EE3900 Gate Assignment - 3

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Download latex-tikz codes from

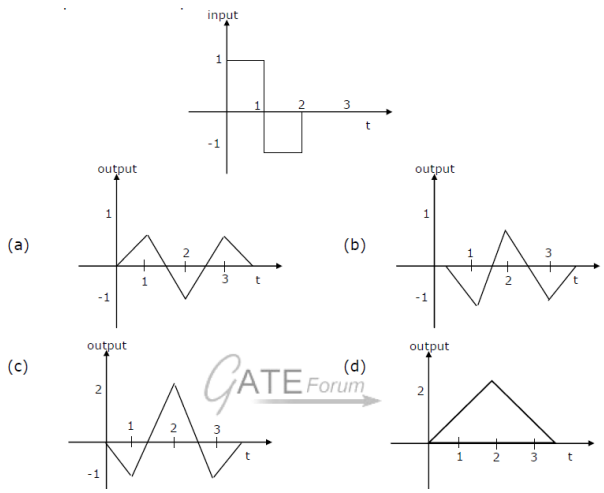
https://github.com/adhvik24/EE3900/blob/main/Gate_A3/main.tex

Download python codes from

https://github.com/adhvik24/EE3900/blob/main/Gate_A3/plot.py

1 EC 2005/Q.71

A signal as shown in figure is applied to a matched filter. Which of the following does represent the output of this matched filter ?



2 SOLUTION

Lemma 2.1. *The unit step signal, u(t), is given by:*

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & otherwise \end{cases} \tag{2.0.1}$$

On time-shifting u(t) by T, we get:

$$u(t-T) = \begin{cases} 1 & t \geq T \\ 0 & otherwise \end{cases} \tag{2.0.2}$$

Lemma 2.2. *Convolution of u(t-a) and u(t-b) is,*

$$u(t-a) * u(t-b) = \int_{-\infty}^{\infty} u(\tau - a)u(t-\tau-b)d\tau$$

$$u(t-a) * u(t-b) = \begin{cases} t-b-a & t \geq a+b \\ 0 & otherwise \end{cases}$$

For a input x(t) the matched filter's impulse response h(t) is,

$$h(t) = x(T-t) \tag{2.0.3}$$

The input signal x(t) is,

$$x(t) = u(t) - 2u(t-1) + u(t-2) \tag{2.0.4}$$

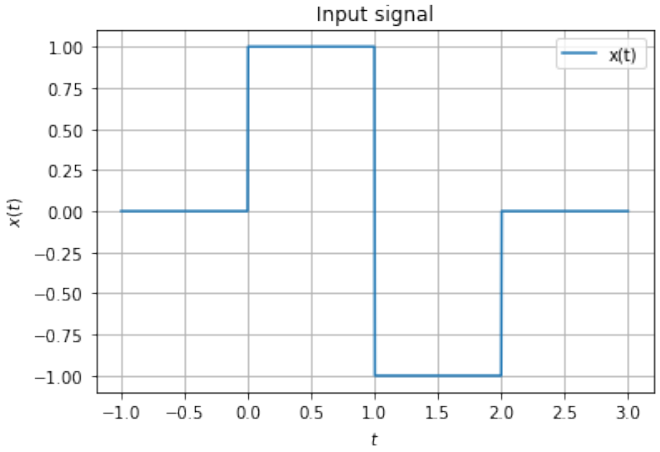


Fig. 1: input signal

Using (2.0.3), The impulse response for the matched filter of the input signal is h(t),

$$h(t) = x(2-t) \tag{2.0.5}$$

$$\implies h(t) = x(2-t) = u(2-t) - 2u(1-t) + u(-t)$$

$$\therefore h(t) = -u(t) + 2u(t-1) - u(t-2) = -x(t)$$

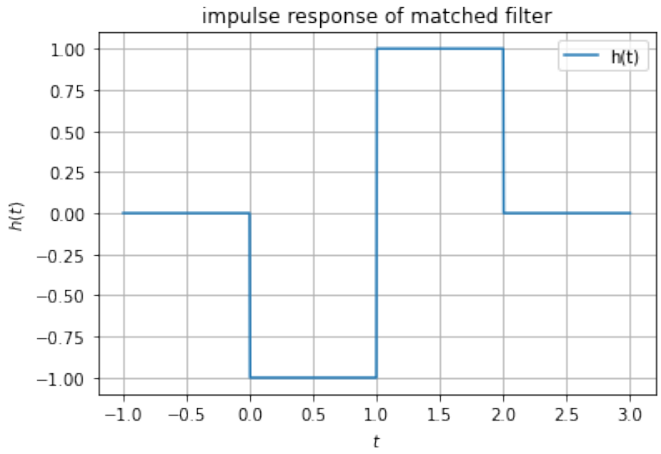


Fig. 2: impulse response of matched filter

Now, The corresponding output y(t) is given by,

$$\begin{aligned} y(t) &= h(t) * x(t) \\ &= -(u(t) - 2u(t-1) + u(t-2)) * \\ &\quad (u(t) - 2u(t-1) + u(t-2)) \\ &= -(u(t) * u(t) - 4u(t-1) * u(t) + 2u(t-2) * u(t)) - \\ &\quad (4u(t-1) * u(t-1) - 4u(t-1) * u(t-2)) - \\ &\quad (u(t-2) * u(t-2)) \end{aligned} \tag{2.0.6}$$

Using 2.2, We will get

$$y(t) = \begin{cases} 0 & t < 0 \\ -t & 0 \le t < 1 \\ 3t - 4 & 1 \le t < 2 \\ 8 - 3t & 2 \le t < 3 \\ t - 4 & 3 \le t < 4 \\ 0 & t \ge 4 \end{cases} \tag{2.0.7}$$

Therefore, Option(C) is the correct option.

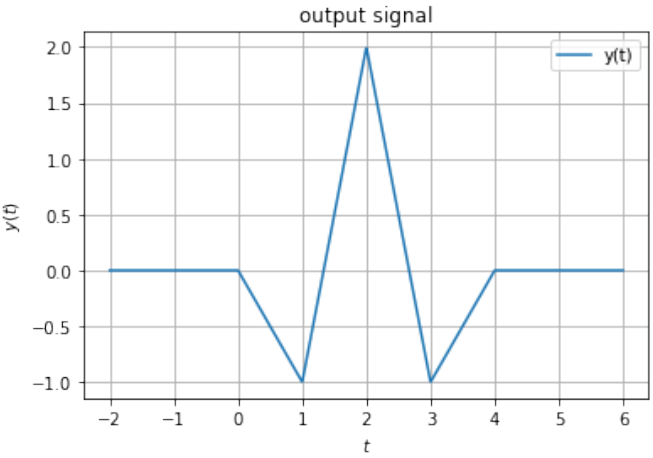


Fig. 3: output signal