EE3900 Assignment - 5

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Download latex-tikz codes from

https://github.com/adhvik24/EE3900/blob/main/Assignment_5/Assignment5.tex

Download python codes from

https://github.com/adhvik24/EE3900/blob/main/Assignment_5/codes/a_5.py

1 Quadratic Forms/Q 2.16

Find the zeroes of the quadratic polynomial $x^2+7x+10$ and verify the relationship between the zeroes and the coefficients.

2 Solution

Lemma 2.1. *A general polynomial equation $p(x,y)$ of degree 2 is given by :*

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \tag{2.0.1}$$

The vector equation of $p(x,y)$ is given by :

$$\mathbf{x}^{\mathbf{T}} \begin{pmatrix} A & \frac{B}{2} \\ \frac{B}{2} & C \end{pmatrix} \mathbf{x} + \begin{pmatrix} D & E \end{pmatrix} \mathbf{x} + F = 0 \tag{2.0.2}$$

And for a quadratic polynomial we have :

$$B = 0 \tag{2.0.3}$$

$$C = 0 \tag{2.0.4}$$

$$E = 0 \tag{2.0.5}$$

If we take $A = 1$, we have :

$$\textit{Sum of zeroes} = -D \tag{2.0.6}$$

$$\textit{Product of zeroes} = F \tag{2.0.7}$$

The given equation can be written as,

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} \frac{7}{2} & -\frac{1}{2} \end{pmatrix} \mathbf{x} + 10 = 0 \tag{2.0.8}$$

where,

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{2.0.9}$$

Substituting (2.0.9) in (2.0.8),

$$\begin{pmatrix} x \\ y \end{pmatrix}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 2 \begin{pmatrix} \frac{7}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 10 = 0 \tag{2.0.10}$$

$$\implies \mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \ \mathbf{u} = \begin{pmatrix} \frac{7}{2} \\ -\frac{1}{2} \end{pmatrix}, \ f = 10 \tag{2.0.11}$$

For obtaining the affine transformation, we use

$$\mathbf{x} = \mathbf{P}\mathbf{y} + \mathbf{c} \tag{2.0.12}$$

The corresponding eigenvalues of $\mathbf{V}$ are

$$\lambda_1 = 0, \ \lambda_2 = 1 \tag{2.0.13}$$

$$\implies \mathbf{D} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.14}$$

The corresponding eigen vectors are

$$\mathbf{p}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \ \mathbf{p}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.15}$$

$$\implies \mathbf{P} = \begin{pmatrix} \mathbf{p}_1 & \mathbf{p}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{and } \mathbf{c} = 10 \tag{2.0.16}$$

Solving the equation,

$$x^2 + 7x + 10 = 0 \tag{2.0.17}$$

$$\implies \left(x + \frac{7}{2}\right)^2 = \frac{9}{4} \tag{2.0.18}$$

$$\implies \left(x + \frac{7}{2}\right) = \pm\frac{3}{2} \tag{2.0.19}$$

$$\implies x = -2, -5 \tag{2.0.20}$$

Verifying the relationship between the zeroes and coefficients. By comparing (2.0.8) with (2.0.2),

$$\implies \text{sum of the zeroes} = -7 = -D \tag{2.0.21}$$

$$\text{product of zeroes} = 10 = F. \tag{2.0.22}$$

$\therefore$ The zeroes of equation $x^2 + 7x + 10$ are -2, -5.

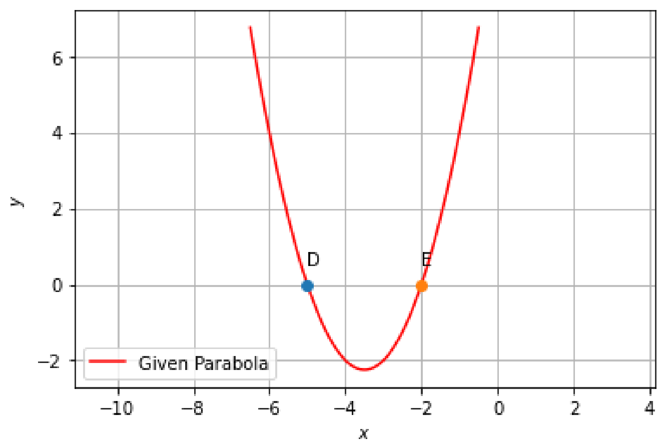

Fig. 1: Quadratic polynomial $x^2 + 7x + 10$