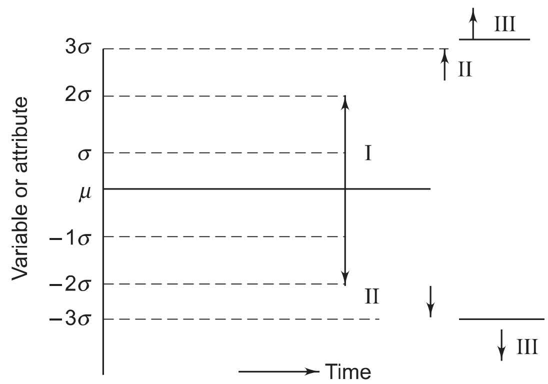


Figure 12.5 Three Zones in Control Charts

CONTROL CHART FOR VARIABLE DATA

We will look at the following types of control charts using variable data.

- x bar and range chart (we can also plot standard deviation instead of range)
- x and moving range chart

X Bar and Range Chart

We plot this chart in the following manner. Samples from regular production are picked up at random. The minimum number of samples or sample size should be greater than or equal to four to get consistent results. Usually the samples are picked up at regular intervals in a shift or in a day or more. Usually 25 such samples are picked up. Measurements are to be carried out on a selected parameter and then listed sub-group wise. For each sub-group, the average and range are to be calculated.

Example 12.1

A jewellery shop was making 10 grams gold coins for sale. At regular intervals, four coins each were selected at random. The weights of the gold coins were measured. The table below indicates the measured values of 25 sub-groups of gold coins (called x). Against each column, the x bar values were calculated, which are nothing but the average of the four values. Just following that is the range of each sub-group, which is nothing but the difference between the largest value and the lowest value.

Values of gold coin — x

Sub group	1	2	3	4	5	6	7	8	9	10	11	12	13
	10.1	9.9	10.2	10	10.6	10	10	10.1	10.3	10.1	9.8	10.1	10
	9.9	10	10.1	9.8	9.9	9.9	9.8	9.9	10	10	10.1	10.2	9.6
	9.9	9.9	9.9	10.2	10.2	10.1	10.1	9.9	9.9	10	10	10	10.2
	10	10.1	10	10	10	9.9	9.9	9.9	10	10.1	10.1	10	10.1

Calculated x bar

9.975	9.975	10.05	10	10.175	9.975	9.95	9.95	10.05	10.05	10	10.075	9.975

Calculated range

0.2	0.2	0.3	0.4	0.7	0.2	0.3	0.2	0.4	0.1	0.3	0.2	0.6

Values of gold coin — x

Sub group	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
	10.2	9.9	10.3	9.9	10.6	10.2	9.8	10.1	10.3	10.1	9.8	10.1
	9.8	10	10.1	9.8	9	9.9	9.8	9.9	10.1	9.9	10.1	10.2
	9.7	10	9.2	10.2	10.2	10.1	10.1	9.9	9.9	10	9.7	9.6
	10	10.1	10	10.1	9.8	9.9	9.9	10.1	10	10.1	10.1	10

Calculated x bar

9.925	10	9.9	10	9.9	10.025	9.9	10	10.075	10.025	9.925	9.975

Calculated range

0.5	0.2	1.1	0.4	1.6	0.3	0.3	0.2	0.4	0.2	0.4	0.6

Now, we have to calculate the average range —R bar. This is nothing but the average of the ranges of the 25 sub-groups.

$$R \text{ bar} = 0.412$$

Now, we have the calculate the Upper Control Limit (UCL) for the R chart.

$$\text{UCL} = \text{D4} \times R \text{ bar}$$

The Lower Control Limit (LCL) is calculated using the following formula:

$$\text{LCL} = \text{D3} \times R \text{ bar}$$

We have to get the values of D3 and D4 from Table C. Since D3 is zero for a sample size of 4, LCL will also be equal to zero.

UCL = D4 × Rbar = 0.9394, LCL=0

Now, we plot the R chart, which is given in Fig.12.6:

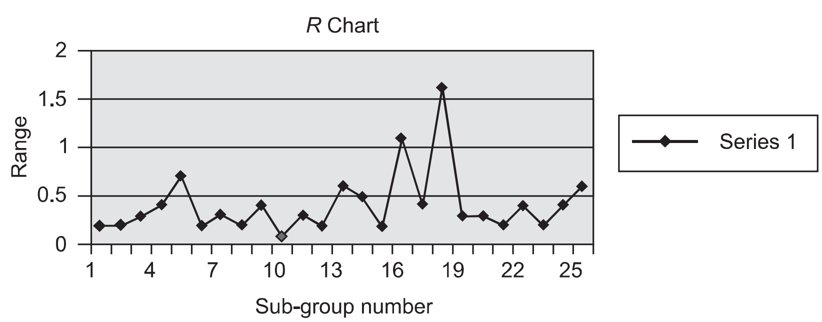


Figure 12.6

We now find that the range R in the chart exceeds the control limits in respect of two samples namely sub-group number 16 and 18. This could be due to assignable causes or in some cases measurement errors. Therefore, these two sub-groups are to be eliminated. Data after removing outliers are given below:

First revision after eliminating outliers

	10.1	9.9	10.2	10	10.6	10	10	10.1	10.3	10.1	9.8	10.1
	9.9	10	10.1	9.8	9.9	9.9	9.8	9.9	10	10	10.1	10.2
	9.9	9.9	9.9	10.2	10.2	10.1	10.1	9.9	9.9	10	10	10
	10	10.1	10	10	10	9.9	9.9	9.9	10	10.1	10.1	10
$\bar{x}$:	9.975	9.975	10.05	10	10.175	9.975	9.95	9.95	10.05	10.05	10	10.075
R :	0.2	0.2	0.3	0.4	0.7	0.2	0.3	0.2	0.4	0.1	0.3	0.2

	10	10.2	9.9	9.9	10.2	9.8	10.1	10.3	10.1	9.8	10.1
	9.6	9.8	10	9.8	9.9	9.8	9.9	10.1	9.9	10.1	10.2
	10.2	9.7	10	10.2	10.1	10.1	9.9	9.9	10	9.7	9.6
	10.1	10	10.1	10.1	9.9	9.9	10.1	10	10.1	10.1	10
$\bar{x}$:	9.975	9.925	10	10	10.025	9.9	10	10.075	10.025	9.925	9.975
R :	0.6	0.5	0.2	0.4	0.3	0.3	0.2	0.4	0.2	0.4	0.6

We will now calculate R bar and UCL and LCL.

R bar = 0.3304

UCL = D4 × Rbar = 0.7534, LCL = 0

The R chart is plotted again and given in Fig.12.7:

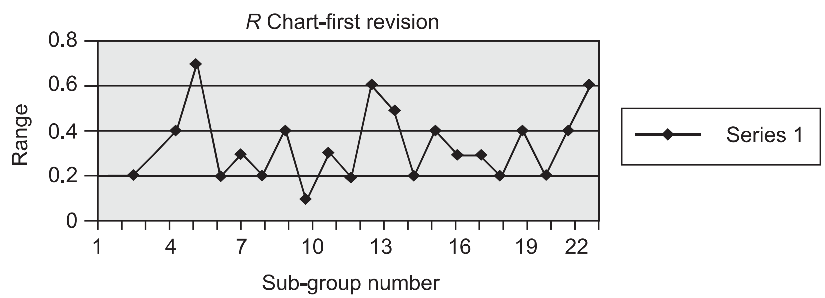


Figure 12.7

Now, there are no outliers. Therefore, the R bar (central value) and UCL and LCL are noted and frozen. It is now time to plot x bar chart. The x bar chart is nothing but a graph of x bar on the y-axis and sub-group numbers in the x-axis. The calculation of x bar bar (grand average) and UCL and LCL for the x bar chart are given below:

Grand average (x bar bar) = 10.002

UCL for x bar = x bar bar + A2 × R bar = 10.243

LCL for x bar = x bar bar – A2 × R bar = 9.761

The grand average is the average of the 23 sub-groups (since we have removed two outliers in the range chart). The values for A2 is to be obtained from Table B.

The x bar chart is given in Fig.12.8.

Let us now summarize the steps involved for arriving at $\bar{x}$ and R charts.

1. Decide on the parameter to be controlled.

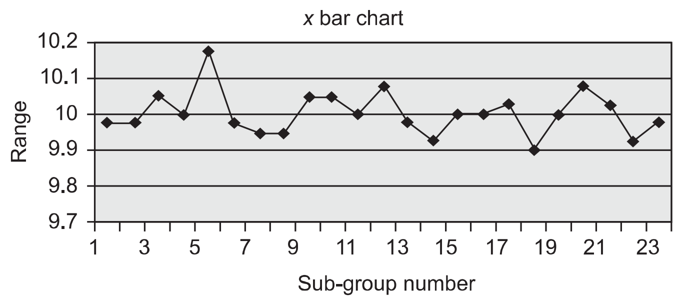


Figure 12.8

2. Pick up at least 4 samples at a time and measure the performance on the selected parameter as per step 1; pick up the samples 25 times.
3. Find $\bar{x}$ and R for each subgroup.
4. Find $\bar{R}$ as an average of R of the 25 subgroups.
5. Calculate UCL and LCL for R chart, taking D4 and D3 from Table C.
   UCL R = D4 *R* bar
   LCL R = D3 *R* bar
6. Plot the *R*-values.
7. Check whether any *R*-values lie outside UCL or LCL.
8. If all *R*-values are within limits, go to step 11.
9. If upto two *R*-values are outside the limits, eliminate the respective subgroups and repeat steps 4 to 8.
10. If more than two *R*-values lie outside limits initially or if some values go out of limits after recalculation in step 9 stop. Find out special causes and take action to improve the process. Start all over again after the process is stable.
11. The grand average *x* bar bar is the average of the remaining sub-groups. Calculate $\bar{\bar{x}}$ excluding the subgroups, if any whose *R* were out of limits.
12. Calculate LCL, UCL, action limits as well as warning limits for $\bar{\bar{x}}$ and *R* Charts.
    UCL *x* bar = *x* bar bar + A2 *R* bar.
    LCL *x* bar = *x* bar bar – A2 *R* bar.

    Now, plot the *x* bar chart. Check that sample averages ($\bar{x}$) of all the sub-groups lie within the control limits.

Now, let us take one more example of *x* bar and *R* control chart.

Example 12.2

A manufacturer of 300 grams weight wants to carry out a process capability study. For this purpose, he has picked up 25 sub-groups of 4 samples of weights manufacturing over the period of 2 days. Values of 300 grams weights are given:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
301	299	302	300	306	300	300	301	303	301	295	301	300
299	300	301	298	299	299	298	299	300	294	301	302	296
299	299	299	302	302	301	301	299	295	310	300	300	302
300	301	300	302	307	300	301	302	300	302	312	301	304

The corresponding *x* bar and the range of each group are given below:

x bar												
299.75	299.75	300.5	300.5	303.5	300	300	300.25	299.5	301.75	302	301	300.5
range												
2	2	3	4	8	2	3	3	8	16	17	2	8

Values of 300 grams weights – x

	14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
	302	299	303	299	306	302	298	301	303	301	302	301
	298	300	301	298	298	299	298	299	301	299	303	301
	297	300	298	302	302	301	301	299	299	300	302	301
	304	301	301	301	304	304	302	302	301	301	302	302
x bar												
	300.25	300	300.75	300	302.5	301.5	299.75	300.25	301	300.25	302.25	301.25
range												
	7	2	5	4	8	5	4	3	4	2	1	1

The *R* bar, UCL and LCL for the above data are calculated and given below:

R bar = 4.96

UCL = D4 × Rbar = 11.309, LCL = 0

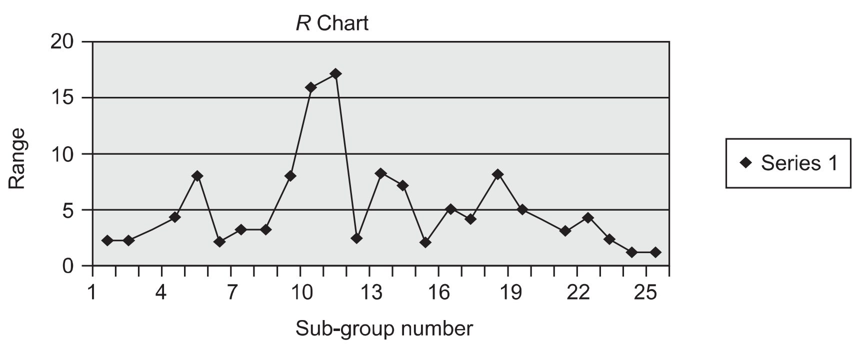


Figure 12.9

It is found that the range of sub-groups 10 and 11 are beyond the UCL. Therefore, these two outliers are eliminated and given below:

First revision after eliminating outliers

	301	299	302	300	306	300	300	301	303	301	300	302	299
	299	300	301	298	299	299	298	299	300	302	296	298	300
	299	299	299	302	302	301	301	299	295	300	302	297	300
	300	301	300	302	307	300	301	302	300	301	304	304	301
$\bar{x}$:	299.75	299.75	300.5	300.5	303.5	300	300	300.25	299.5	301	300.5	300.25	300
R:	2	2	3	4	8	2	3	3	8	2	8	7	2

	303	299	306	302	298	301	303	301	302	301
	301	298	298	299	298	299	301	299	303	301
	298	302	302	301	301	299	299	300	302	301
	301	301	304	304	302	302	301	301	302	302
$\bar{x}$:	300.75	300	302.5	301.5	299.75	300.25	301	300.25	302.25	301.25
R:	5	4	8	5	4	3	4	2	1	1

The *R* bar, UCL and LCL for the range chart calculated after revision are given below:

R bar = 3.9565

UCL = D4 $\times$ *R*bar = 9.0209   LCL = 0

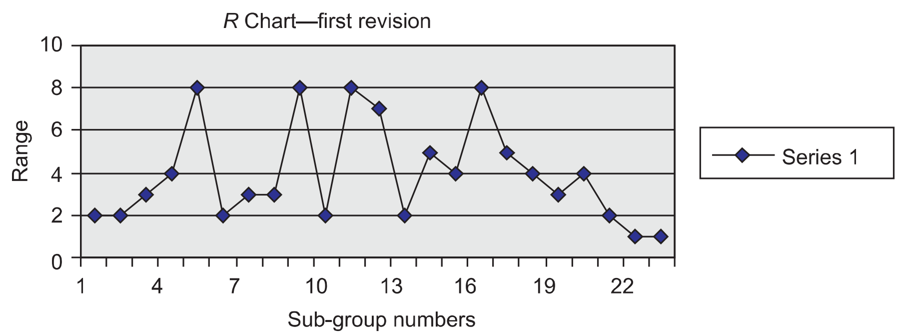


Figure 12.10

Now, there are no outliers. Hence, we will now proceed to plot the *x* bar chart. But, before that we have to calculate the grand average and the control limits. The calculation is given below:

x bar bar = 300.65

UCL = *X* bar bar + A2 $\times$ R bar = 303.54

LCL = *X* bar bar – A2 $\times$ *R* bar = 297.76

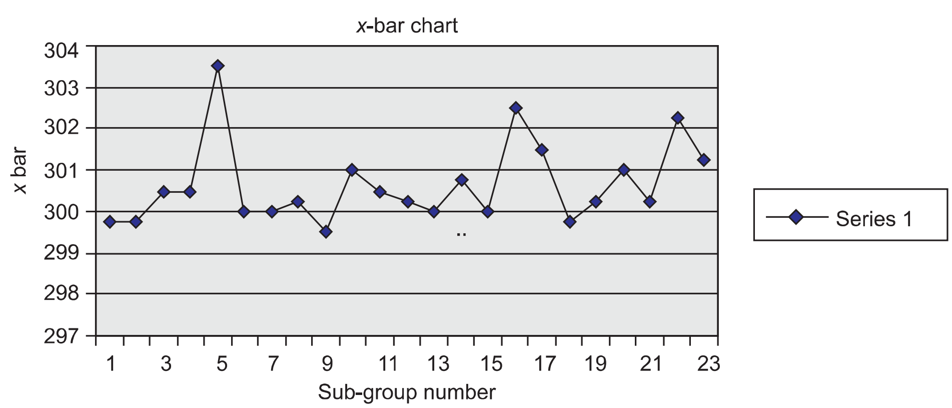


Figure 12.11

In the *x* bar chart also, there are no outliers.

x bar and *R* charts are quite useful for study of process performance when we collect data as variables. The control chart indicates whether the process is in control. It indicates clearly the variation due to assignable causes. The points we were eliminating in the subsequent iterations in the worked examples can be due to assignable causes or error in measurements.

Warning and Action Zones

The warning zone starts at $\bar{\bar{x}} \pm 2\ \sigma/\sqrt{n}$ or

$\bar{\bar{x}} \pm 2/3\ A_2\ R$ bar

The action zone starts at $\bar{\bar{x}} + \sigma/\sqrt{n}$ or

$\bar{\bar{x}} \pm A_2\ R$ bar

Thus the action zone starts at UCL and LCL in the average chart. The warning line starts at 2/3 of UCL and LCL.

Example 12.3

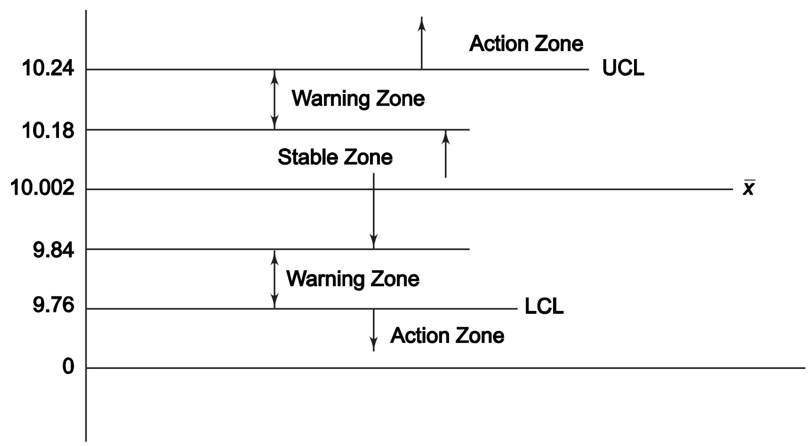
In our worked example 12.1, in the $\bar{x}$ chart action zone lies above 10.24 and below 9.76.

The warning zone starts at 10.18.

Similarly, the lower warning limit starts at 9.84.

Therefore, when the results fall above 10.24, but below 9.76, then we have to analyze and take action.

The warning and action zones for $\bar{x}$ chart are illustrated in Fig. 12.12:



Warning and Action Zones

Figure 12.12

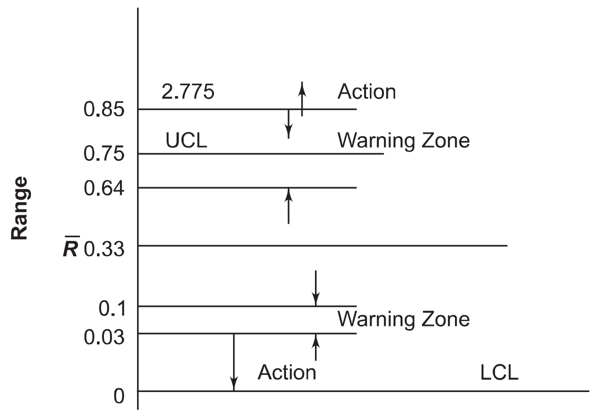
Control limits for Range

	Known R	Known σ
Upper action line	$D^{1}_{\cdot 001}\ \overline{R}$	$D_{0.001}\sigma$
Lower action line	$D^{1}_{\cdot 999}\ \overline{R}$	$D_{0.999}\ \sigma$
Upper warning line	$D^{1}_{\cdot 025}\ \overline{R}$	$D_{0.025}\ \sigma$
Lower warning line	$D^{1}_{\cdot 975}\ \overline{R}$	$D_{0.975}\ \sigma$

The values of constants can be obtained from Table C. For the worked example 12.1:

Upper action line = $2.57 \times 0.33 = 0.85$
Lower action line = $0.1 \times 0.33 = 0.03$
Upper warning line = $1.93 \times 0.33 = 0.64$
Lower warning line = $0.29 \times 0.33 = 0.1$
We know UCL = 0.75, LCL = 0.

The control lines for the worked example are given in Fig. 12.13:



***R* Chart**

Figure 12.13

Let us look at another example to reconfirm our understanding.

Example 12.4
The peanut manufacturer has observed the following at the end of the day.
No. of sub-groups = 20
Size of sub-group = 5

$$\Sigma \bar{x} = 6000$$
$$\Sigma R = 40$$

Draw the control limits for $\bar{x}$ and Range.

Step 1 : Find out $\bar{\bar{x}}$

$$\bar{\bar{x}} = 6000/20 = 300$$

Step 2 : Find out $\bar{R}$

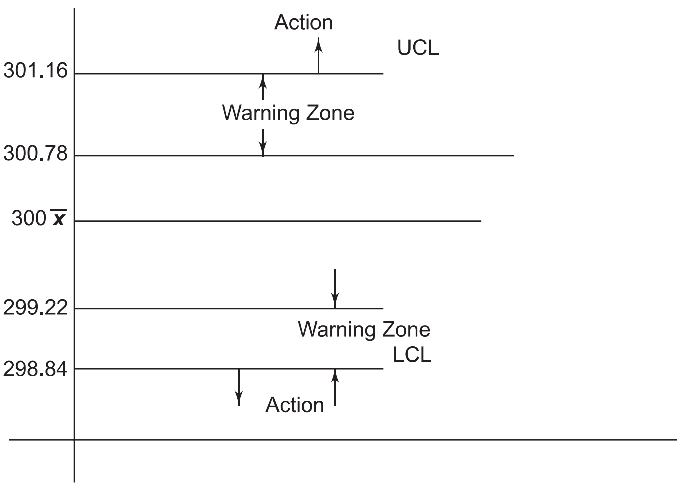
$$\bar{R} = 40/20 = 2$$

$\bar{x}$ Chart

$$\begin{aligned} \text{UCL} &= \bar{\bar{x}} + A_2 \bar{R} \\ &= 300 + 0.58 \times 2 \\ &= 301.16 \\ \text{LCL} &= 300 - (0.58 \times 2) \\ &= 298.84 \end{aligned}$$

Upper warning line $= \bar{\bar{x}} + 2/3\ A_2\ \bar{R}$
$= 300 + 0.39 \times 2 = 300.78$
Lower warning line $= 300 - 0.78$
$= 299.22$

Now we can draw the control limits for $\bar{x}$



$\bar{x}$ Chart

Figure 12.14

$\bar{R}$ Chart

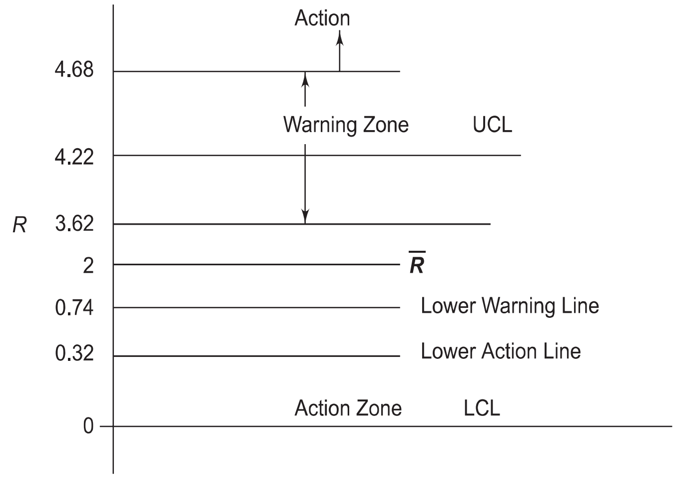
UCL = D4 $\bar{R}$ = $2.11 \times 2 = 4.22$
Upper Action Line = $2.34 \times 2 = 4.68$
Upper Warning Line = $1.81 \times 2 = 3.62$
Lower Warning Line = $0.37 \times 2 = 0.74$
Lower Action Line = $0.6 \times 2 = 0.32$
LCL = 0, since D3 = 0

The control limits for *R* Chart are shown in Fig. 12.15:

Indications of Abnormality in the Process The process is said to be out of control, when the performance lies outside $\pm 3\ \sigma$ limits (UCL or LCL) in the *x* bar chart. The performance of the variable should preferably lie between $\pm 2\ \sigma$. However, the performance of some patterns as given below calls for immediate action to analyze the causes and taking corrective actions:

Performance within ± 2 σ

1. Seven consecutive points showing a continuously increasing or decreasing pattern.
2. Seven points remaining close to the central line.
3. Seven consecutive values showing either a continuous upward trend or downward trend.
4. Values showing a cyclic pattern.



***R* Chart**

Figure 12.15

Performance in the warning zone (outside ± 2 σ, but within ± 3 σ)

5. No incidence of two consecutive points in the warning zone.
6. Not more than four points in warning zone.

PROCESS CAPABILITY INDICES

The fundamental requirement of any process is that it should be stable first. Stability is indicated by consistent performance of the process within the limits set. Only variations allowed are the common cause variations. Therefore, statistical control implies performance of the process within the set limits. Only thereafter, the process will be predictable. Therefore, capability can be assessed only on a process, which is consistently stable over a period of time.

The process is abnormal when there are variations due to special or assignable causes. The abnormal process is out of control. When there are special causes, the process should be stopped to find out the special causes and eliminating them.

We will study the capability of the process, which has variations only due to common causes. The process capability indices are deduced to find a relationship between variations due to common causes and tolerance.

Recall the areas under the normal curve. We know that the area under the curve between ± 2 σ is 95.45%. This means 4.55% of the curve is outside this total 4 σ limits. If the tolerance of the process is equal to ± 2 σ, then 4.55% of the products will be defective. If the tolerance is equal to ± 3 σ, then 0.27% of the products will be defective. The tolerance limits are actually set by customer and what is under the manufacturer's control is the process performance and statistical control limits.

The process capability has two indices namely *Cp* and *Cpk*.

Cp

Cp = USL – LSL/6 σ

Where USL : Upper Specification Limit
LSL : Lower Specification Limit

$$\sigma = \bar{R}/d_2$$

Let us consider various possibilities with regard to *Cp*.

$Cp < 1$: This means that the normal curve extends beyond $\pm 3\,\sigma$ limits (UCL and LCL). Therefore, the process is incapable. When $Cp > 1$, it becomes increasingly capable, provided the grand average is maintained at (USL + LSL)/2.

Cpk

Cp can be applied only when the process is correctly centered about the mid specification since it takes into account the precision with the total tolerance. To know how well a process is performing both accuracy and precision should be compared. C_{pk} takes into account both the degree of random variation and accuracy. This is the most popular process capability index.

$$C_{pk} = \text{minimum of } (\text{USL} - \bar{\bar{x}}/3\,\sigma,\ \bar{\bar{x}} - \text{LSL}/3\,\sigma)$$

When $C_{pk} <= 1$, the variations and centering may cause infringing into one of the tolerance limits and hence process not capable. If the process means coincides with mid specifications, then both C_{pk} and *Cp* will give the same value. C_{pk} gives additional information about the centering. Therefore, it is also called process performance index. Hence, increasing value of C_{pk} means that the process is increasingly becoming capable.

Example 12.5

Look at worked example 12.1 after eliminating outliers.

Let USL = 10.5 and LSL = 9.5

$$\bar{\bar{x}} = 10.002$$

$$\bar{R} = 0.3304$$

$$\sigma = \bar{R}/d_2 \text{ (take value of } d_2 \text{ from Table B)}$$

$$= 0.3304/2.059 = 0.13$$

$$Cp = 10.5 - 9.5/6\ (0.13) = 1.28$$

$$\text{USL} - \bar{\bar{x}} = 10.5 - 10.002 = 0.498$$

$$\bar{\bar{x}} - \text{LSL} = 10.002 - 9.5 = 0.502$$

Therefore, $C_{pk} = 0.498/3\ (0.13) = 1.27$

Since C_{pk} is almost equal to C_p, we can conclude that the process is nearly centered. Since they are greater than 1, it is capable provided we maintain grand average at mid-specification limits.

Example 12.6

Let the specification limit be 300 ± 10 for data is Example 12.2

$$\bar{\bar{x}} = 300.65$$

$$\sigma = \bar{R}/\mathrm{d}_2 = 3.9565/2.059 = 1.9$$

$$C_p = 20/6 \times 1.9 = 1.8$$

$$\bar{\bar{x}} - \text{LCL}/3 \times 1.9 = 300.65 - 290/5.7 = 1.9$$

Or

$$USL - \bar{\bar{x}}/5.7 = 310 - 300.65 / 5.7 = 1.7$$

$$\therefore \quad C_{pk} = 1.7$$

The process is capable but not centered as the data also reveals.

Example 12.7

Given sample size = 4 and 25 samples were considered. The process is under control.

$$\bar{\bar{x}} = 105$$
$$USL = 110$$
$$LSL = 90$$
$$\sigma = 2$$
$$Cp = 20/6 \times 2 = 1.66$$

$USL - \bar{\bar{x}}$ is the lowest.

Therefore, $$C_{pk} = 110 - 105/3 \times 2 = 5/6 = 0.83$$

Since, Cp is 1.66, the process could be highly capable. But $Cp = 0.82$ indicates that the capability has been reduced due to shifting of the mean to the right. There may be processes, which are under statistical control, meaning that the variation is due to random causes. But the process may not be capable as inferred from the C_{pk} value. This may be overcome by widening the tolerance limits, provided the customer accepts it. If not, the rejects will be high which have to be weeded out before supply. Irrespective of sample size, $C_{pk} < 1$ indicates an incapable process. If C_{pk} greater than 1, but upto 2, we have to confirm the process capability through more samples. $C_{pk} > 3$ is definitely a capable process.

X and *MR* Chart

$\bar{x}$ and R chart requires at least four items in each sub-group. In some industries such as chemical industry or when the cost of samples is high, we can have only one item per subgroup. In such cases, we cannot find the range R. In such situations, we can use moving range (*MR*) chart. Moving range is the difference between the value and the one immediately preceding it. In this case, we use the *X* and *MR* chart. Thus, there are two sets of data, one is the measurement called '*X*' and the other is the moving range called '*MR*'.

The parameters of *X* and *MR* charts are given below:

Parameter	***X***	***MR***
Centre	$\bar{x}$	$\overline{MR}$
UCL	$\bar{x} + 3\ (\overline{MR}/1.13)$	$\overline{MR} \times 3.27$
LCL	$\bar{x} - 3\ (\overline{MR}/1.13)$	0

Example 12.8

The 11 subgroups of 1 sample each had values as indicated below:

100 101 100 102 100 99 100 98
99 100 101

Calculate control limits for *MR* chart.

Solution

Let us first calculate the average of the values $\bar{x}$ = 100

We will get 10 moving ranges for 11 subgroups or values. The absolute value of the difference between one value and the next is to be calculated. Thus *MR* will be:

1	1	2	2	1	1	2	1	1	1

$$\therefore \quad \overline{MR} = 13/10 = 1.3$$

$$\text{UCL MR} = \overline{MR} \times 3.27$$
$$= 1.3 \times 3.27 = 4.251$$
$$\text{LCL } MR = 0$$

Now, we can calculate control limits for the *X* chart.

$$\text{UCL} = \bar{x} + 3\,(\overline{MR}/1.13)$$
$$= 100 + 3\,(1.3)/1.13 = 103.4$$
$$\text{LCL} = \bar{x} - 3(\overline{MR}/1.13)$$
$$= 96.6$$

Now we can plot the values in *X* and *MR* Chart.

Example 12.9

The temperatures of a steam bath were recorded and shown below (*x* values). Plot *x* and *MR* chart,

x values

100	101	100	102	103	101	100	99	101	102	103	102	101	100
101	102	102	103	101	99	98	100	101	102				

The moving range is the difference between the adjacent values (*MR*). The moving ranges are calculated and given below:

MR values

1	1	2	1	2	1	1	2	1	1	1	1	1	1
1	0	1	2	2	1	2	1	1					

Then, we calculate *X* bar, which is the average of the x values. The average of *MR* values which is *MR* bar. The control limits for *MR* charts are given below:

UCL = *MR* bar x D4

Note:

Since the sample size is one always, in the moving range chart, we take D4 as 3.27 corresponding to a sample size of 2.

LCL = 0

For *x* chart,

UCL = *x* bar + 3 (*MR* bar/1.13)

LCL = *x* bar – 3 (*MR* bar/1.13)

The charts are given in Fig.12.16:

UCL *MR*	3.9809
LCL	0

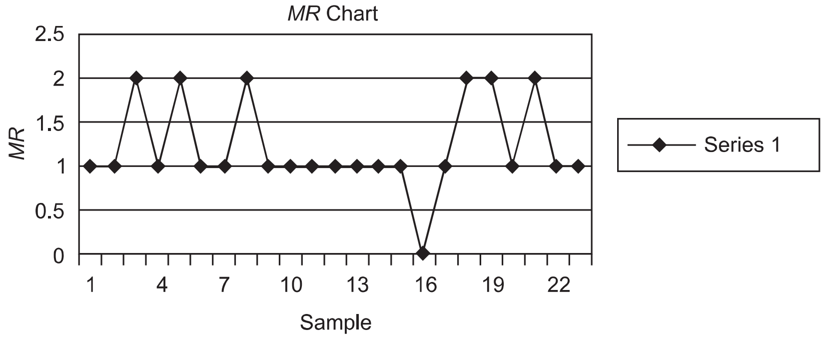


Figure 12.16

Since there are no outliers, we do not have to eliminate any points. Now, we can plot x chart and see whether there are any outliers.

x bar = 101
MR bar = 1.2174
UCL *X* = 104.23
LCL *X* = 97.768

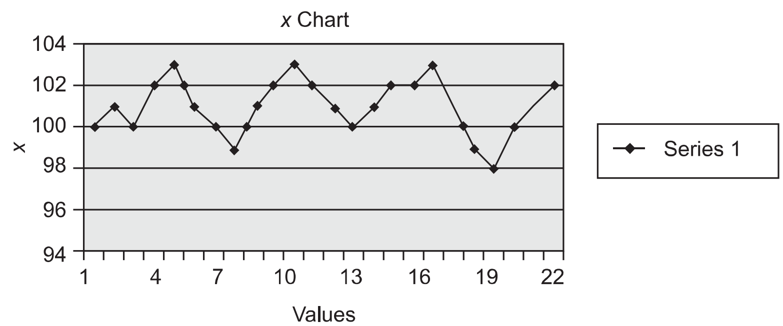


Figure 12.17

This chart is useful in the following situations:

- Samples are more expensive
- The data is accumulated slowly over a period of time
- Each lot consists of only one sample

CONTROL CHARTS BY ATTRIBUTES

The control chart by variables is quite useful to study the performance against individual characteristics of products. The control charts are to be plotted for each quality characteristics. An entity may contain hundreds of quality characteristics. This means that we need to plot one chart for each characteristic. This may neither be feasible nor desirable in the normal circumstances. Hence, this chart is more expensive. At the same time, the usefulness of the chart should not be undermined because this gives micro level analysis of the quality of the products manufactured. This can be used selectively.

When a quality problem is found and if the organization wants to carry out further analysis, then specific characteristics of the entity can be measured and controlled using *X* bar and *R* control charts.

Usually, the organizations collect data on the number of non-conforming products. This data is readily available. Hence, it is rather easier to prepare control charts by attributes. The attribute charts are likely to be less expensive. Furthermore, the top management will be interested in the overall results and hence attribute charts fit the bill. Thus, the control chart by attributes could be the starting point in every organization. Here, we count the occurrences of failures or defects in a product or number of defectives, percentage of non-conforming products, etc. The *x* bar and *R* chart is based on normal frequency distribution. However, the attributes charts are based on distributions as given below:

Binominal Distribution	**Poisson Distribution**
np chart	*c* chart
p chart	*u* chart

Let us now look at the charts based on binominal distribution. Here, we count the number of non-conforming products, i.e. the number of defectives. The *np* chart helps us to analyze the quality of the process by looking at the number of non-conforming entities. The *p* chart on the other hand looks at the fraction rejected due to non-conformances to specifications or fraction defectives.

Let us first look at the *np* chart.

np Chart

The *np* chart is used when the sample size is constant. Such conditions occur in manufacturing organizations. The organization may carry out inspection of constant size of products and records the number of defective units in each sample.

Example 12.10

The following table indicates the number of defectives in a toilet soap manufacturing organization. Sample size is constant and equal to 100. The steps involved in finding out the control limits are given below:

- Collect data about the number of defective units in each sample. The sample size may be 100 or any convenient number, which is called *n*. The number of defectives in each subgroup is called *np*.
- Collect these data periodically say 25 times. The number of samples, i.e. the number of sub-groups = *N*.
- Calculate the average number of defectives – *np* bar, which is equal to $\Sigma np/N$.
- Calculate *p* bar, which is the average fraction defective. This is equal to *np* bar/*n*.
- Now, the central line is *np* bar.
- The control limits are *np* bar +/– 3 × √*np* bar (1–*p* bar)

np = defectives	0	1	5	6	4	2	8	2	1	5	3	4	6	1	0	5	7	2	6	1

np bar = 3.45
p bar = 0.0345
UCL = 8.925288
LCL = 0

The *np* chart plotted as per Fig. 12.18 is a graph of fraction defective vs. time or the sample number. We can also plot percent defective chart using the same data. The differences lie in the following:

- Percent defective is 100 × fraction defective
- In the control limits, we will have (100 – *p* bar) instead of (1 – *p* bar).

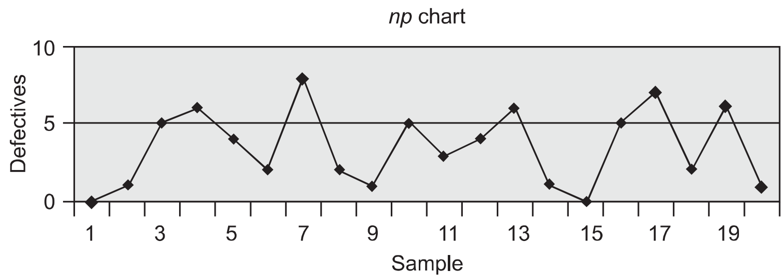


Figure 12.18

p chart

In the above example, if the sample size is not constant, then we can plot a *p* chart. The *p* chart is more popular and is widely used. This chart is used to plot the fraction rejected. The fraction-rejected *p* is defined as the ratio of number of non-conforming items to the total number of items inspected. As the name suggests, it is a fraction less than 1.

The *p* chart can also be plotted as per cent rejected which will be hundred times *p*.

The steps involved in finding out the control limits are given below:

- Collect the data about the number of defective units in each sample of unequal size. The sample size may be noted along with the number of defective units.
- Calculate *p* (per cent or fraction defective). When the number of defects are divided by the number of samples and expressed as a fraction or a percentage, it is called *p*.
- The individual size of samples is equal to *n*. *n* is not a constant and it may vary from sample to sample.
- Collect these data periodically say 25 times. The number of samples is equal to *N*.
- Average sample size is equal to $\Sigma n / N$.
- Average fraction or per cent defective, *p* bar is equal to $\Sigma p/N$.
- Now, the central line is *p* bar.
- The control limits for fraction defective chart are *p* bar ± 3 × √*p* bar(1–*p* bar)/*n* bar.
- The control limits for per cent defective chart are *p* bar ± 3 × √*p* bar(100–*p* bar)/*n* bar.
- If the lower control limit is less then zero, then LCL = 0.

Example 12.11

In a cell phone manufacturing plant samples were taken from each day's production and tested and number of defectives found each day was recorded. It is given below:

No of defectives	2	1	5	1	4	5	2	3	1	0	0	2	5	4	1	3	2	1	5	0
Sample size	50	55	80	70	90	60	72	80	90	50	81	92	55	63	70	59	58	62	70	75
percent defectives	4	1.8	6.3	1.4	4.4	8.3	2.8	3.8	1.1	0	0	2.2	9.1	6.3	1.4	5.1	3.4	1.6	7.1	0

Average sample size = 69
p bar = 3.51224

UCL = 10.15595
LCL = 0

The *p* chart follows.

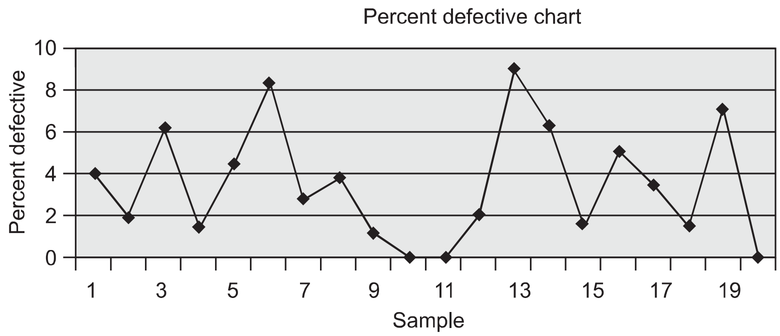


Figure 12.19

The selection of a p chart or np chart depends on the convenience. If the number of items inspected varies from sample to sample, then p chart is ideal. However, if the sample size is constant, then np chart will be useful.

c CHART

The control chart by variables namely X bar and R chart is plotted for chosen quality characteristics, which is measured on the continuous scale. The p or np charts are useful for characterizing the number of defectives. The control charts for non-conformities is called c chart. It is useful when a non-conforming item contains one or more non-conformances, i.e. it contains one or more defects. When it is necessary to study the total number of non-conformances in a product or a group of equal number of similar products, we can use the control chart techniques based on Poisson distribution. The c and u charts are based on Poisson distribution. The c chart is applicable when the number of products inspected is a constant. Usually it is one item. The variable c is the number of defects found in the constant sample size. When the sample size varies, i.e. when the opportunity for occurrence of non-conformities change from sample to sample then a u chart is used.

c charts are used for understanding the number of defects in a specific portion of the population. For instance, this can be used to analyze the number of defects in a specified area like 10 cm × 10 cm in a printed circuit board. The requirement is that the samples should have the same area or dimensions. For instance, the contamination in one litre of water can be studied using a c chart by drawing samples periodically.

The steps involved in finding out the control limits are:

- Collect data about the number of defects per unit of the same size. Collect 25 such samples. Call them c.
- Calculate c bar, which is $\Sigma c/n$.
- The control limits for c are c bar ± 3 × $\sqrt{c}$ bar.

Example 12.12

An automobile company has a painting section. The number of defects were counted in an area of 1 metre by 1 metre of the chassis. The number of defects for the same area in different samples is listed in table below. Plot a c chart.

C, no. of defectives	0	1	5	6	4	2	8	2	1	5	3	4	6	1	0	5	7	2	6	1

Solution

c bar = 3.45
UCL = 9.022253
LCL = –2.12225 (treat as zero)

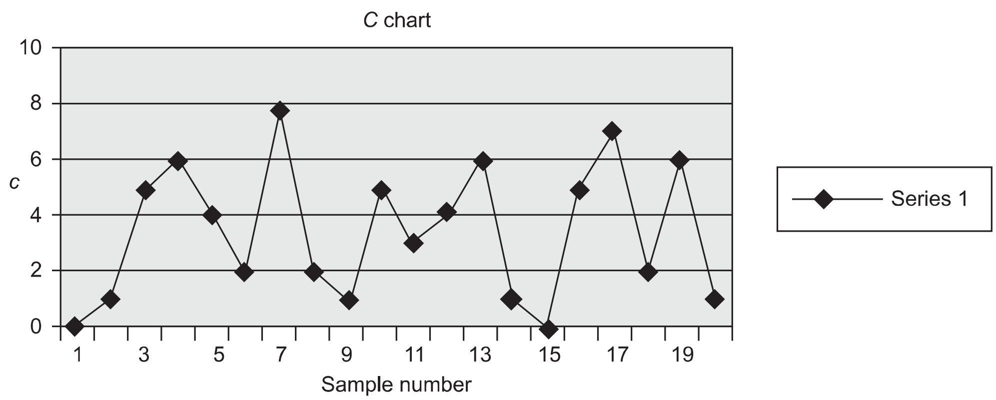


Figure 12.20

u Chart

The u chart is quite useful in software industry. It is more flexible than the c chart. In a u chart, the unit inspected is not a constant. Therefore, we normalize the defects for a standard unit.

Example 12.13
The number of defects found in the software program and the total number of lines of program are given in the table below. Plot a u chart.

No. of defectives	2	1	5	1	4	5	2	3	1	1	1	2	5	4	1	3	2	15	2
Sample size (kilo lines of code)	5	5	5	5	4	4	4	4	4	4	4	4	4	5	5	5	5	55	5

Solution
The number of defects and sample size are indicated in the above table. We have to normalize the above by finding out defects per kilo line of code. The normalized defects per unit are given below. The central line is u bar.

u bar = $\Sigma u/N$
where N = No. of samples

However, we have a difficulty in finding out the control limits. Here, we will have multiple control limits depending upon each sample size. The general formula is:
Control Limit = u bar ± 3 × $\sqrt{u \text{ bar}/n}$

Therefore, while drawing the control limits corresponding to each sample size, we have to determine both UCL and LCL for the size of the sample. In the above example, there are only two sample sizes. Therefore, we have to calculate the control limits for the sample size of 4 and 5 separately. Once this is done, we can plot the u as well as the control limits.

Defects per unit *u* 0.4 0.2 1 0.2 1 1.3 0.5 0.8 0.3 0.3 0.3 0.5 1.3 0.8 0.2 0.6 0.4 0.2 1 0.4

U bar	0.57
UCL for 5 samples	1.6
LCL for 5 samples	–1 treated as zero
UCL for 4 samples	1.7
LCL for 4 samples	–1 treated as zero

The *u* chart is given below along with the control limits.

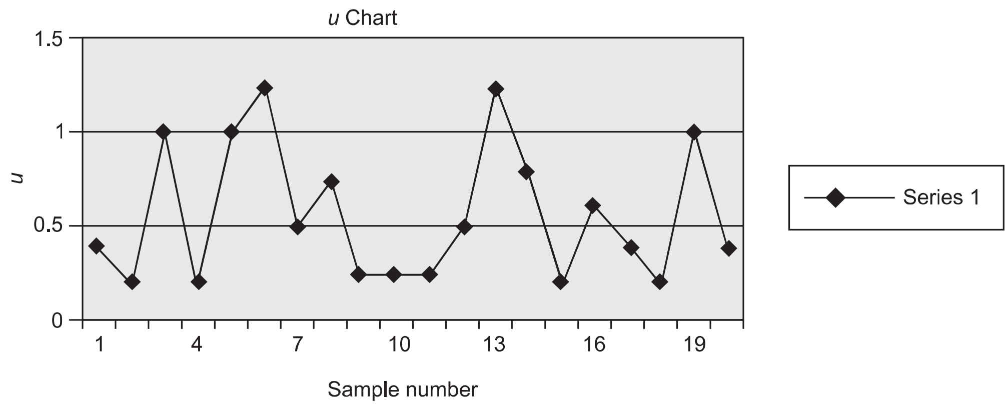


Figure 12.21

Comparison between control chart by variables and by attributes

	Variable Charts	*Attribute Charts*
Chart type	*X* bar and *R* or σ , *x* and *MR*	*np, p, c, u*
Type of data	Measured on a continuous scale	Number of defectives or defects
Advantages	Useful for improvement of product quality characteristics	Data collection is easy and economical
	Detailed information about chosen characteristics of a process or product	Provides a holistic view
	Focused on specific parameters	Data readily available in every organization
	Suitable for detailed analysis	Easily understandable since it is pass or fail criteria only
Disadvantages	Cannot be used with go/no go type inspection	Not suitable for control and improvement of individual parameters of processes or products
	Expensive	Severity of defects not visible
		Not detailed enough
Application	Control and improvement of individual parameters of processes such as temperature, dust, etc. and every characteristics of products such as length, noise, gain, etc.	Control of number of defects in a unit or number of defectives—usually product oriented.

The need for control charts:

- Provides information about process capability
- Provides a basis for predicting future performance
- Provides a basis for measuring improvement
- Control charts help in getting the process under control and thereby prevent defects from occurring.
- In accordance with Juran's chain reaction, the above leads to improving quality, productivity and increased market share.
- Prevents unnecessary process adjustments and enables necessary process adjustments
- Provides a lot of diagnostic information
- Helps in identifying assignable causes immediately
- Gives feedback to the process owners
- Helps the management to keep the processes under control and effect savings
- Enables highest ROI.

The summary of control charts discussed in this chapter is:

Variables

Chart Name	*Measures plotted*	*Centre line*	*Control limits*	*Remarks*
x bar	Average of individual samples—*x* bar	*x* bar bar	*x* bar bar ± A2 × *R* bar	*x* bar bar Σ *x*bar/*N* (*N* = No. of samples)
R	Range of each sample—*R*	*R* bar	UCL = D4 × *R* bar LCL = D3 × *R* bar	*R* bar = Σ*R*/*N* *N* = No. of samples
x	Individual values—*x*	*x* bar	*x* bar ± 3 (*MR* bar/1.13)	*x* bar = Σ*x*/*n*
MR	Moving range—*MR*	*MR* bar	UCL = *MR* bar × 3.27 LCL = 0	*MR* bar = Σ*MR*/*n*-1 (n = no. of values)

Attributes

Chart Name	*Measures plotted*	*Centre line*	*Control limits*	*Remarks*
np	*np* – number of defectives in samples of constant size of n	*np* bar	*np* bar ± 3 √*np* bar (1–*p* bar)	*n* – sample size *N* – No. of samples *np* – defects in the individual sub-groups *np* bar = Σ*np*/*N* *p* bar = *np* bar/*n*
p	*p* – fraction defectives in samples of varying sizes	*p* bar	*p* bar ± 3 √*p* bar (1–*p* bar)/*n* bar	*p* bar = Σ *p*/*N* *N* = No. of samples in the data *n* bar = average sample size

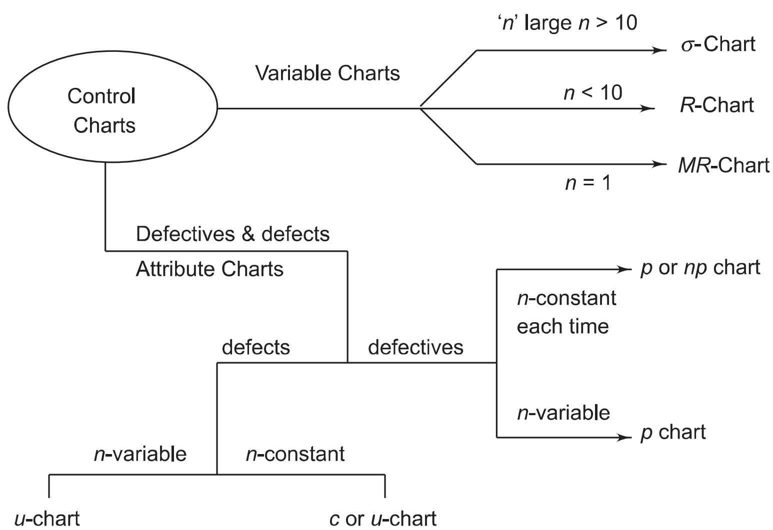
(Contd.)

(Contd.)

Chart Name	*Measures plotted*	*Centre line*	*Control limits*	*Remarks*
c	*c* – No. of defects or deviations in samples of constant size	*c* bar	*c* bar ± 3 √*c* bar	*c* bar - average number of defects in sample of constant size
u	*u* – No. of defects per unit in the samples of variable sizes	*u* bar	*u* bar ± 3 √*u* bar/*n* For each unique *n* we have to calculate the limits	*u* – defects per sample *u* bar = Σ*u*/*N* *n* – sample size *N* – No. of samples

GUIDANCE FOR SELECTION OF CHARTS

Guidance for selection of appropriate control chart is given in Fig.12.22:



(One defective module can have many defects)

Figure 12.22

SUMMARY

This chapter gives the most important tools to assess the process capability. The aim of TQM is continuous improvement of every process in the organization. The central limit theorem states that the sample averages distribute more closely around μ than the individual values of the population. Irrespective of the underlying distribution of the population, if four or more samples are drawn at random from the lot, the sample averages will be normally distributed. That is the basis for control charts. Control charts are plots of process performance with time. There are two types of control charts :

- by variables
- by attributes

When we measure a quality characteristics on a continuous scale, we use control chart by variables. On the contrary, when we count the number of defectives, we use control chart by attributes. We discussed $\bar{x}$ and R as well as x and MR charts belonging to the first category. R is a measure of variations. There are three zones in the control chart namely, stable zone, warning zone and action zone. We first plot the R chart. The chart is studied to see any assignable causes or measurement errors. Such points are eliminated and the R chart is revised to get the values of $\overline{R}$ and the three zones. Thereafter, $\bar{x}$ chart is made. We usually take at least four samples each and carry out the analysis after measuring 25 such subgroups. When it is expensive to test samples of size more than 1, we plot MR and x chart. Here, the moving range (MR) is the difference between the value and the one preceding it. Then, we discussed about the use of the following 4-control chart by attributes, np, p, c and u.

The np chart is used when the sample size is constant. np represents the number of defectives in each subgroup. p chart is used when the sample size is not constant. The charts can depict both the fraction defective and percent defective. c charts are used for controlling the variations in the number of defects in a specific portion of the population of constant size or unit like the number of defects in one square cm of a silicon wafer. On the contrary, a u charts can be used when the sample size or unit is varying. The control charts facilitate the following:

- Understand process performance
- Correct special cause variation
- Reduce common cause variation
- Improve the process continuously

REVIEW QUESTIONS

I. Choose the most appropriate answer.

1. Rectangular distribution is
   (a) Normal (b) Uniform
   (c) Gaussian (d) All the above
2. Warning zone
   (a) Ends at 2 σ (b) Starts at 2 σ
   (c) Starts at 3 σ (d) None of the above
3. Variable control charts include
   (a) c (b) np
   (c) σ (d) None of the above
4. Range chart has
   (a) UCL (b) LCL
   (c) Mean Range (d) All the above

5. Centering of data is taken into account while calculating
   (a) *Cp* (b) *Cpk*
   (c) USL (d) None of the above
6. The upper control limit of *X* bar chart whose details are given below:
   X bar bar = 0.5, *R* bar = 0.002, $n = 3$,
   (a) 0.503 (b) 0.502
   (c) 0.505 (d) None of the above
7. What is the UCL of the range chart where $n = 5$ and $R = 0.004$
   (a) 0.0085 (b) 0.007
   (c) 0.0065 (d) None of the above
8. What is the UCL of a *p* chart, (fraction defective) if a *p* bar = 0.05 and $n = 30$?
   (a) 0.09 (b) 0.14
   (c) 0.169 (d) None of the above
9. What is the UCL of a *c* chart, if *c* bar = 20?
   (a) 35 (b) 33.4
   (c) 40 (d) None of the above
10. What is the LCL of a *u* chart, if *u* bar = 12 and $n = 30$?
    (a) 0 (b) 1.1
    (c) 10.1 (d) None of the above
11. What is the *Cp* index given that sigma = 0.001, USL = 0.758 and LSL = 0.75?
    (a) 1.33 (b) 0.75
    (c) 1 (d) None of the above
12. What is the *Cpk* for the data in the above question, if *X* bar = 0.755?
    (a) 1.33 (b) 1.1
    (c) 1 (d) None of the above
13. A process has USL of 100 and no LSL. If *X* bar bar is 32 and standard deviation is 10, what is *Cpk*?
    (a) 1.27 (b) 2.27
    (c) 1.33 (d) None of the above
    Note : One sided tolerance, LSL, is not zero, it can be any value less than USL and hence not known exactly.
14. *R* bar indicates
    (a) Centering of the process (b) Percentage defectives
    (c) Variation (d) None of the above
15. *X* bar bar indicates
    (a) Centering of the process (b) Percentage defectives
    (c) Variable (d) None of the above
16. The control charts for percent defectives is called
    (a) *c* chart (b) *X* bar and *R* chart
    (c) *np* chart (d) None of the above

II True or False

1. *R* chart is control chart by attributes
2. Stable zone ends at 1 σ

3. In $\bar{x}$ and *R* chart, we have to start with $\bar{x}$ chart.
4. Even if 15 out of 25 subgroups exceed the UCL, process is stable
5. Seven points closer to central line means that the process is not stable
6. *Cpk* takes into account both precision and accuracy
7. *Cp* < 1 means a process is not capable
8. *MR* chart requires sample size of four
9. We plot percent defectives in *np* chart.

III Match The Following

A	B
c	Sample size 1
u	Number of defects in portion of the population
np	Percent defectives
p	Number of defects
MR	Along with $\bar{x}$
r	Number of defects in products of varying sample size

IV Explain Briefly

1. Differences between variables and attributes
2. Difference between *c* and *u* charts
3. Difference between *R* and *MR* charts
4. Differences between *p* and *np* charts
5. Three zones of control charts
6. Steps involved in plotting $\bar{x}$ and *R* chart
7. Steps involved in plotting *p* chart
8. Steps involved in plotting *MR* chart
9. Steps involved in plotting *u* chart
10. Central limit theorem
11. Differences between *Cp* and *Cpk*.
12. Comment on the following indices obtained on production of the same lot.

Cp	*Cpk*
1.5	1.5
2.0	0.7
0.5	0.5
2.5	0.5

V. Practice Problems

1. In a bolt manufacturing organization, 100 samples were drawn at random at periodic intervals. The number of defective samples in each subgroup is given below. Plot *np* chart and a *p* chart for the process.

np = defectives	1	1	1	3	4	2	3	2	1	3	3	1	2	1	2	1	2	2	1	1

2. In a biscuit manufacturing company, 200 samples were drawn at random. The number of defectives is given below. Plot a p chart and a np chart for the biscuit manufacturing process.

np = defectives	11	1	10	11	4	8	3	9	1	10	3	7	2	6	2	5	7	2	9	1

3. A tile manufacturer inspected each operator's output and noted the number of defectives and sample size. The data is given below. Plot a p chart.

No. of defectives	2	1	5	1	4	5	2	3	1	0	0	2	5	4	1	3	2	1	5	0
Sample size	120	56	278	311	123	254	17	35	45	67	56	92	235	123	70	45	58	62	278	75

4. A glassware production gave the following defectives on sample size indicated therein. Plot a p chart.

No. of defectives	2	1	5	1	4	5	2	3	1	0	0	2	5	4	1	3	2	1	5	0
Sample size	17	21	31	11	25	35	17	34	45	25	22	23	33	34	34	25	21	21	45	10

5. A TV receiver manufacturer measured the number of defects in each TV receiver on final inspection. Plot a c chart and a u chart for the assembly line.

No. of defects	12	11	7	6	5	4	3	2	1	5	4	6	7	8	9	5	7	8	6	7

6. A carpet manufacturer inspected and counted the number of defects in each carpet that was manufactured. The number of defectives and the sample size are listed below. Plot a p chart.

No. of defectives	2	1	5	1	4	5	2	3	1	1	1	2	5	4	1	3	2	1	5	2
Sample size	25	25	25	18	18	18	18	14	14	14	12	12	20	20	20	20	18	18	18	18

7. In a smithy the defects in axes made were counted and are indicated below along with the sample size. Plot a u chart.

No. of defective	12	9	5	7	8	8	8	7	8	8	9	10	5	4	1	8	7	9	8	5
Sample size	25	25	25	18	18	18	18	14	14	14	12	12	20	20	20	20	18	18	18	18

8. The specification for the width of table was 100 cms. The actual width measured on the samples is given below. Plot a X and MR chart.

99 101 100 102 98 101 100 99 101 100 103 102 99 100 101 98 102 98 101 99 98 100 101 99

9. A company manufacturing speedometers, picked up 4 samples each 25 times in a shift. For a setting of 10 Kmph, the values indicated by the speedometer samples are given below. Plot X bar and R chart. Draw the Action and Warning lines as well as the control limits.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
10.1	9.8	10.2	10	9.8	10	9.8	10.1	10.3	9.8	9.8	9	10
9.9	10	10.1	9.8	9.9	9.9	9.8	9.9	10	10	10.1	10.2	9.6
10	10	10	10.2	10.2	10.1	10.1	10	9.9	10	10	10	10
10	10.1	10	10	10	9.9	9.9	9.9	10	10.1	10.1	10	10.1

14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
10.2	9.9	10.3	9.9	10.6	10.2	9.8	10.1	10.3	10.1	9.8	10.1
9.8	10	10.1	9.8	9	9.9	9.8	9.9	10.1	9.9	10.1	10.2
10	10	10	10	10	10	10	10	10	10	10	10
10	10.1	10	10.1	9.8	9.9	9.9	10.1	10	10.1	10.1	10

10. 4 samples each of stopwatches were picked up at random from the assembly line. 25 times the samples were picked up. The indication of the speedometer for a true value of 300 seconds were checked and noted. The values indicated by the various samples are given below. Plot *X* bar and *R* chart. Draw the Action and Warning lines as well as control limits.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
301	299	302	300	306	300	300	301	303	301	295	301	300	302	299
288	300	301	298	268	299	298	299	300	300	301	302	296	298	300
299	299	299	302	302	301	301	299	295	310	300	300	302	297	300
300	301	300	302	307	300	301	302	300	302	300	301	304	304	301

16	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
303	299	306	302	298	301	303	301	302	301
301	298	298	299	298	299	301	299	303	301
298	302	302	301	301	299	299	300	302	301
301	301	304	304	302	302	301	301	302	302

11. In Problems 1 and 2, if the data the corresponds to the number of defects, calculate a *c* chart.

Table A: Proportional under the Tail of the Normal Distribution

$Z = \dfrac{(x-\mu)}{\sigma}$	*.00*	*.01*	*.02*	*.03*	*.04*	*.05*	*.06*	*.07*	*.08*	*.09*
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0216	.0211	.0206	.0201	.0197	.0192	.0187	.0183
2.1	.0179	.0174	.0170	.0165	.0161	.0157	.0153	.0150	.0146	.0142

(*Contd.*)