let X be a random variable denotes the desired outcome,

$$X = X_1 + X_2 + X_3 \quad (2.0.2)$$

$$\Rightarrow X \in \{3, 4, \dots, 18\} \quad (2.0.3)$$

We also know that when two dies are rolled the probability mass function of a random variable Y is

(Where Y denotes the sum of values appeared on each die $\Rightarrow Y = X_1 + X_2$)

$$P_Y(n) = \begin{cases} 0 & n < 1 \\ \frac{n-1}{36} & 2 \leq n \leq 7 \\ \frac{13-n}{36} & 7 < n \leq 12 \\ 0 & n > 12 \end{cases} \quad (2.0.4)$$

$$P_Y(n) = \Pr(X_1 + X_2 = n) \quad (2.0.5)$$

We have to find $P_X(5) = \Pr(X_1 + X_2 + X_3 = 5)$

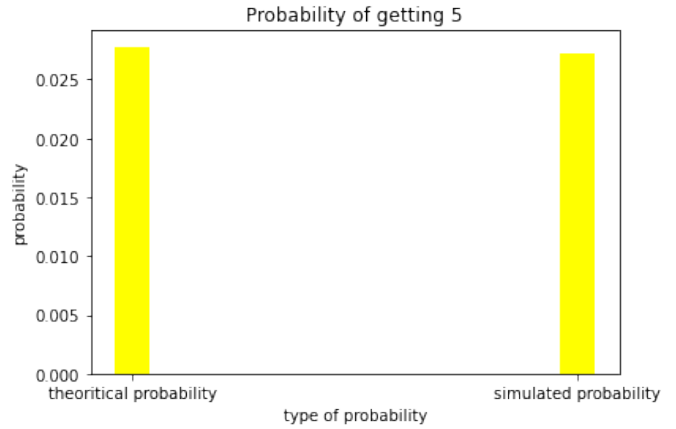


Fig. 1: Probability of getting sum of 5

$$\begin{aligned} P_X(5) &= \Pr(X_1 + X_2 + X_3 = 5) \\ &= \Pr(X_1 + X_2 = 5 - X_3) \\ &= \sum_{k=1}^3 \Pr(X_1 + X_2 = 5 - k | X_3 = k) P_{X_3}(k) \end{aligned} \quad (2.0.6)$$

After unconditioning, As X_1, X_2, X_3 are independent,

$$\begin{aligned} \Pr(X_1 + X_2 = 5 - k | X_3 = k) P_{X_3}(k) \\ = \Pr(X_1 + X_2 = 5 - k) P_{X_3}(k) \end{aligned} \quad (2.0.7)$$

Using (2.0.7) and (2.0.1) in (2.0.6),

$$\begin{aligned} P_X(5) &= \Pr(X_1 + X_2 + X_3 = 5) \\ &= \sum_{k=1}^3 \Pr(X_1 + X_2 = 5 - k | X_3 = k) P_{X_3}(k) \\ &= \frac{1}{6} \sum_{k=1}^3 \Pr(X_1 + X_2 = 5 - k) \\ &= \frac{1}{6} \sum_{k=1}^3 P_Y(5 - k) \end{aligned}$$

$$\therefore P_X(5) = \frac{1}{6}(P_Y(4) + P_Y(3) + P_Y(2)) \quad (2.0.8)$$

Using (2.0.4) in (2.0.8),

$$P_X(5) = \frac{1}{6}\left(\frac{3}{36} + \frac{2}{36} + \frac{1}{36}\right) \quad (2.0.9)$$

$$\therefore P_X(5) = \frac{1}{36} \quad (2.0.10)$$

Therefore the probability of getting a sum of 5 when three fair dies are rolled is $\frac{1}{36}$.

Ans: Option (D)