EE3900 Assignment - 4

Adhvik Mani Sai Murarisetty - AI20BTECH11015

Download latex-tikz codes from

https://github.com/adhvik24/EE3900/blob/main/Assignment_4/Assignment4.tex

Download python codes from

https://github.com/adhvik24/EE3900/blob/main/Assignment_4/Assignment4.py

1 Linear forms qn 2.17

A line perpendicular to the line segment joining the points $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}2\\3\end{pmatrix}$ divides it in the ratio 1 : n. Find the equation of the line.

2 SOLUTION

Let $\mathbf{M}$ be the point that divides the two points $\mathbf{A} = \begin{pmatrix}1\\0\end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix}2\\3\end{pmatrix}$ in ratio $1 : n$.

$$\mathbf{M} = \frac{n\mathbf{A}+\mathbf{B}}{n+1} = \frac{n\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}2\\3\end{pmatrix}}{n+1} \tag{2.0.1}$$

$$\implies \mathbf{M} = \frac{1}{n+1}\begin{pmatrix}n+2\\3\end{pmatrix}$$

The direction vector of line AB is

$$\begin{pmatrix}1\\0\end{pmatrix}-\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}-1\\-3\end{pmatrix} \tag{2.0.2}$$

The direction vector of line AB is normal vector of perpendicular line. Then

$$\mathbf{n} = \begin{pmatrix}-1\\-3\end{pmatrix} \tag{2.0.3}$$

The equation of line in terms of normal vector is then obtained as

$$\mathbf{n}^T(\mathbf{x}-\mathbf{M}) = 0 \tag{2.0.4}$$

$$\implies \begin{pmatrix}-1 & -3\end{pmatrix}\left(\mathbf{x}-\frac{1}{n+1}\begin{pmatrix}n+2\\3\end{pmatrix}\right) = 0 \tag{2.0.5}$$

$$\therefore \begin{pmatrix}-1 & -3\end{pmatrix}\mathbf{x} = \frac{-n-11}{n+1} \tag{2.0.6}$$

We got equation of the line perpendicular to line segment joining points $\mathbf{A}$ and $\mathbf{B}$ and dividing them in the ratio $1 : n$.
For plotting let us take $n = 2$, Then the perpendicular line equation will be as,

$$\begin{pmatrix}-1 & -3\end{pmatrix}\mathbf{x} = \frac{-13}{3} \tag{2.0.7}$$

And the graph looks like,

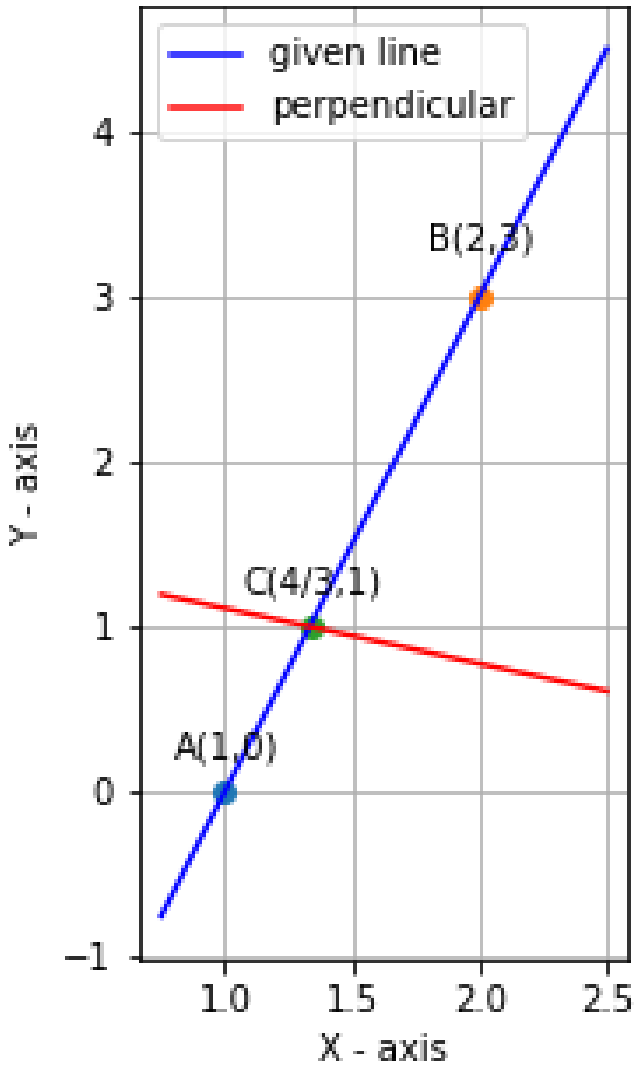

Fig. 1: graphical interpretation