# Assignment 2

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Download all python codes from

https://github.com/adhvik24/AI1103_PROBABILITY_ASSIGN2/tree/main/codes

and latex-tikz codes from

https://github.com/adhvik24/AI1103_PROBABILITY_ASSIGN2/blob/main/AI1103_Assignment2.tex

## 1 Gate Problem No. 15

A random variable X has probability density function $f(x)$ as given below:

$$f(x) = \begin{cases} a + bx & 0 < x < 1 \\ 0 & otherwise \end{cases} \tag{1.0.1}$$

If the expected value $E(X) = \frac{2}{3}$ , then $\Pr(X < 0.5)$ is..........

## 2 Solution

We know that the total probability is one,

$$\int_{-\infty}^{\infty} f(x)\, dx = 1 \tag{2.0.1}$$

Using (1.0.1) in (2.0.1),

$$\int_{0}^{1} (a + bx)\, dx = 1 \tag{2.0.2}$$

$$\left[ax + \frac{bx^2}{2}\right]_0^1 = 1 \tag{2.0.3}$$

$$\left(a + \frac{b}{2}\right) - 0 = 1 \tag{2.0.4}$$

$$\implies a + \frac{b}{2} = 1 \tag{2.0.5}$$

We know that expectation value of X,

$$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx \tag{2.0.6}$$

Using $E(X) = \frac{2}{3}$ and (1.0.1) in (2.0.6), we get

$$\frac{2}{3} = \int_0^1 x(a + bx)\, dx \tag{2.0.7}$$

$$= \int_0^1 ax + bx^2\, dx \tag{2.0.8}$$

$$= \left[\frac{ax^2}{2} + \frac{bx^3}{3}\right]_0^1 \tag{2.0.9}$$

$$= \frac{a}{2} + \frac{b}{3} - 0 \tag{2.0.10}$$

$$\implies \frac{a}{2} + \frac{b}{3} = \frac{2}{3} \tag{2.0.11}$$

By solving (2.0.5) and (2.0.11), we get

$$a = 0 \text{ } and \text{ } b = 2. \tag{2.0.12}$$

Using values of $a$ and $b$ in (1.0.1), we get

$$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & otherwise \end{cases} \tag{2.0.13}$$

The graph of PDF of X is 1

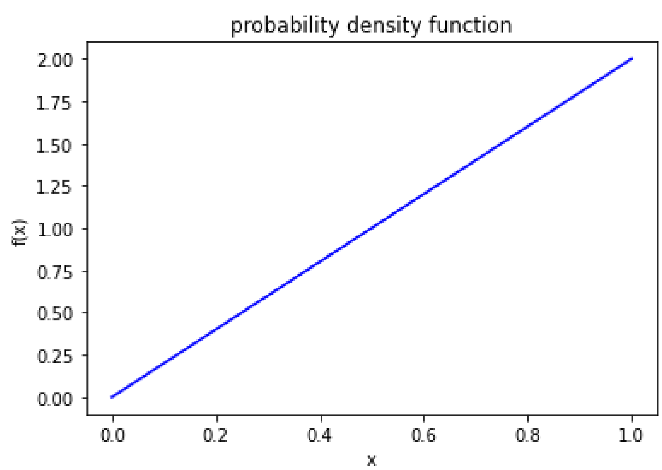


Fig. 1: Probability Density Function (PDF) of X

Let $F_X(x)$ be the cumulative distribution function of random variable X.

$$F_X(x) = \int_{-\infty}^{x} f(x)\, dx \tag{2.0.14}$$

$F_X(x)$ can be obtained from the uniform distribution of a random variable U on (0,1) and let U=$X^2$.

$$0 < U < 1 \tag{2.0.15}$$

As for random variable X also,

$$0 < F_X(x) < 1 \tag{2.0.16}$$

This similarity between U and $F_X(x)$ is used to generate the random variable X from U.

$$\begin{aligned} F_X(x) &= \Pr(X < x) & (2.0.17)\\ &= \Pr\left(\sqrt{U} < x\right) & (2.0.18)\\ &= \Pr\left(U < x^2\right) & (2.0.19)\\ &= F_U(x^2) & (2.0.20)\end{aligned}$$

From uniform distribution,
The graph of Probability Density Function (PDF) of U is 2

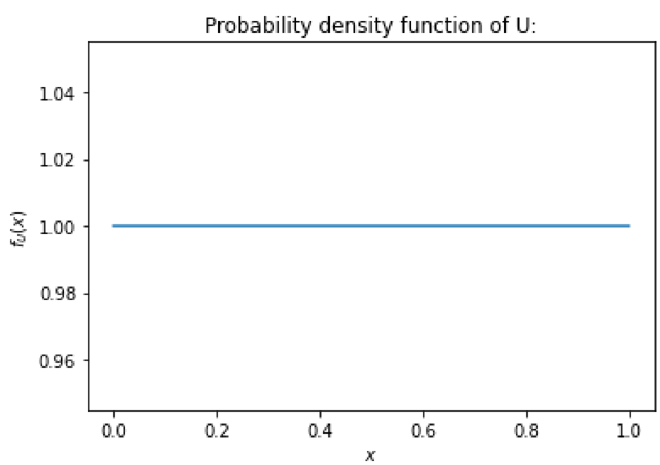


Fig. 2: Probability Density Function (PDF) of U

$$F_U(x) = \begin{cases} 0 & x \le 0 \\ x & 0 < x < 1 \\ 1 & x \ge 1 \end{cases} \tag{2.0.21}$$

Using (2.0.21) in (2.0.20),
Cumulative distribution function (CDF) of random variable X is,

$$F_X(x) = \Pr(X < x) = \begin{cases} 0 & x \le 0 \\ x^2 & 0 < x < 1 \\ 1 & x \ge 1 \end{cases} \tag{2.0.22}$$

The graph of Cumulative distribution function (CDF) of random variable X is 3

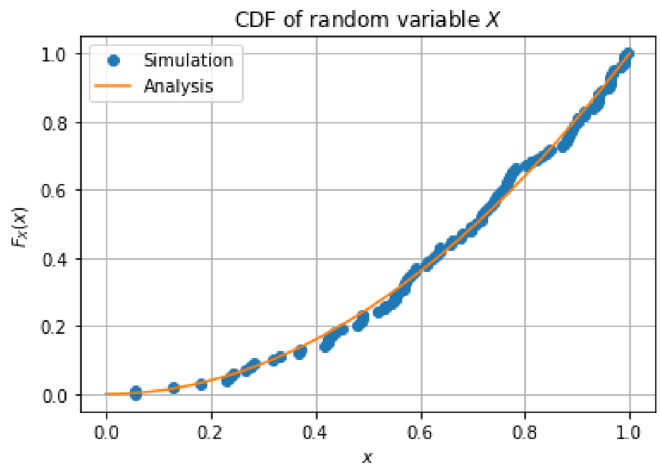


Fig. 3: Cumulative Density Function (CDF)

Now we have to find $\Pr(X < 0.5)$,Using (2.0.22),

$$\Pr(X < 0.5) = (0.5)^2 \tag{2.0.23}$$

$$\implies \Pr(X < 0.5) = 0.25 \tag{2.0.24}$$