

EE3900 Gate Assignment - 3

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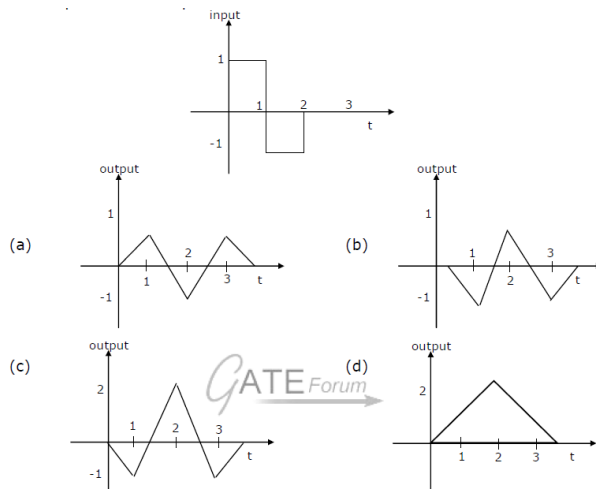
https://github.com/adhvik24/EE3900/blob/main/Gate_A3/main.tex

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1 EC 2005/Q.71

A signal as shown in figure is applied to a matched filter. Which of the following does represent the output of this matched filter ?



2 SOLUTION

Lemma 2.1. The unit step signal, $u(t)$, is given by:

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.1)$$

On time-shifting $u(t)$ by T , we get:

$$u(t - T) = \begin{cases} 1 & t \geq T \\ 0 & \text{otherwise} \end{cases} \quad (2.0.2)$$

Lemma 2.2. Convolution of $u(t-a)$ and $u(t-b)$ is, (assume $a < b$)

$$u(t-a) * u(t-b) = \int_{-\infty}^{\infty} u(\tau-a)u(t-\tau-b)d\tau$$

$$u(t-a) * u(t-b) = \begin{cases} t-b-a & t \geq a+b \\ 0 & \text{otherwise} \end{cases}$$

For an input $x(t)$ the matched filter's impulse response $h(t)$ is,

$$h(t) = x(T-t) \quad (2.0.3)$$

The input signal $x(t)$ is,

$$x(t) = u(t) - 2u(t-1) + u(t-2) \quad (2.0.4)$$

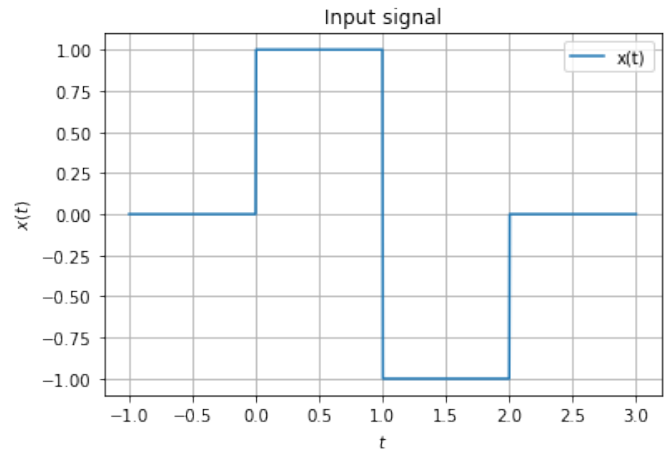


Fig. 1: input signal

Using (2.0.3), The impulse response for the matched filter of the input signal is $h(t)$,

$$h(t) = x(2-t) \quad (2.0.5)$$

$$\begin{aligned} \Rightarrow h(t) &= x(2-t) = u(2-t) - 2u(1-t) + u(-t) \\ \therefore h(t) &= -u(t) + 2u(t-1) - u(t-2) = -x(t) \end{aligned}$$

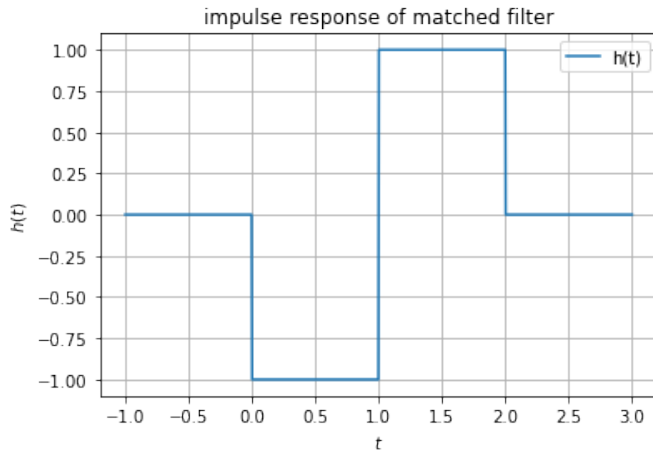


Fig. 2: impulse response of matched filter

Now, The corresponding output $y(t)$ is given by,

$$\begin{aligned}
 y(t) &= h(t) * x(t) \\
 &= -(u(t) - 2u(t-1) + u(t-2)) * \\
 &\quad (u(t) - 2u(t-1) + u(t-2)) \\
 &= -(u(t) * u(t) - 4u(t-1) * u(t) + 2u(t-2) * u(t)) - \\
 &\quad (4u(t-1) * u(t-1) - 4u(t-1) * u(t-2)) - \\
 &\quad (u(t-2) * u(t-2)) \quad (2.0.6)
 \end{aligned}$$

Using 2.2, We will get

$$y(t) = \begin{cases} 0 & t < 0 \\ -t & 0 \leq t < 1 \\ 3t - 4 & 1 \leq t < 2 \\ 8 - 3t & 2 \leq t < 3 \\ t - 4 & 3 \leq t < 4 \\ 0 & t \geq 4 \end{cases} \quad (2.0.7)$$

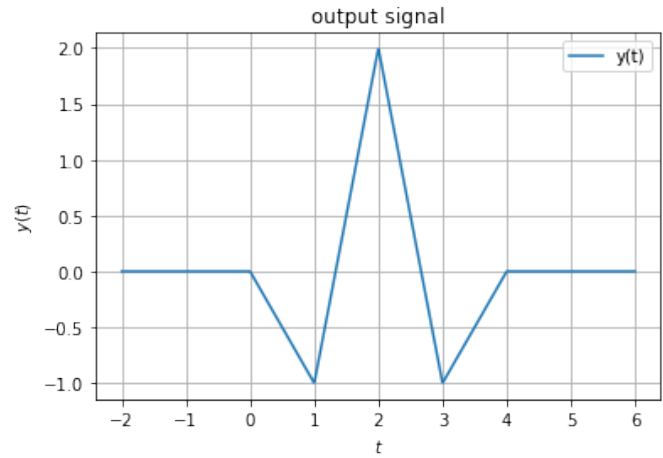


Fig. 3: output signal

Therefore, Option(C) is the correct option.