Quiz1

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Download latex-tikz codes from

https://github.com/adhvik24/EE3900/blob/main/quiz1/main.tex

Download python codes from

https://github.com/adhvik24/EE3900/blob/main/quiz1/plot.py

Problem 2.27(System B)

(2.27(System B)) Three systems A, B, and C have the inputs and outputs indicated in Figure P2.27 - 1. Determine whether each system could be LTI. If your answer is yes, specify whether there could be more than one LTI system with the given input-output pair. Explain your answer.

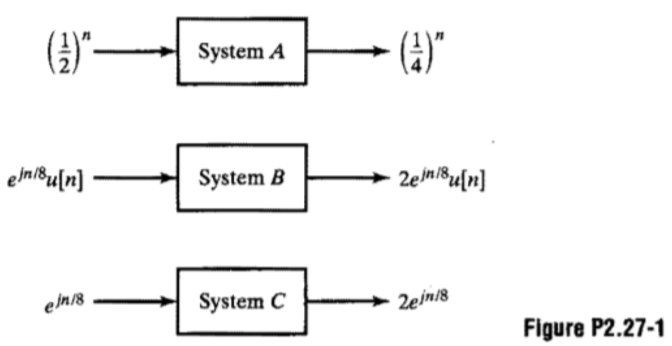

Fig. 1: Systems

Solution

System B:
The input signal x[n] is,

$$x[n] = e^{\frac{jn}{8}}u[n] \tag{0.0.1}$$

The output signal y[n] is,

$$y[n] = 2e^{\frac{jn}{8}}u[n] \tag{0.0.2}$$

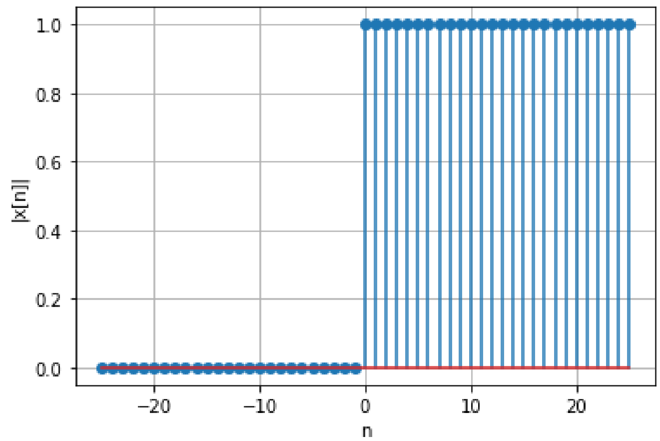

Fig. 2: Amplitude of x[n]

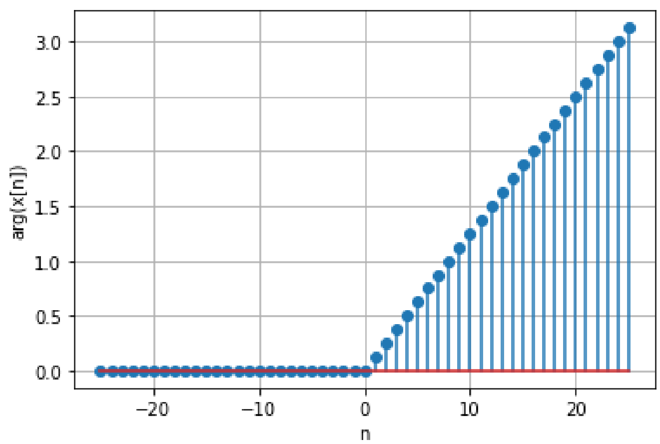

Fig. 3: Phase of x[n]

Then the fourier transform of x[n] is,

$$X\left(e^{j\omega}\right) = \sum_{n=-\infty}^{n=\infty} x[n]e^{-j\omega n} \tag{0.0.3}$$

$$= \sum_{n=-\infty}^{n=\infty} e^{\frac{jn}{8}}u[n]e^{-j\omega n} \tag{0.0.4}$$

$$= \sum_{n=0}^{n=\infty} e^{\frac{jn}{8}}e^{-j\omega n} \tag{0.0.5}$$

$$= \sum_{n=0}^{n=\infty} e^{-j\left(\omega-\frac{1}{8}\right)n} \tag{0.0.6}$$

$$\implies X\left(e^{j\omega}\right) = \frac{1}{1-e^{-j\left(\omega-\frac{1}{8}\right)}} \tag{0.0.7}$$

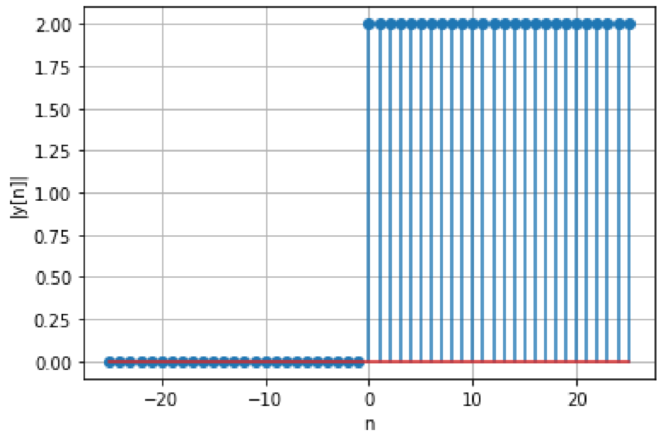

Fig. 4: Amplitude of y[n]

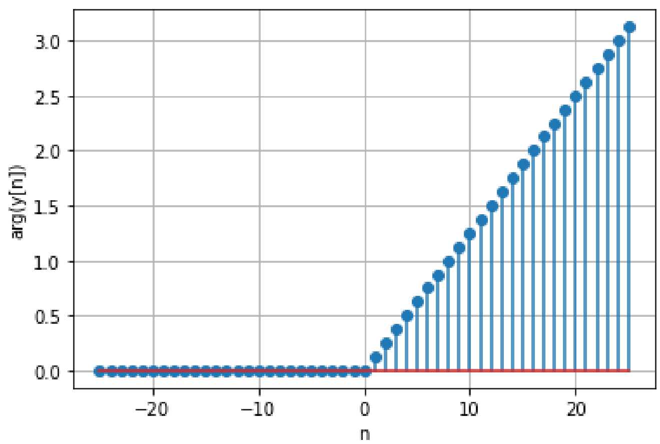

Fig. 5: Phase of y[n]

As y[n]=2x[n], Then the fourier transform of y[n] is,

$$Y\left(e^{j\omega}\right) = \frac{2}{1 - e^{-j\left(\omega - \frac{1}{8}\right)}} \tag{0.0.8}$$

Then the frequency response of the system is,

$$H\left(e^{j\omega}\right) = \frac{Y\left(e^{j\omega}\right)}{X\left(e^{j\omega}\right)} \tag{0.0.9}$$

$$= 2 \tag{0.0.10}$$

$\implies$ As, the frequency response is constant we will get the output signal as th scaled version of the input signal. Here the relation will be like,

$$y[n] = 2x[n] \tag{0.0.11}$$

Thus we can say that the system is a LTI system.
PROOF:

Definition 1. ***Linear*** *The response to an arbitary linear combination of input signals is always the same linear combinations of the individual responses to these signals*

$$x_1[n] \implies y_1[n] = 2x_1[n] \tag{0.0.12}$$

$$x_2[n] \implies y_2[n] = 2x_2[n] \tag{0.0.13}$$

$$ax_1[n] + bx_2[n] \implies 2\left(ax_1[n] + bx_2[n]\right) \tag{0.0.14}$$

$$\therefore ax_1[n] + bx_2[n] \implies ay_1[n] + by_2[n] \tag{0.0.15}$$

As this system obeys both law of addition and law of homogenity, the given system is linear.

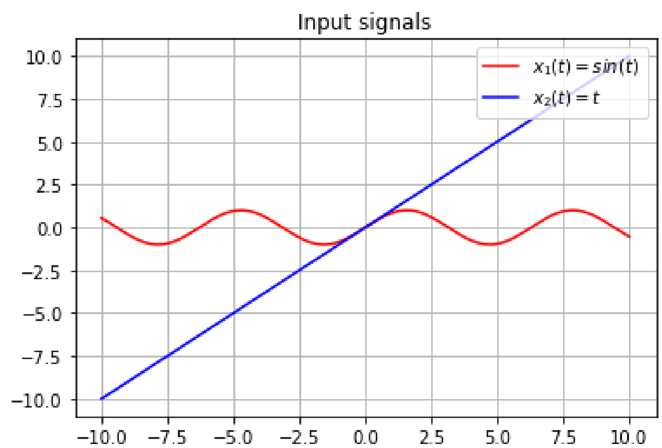

Fig. 1: Example inputs

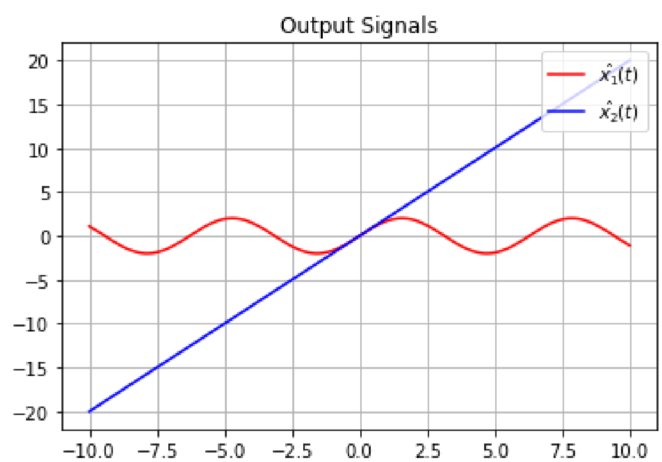

Fig. 2: Outputs for example inputs

Definition 2. ***Time Invariant*** *The response to an arbitrary translated set of inputs is always the response to the original set, but translated by the*

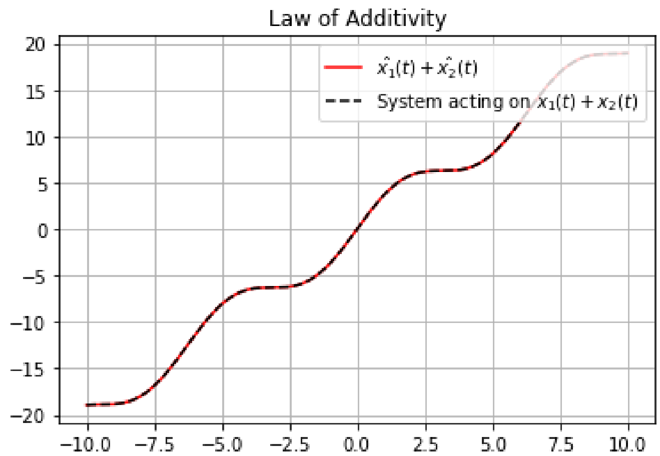

Fig. 3: Law of additivity

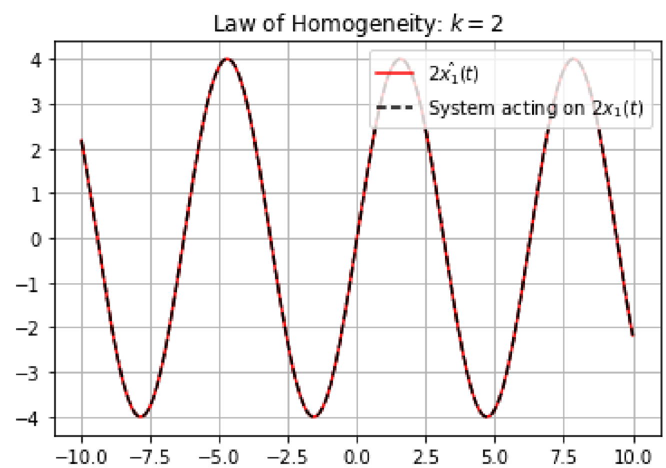

Fig. 4: Law of homogenity

same amount.
If

$$x[n] \implies y[n] \tag{0.0.16}$$

then

$$x[n-n_0] \implies y[n-n_0] \tag{0.0.17}$$

for all x and n_0.

Here

$$x[n] \implies y[n] = 2x[n] \tag{0.0.18}$$

adding time delay(n_0) to the output signal

$$2x[n] \implies 2x[(n-n_0)] \tag{0.0.19}$$

adding time delay(n_0) to the input signal

$$x[n] \implies x[n-n_0] \tag{0.0.20}$$

Now the ouput signal

$$x[n-n_0] \implies 2x[n-n_0] \tag{0.0.21}$$

As 0.0.19 and 0.0.21 are same, the given signal is time invariant.

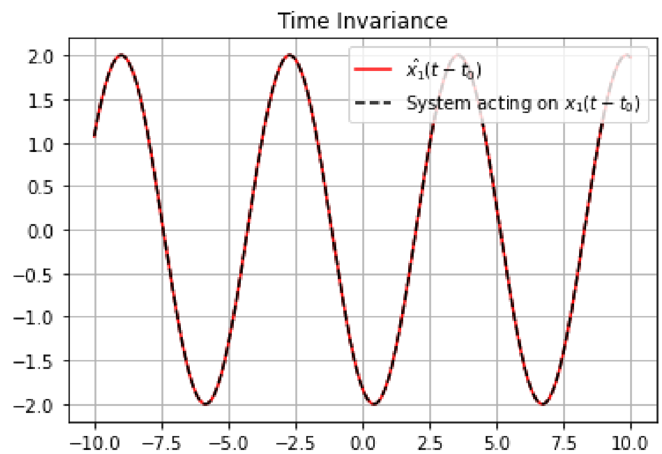

Fig. 1: Time invariant

$\implies$ The system is a **LTI system** and it is **unique**. **Also verified in python**.