NATIONAL INSTITUTE OF TECHNOLOGY CALICUT

EC3093D Digital Signal Processing Lab

Winter Semester 2023-24

Experiment No. 6: IIR Filter design (Tool – MATLAB)

Part A

(Realization of Digital Filters, Magnitude Response and Phase Response, Design using Bilinear Transformation and Impulse Invariant Design Methods)

1. Write MATLAB codes for obtaining the
   (i) parallel form realization and
   (ii) cascade form realization

   for the following transfer functions:

   a. $H(z) = \frac{0.4(1-z^{-2})}{1+1.2z^{-1}+0.32z^{-2}}$
   b. $H(z) = \frac{z^2-0.16}{z^2+1.1z^1+0.18}$ (10 Marks)

2. Given each of the following digital transfer functions,
   (i) $H(z) = \frac{0.35(1+z^{-1})}{1-0.3z^{-1}}$
   (ii) $H(z) = \frac{0.1(1-z^{-1})}{1+0.8z^{-1}}$
   (iii) $H(z) = \frac{0.7(1-z^{-2})}{1-1.26z^{-1}+0.4z^{-2}}$
   (iv) $H(z) = \frac{0.8(1-1.6z^{-1}+z^{-2})}{1-1.28z^{-1}+0.6z^{-2}}$
   (v) $H(z) = \frac{0.5+0.7z^{-1}+z^{-2}}{1+0.7z^{-1}+0.5z^{-2}}$

   a. Plot the magnitude response and phase response for each transfer function.

   b. Identify the corresponding filter type, such as lowpass, highpass, bandpass, bandstop or allpass. (20 Marks)

3. Given a lowpass prototype $H_P(s) = \frac{1}{s+1}$, determine each of the following analog filters and plot their magnitude responses from 0 to 200 radians per second.
   (i) The lowpass filter with a cutoff frequency of 40 radians per second.
   (ii) The highpass filter with a cutoff frequency of 40 radians per second.

(iii) The bandpass filter with a center frequency of 100 radians per second and bandwidth of 20 radians per second.

(iv) The bandreject filter with a center frequency of 100 radians per second and bandwidth of 20 radians per second. (10 Marks)

4. Given a lowpass prototype $H_P(s) = \frac{1}{s+1}$ with a cutoff frequency of 1 rad/sec, use BLT to design the corresponding digital IIR filters given below. Use MATLAB to plot the magnitude response and phase response of the designed filters.
   (i) Lowpass filter with a cutoff frequency of 50 Hz, assuming a sampling rate of 300 Hz.
   (ii) Highpass filter with a cutoff frequency of 50 Hz, assuming a sampling rate of 300 Hz.
   (iii) Bandpass filter with a lower cutoff frequency of 60 Hz, an upper cutoff frequency of 80 Hz, and a sampling rate of 350 Hz.
   (iv) Bandstop filter with a lower cutoff frequency of 60 Hz, an upper cutoff frequency of 80 Hz, and a sampling rate of 350 Hz. (30 Marks)

5. Use impulse-invariant method for designing the digital filters $H(z)$ from the corresponding Laplace transfer functions $H(s)$ given below. Use sampling rate f_s = 10 Hz. Also, plot the magnitude frequency response and the phase frequency response with respect to $H(s)$ and $H(z)$ for the frequency range from 0 to $\frac{f_s}{2}$ Hz.
   (i) $H(s) = \frac{3}{s+3}$
   (ii) $H(s) = \frac{1}{s^2+3s+2}$
   (iii) $H(s) = \frac{s}{s^2+4s+5}$ (30 Marks)

Part B

(MATLAB functions, Digital Butterworth and Chebyshev Filter Design, Alternate IIR Filter Design, Lattice Structure)

1. **MATLAB functions**: Familiarize various MATLAB functions e.g. freqs, freqz etc. Plot the responses of the filters in Part A using MATLAB functions and compare the responses.

2. **Digital Butterworth Filter Design:** Lowpass Butterworth filters are all-pole filters characterized by the magnitude-squared frequency response

$$|H(\Omega)|^2 = \frac{1}{1+(\Omega/\Omega_C)^{2N}} = \frac{1}{1+\epsilon^2(\Omega/\Omega_P)^{2N}}$$

where N is the order of the filter, Ω_C is the cutoff frequency or the -3-dB frequency, Ω_P is the passband edge frequency, and $\frac{1}{1+\epsilon^2}$ is the band-edge value of $|H(\Omega)|^2$. If A_P dB be the passband ripple and A_S dB be the stopband attenuation then it can be written that

$$A_P \text{ dB} = -20\, log_{10} \frac{1}{\sqrt{1+\epsilon^2}}$$

$$A_S \text{ dB} = -20\, log_{10} \frac{1}{\sqrt{1+\epsilon^2(\Omega_S/\Omega_P)^{2N}}}$$

ϵ and N can be calculated from the above equations.

- Study the design of Butterworth Filters using BLT Method.
- Design a digital lowpass Butterworth filter with 3 dB attenuation at 1.5 kHz, 10 dB stopband attenuation at 3 kHz and sampling frequency of 8,000 Hz. Use MATLAB to plot the magnitude and phase responses.
- Explore the MATLAB functions buttord, butter etc.

3. **Digital Chebyshev Filter Design:** There are two types of Chebyshev filters. Type I Chebyshev filters are all-pole filters that exhibit equiripple behavior in the passband and a monotonic characteristic in the stopband. On the other hand, the family of type II Chebyshev filters contains both poles and zeros and exhibits a monotonic behavior in the passband and an equiripple behavior in the stopband. The zeros of this class of filters lie on the imaginary axis in the s -plane. The magnitude squared of the frequency response characteristic of a type I Chebyshev filter is given as

$$|H(\Omega)|^2 = \frac{1}{1+\epsilon^2 T_N^2(\frac{\Omega}{\Omega_P})}$$

where ϵ is a parameter of the filter related to the ripple in the passband and $T_N(x)$ is the Nth-order Chebyshev polynomial defined as

$$T_N(x) = \begin{cases} cos(N\, cos^{-1}\, x), & |x| \leq 1 \\ cosh(N\, cosh^{-1}\, x), & |x| > 1 \end{cases}$$

As before, if A_P dB be the passband ripple and A_S dB be the stopband attenuation then it can be written that

$$A_P \text{ dB} = -20\, log_{10} \frac{1}{\sqrt{1+\epsilon^2}}$$

$$A_S \text{ dB} = -20\, log_{10} \frac{1}{\sqrt{1+\epsilon^2 T_N^2(\frac{\Omega_S}{\Omega_P})}}$$

ϵ and N can be calculated from the above equations.

- Study the properties of the Chebyshev polynomials.
- Study the design of Butterworth Filters using BLT Method.
- Design a first-order digital highpass Chebyshev filter with a cutoff frequency of 3 kHz and 1 dB ripple on passband using a sampling frequency of 8,000 Hz. Use MATLAB to plot the magnitude and phase responses.
- Explore the MATLAB functions cheb1ord, cheby1 etc.

4. **Pole-Zero Placement Method for IIR Filter Design:**
   - Study the basics of IIR filter design using Pole-Zero Placement Method.
   - What are the constraints that should be imposed while designing the filters?

5. **All-Pole Lattice Structure:**
   - Study the All-Pole Lattice Structure.
   - Compare the lattice structure with the direct form structure in terms of the number of delays and multipliers.

References:

1. John G. Proakis, Dimitris G. Manolakis, Digital Signal Processing: Principles, Algorithms and Applications, 4th Edition, Pearson India, 2007.
2. Oppenheim A. V., Schafer R. W, Digital Signal Processing, Pearson India, 2015.
3. Mitra S. K., Digital Signal Processing: A Computer Based Approach, McGraw-Hill Publishing Company, 2013.