Section 5

Q2) Binary communication channel

The per bit error rate over a certain binary communication channel is 10^{-10}. No other statistics are know about the channel or the data.

ii) b) Find an upper bound on the probability that a block of 1000 bits has 10 or more erroneous bits.

A2b) According to the question information given to us is the error bit rate of a communication channel Y is given.

Bit Error Rate (BER) $=10^{-10}$

To calculate the number of erroneous bits in a lock of 1000 bits, let us take the special case of binomial probability distribution that is the Bernoulli probability distribution.

Let the Bernoulli random variable X ranging from 1 to 1000(block is of 1000 bits) representing the number of erroneous bits have a common probability mass function (PMF) as:

$$X_i = \begin{cases} 1 & \text{with probability } 10^{-10} \\ 0 & \text{with probability } 1 - 10^{-10} \end{cases}$$

Then the total number of erroneous bits can be calculated with E(N), where N is the number of total bits, (N=1000)

$$\mathrm{E}(N) = \mathrm{E}\left[\sum_{i=1}^{1000} X_i\right] = \sum_{i=1}^{1000} \mathrm{E}(X_i) = 1000\mathrm{E}(X_i) = 1000 \cdot 10^{-10} = 10^{-7}.$$

To calculate the upper bound on the probability so that a block of 1000 bits has 10 or more erroneous bits can be calculated with the help of Markov Inequality.

Markov Inequality states that, in probability theory, **Markov's inequality** gives an upper bound for the probability that a non-negative function of a random variable is greater than or equal to some positive constant.

$$\mathrm{P}\{N \geq 10\} \leq \frac{\mathrm{E}(N)}{10} = 10^{-8}.$$

Therefore the upper bound is 10^{-8}.