

## **PROPERTIES OF DFT**

### **Aim**

To perform the following properties of DFT:

1. Linearity
2. Convolution
3. Multiplication
4. Parsevals theorem

### **Theory**

#### 1. Linearity

The linearity property of DFT states that the DFT of a linear weighted combination of two or more signals is equal to similar linear weighted combination of the DFT of individual signals.

Let,

$$\text{DFT}\{x_1(n)\}=X_1(K) \text{ \& \text{DFT}\{x_2(n)\}=X_2(K)}$$

then,

$$\text{DFT}[a_1x_1(n)+a_2x_2(n)]=a_1X_1(K)+a_2X_2(K) \text{ where } a_1 \text{ \& } a_2 \text{ are constants}$$

#### 2. Convolution

The Circular Convolution of two N-Point Sequences  $x_1(n)$  &  $x_2(n)$  is defined as,

$$x_1(n) \otimes x_2(n) = \sum_{m=0}^{N-1} x_1(n) x_2((n-m))_N$$

The Convolution Property of DFT says that, the DFT of circular convolution of two sequences is equivalent to product of their individual DFTS. Let,

$$\text{DFT}[x_1(n)]=X_1(K) \text{ and } \text{DFT}(x_2(n))=X_2(K)$$

then, By Convolution property,

$$\text{DFT}(x_1(n) \otimes x_2(n))=X_1(K) X_2(K)$$

#### 3. Multiplication

The Multiplication Property of DFT says that the DFT of product of two discrete time sequences is equivalent to circular convolution of DFT's of the individual sequences scaled by the factor of  $1/N$ . If,

$$\text{DFT}(x(n))=X(K)$$

then,

$$\text{DFT}(x_1(n) x_2(n))=1/N[X_1(K)\otimes X_2(K)]$$

#### 4. Parseval theorem

Let  $\text{DFT}(x_1(n))=X_1(K)$  &  $\text{DFT}(x_2(n))=X_2(K)$  then by Parseval' theorem

$$\sum_{n=0}^{N-1} x_1(n) x_2^*(n) = 1/N \sum_{k=0}^{N-1} X_1(K) X_2^*(K)$$

### Program

#### 1)Linearity property

```
clc;
clear all;
close all;
x1=[1 2 3 4];
x2=[2 1 2 1];
a1=2;
a2=3;
x1k=fft(x1);
x2k=fft(x2);
lhs=(a1*x1k)+(a2*x2k);
lhsk=fft(lhs);
disp('LHS=');
disp(lhsk);
rhsk=(a1*x1k)+(a2*x2k);
disp("RHS=");
disp(rhsk);
```

#### 2)Convolution property

```
clc;
```

```

close all;
clear all;
x1=[1 2 3 4];
x2= [2 1 2 1];
N=max(length(x1), length(x2));
x1new=[x1 zeros(1, N-length(x1))];
x2new=[x2 zeros(1, N-length(x2))];
X1= fft(x1new);
X2 =fft(x2new);
circular_conv_time =cconv(x1new, x2new, N);
product_freq =ifft(X1 .*X2);
disp("x1(n) cconv x2(n):");
disp(circular_conv_time);
disp("IDFT(X1(k)*X2(k)):");
disp(product_freq);

```

### **3)Multiplication property**

```

clc;
close all;
clear all;
x1 = [1 2 1 2];
x2 =[1 2 3 4];
N=max(length(x1), length(x2));
x1new=[x1 zeros(1, N-length(x1))];
x2new=[x2 zeros(1, N-length(x2))];
product_time=x1new.*x2new;
dft_product_time=fft(product_time);
X1=fft(x1new);
X2=fft(x2new);

```

```

Y=cconv (X1,X2, N);
disp("DFT(x1(n)*x2(n))");
disp(dft_product_time);
disp("X1(k)circonvX2(k)/N:");
disp(Y./N);

```

#### 4)Parsevals theorem

```

clc;
clear all;
close all;
x1=[1 2 1 1];
x2=[1 2 3 4];
N =max(length(x1), length(x2));
x1new=[x1 zeros(1, N-length(x1))];
x2new=[x2 zeros(1, N-length(x2))];
time_domain_value=sum(x1new.*conj (x2new));
freq_domain_value = sum(fft(x1new).*conj (fft(x2new)))/ N;
disp("Sum{n:0->N-1; x1(n) *conj(x2(n))}");
disp(time_domain_value);
disp("Sum{k:0->N-1;X1(k)*conj(X2(k))}/N:");
disp(freq_domain_value);

```

#### Result

Performed various properties of dft and verified the outputs.

## Observation

### 1)Linearity property

LHS=

$$38.0000 + 0.0000i -4.0000 + 4.0000i \quad 2.0000 + 0.0000i -4.0000 - 4.0000i$$

RHS=

$$38.0000 + 0.0000i -4.0000 + 4.0000i \quad 2.0000 + 0.0000i -4.0000 - 4.0000i$$

### 2)Convolution property

$x_1(n)$  cconv  $x_2(n)$ :

$$14 \quad 16 \quad 14 \quad 16$$

IDFT( $X_1(k)*X_2(k)$ ):

$$14 \quad 16 \quad 14 \quad 16$$

### 3)Multiplication property

DFT( $x_1(n)*x_2(n)$ ):

$$16.0000 + 0.0000i \quad -2.0000 + 4.0000i \quad -8.0000 + 0.0000i \quad -2.0000 - 4.0000i$$

$X_1(k)$  circonv  $X_2(k)/N$ :

$$16.0000 + 0.0000i \quad -2.0000 + 4.0000i \quad -8.0000 + 0.0000i \quad -2.0000 - 4.0000i$$

### 4)Parsevals theorem

$\sum_{n:0 \rightarrow N-1} x_1(n) * \text{conj}(x_2(n))$ :

$$12$$

$\sum_{k:0 \rightarrow N-1} X_1(k) * \text{conj}(X_2(k)) / N$ :

$$12$$