EE3900 Gate Assignment - 4

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Download latex-tikz codes from

https://github.com/adhvik24/EE3900/blob/main/Gate_A4/main.tex

Download python codes from

https://github.com/adhvik24/EE3900/blob/main/Gate_A4/codes/plot.py

Problem

(GATE EC-1997 Qn 5.2) If the fourier transform of the deterministic signal $x(t)$ is $X(f)$, then

1) The fourier transform of $x(t-2)$ is
   a) $X(f)e^{-j4\pi f}$
   b) $X(2f)$
   c) $2X(2f)$
   d) $X(f-2)$
2) The fourier transform of $x(\frac{t}{2})$ is
   a) $X(f)e^{-j4\pi f}$
   b) $X(2f)$
   c) $2X(2f)$
   d) $X(f-2)$

Solution

1) By the time shifting property of Fourier transform,

$$x(t-t_0) \overset{\mathcal{F}}{\rightleftharpoons} e^{-j2\pi f t_0} X(f) \tag{0.0.1}$$

$$x(t-2) \overset{\mathcal{F}}{\rightleftharpoons} e^{-j4\pi f} X(f) \tag{0.0.2}$$

Let $x(t) = \text{rect}(t)$ and $\text{sinc}(t) = \frac{\sin(t)}{t}$

$$X(f) = \text{sinc}(f) \tag{0.0.3}$$

For $x(t) = \text{rect}(t-2)$,

$$e^{-j4\pi f} X(f) = e^{-j4\pi f} \text{sinc}(f) \tag{0.0.4}$$

The correct option is (a).

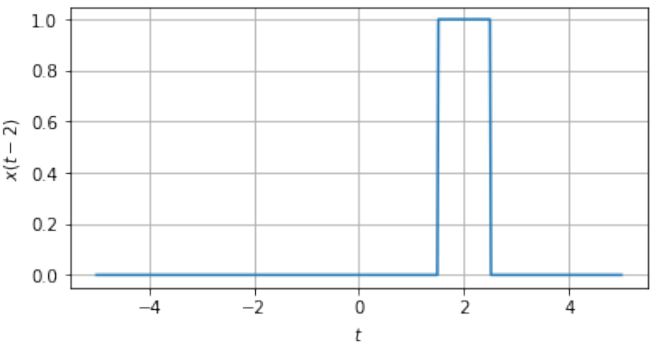

Fig. 1: Plot of rect(t-2)

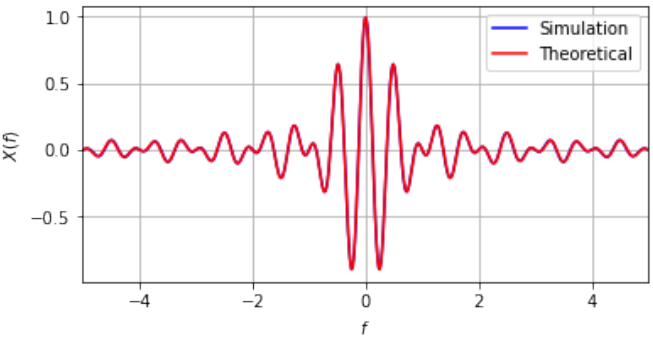

Fig. 2: Fourier transform Simulation v/s Theoretical

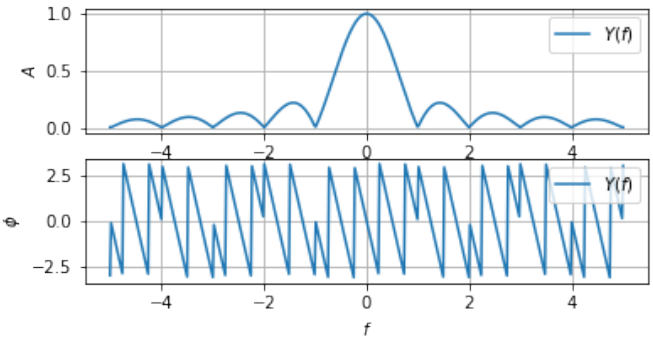

Fig. 3: Amplitude and phase v/s frequency plot

2) By the time scaling property of Fourier transform,

$$x(\alpha t) \overset{\mathcal{F}}{\rightleftharpoons} \frac{1}{|\alpha|} X\left(\frac{f}{|\alpha|}\right) \tag{0.0.5}$$

$$x(\frac{t}{2}) \overset{\mathcal{F}}{\rightleftharpoons} 2X\left(2f\right) \tag{0.0.6}$$

Let $x(t) = \text{rect}\left(t\right)$ and $\text{sinc}(t) = \frac{\sin(t)}{t}$

$$X(f) = \text{sinc}\left(f\right) \tag{0.0.7}$$

For $x(t) = \text{rect}\left(\frac{t}{2}\right)$,

$$2X\left(2f\right) = 2\,\text{sinc}\left(2f\right) \tag{0.0.8}$$

The correct option is (c).

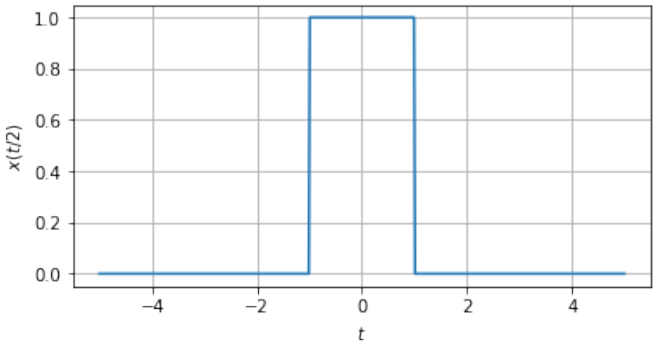

Fig. 4: Plot of rect$(\frac{t}{2})$

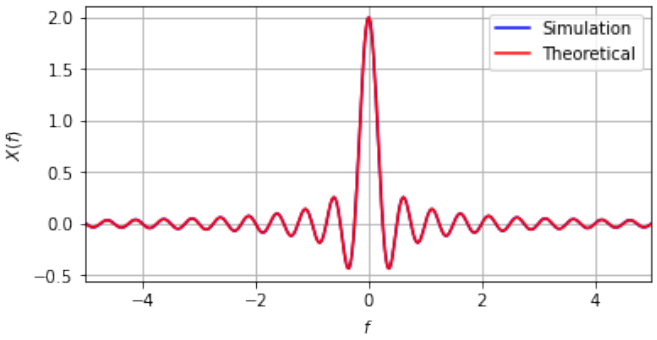

Fig. 5: Fourier transform Simulation v/s Theoretical