Section 5

Q1) 100 balls are tossed independently and at random into 50 bins. Let X be a random variable representing the number of empty boxes. Find the expected value of random variable X, i.e. E(X).

A1) We are given a total of 100 balls and 50 empty bins. According to the question we have to take the following assumptions:

a. All the balls are identical
b. All the bins are identical
c. Not all bins remain empty after all the balls are tossed.

Taking the Discrete Random Variable (X), where 'X' is the number of bins that remain empty we calculate the value of P(X<50) where 'X' vary from 1 to 50.

For X=0,

$$P(0)=\frac{(50)^{100}}{(50)^{100}} = 1.00000$$

For X=1,

$$P(1)= \frac{(49)^{100}}{(50)^{100}} = 0.13261$$

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For X=49,

$$P(49)= \frac{(1)^{100}}{(50)^{100}} = 1.2676E\text{-}170$$

Similarly, by varying X from 1 to 49 , we get the above mentioned calculations by running the following code on python

```
PX=[ ]

for j in range (0,49);
        PX = j*(pow((50-i),100)/pow(50,100))
print(PX)
```

To calculate the expected value we know that, The expected value of random variable *X* is the weighted average of the value of random variable 'X' with the weights being probabilities $P(X_i)$ and is often written as E(*X*)

```
EXP = 0

for i in range (0,49);
        EXP = EXP + i*(pow((50-i),100)/pow(50,100))
print(EXP)
```

Which comes out to be E(X<50) =0.173633.