



A network perspective of the stock market

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ABSTRACT

Complex networks are constructed to study correlations between the closing prices for all US stocks that were traded over two periods of time (from July 2005 to August 2007; and from June 2007 to May 2009). The nodes are the stocks, and the connections are determined by cross correlations of the variations of the stock prices, price returns and trading volumes within a chosen period of time. Specifically, a winner-take-all approach is used to determine if two nodes are connected by an edge. So far, no previous work has attempted to construct a full network of US stock prices that gives full information about their interdependence. We report that all networks based on connecting stocks of highly correlated stock prices, price returns and trading volumes, display a scalefree degree distribution. The results from this work clearly suggest that the variation of stock prices are strongly influenced by a relatively small number of stocks. We propose a new approach for selecting stocks for inclusion in a stock index and compare it with existing indexes. From the composition of the highly connected stocks, it can be concluded that the market is heavily dominated by stocks in the financial sector.

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1. Introduction

Fluctuations of stock prices are not independent, but are highly inter-coupled with strong correlations with the business sectors and industries to which the stocks belong. Recently, analyses based on network models have been proposed for studying the correlations of stock prices, including the works of Mantegna (1999), Vandewalle et al. (2001), Bonanno et al. (2001), Bonanno et al. (2003), Bonanno et al. (2004), and Onnela et al. (2003). The usual approach involves a procedure of finding correlation between each pair of time series of stock prices, and a subsequent procedure of constructing a network that connects the individual stocks based on the levels of correlation. The resulting networks are usually very large and their analysis is rather complex. In much of the previous work, such as Onnela et al. (2003) and Onnela et al. (2004), networks of relatively small size were constructed, and specific filtering processes were applied to further reduce the complexity. In particular, the method of Minimal Spanning Tree (MST) has been used for filtering networks, resulting in simpler forms of graphs that can facilitate analysis. The MST reduction is a topology based approach, which removes edges drastically by retaining only those that fit the MST criterion. With reduced complexity, Vandewalle et al. (2001) observed a scalefree degree distribution in MST filtered networks of US stock prices. The topological change in the MST structure of networks of US stock prices has also been studied by Onnela et al. (2003) who found variation in the value of the power-law exponent of the scalefree degree distribution of the MST filtered networks for “business as usual” and “crash” periods. Notwithstanding, the reduction of complexity by introducing MST filtering to correlation-based networks, essential information about the internal structure is inevitably lost. In order to retain more information about the networks, less drastic filtering may be applied, e.g., using Planar Maximally Filtered Graph (PMFG), as proposed by Tumminello et al. (2005). However, both MST and PMFG suffer substantial loss of information as edges of high

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correlations are often removed while edges of low correlations are retained just because of their topological conditions fitting the topological reduction criteria. The usefulness of such MST or PMFG filtered networks is thus greatly reduced, especially in respect of their ability to identify the levels of correlation among stock prices.

To the best of our knowledge, no previous work has attempted to construct a full network of correlation-based connections for US stocks, which retains all information of the internal structure that reflects the interdependence of the stock prices. Our calculation of cross correlation values is similar to that adopted by Onnela et al. (2004) but here we use a winner-take-all approach in establishing edges of the network, which makes binary decision on connecting two stock prices according to the truth value of their cross correlation being larger than a threshold value. Specifically, we examine the closing prices of stocks which were traded over a chosen period of time. We first consider all 19807 stocks (out of 51835) which were traded each trading day from July 1, 2005, to August 30, 2007 (Set 1 data). Our aim is to construct networks that connect stock prices having similar variation profiles over the period of time being considered. Basically we examine the time series of the daily stock prices and establish connections between any pair of stocks. If the *cross correlation* of the time series of the daily stock prices of two stocks is greater than a threshold (e.g., 0.9), we consider that the two stocks are “connected”. This simple winner-take-all criterion for establishing connections can also be applied to daily price returns, daily trading volumes, etc. in addition to daily closing prices. For instance, when cross correlation is taken between two time series of price returns, a different network can be formed. We will show in this paper that the full networks of stock prices, price returns and volumes are scalefree, and will report the power-law exponents along with a number of network parameters found from the US stocks that were traded in the period stated above. The same analysis, when performed for an alternative period of time from June 1, 2007 to May 30, 2009 (Set 2 data), shows consistent qualitative properties of the networks.

Presently, stock market indexes, e.g., Standard & Poor’s 500 Index, Dow Jones Indexes, and Nasdaq Indexes, are used to gauge the market variations and the levels of market capitalization. Because power-law distributions have been found in the stock prices, we know that a small number of stocks are having strong influence over the entire market, and we therefore propose that stocks corresponding to nodes of high degrees can be used to compose a new index that can naturally and adequately reflect the market variation. We will evaluate the correlation of this new index with other existing indexes. Again, the two sets of data corresponding to two different 2-year periods give consistent results.

It is worth noting that correlation analysis is not uncommon in finance. In fact, correlation analysis in asset allocation and risk management has been widely studied, leading to the derivation of many practical portfolio selection and asset allocation models that have been successfully used in real-world applications. Correlation analysis is also important in portfolio optimization for both international and domestic market investments. Some earlier studies have shown that diversification benefits can be affected during times of greater volatility on financial markets. Thus, correlation analysis has played an important role in establishing the link between diversification benefits and market volatility. For instance, Longin and Solnik (2001) have analyzed such correlation, leading to the conclusion that turbulent market conditions would result in a reduction in the benefits from portfolio diversification. These earlier results stimulated further research on the characteristics of the correlations. For example, the work of Campbell et al. (2008) has led to better understanding of the “correlation structure” or correlation distribution.

We will begin with a quick review of scalefree networks in Section 2. We will then introduce the connection criterion and the network construction procedure in Section 3. Results on the network properties will be presented in Section 4, and the scalefree distribution further analyzed in Section 5. A degree-based index will be introduced in Section 6 and compared with existing indexes in terms of statistical correlations. Some conclusions will be drawn in Section 7.

2. Complex networks approach

The study of complex networks in physics has aroused a lot of interest across a multitude of application areas. A key finding is that most networks involving man-made couplings and connection of people are naturally connected in a scalefree manner, which means that the number of connections follows a power-law distribution. Scalefree power-law distribution is a remarkable property that has been found across of a variety of connected communities.¹ This property has also been shown to be a key to optimal performance of networked systems in Zheng et al. (2006).

A network is usually defined as a collection of “nodes” connected by “links” or “edges”. If we consider a network of stock prices, then the nodes will be the individual stocks and a link between two nodes denotes that the two stocks being connected display some “similarity”. The number of links emerging from and converging at a node is called the “degree” of that node, usually denoted by k . So, we have an average degree for the whole network. The key concept here is the distribution of k . This concept can be mathematically presented in terms of probability density function. Basically, the probability of a node having a degree k is $p(k)$, and if we plot $p(k)$ against k , we get a distribution function. This distribution tells us about how this network of stock prices are connected. Recent research has provided concrete evidence that networks with man-made couplings and/or human connections follow power-law distributions, i.e., the log of $p(k)$ vs. the log of k being a straight line whose gradient is defined as the *characteristic exponent*, as explained in Ravid and Rafaeli (2004) and Csanyi and Szendroi (2004). Such networks are termed *scalefree networks*.

¹ Pioneering works include Barabasi and Albert (1999), Newman (2001), Strogatz (2001), Csanyi and Szendroi (2004), and Ravid and Rafaeli (2004).

3. Network construction

We first consider the Set 1 data and the resulting networks of 19807 nodes. Specifically, each node corresponds to one of the stocks traded between July 1, 2005 to August 30, 2007. An illustration is shown in Fig. 1. For each pair of stocks (nodes), we will evaluate the cross correlation of the time series of their daily stock prices, daily price returns and daily trading volumes.

Let $p_i(t)$ be the closing price of stock i on day t and $v_i(t)$ be the trading volume of stock i on day t . Then, the price return of stock i on day t , denoted by $r_i(t)$, is defined as

$$r_i(t) = \ln \left[\frac{p_i(t)}{p_i(t-1)} \right]. \quad (1)$$

Suppose $x_i(t)$ and $x_j(t)$ are the daily prices or price returns or trading volumes of stock i and stock j , respectively, over the period $t = 0$ to $N - 1$. We now compare the two time series with no relative delay. In other words, x_i and x_j are compared from $i = 0$ to $N - 1$ with no relative time shift. The cross correlation between x_i and x_j with no time shift is given by Cohen et al. (2003).

$$c_{ij} = \frac{\sum_t [(x_i(t) - \bar{x}_i)(x_j(t) - \bar{x}_j)]}{\sqrt{\sum_t (x_i(t) - \bar{x}_i)^2} \sqrt{\sum_t (x_j(t) - \bar{x}_j)^2}} \quad (2)$$

where \bar{x}_i and \bar{x}_j are the means of the time series and the summations are taken over $t = 0$ to $N - 1$.

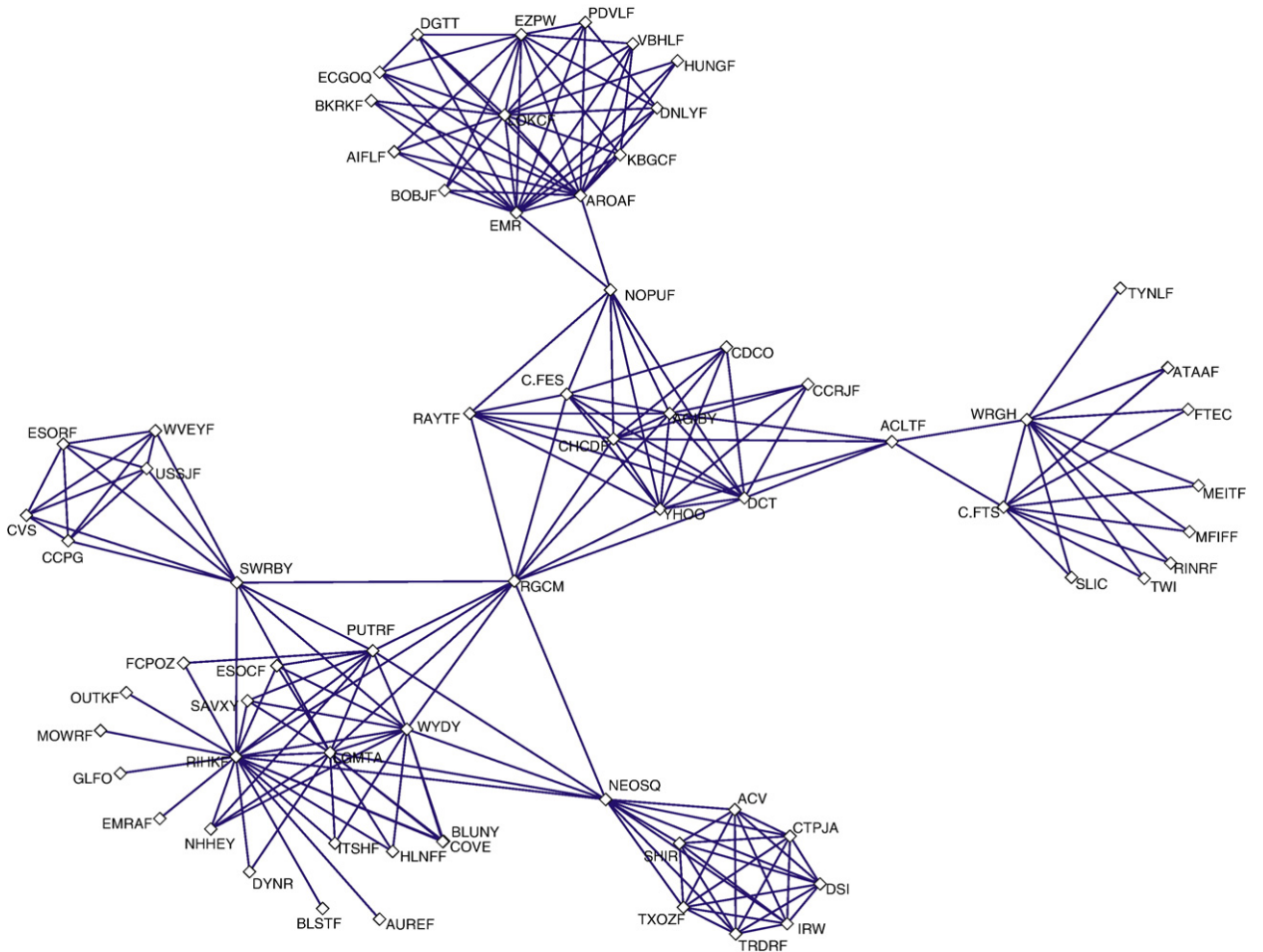


Fig. 1. Partial view of the network of US stock prices, based on Set 1 data, i.e., from the period July 1, 2005, to August 30, 2007. Node labels are stock abbreviations, e.g., YHOO is Yahoo! Inc., ACLTF is ATCO Ltd CL II VTG C, AGIBY is Anglo Irish Bank Corporation PLC, etc.

Table 1

Network parameters from US stock networks constructed from daily closing prices using a winner-take-all connection criterion based on Set 1 data, i.e., for the period July 1, 2005, to August 30, 2007.

Parameters	$\rho = 0.85$	$\rho = 0.90$	$\rho = 0.95$
Number of nodes N	19,807	19,807	19,807
Number of connections L	4,652,650	1,495,250	143,181
Average shortest length s	3.375	3.954	4.995
Diameter D	16	18	30
Average clustering coefficient C	0.421	0.302	0.148
Average degree K	469.80	150.98	14.46
Power-law exponent γ	0.778	1.075	0.992
Mean fitting error	6.26e−7	4.26e−7	1.65e−7

In defining our criterion for connecting a pair of nodes, we need a threshold value for the cross correlation. Since cross correlation is a measure of similarity and its value is between 0 and 1, we simply choose a positive fractional value as the threshold. Suppose the threshold is ρ . Then, the connection criterion for stock i and stock j is simply written as

$$C_{ij} > \rho. \quad (3)$$

4. Measured network parameters

We begin with relatively large values of ρ as our objective is to construct stock networks that reflect connections of highly correlated stock price time series. The total duration of Set 1 data is 564 trading days (from July 1, 2005 to August 30, 2007). It is found that the degree distributions display scalefree characteristics when ρ is sufficiently large. Applying the least squares method with data points in the straight line segment of the log–log degree distribution plots, the power-law exponent is found to vary between 1 and 3. We also calculate the mean fitting error to examine the fitness of the power-law distribution over the data points. Specifically, suppose the distribution of $p(k)$ vs. k has been approximated by a power-law function $P(k) = \alpha e^{-\gamma k}$, and the values of α and γ can be found from any fitting method. Here, we define *fitting error*, $\varepsilon_{\text{fitting}}$, as follows:

$$\varepsilon_{\text{fitting}} = \sum_k |p(k) - \alpha e^{-\gamma k}| \quad (4)$$

where α and γ are found from the least squares method. In addition, we have calculated the number of connections L , average shortest length s , average clustering coefficient C , average degree K , and the power-law exponents. Tables 1, 2 and 3 show the results for networks based on closing prices, price returns and trading volumes, respectively. Fig. 2 illustrates the power-law degree distribution for $\rho = 0.9$.

For ρ below about a certain value, the power law distribution becomes blur, i.e., the mean fitting error increases. The networks thus constructed do not show clear scalefree characteristics. This result should be expected since with small ρ , the network tends to be randomly connected, and in the extreme case of $\rho = 0$, the network is fully connected. To determine how well the degree distribution approximates a power-law distribution, we compute the fitting error of the power-law distribution and note its variation as the value of ρ reduces. This simple test is sufficient for the purpose of finding the relative relationship of the “scalefree-ness” of the degree distribution with the choice ρ . The variations of the fitting errors with respect to ρ are plotted in Fig. 3, which clearly demonstrates that the degree distribution is scalefree for sufficiently large values of ρ .

It should be noted that the entire analysis described above can be repeated for Set 2 data, i.e., for the period June 1, 2007 to May 30, 2009. As no observable difference in the results based on Set 2 data has been observed, we omit the details here. Moreover, it is worth noting that the period of time chosen for calculation of correlation is about two years in this study, and we may conclude that such a duration of time is adequate for providing meaningful correlation calculations. In other words, the networks constructed over such a duration of time are generally robust.

Table 2

Network parameters from US stock networks constructed from daily price returns using a winner-take-all connection criterion based on Set 1 data.

Parameters	$\rho = 0.70$	$\rho = 0.80$	$\rho = 0.90$
Number of nodes N	19,807	19,807	19,807
Number of connections L	15,785	6675	2359
Average shortest length s	2.946	2.290	2.043
Diameter D	20	7	8
Average clustering coefficient C	0.104	0.058	0.238
Average degree K	1.594	0.674	0.238
Power-law exponent γ	2.019	3.067	2.920
Mean fitting error	16.07e−5	8.29e−5	2.78e−6

Table 3

Network parameters from US stock networks constructed from daily trading volumes using a winner-take-all connection criterion based on Set 1 data.

Parameters	$\rho = 0.70$	$\rho = 0.80$	$\rho = 0.90$
Number of nodes N	19,807	19,807	19,807
Number of connections L	256,046	167,340	96,203
Average shortest length s	4.542	4.927	7.165
Diameter D	21	19	19
Average clustering coefficient C	0.260	0.194	0.140
Average degree K	25.854	16.897	9.714
Power-law exponent γ	1.374	1.285	1.5933
Mean fitting error	$1.33e-5$	$2.56e-6$	$2.50e-7$

5. Stock network is scalefree

The properties of the stock networks constructed on the basis of cross correlation of stock prices are dependent upon the choice of the threshold ρ . We generally observe that the total number of connections increases with decreasing ρ , and as ρ approaches 0, the network becomes fully connected, as expected. The power-law degree distribution holds for large ρ , and becomes blur as ρ decreases, which is again consistent with the fact that the network becomes effectively more fully connected as ρ decreases (Fig. 3).

We are particularly interested in the case where ρ is high as the network so formed would connect stocks of closely resembling daily price fluctuations. As we have shown earlier, the stock network is scalefree and displays clear power-law degree distributions. Thus, we may conclude that stocks having close resemblance with a large number of other stocks are relatively few. This transpires that the stock market is essentially influenced by a relatively small number of stocks, and hence we may introduce an index that reflects on the performance of the stock market based on a small number of stocks that have a relatively high number of connections. In other words, an index can be defined by the stocks of high degrees.

6. Degree-based indexes: indexes that probe dominant stocks

From the above winner-take-all correlation-based networks, we can identify stocks that have the highest degrees. These stocks have the largest numbers of connections with themselves and other stocks in the market. At this point, we should reiterate that we refer to highly connected stocks as those resembled by most other stocks in terms of their time series of closing prices and price returns, and that because of the scalefree property of the degree distribution, a relatively small number of stocks are highly connected. On the basis of the top 10% most highly connected stocks, we select those whose share information is fully available from Set 1 data, i.e., for the period from July 1, 2005 to August 30, 2007. New indexes are computed using the market capitalization formula,² i.e.,

$$\text{Index} = \frac{\sum_i [\text{price}_i \times \text{number of shares}_i]}{\text{total market value of stocks during base period}} \quad (5)$$

In selecting the network, we choose ρ that gives about 500 stocks out of the top 10%. For the network based on closing prices, we choose $\rho = 0.9$, and for the network based on price returns, we choose $\rho = 0.5$. From the top 10% highly connected stocks, we get 330 stocks with full share information for the closing price network, and 486 stocks with full share information for the price return network. These stocks will be used to compute indexes, as mentioned above. Moreover, for simplicity, we do not normalize the new indexes by introducing different normalizing divisors, and simply take the divisors as unity. Thus, the ranges of the indexes are not the same. Nonetheless, calculations of the correlations of different pairs of indexes automatically perform the normalization. Fig. 4 shows the time series of the new degree-based indexes, Dow Jones index, Standard & Poor's 500 index, and Nasdaq Composite index for the 564 trading days from Set 1 data, i.e., for the period July 1, 2005 to August 30, 2007.

6.1. Correlations between the indexes

In order to establish any correlation between the new degree-based indexes and other existing indexes, we resort to the use of formal hypothesis tests. Specifically, we set up a null hypothesis $H: \rho_0 = 0$, i.e., there is no correlation between time series of the new degree-based indexes and the existing indexes such as Dow Jones, Standard & Poor's 500 and Nasdaq. The null hypothesis is rejected if ρ_0 surpasses the critical value given the significance level $\alpha = 0.001$. Therefore, any correlation with p -value larger than 0.001 would be rejected. The Spearman's correlations³ are shown in Table 4. See Hollander and Wolfe (1973) for a detailed

² See <http://www.investopedia.com/ask/answers/05/sp500calculation.asp> for a description of the market capitalization formula.

³ While the usual correlation (Pearson correlation) indicates the strength of a linear relationship between two variables, its value alone may not be sufficient to evaluate the relationship under study, especially in the case where the assumption of normality is invalid. The Spearman's correlation makes no assumption about the frequency distribution of the variables, and is thus more suited for our purpose.

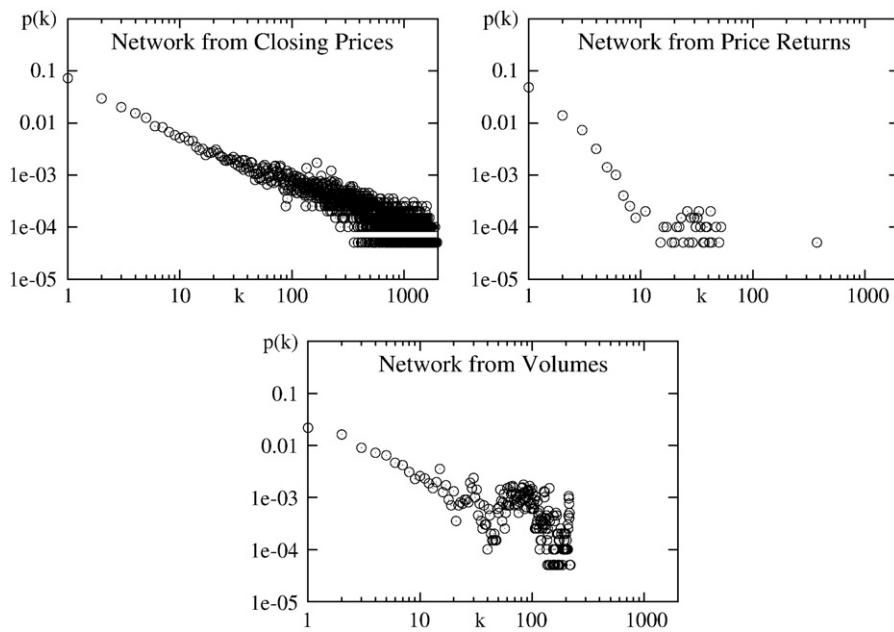


Fig. 2. Scalefree degree distribution of networks formed by a winner-take-all connection criterion with $\rho = 0.9$ applied to cross correlation of daily closing prices (upper left panel); daily price returns (upper right panel); and daily trading volumes (lower panel), using Set 1 data.

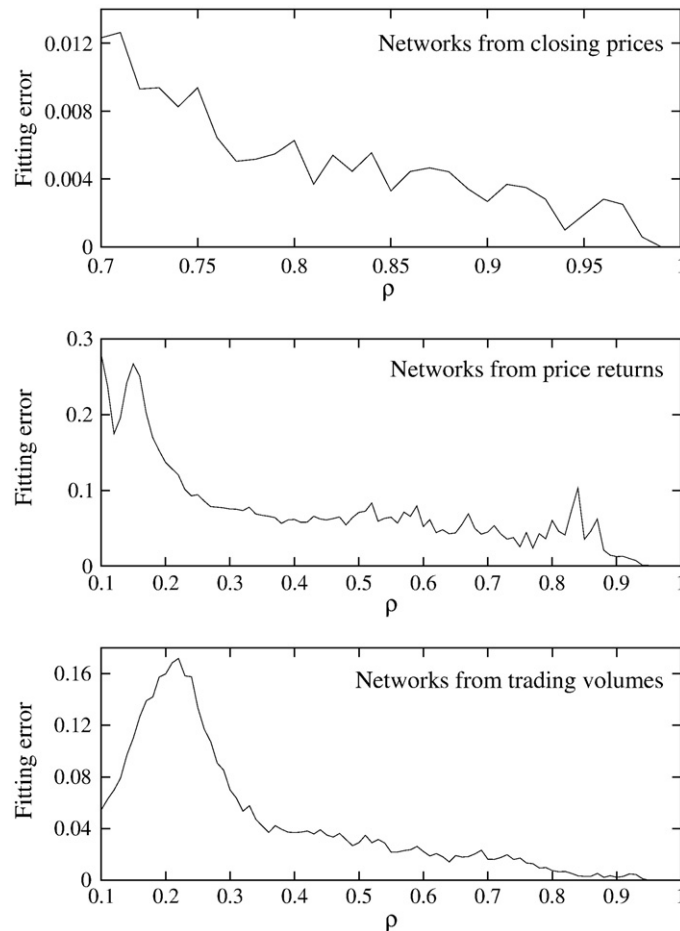


Fig. 3. Fitting error vs. threshold ρ for networks formed from closing prices (upper left panel); daily price returns (upper right panel); and daily trading volumes (lower panel), using Set 1 data. This generally shows that the degree distribution is only scalefree when ρ is large. For small ρ , the networks tend to be fully connected and do not display scalefree distribution.

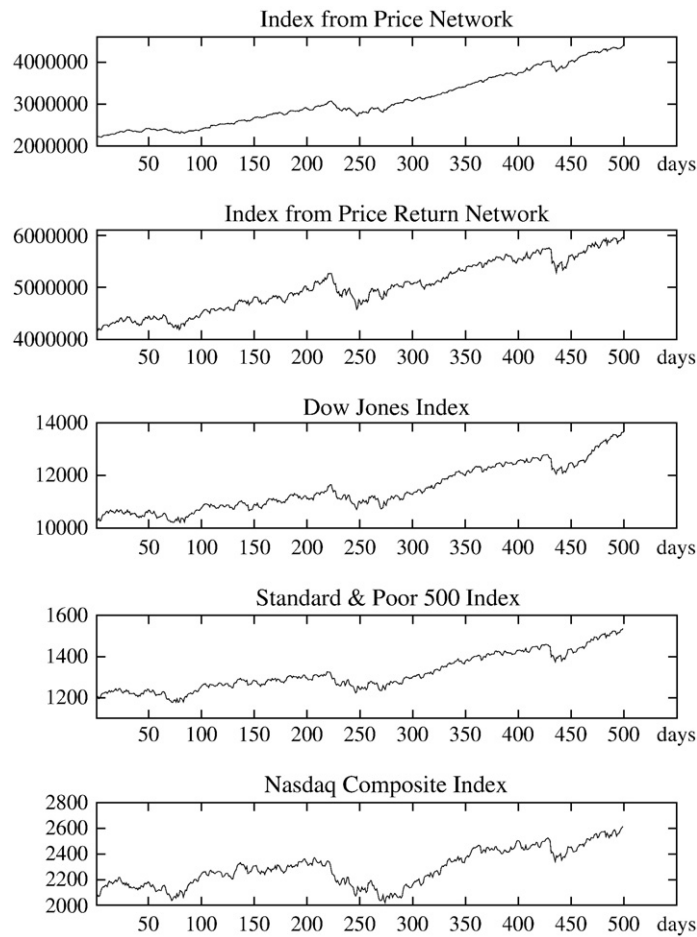


Fig. 4. Comparison of new degree-based indexes based on the closing price network with (first upper panel) and price return network (second upper panel), with Dow Jones index, S&P500 index and Nasdaq Composite index. The new indexes are computed using Eq. (5) and from the top 10% of most highly “connected” stocks.

description of statistical tests. Here, we clearly see that the correlations between the degree-based indexes and the existing indexes are statistically significant.

7. Discussions

Although the degree-based indexes are found to be correlated with other existing indexes, the basis of their compositions is completely different. The new indexes are defined in terms of highly connected stocks, and no human intervention is involved. They reflect the market trend in terms of the performance of the majority of stocks. Because the degree distribution of the stock network is scalefree, relatively few highly connected stocks are sufficient to represent the majority of stocks. In other words, the scalefree property has been exploited to ensure a high level of representation of the degree-based indexes. We may also claim that

Table 4

Spearman's correlations between new degree-based indexes and other existing indexes, with p -value < 0.001 .

Degree-based indexes		
	Closing price network	Price return network
Dow Jones	0.9838 (for Set 1)	0.9885 (for Set 1)
	0.9869 (for Set 2)	0.8091 (for Set 2)
Standard & Poor's 500	0.9657 (for Set 1)	0.9799 (for Set 1)
	0.9879 (for Set 2)	0.8138 (for Set 2)
Nasdaq	0.8501 (for Set 1)	0.8754 (for Set 1)
	0.9449 (for Set 2)	0.8175 (for Set 2)

Table 5

Comparison of compositions of new degree-based index and S&P500.

Degree-based index based on price return network			
Composition by sectors	S&P500	Set 1 Data (July 2005 to August 2007)	Set 2 Data (June 2007 to May 2009)
Basic materials	6.0%	3.6%	4.8%
Capital goods	3.8%	1.2%	3.8%
Conglomerates	1.8%	0.8%	0.4%
Consumer cyclical	0.4%	0.2%	1.2%
Consumer non-cyclical	7.8%	0.6%	1.2%
Energy	7.0%	2.2%	5%
Financial	18.6%	68.0%	56.8%
Healthcare	10.0%	0.2%	2.4%
Services	21.0%	15.0%	16.6%
Technology	15.6%	0.8%	3.4%
Transportation	1.8%	0.2%	0.6%
Utilities	6.2%	4.8%	0%
Unclassified	0%	2.4%	3.8%

since the stock market is scalefree, our proposed indexes are highly indicative of the market performance because the selected representative stocks are the most highly connected stocks. On the other hand, existing indexes reflect the performance of stock markets on the basis of stocks that are “selected” from different business and industrial sectors according to criteria defined by the publishing institutions. The bases of selection for the degree-based indexes and existing indexes are different. Table 5 shows a comparison of the compositions of the degree-based indexes and the S&P500 index. It is interesting to note that while the composition of the S&P500 index is generally consistent with that of the degree-based indexes, the degree-based indexes are clearly more biased by stocks in the financial sector. This clearly suggests that the stock market is actually dominated by the financial sector. A graphical view is shown in Fig. 5.

8. Conclusion

Complex networks have been constructed for US stocks (all the US stocks that were traded each trading day over two selected period of time, from July 1, 2005 to August 30, 2007 and from June 1, 2007 to May 30, 2009. To the best of our knowledge, this is the first attempt of constructing such a large scale stock network. The construction procedure is based on connecting any two stocks whose daily price time series are “similar” in terms of cross correlation evaluated over a period of time. It has been found that the networks formed by connecting stocks of highly correlated closing prices, price returns and trading volumes are all *scalefree*. Some network parameters have been calculated. This result suggests that a relatively small number of stocks are exerting much of the influence over the majority of stocks. New indexes that reflect on the performance of the majority stock may be defined based on a relatively small number of highly connected stocks. It has been found that the market is actually heavily influenced by stocks in the financial sector.

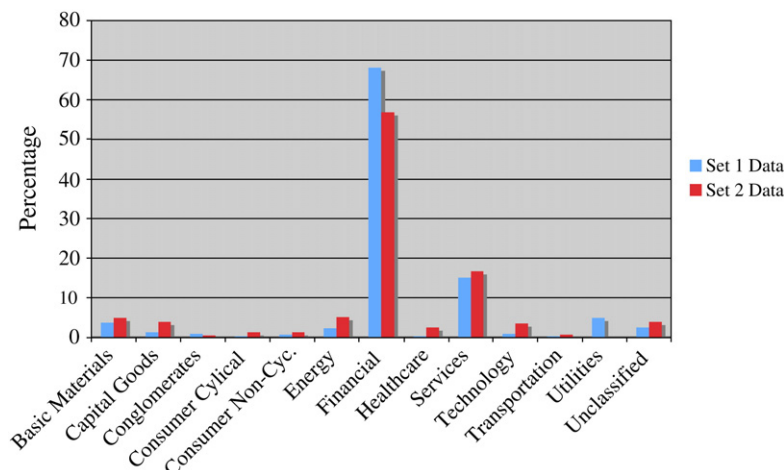


Fig. 5. Bar chart of stock composition comparison. Two sets of data show consistent result that the top highly connected stocks are dominated by the financial sector. Refer to Table 5 for data.

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References

- Barabasi, A.-L., Albert, R., 1999. Emergence of scaling in random networks. *Science* 286, 509–512.
- Bonanno, G., Lillo, F., Mantegna, R.N., 2001. High-frequency cross-correlation in a set of stocks. *Quantit. Finan.* 1, 96–104.
- Bonanno, G., Caldarelli, G., Lillo, F., Micciché, S., Vandewalle, N., Mantegna, R.N., 2004. Networks of equities in financial markets. *Euro. Phys. J. B* 38, 363–371.
- Bonanno, G., Caldarelli, G., Lillo, F., Mantegna, R.N., 2003. Topology of correlation-based minimal spanning trees in real and model markets. *Phys. Rev. E* 68, 046103.
- Campbell, R.A.J., Forbes, C.S., Koedijk, K.G., Kofman, P., 2008. Increasing correlations or just fat tails? *J. Empirical Finance* 15, 287–309.
- Cohen, J., Cohen, P., West, S.G., Aiken, L.S., 2003. *Applied Multiple Regression/Correlation Analysis for the Behavioral Sciences*, 3rd ed. Lawrence Erlbaum Associates, Hillsdale, NJ.
- Csanyi, G., Szendroi, B., 2004. Structure of a large social network. *Physical Review E* 69, 036131-1-5.
- Hollander, M., Wolfe, D.A., 1973. *Nonparametric Statistical Methods*. Wiley, New York.
- Longin, F., Solnik, B., 2001. Extreme correlation of international equity markets. *J. Finance* 56 (2), 649–676.
- Mantegna, R.N., 1999. Hierarchical structure in financial markets. *Euro. Phys. J. B* 11, 193–197.
- Newman, M.E.J., 2001. Scientific collaboration networks I: Network construction and fundamental results. *Phys. Rev. E* 64, 016131-1-8.
- Onnela, J.P., Chakraborti, A., Kaski, K., 2003. Dynamics of market correlations: taxonomy and portfolio analysis. *Phys. Rev. E* 68, 056110.
- Onnela, J.P., Kaski, K., Kertesz, J., 2004. Clustering and information in correlation based financial networks. *Euro. Phys. J. B* 38, 353–362.
- Ravid, G., Rafaeli, S., 2004. Asynchronous discussion groups as small world and scale free networks. *Peer-Reviewed J. Internet* 9 (9) 2004.
- Strogatz, S.H., 2001. Exploring complex networks. *Nature* 410, 268–276.
- Tumminello, M., Aste, T., Di Matteo, T., Mantegna, R.N., 2005. A tool for filtering information in complex systems. *Proc. Natl. Acad. Sci. USA* 102, 10421–10426.
- Vandewalle, N., Brisbois, F., Tordoir, X., 2001. Self-organized critical topology of stock markets. *Quantit. Finan.* 1, 372–375.
- Zheng, X., Lau, F.C.M., Tse, C.K., 2006. Study of LPDC codes built from scale-free networks. *Proc. Int. Symp. Nonlinear Theory and Its Applications*, Bologna, Italy, pp. 563–566.