EC3093D Digital Signal Processing Lab
Winter Semester 2023-24

Experiment 4: FFT and Z-Transform

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General instructions

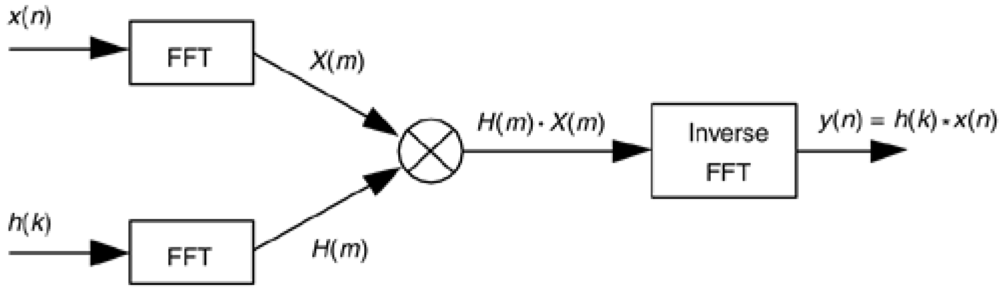
- Use Matlab for all problems
- **Report evaluation** will be **based on all the problems** in this experiment.
- **Lab output evaluation** will be **based only on** the problems or parts of problems which are marked as **'Do in lab'**
- **Report** needs to be submitted (as one pdf file per group) via Eduserver (before deadline) and will be evaluated out of a *maximum* of 200 marks, and split-up is given alongside the problems.
    - For each problem, the report should cover
        1. the problem (copy from this document)
        2. the result depicted using necessary figure(s) with proper title, legends, axis labels etc with readable font sizes.
        3. a **brief** discussion of the results, highlighting only the salient/interesting points.
- **Lab output** will be evaluated out of a *maximum* of 100 marks, and split-up is given alongside the problems.
    - the result depicted using figure(s) with proper title, legends, axis labels etc with readable font sizes.
- You need to be familiar with all the problems. **Experiment quiz will cover all problems**.
- You can get up to 10% extra marks by solving the Bonus question and submitting along with the report
- Needless to say, no built-in commands in Matlab should be used, except when specifically allowed in the description of the question.
- For queries regarding this experiment, contact anup.aprem@nitc.ac.in

Problems
Part - A

1. **FFT**: Implement the radix-2 FFT algorithm to compute the N-point DFT $X[k]$ of a sequence $x[n]$ of length N. Assume $N = 2^m$. Make 'N' a variable. Modify to compute the N -point IDFT $x[n]$ of $X[k]$. **Do in lab (25 marks)**
2. Test the FFT program in (1) by computing DFT for
    a. A rectangular signal
    b. sinusoidal signal
    c. Unit impulse. **Do in lab (10 marks)**

   Try to reconstruct the signals back from the spectrum using your IDFT program. Observe the effect of zero padding using different number of samples, while plotting the spectrum.

3. **Fast linear convolution using FFT**
    a. Use *audioread*, read the sample audio waveform *acoustic.wav* and the impulse response of a cathederal *impulse_cathederal.wav* (the impulse response models the reverberation that you typically observe in a cathederal).
    Perform convolution (Use the code you have written in Experiment 1!) between the audio signal and the impulse response. Find the time required to perform the convolution. **Do in lab (5 marks)**
    b. Another approach to do convolution is by using FFT as follows.



    Note here that the input $x[n]$ and $h[n]$ should be appropriately zero-padded (as done in Experiment 3).
    Find the time required to perform convolution using this approach, using the FFT code that you have written. Is there any difference, in the time required to perform convolution, between the FFT approach and using straightforward convolution? What are your inferences? **Do in lab (20 marks)**
    c. Real time implementation: Audio signals are typically quite long (though in our example, this is not the case). Hence, for real time implementation, processing should happen in segments.
    Two such approaches are: Overlap-add and Overlap-save. Implement either overlap-add or overlap-save method for performing convolution. Modify the algorithm to use the FFT based filtering as in (b) to perform convolution. Compare the execution time for the different approaches as a function of block size (a parameter in overlap-add/overlap-save method). **Bonus (10 %)**

<u>Part – B</u>

1. Write a MATLAB program to compute and display the poles and zeros of a transfer function.

    To do this, first compute and display the factored form of the transfer function. Then, generate the pole-zero plot of a transfer function, that is as a ratio of two polynomials in z^{-1}. Using this program, analyze the z-transform of

$$H(z) = \frac{2 + 5z^{-1} + 9z^{-2} + 5z^{-3} + 3z^{-4}}{5 + 45z^{-1} + 2\,z^{-2} + z^{-3} + z^{-4}}$$

Do in lab (20 marks)

2. From the pole-zero plot generated in Qn (1), determine the possible ROCs. Can you tell from the pole-zero plot whether or not the DTFT exists? Is the system stable if it is causal? **Do in lab (20 marks)**

3. Using zp2tf, determine the rational form of a z transform whose zeros are at $s_1 = 0.3$, $s_2 = 2.5$, $s_3 = -0.2 + j0.4$, and $s_4 = -0.2 - j0.4$; the poles are at $p_1 = 0.5$, $p_2 = -0.75$, $p_3 = 0.6 + j\,0.7$, and $p_4 = 0.6 - j\,0.7$; and the gain constant $k = 3.9$. **Homework (25 marks)**

4. Using impz() determine the first 10 samples of the inverse Z transform of
$$H(z) = \frac{z^{-1}}{3 - 4z^{-1} + z^{-2}}$$
Using residuez obtain the partial fraction expansion of $H(z)$. From the partial fraction expansion, write down the closed form expression of the inverse Z transform (assuming causal). Evaluate the first 10 samples of the closed form expression for $h[n]$ using Matlab and compare with the result obtained using impz. **Homework (25 marks)**

5. Using residuez convert back the partial fraction expression for $H(z)$ in Part (4) to the rational function form. **Homework (25 marks)**

6. Determine the zeros for the systems $H_1(z) = 6 + z^{-1} - z^{-2}$ and $H_2(z) = 1 - z^{-1} - 6z^{-2}$, and indicate whether the system is minimum-phase, maximum-phase or mixed-phase system. Show responses. **Homework (25 marks)**

Background Reading & Quiz

1. FFT- Decimation in Time, Decimation in Frequency
2. Linear convolution, Circular convolution
3. Overlap-add, Overlap-save
4. Direct z-transform
5. Region of Convergence
6. Inverse z-transform
7. Transfer function
8. Poles and Zeros
9. Minimum Phase, Maximum Phase, Mixed-Phase systems