EE3900 Assignment - 2

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1 MATRIX QN 2.70

Examine the consistency of the system of given equations:

$$x + 2y = 2$$
$$2x + 3y = 3$$

2 SOLUTION

If solution exists for the given system of equations then they are said to be consistent, otherwise they are inconsistent.

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} \vec{X} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \qquad (2.0.1)$$

The augmented matrix for the above equation and row reducing as follows

$$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 3 & 3 \end{pmatrix} \xleftarrow{R_2 \to R_2 - 2R_1} \begin{pmatrix} 1 & 2 & 2 \\ 0 & -1 & -1 \end{pmatrix} \qquad (2.0.2)$$

$$\xleftarrow{R_1 \to R_1 + 2R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \end{pmatrix} \qquad (2.0.3)$$

$$\implies \text{Rank} \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} = \text{Rank} \begin{pmatrix} 1 & 2 & 2 \\ 2 & 3 & 3 \end{pmatrix} = 2 \qquad (2.0.4)$$

Here, $Rank(A) = Rank(A|B)$. Therefore, the system is consistent. Also, there exist a unique solution as $Rank(A) = n$ (number of unknown).
From equation (2.0.3), we get:

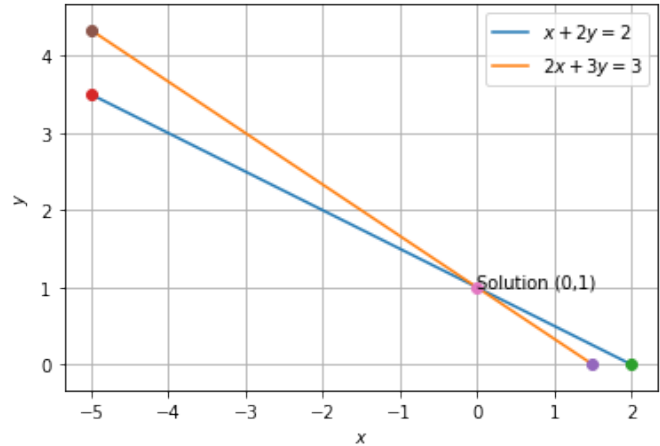$$\vec{X} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (2.0.5)$$

Fig. 1: Lines and their intersection denoting the solution

Plotting the lines and the intersection point in Fig.(1)
$\therefore$ The given system of equation is consistent with unique solution of,

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$