# Imaging Mechanism for Dual-Head Detectors

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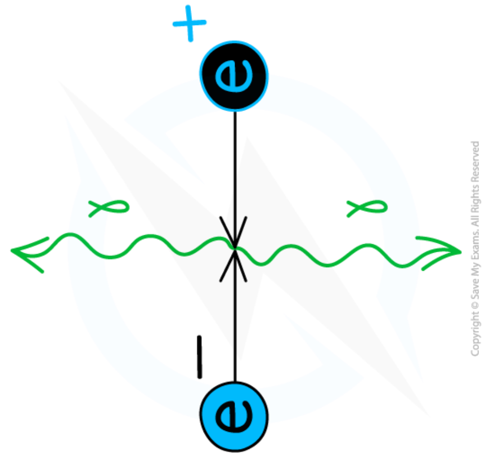
# The Basic Principle of PET

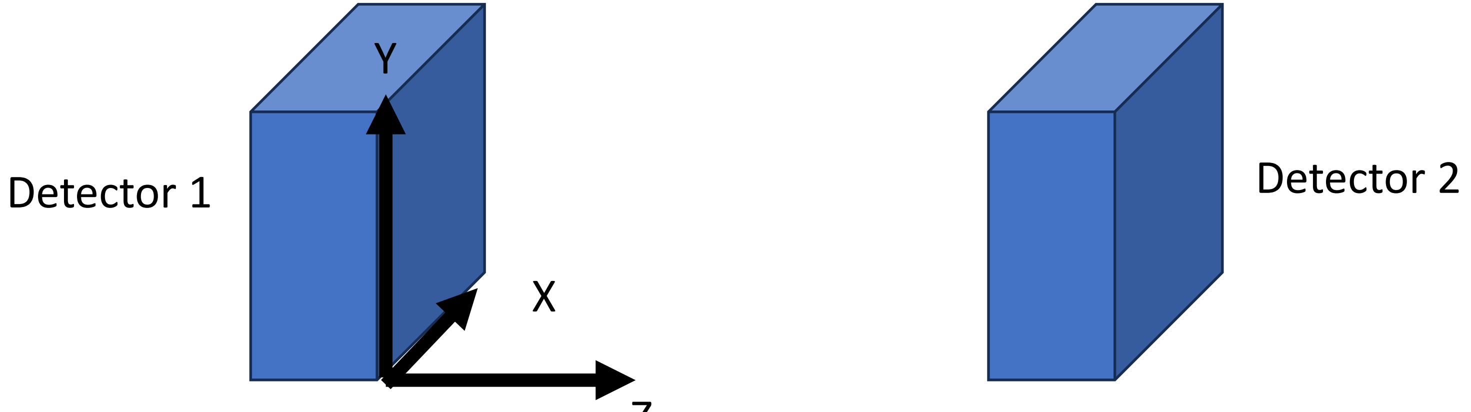
Positron Emission Tomography(PET) is an imaging technique that exploits the pair annihilation reaction to determine the location of a positron emitting radioisotope. In a pair annihilation reaction, a positron combines with an electron emitting two gamma photons in the opposite directions along a straight line. The goal of PET is to determine the location of the radioisotope by detection of gamma ray photons generated in a reaction.



# Detector Setup

Our detector setup has two planar detectors placed facing each other at a separation of 40 cm from each other. Each detector has dimensions of 50 mm X 50 mm.

For our convenience ,we choose our origin to be the lower left corner of the detector placed on the left and we align our axes so that our detectors lie in the z=0 and z=40cm planes. Along the x and y axes our detectors extend from x=0 to x=50 along the x axis and y=0 to y=50 along the y axis.



# Statement of the Problem

The problem of computation of the location of the source from line intersection data is an inverse problem. The problem can be stated as:

$$\bar{g_i} = \int\int\int_{\Omega} \lambda(x, y, z) h_i(x, y, z) dx dy dz \qquad (1)$$

where $g_i$ is the $i^{th}$ measurement,
$\lambda(x, y, z)$ is the spatial distribution of the radio isotope,
$h_i$ is the contribution to measurement i by an atom placed at (x,y,z), and
$\Omega$ represents our Field of View(FOV).
The aim of PET is to compute $\lambda$ given $g_i$s. The above model accounts for random coincidences and other physical effects in the $h_i$ term, $h_i$ is zero out of the field of view. In an ideal case $h_i$ takes either 0 or 1 depending on (x,y,z).

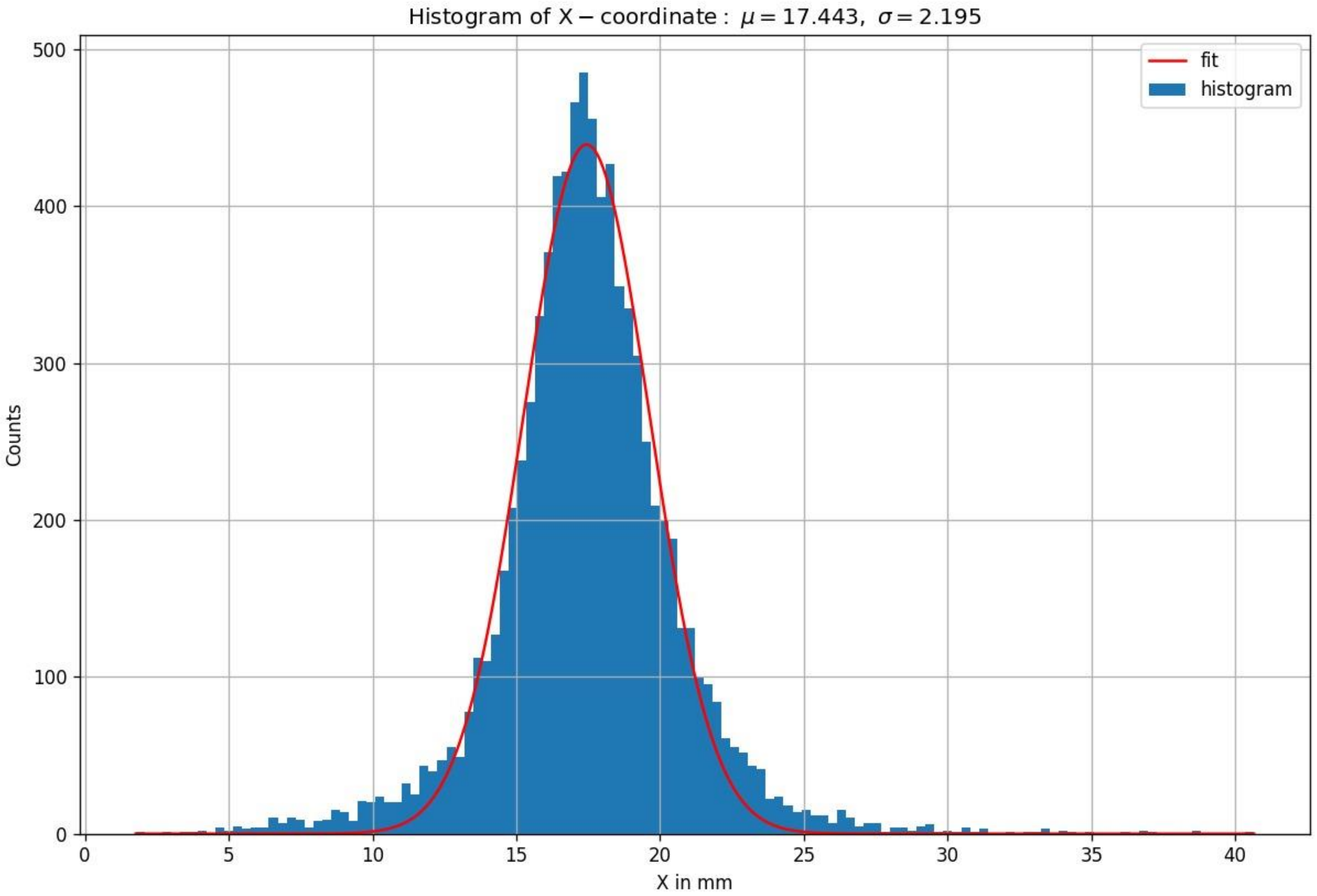
# Initial Approach

Our detector setup provides us with two points (one on each detector) that are detected at almost the same time, giving us the line of reaction. The very first approach to obtain the source location can be by taking all intersections among all the lines of reaction recorded. Such a model suffers due to lack of data as an exact intersection between two lines is very rare in a case where small errors in observation shift the lines by a considerable system.

# Modification of the initial approach

Since the problem with our initial approach was due to reduced number of intersection points, In our next approach we try to overcome this by considering any two lines as intersecting whenever they get closer to each other than a certain fixed distance. In our case, we considered any two lines as intersecting, if the shortest distance between them was less than or equal to $0.1 \times 10^{-3}$ mm. This led to more number of points and hence increased resolution. The model was able to point out sources with a resolution of approximately 4 mm along the x and y axis. The resolution along the z axis was poor( roughly 6 cm). The model was able to differentiate sources kept 10 mm apart along the x-axis.

# Results of the modified approach



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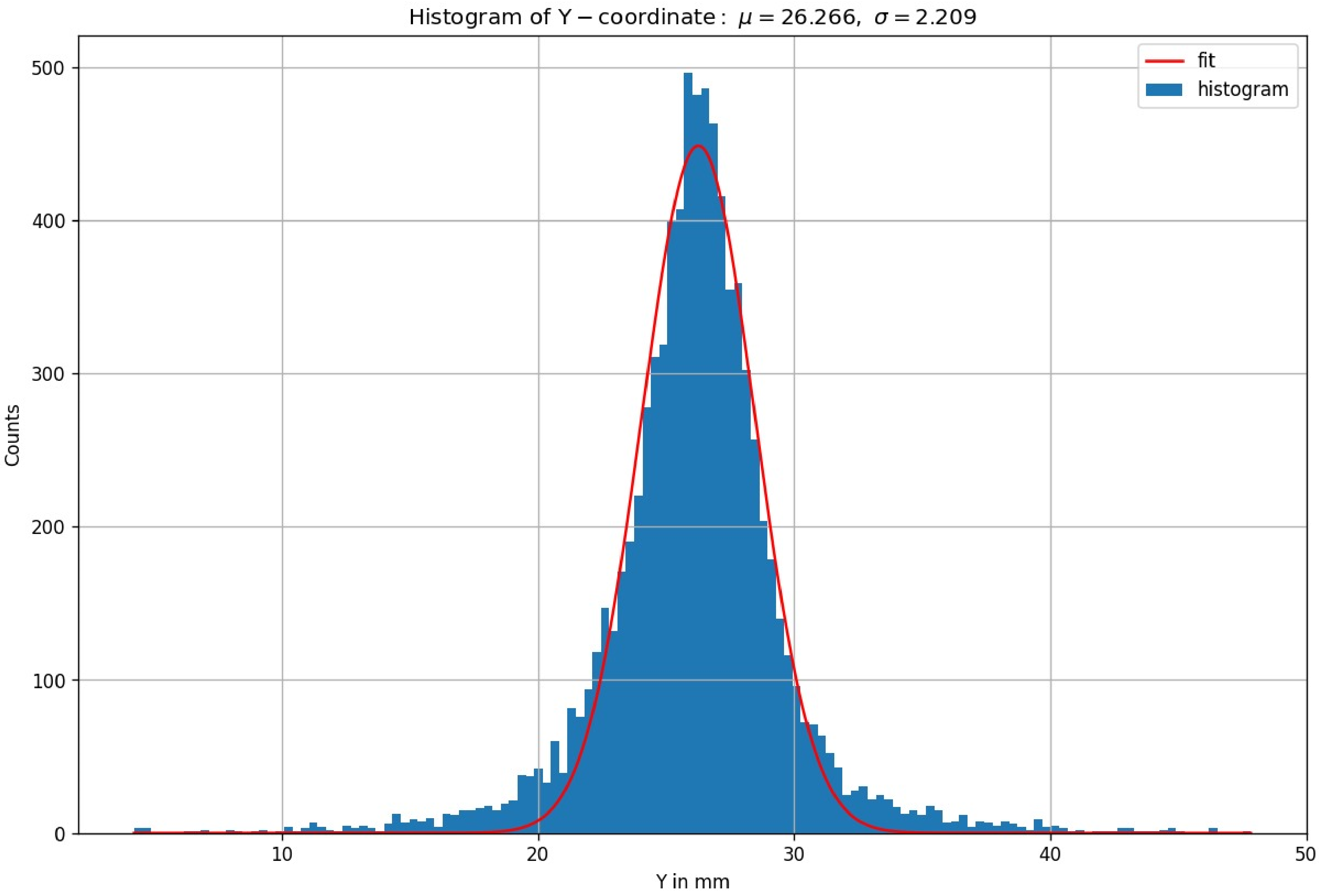
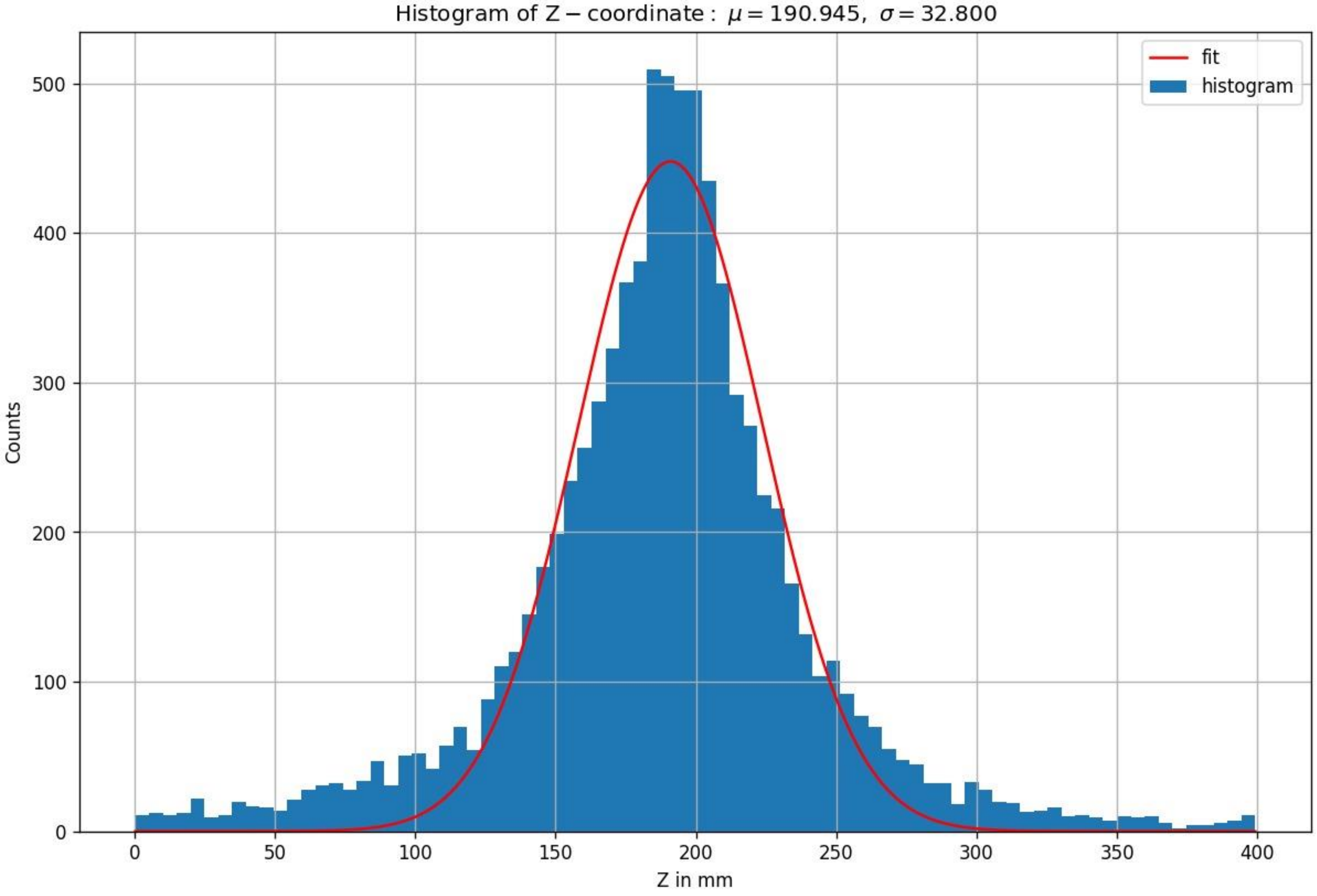


# Results of the modified approach

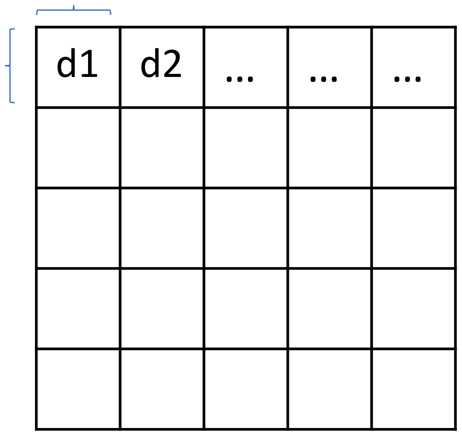


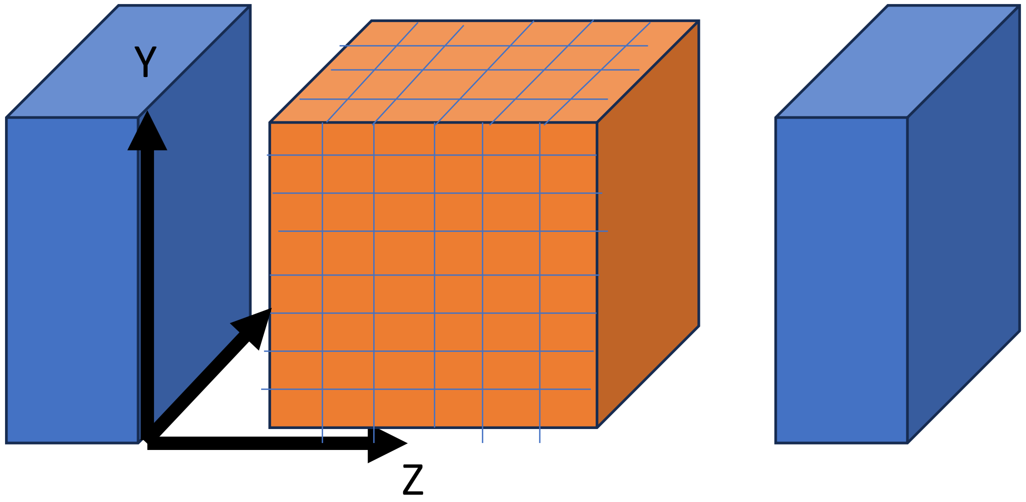
# Other Modifications to our previous approach

- We can use random sampling within our collection of lines to increase resolution of our source. This can be helpful for cases where we have multiple sources and some the data points corresponding to some sources are larger in number than others
- We can treat our detector surfaces as an array of smaller detectors, and we can reduce some noise in our final measurements by eliminating those pairs of the smaller detectors that lie outside of the 95% of (LOR) data.
- We can have another approach by again considering our detectors as an array of smaller detectors and for each pair of detectors we can determine the number of LORs recorded in them, we then take line intersections for all lines passing through the centers of every pair of smaller detectors, and then multiplying by the density of LORs recorded in each pair of detectors to obtain the final line intersection data.

# MLEM  for PET

- One can approach the problem from a discrete point of view.
- One can divide our field of view into small voxels and try to estimate the distribution of the source among the voxels.
- We assume an initial distribution of the source, for a large enough dataset and a reasonable choice of probabilities , we should expect our model to predict accurately.



# Notations

Let $\lambda(x, y, z)$ represent the distribution of the source at the location (x,y,z). We wish to estimate lambda in our field of view(FOV). We divide our FOV into blocks of fixed size and indexed by b=1,2,3,4.... . We also pair up detector slices with each other and index a pair of detectors on opposite detector pairs by d=1,2,3,4,5,...... .Let $n^*(d)$ represent the total number of coincidences detected by a detector pair and let p(b,d) represent the probability that a source located inside block b is detected by detector pair d. We denote by $\lambda(b)$ the integral of $\lambda(x, y, z)$ on the block b. Let n(b,d) represent the number of emissions in box b detected in tube d.

In our discretized model, where we have our FOV divided into discrete voxels, calculating $\lambda(b)$ for all b is our aim.

# MLE calculation

We assume that n(b,d) are independent poisson variables with mean

$$E[n(b,d)] = \lambda(b,d) = \lambda(b)p(b,d) \tag{1}$$

. Thus,

$$P[n(b,d) = k] = e^{-\lambda(b,d)} \frac{\lambda(b,d)^k}{k!}. \tag{2}$$

Thus, the likelihood function is given by

$$L(\lambda) = P(n^*|\lambda) = \sum_A \prod_{\substack{(b=1,2,3,..B)\\(d=1,2,3,...D)}} e^{(-\lambda(b,d)\frac{\lambda(b,d)^{n(b,d)}}{n(b,d)!})} \tag{3}$$

where $n^*$ represents our observed dataset.Note that $n^*(d) = \sum_{b=1}^{B} n(b,d)$ for d=1,2,3,4.,D and $n(b) = \sum_{d=1}^{D} n(b,d)$

# EM algorithm(from [1])

The EM algorithm can be applied to an exponential family of the form

$$f(x|\Phi) = b(x)exp(\Phi t(x)^T)/a(\Phi). \tag{1}$$

Given this form the EM algorithm can be proceeded in the following two steps applied iteratively.

- E-step: Estimate complete sufficient statistics t(x) by finding

$$t^{(p)} = E(t(x)|y, \Phi^{(p)}) \tag{2}$$

- M-step: Determine $\Phi^{(p+1)}$ as the solution of

$$E[t(x)|\Phi] = t^{(p)} \tag{3}$$

# EM algorithm for our case

We have our E-step as

$$E[n(b)|\lambda] = E[\sum_d n(b,d)|\lambda] = \sum_d \frac{n^*(d)\lambda(b,d)}{\sum_{b'} \lambda(b')p(b',d)} \qquad from[2] \qquad (1)$$

Then the M-step is given as

$$E[n(b)|\lambda^{new}] = \sum_d \frac{n^*(d)\lambda^{(old)}(b,d)}{\sum_b' \lambda^{(old)}(b')p(b',d)} \qquad (2)$$

$$=> \lambda^{(new)}(b) = \sum_d \frac{n^*(d)\lambda^{(old)}(b,d)}{\sum_b' \lambda^{(old)}(b')p(b',d)} = \sum_d \frac{n^*(d)\lambda^{(old)}(b)p(b,d)}{\sum_b' \lambda^{(old)}(b')p(b',d)} = \lambda^{(old)}(b) \sum_d \frac{n^*(d)p(b,d)}{\sum_b' \lambda^{(old)}(b')p(b',d)} \qquad (3)$$

Thus finally we obtain our iterative algorithm as,

$$\lambda^{(new)}(b) = \lambda^{(old)}(b) \sum_d \frac{n^*(d)p(b,d)}{\sum_b' \lambda^{(old)}(b')p(b',d)} \qquad (4)$$

# A calculation of relative error along the Z-coordinate

We have

$$\Delta z^2 = (\frac{\partial f}{\partial x_1})^2 \Delta x_1^2 + (\frac{\partial f}{\partial x_2})^2 \Delta x_2^2 \qquad (1)$$

$$= (\tfrac{z_0 x_2}{(x_1+x_2)^2})^2 \Delta x_1^2 + (\tfrac{z_0 x_1}{(x_1+x_2)^2})^2 \Delta x_2^2$$

$$= z_0^2 (\tfrac{x_1^2+x_2^2}{(x_1+x_2)^4}) \Delta x_2^2$$

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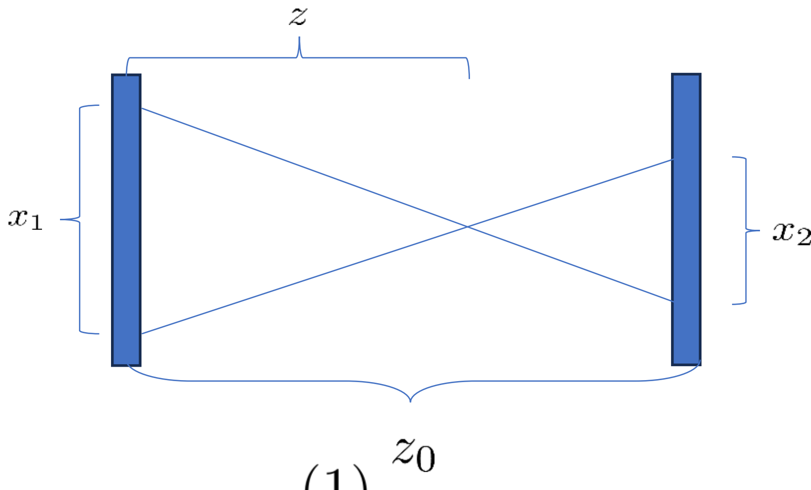
as $\Delta x_1^2 = \Delta x_2^2 = 100$ and $z_0 = 400$



(1)

Where z=$(\frac{z_0 x_1}{x_1+x_2}) = f(say)$ Thus,

$$\Delta z^2 = (40000)^2 \frac{(x_1^2 + x_2^2)}{(x_1 + x_2)^4}$$

for x1=x2=20 we have $\Delta z^2 = 5000$ i.e. $\Delta z \approx 70mm$

# References Used

[1]Dempster, Arthur P., Nan M. Laird, and Donald B. Rubin. "Maximum likelihood from incomplete data via the EM algorithm." *Journal of the royal statistical society: series B (methodological)* 39.1 (1977): 1-22

[2]Shepp, Lawrence A., and Yehuda Vardi. "Maximum likelihood reconstruction for emission tomography." *IEEE transactions on medical imaging* 1.2 (1982): 113-122

[3]Bailey, Dale L., et al. *Positron emission tomography*. Vol. 2. London: Springer, 2005

[4] Wernick, Miles N., and John N. Aarsvold. *Emission tomography: the fundamentals of PET and SPECT*. Elsevier, 2004