

EE3900 Assignment - 5

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Download latex-tikz codes from

https://github.com/adhvik24/EE3900/blob/main/Assignment_5/Assignment5.tex

Download python codes from

https://github.com/adhvik24/EE3900/blob/main/Assignment_5/codes/a_5.py

1 QUADRATIC FORMS/Q 2.16

Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$ and verify the relationship between the zeroes and the coefficients.

2 SOLUTION

Lemma 2.1. A general polynomial equation $p(x, y)$ of degree 2 is given by :

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \quad (2.0.1)$$

The vector equation of $p(x, y)$ is given by :

$$\mathbf{x}^T \begin{pmatrix} A & \frac{B}{2} \\ \frac{B}{2} & C \end{pmatrix} \mathbf{x} + (D \ E) \mathbf{x} + F = 0 \quad (2.0.2)$$

And for a quadratic polynomial we have :

$$B = 0 \quad (2.0.3)$$

$$C = 0 \quad (2.0.4)$$

$$E = 0 \quad (2.0.5)$$

If we take $A = 1$, we have :

$$\text{Sum of zeroes} = -D \quad (2.0.6)$$

$$\text{Product of zeroes} = F \quad (2.0.7)$$

The given equation can be written as,

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + (7 \ 0) \mathbf{x} + 10 = 0 \quad (2.0.8)$$

where,

$$\mathbf{x} = \begin{pmatrix} x \\ 0 \end{pmatrix} \quad (2.0.9)$$

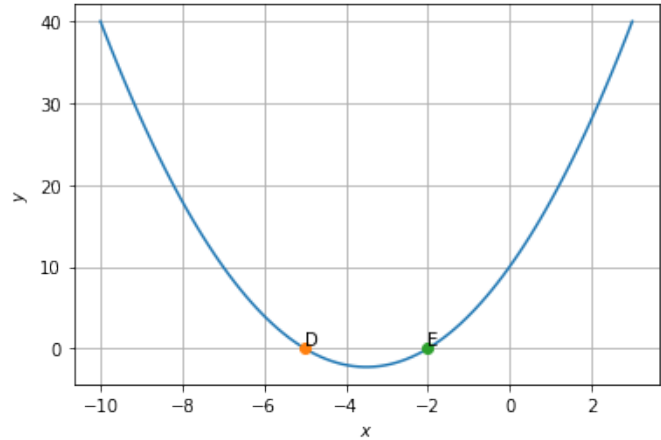


Fig. 1: Quadratic polynomial $x^2 + 7x + 10$

Substituting (2.0.9) in (2.0.8),

$$\begin{pmatrix} x \\ 0 \end{pmatrix}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} + (7 \ 0) \begin{pmatrix} x \\ 0 \end{pmatrix} + 10 = 0 \quad (2.0.10)$$

$$\Rightarrow x^2 + 7x + 10 = 0 \quad (2.0.11)$$

$$\Rightarrow \left(x + \frac{7}{2}\right)^2 = \frac{9}{4} \quad (2.0.12)$$

$$\Rightarrow \left(x + \frac{7}{2}\right) = \pm \frac{3}{2} \quad (2.0.13)$$

$$\Rightarrow x = -2, -5 \quad (2.0.14)$$

Verifying the relationship between the zeroes and coefficients. By comparing (2.0.8) with (2.0.2),

$$\Rightarrow \text{sum of the zeroes} = -7 = -D \quad (2.0.15)$$

$$\text{product of zeroes} = 10 = F. \quad (2.0.16)$$

\therefore The zeroes of equation $x^2 + 7x + 10$ are -2, -5.