EE3900 Gate Assignment - 1

Adhvik Mani Sai Murarisetty - AI20BTECH11015

Download latex-tikz and python codes from

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https://github.com/adhvik24/EE3900/blob/main/
    Gate_A1
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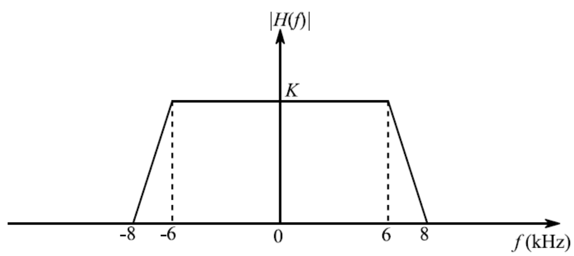
1 Gate EC 2018 Qn 54

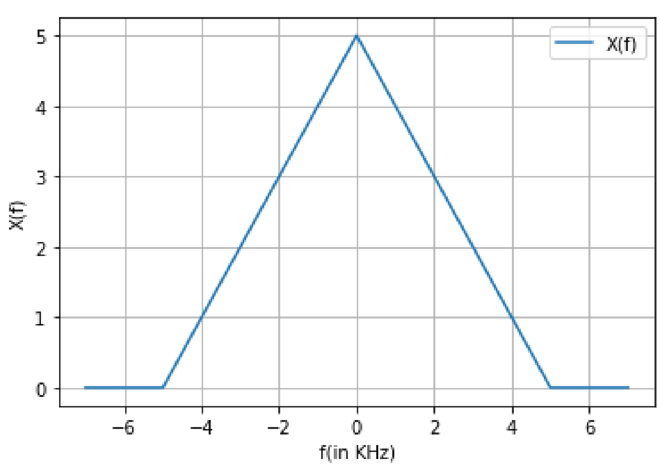
A band limited low-pass signal x(t) of bandwidth 5 kHz is sampled at a sampling rate f_s. The signal x(t) is reconstructed using the reconstruction filter H(f) whose magnitude response is shown below:



The minimum sampling rate f_s (in kHz) for perfect reconstruction of x(t) is

2 SOLUTION

As x(t) is a band limited low-pass signal of bandwidth 5kHz.
Let our X(f) be look like,



As for sampling a signal we will multiply a unit impulse train sampled at a sampling frequency with the signal. So, fourier transform of sampled signal will be the convolution of input's X(f) and delta function.
After sampling x(t) at a sampling rate of f_s, Then it sampled signal's X(f) is,

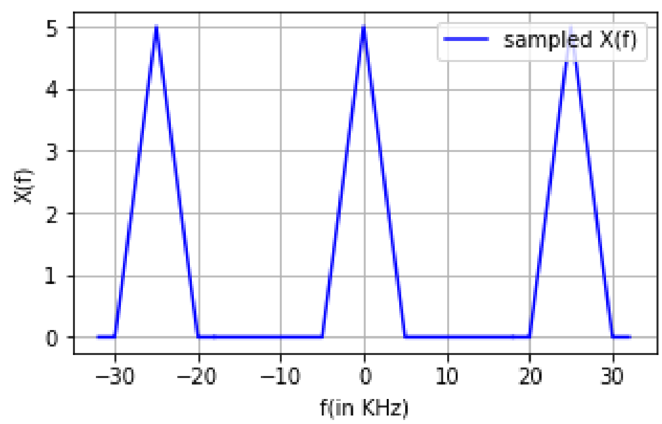


Fig. 1: Sampled signal's X(f)

Note: Here for plotting f_s has been taken as 25kHz.
Therefore, The condition for perfect reconstruction of x(t) from s(t) using filter H is,

$$f_m \leq f_H \leq f_s - f_m$$

Where f_m is the maximum component frequency of x(t), f_H is that of filter and f_s is the sampling frequency.
We know that f_m is 5kHz,f_H is 8kHz and the next sampled part signal starts at f_s-5 kHz.
For perfect reconstruction of x(t) which has been sampled at a rate f_s,

$$f_s - 5 \geq 8$$

So, The possible values of f_s for which reconstruction of x(t) possible is

$$f_s \geq 13$$

$\therefore$ The minimum sampling rate f_s for perfect reconstruction of x(t) is 13kHz.

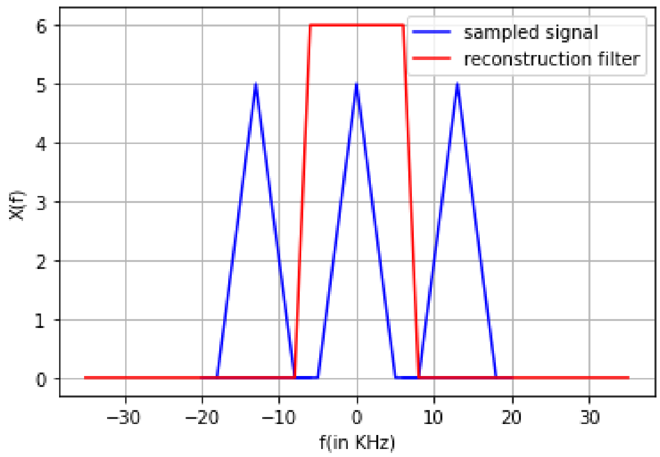


Fig. 2: Applying filter on sampled signal
Note: Here for plotting f_s has been taken as 13kHz.

On applying the given reconstruction filter on the sampled signal looks like, (When f_s = 25kHz)

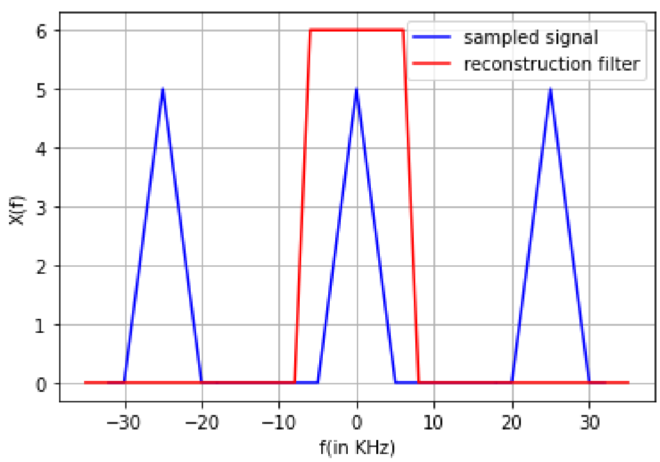


Fig. 3: Applying filter on sampled signal
Note: Here for plotting f_s has been taken as 25kHz.

We are observing a perfect reconstruction of x(t) is possible in the case of f_s=25kHz.
But if we observe when f_s=11kHz, It is not possible to perfect reconstruction of x(t). As it looks like Fig4.

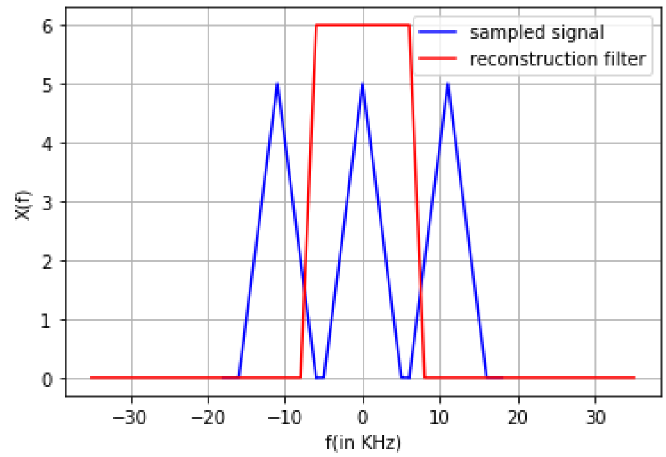


Fig. 4: Applying filter on sampled signal
Note: Here for plotting f_s has been taken as 11kHz.

$\therefore$ We can say that the minimum sampling rate f_s for perfect reconstruction of x(t) is 13kHz.