EE3900 Gate Assignment - 4

Adhvik Mani Sai Murarisetty - AI20BTECH11015

Download latex-tikz codes from

https://github.com/adhvik24/EE3900/blob/main/Gate_A4/main.tex

Download python codes from

https://github.com/adhvik24/EE3900/blob/main/Gate_A4/codes/plot.py

Problem

(GATE EC-1997 Qn 5.2) If the fourier transform of the deterministic signal $x(t)$ is $X(f)$, then

1) The fourier transform of $x(t-2)$ is
   a) $X(f)e^{-j4\pi f}$
   b) $X(2f)$
   c) $2X(2f)$
   d) $X(f-2)$
2) The fourier transform of $x\left(\frac{t}{2}\right)$ is
   a) $X(f)e^{-j4\pi f}$
   b) $X(2f)$
   c) $2X(2f)$
   d) $X(f-2)$

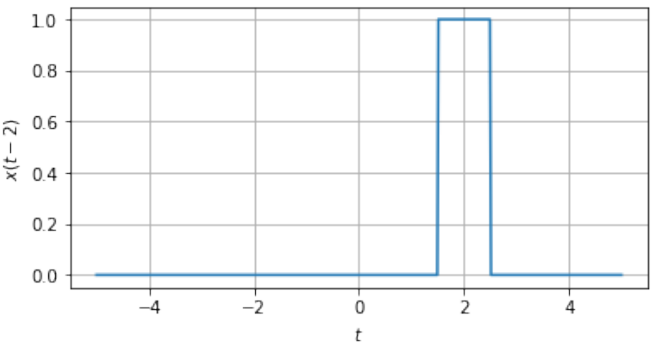

Fig. 1: Plot of rect(t-2)

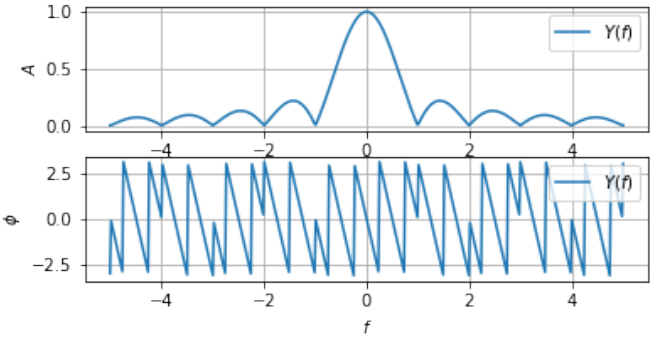

Fig. 2: Amplitude and phase v/s frequency plot

Solution

1) By the time shifting property of Fourier transform,

$$x(t-t_0) \overset{\mathcal{F}}{\rightleftharpoons} e^{-j2\pi f t_0} X(f) \tag{0.0.1}$$

$$x(t-2) \overset{\mathcal{F}}{\rightleftharpoons} e^{-j4\pi f} X(f) \tag{0.0.2}$$

Let $x(t) = \text{rect}(t)$ and $\text{sinc}(t) = \frac{\sin(t)}{t}$

$$X(f) = \text{sinc}(f) \tag{0.0.3}$$

For $x(t) = \text{rect}(t-2)$,

$$e^{-j4\pi f} X(f) = e^{-j4\pi f} \text{sinc}(f) \tag{0.0.4}$$

The correct option is (a).

2) By the time scaling property of Fourier transform,

$$x(\alpha t) \overset{\mathcal{F}}{\rightleftharpoons} \frac{1}{|\alpha|} X\left(\frac{f}{|\alpha|}\right) \tag{0.0.5}$$

$$x\left(\frac{t}{2}\right) \overset{\mathcal{F}}{\rightleftharpoons} 2X(2f) \tag{0.0.6}$$

Let $x(t) = \text{rect}(t)$ and $\text{sinc}(t) = \frac{\sin(t)}{t}$

$$X(f) = \text{sinc}(f) \tag{0.0.7}$$

For $x(t) = \text{rect}\left(\frac{t}{2}\right)$,

$$2X(2f) = 2\,\text{sinc}(2f) \tag{0.0.8}$$

The correct option is (c).

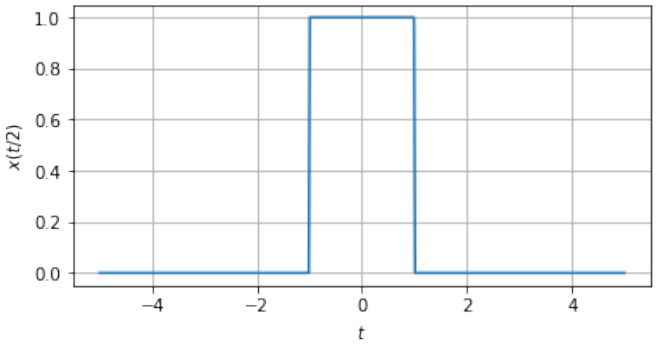

Fig. 3: Plot of rect$\left(\frac{t}{2}\right)$

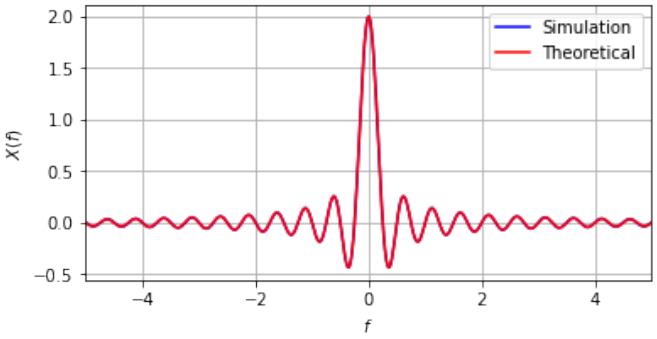

Fig. 4: Fourier transform Simulation v/s Theoretical