Assignment 7

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Download latex-tikz codes from

https://github.com/adhvik24/AI1103-PROBABILITY-AND-RANDOM-VARIABLES/blob/main/ASSIGNMENT_7/AI1103_Assignment7.tex

1 GATE 2021 (ME-SET1), Q.20 (ME section)

Robot Ltd. wishes to maintain enough safety stock during the lead time period between starting a new production run and its completion such that the probability of satisfying the customer demand during the lead time period is 95%. The lead time periods is 5 days and daily customer demand can be assumed to follow the Gaussian (normal) distribution with mean 50 units and a standard deviation of 10 units. Using $\phi^{-1}(0.95)$ =1.64 , where ϕ represents the cumulative distribution function of the standard normal random variable, the amount of safety stock that must be maintained by Robot Ltd. to achieve this demand fulfillment probability for the lead time period is _____ units (round off to two decimal places).

2 SOLUTION

Symbol	definition	value
X	customer demand in lead time	-
X_1	normal R.V denotes daily customer demand	-
μ	Mean of X_1	50
σ	Standard deviation of X_1	10
ϕ	CDF of standard normal R.V	-

TABLE 0: Variables and their definitions

Probability of satisfying customer demand is 0.95. Let Z be a standard normal R.V such that,

$$Z = \frac{X_1 - \mu}{\sigma} \tag{2.0.1}$$

Referring table(0) to use in (2.0.1),

$$Z = \frac{X_1 - 50}{10} \tag{2.0.2}$$

Given that,

$$\phi^{-1}(0.95) = 1.64 \tag{2.0.3}$$

$$\implies \phi(1.64) = 0.95 \tag{2.0.4}$$

$$\phi(1.64) = \Pr(Z \leq 1.64) = 0.95 \tag{2.0.5}$$

$$\implies Z \leq 1.64 \iff \frac{X_1 - 50}{10} \leq 1.64 \tag{2.0.6}$$

$$\implies X_1 - 50 \leq 1.64(10) \tag{2.0.7}$$

$$\therefore X_1 \leq 66.4 \tag{2.0.8}$$

The demand in one day is independent of demand in the other day and the lead time is 5 days.

$$\implies X = 5(X_1) = 5(66.4) = 332 \tag{2.0.9}$$

Therefore the amount of safety stock that must be maintained by Robot Ltd. to achieve this demand fulfillment probability for the lead time period is 332 units.