SPARSE REPRESENTATION BASED ITERATIVE INCREMENTAL IMAGE DEBLURRING

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ABSTRACT

Inspired by the observation that in image restoration, parametric models are extremely specific while pixel-level models are too loose, tending to under or over fit the underlying image respectively, in this paper, we proposed an 'intermediate-language' based method for image deblurring. The solution space is represented at a level higher than the pixel-grid level, while retain an enough degree of freedom (DOF), thus avoids the common local under or over fitting problem. Considering the sparseness property of images, a sparse representation based incremental iterative method is established for blurry image restoration. Comprehensive experiments demonstrate that the framework integrating the sparseness property of images significantly improves the deblurring performance.

***Index Terms*—**Image restoration, sparse representation, image deblurring

1. INTRODUCTION

Image deblurring is a long existing problem in remote sensing, astronomy, etc. If we assume additive white Gaussian noise (AWGN), and linear sensor response, this process can be formulated in matrix-vector from as:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{x}$ and $\mathbf{y}$ are the vectors obtained by lexicographical ordering of original and blurry images respectively, and $\mathbf{H}$ is the Block-Toeplitz with Toeplitz Blocks (BTTB) matrix from PSF (Point Spread Function). Deblurring is to estimate $\mathbf{x}$ from the blurry and noisy observation $\mathbf{y}$ which is an inverse problem and often ill-posed.

Many of the current image deconvolution methods are stem from regularization (Bayesian methods are also of this kind as it is proven that the prior in Bayesian method is equivalent to the regularization term[1]). These kinds of methods work through penalizing the undesired solutions using regularization term. What they do indeed is 'keep away from' the potentially 'bad' solutions. So from this idea, better results may be obtained if 'right' regularization term can be designed. But the design of the regularization term is not so easy and improper designs often result in artifacts. And the abuse of pixel-level for restoration worsens the results. As the pixel-level methods exhibit a vast possible solution space (as every pixel has a DOF), over-fitting or over-interpreting is not uncommon[1], but the parametric models are too specific to generalize (as they have a small number of DOF), so, apparently, an 'intermediate-language' that can describe an image in a higher level is needed. Using the expressive ability of the language as a restriction for the desired solution, it can help overcome the over/under-fitting problems. Some representation methods are explored in the past for image deconvolution. Wavelet representation was used in linear inverse problems in [2] by Donoho; Ref.[3] developed this method using a mirror filter bank representation; Ref.[4-6] using complex wavelet packet for deconvolution. Another noteworthy tread is the combination representation way: Ref.[7] proposed a ForWaRD method, combing Fourier and wavelet method; Stark *et al.* proposed a combination of wavelet and curvelet representation for restoration[8], and more recently, he proposed an morphological method for signal analysis[9].

This paper adopts an 'intermediate-language' for image restoration from the angle of image representation. As this language exhibits small DOF than pixel level model but not so specific as the parametric ones, a better estimation can be expected without severe over/under-fitting problems as the other methods. The rest of the paper is organized as follows. In Section 2, some basics on sparse representation are introduced; in Section 3, we formalize the deblurring problem and derive our algorithm; then we perform some experimentation and analyze the results in Section 4. Finally, we conclude our work in Section 5.

2. SPARSE REPRESENTATION FOR IMAGES

2.1 Sparse Representation

Given an image $\mathbf{x} \in \mathbb{R}^n$, we can represent it as:

$$\mathbf{x} = \mathbf{D}\boldsymbol{\alpha} \tag{2}$$

where $\mathbf{D}$ indicates the dictionary (language) we choose, α is the expression for $\mathbf{x}$ in the selected language. $\mathbf{D}$ is

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$n \times L$ matrix where each column is a word and consists L words in total. Popular selection for $\mathbf{D}$ are : Fourier, wavelet, wavelet packet, cosine packet, and curvelet etc. [8].

Sparse representation means to look for a good expression with fewest words, namely $\|\boldsymbol{\alpha}\|_0 \ll n$. In most applications, the dictionary is selected as a priori from the common orthogonal bases such as Fourier, wavelet etc.[10] or combinations of them[7, 8]. But here we use a different method, which we'll describe in the following section.

2.2 Dictionary Learning

A proper selection for dictionary $\mathbf{D}$ is important for sparse representation as well as restoration. A key concept is morphological diversity[11]. Common languages motioned in the last section exhibit different abilities for expression: Fourier is best candidate for vibrations; wavelet is proper for isotropic properties; curvelet is best for piece-wise smooth away from C^2 contours[8]. As can be seen, although these representations have their advantages in some kind of problems, no single method currently can represent an image desirably and generally. Therefore, it will be benefit if we can design a dictionary $\mathbf{D}$ that is more flexible and adaptive than single representation methods. $\mathbf{D}$ can be generated in two methods[8]:

1) combine some predefined methods to form a mixing language, such as the combination of Fourier and wavelet [7], wavelet and curvelet [8];
2) learn the dictionary adaptively from sample images such as images from similar scenes, noisy images and even blurry images.

This paper adopts the latter method which is more adaptive and flexible. Recent progress in this field such as the K-SVD algorithm can fulfill such tasks. K-SVD method solves the following objective function:

$$\min_{\mathrm{D},\boldsymbol{\alpha}} \|\boldsymbol{\alpha}\|_0 \quad \text{s.t.} \quad \|\mathbf{x} - \mathbf{D}\boldsymbol{\alpha}\|_F^2 \le \lambda \tag{3}$$

where $\|\boldsymbol{\alpha}\|_0$ is L_0 norm defined as $\|\boldsymbol{\alpha}\|_0 = \sum_i \boldsymbol{\alpha}_i^0$ with $0^0 = 0$ while $\|\mathbf{X}\|_F$ is Frobenius norm defined as $\|\mathbf{X}\|_F = \sqrt{\sum_{ij} \mathbf{X}_{ij}^2}$. K-SVD is a direct extension of K-means method. K-means method encodes each sample using a single clustering center, while K-SVD method sparsely encodes the samples using atoms from the dictionary $\mathbf{D}$. K-SVD is consisted of two steps: sparse coding stage searching for the sparse representation for each sample and dictionary update stage revising the dictionary using SVD decomposition. This process can be interpreted as updating the representation and the dictionary in an alternating fashion[12]. In our experiment, we design our dictionary with n=64 and L=256. Dictionary is trained on sharp images with similar contents to the blurry image.

3. SPARSE-ITERATIVE-INCREMENTAL DEBLURRING

We can formalize our algorithm armed with the concepts introduced in former sections. From degradation model (1), we can formulate the problem as the following minimization problem:

$$\min \|\mathbf{y} - \mathbf{Hx}\|_F \tag{4}$$

$$s.t. \quad (\mathbf{x})_i \ge 0 \tag{5}$$

$$\mathbf{x} = \mathbf{D}\boldsymbol{\alpha}, \quad \|\boldsymbol{\alpha}\|_0 \le \lambda \tag{6}$$

where $\mathbf{y}$ is observed blurry and noisy images; $\mathbf{x}$ is the original image; i is the element index in $\mathbf{x}$. We give here only two constrains: non-negative constrains and sparseness constrains. The former one is commonly used in image restoration and is effective in practice; the latter one is our contribution on sparseness constrains. More constrains such as support information can be easily added in to the model.

We propose an incremental algorithm in this paper, which is in an iterative fashion and widely used in learning and image restoration communities[13, 14]. The main algorithm flow is as follows:

1) Prepare dictionary $\mathbf{D}$ by solving Eq.(3);
2) Initialize $\mathbf{x}_0$（$\mathbf{x}_0 = \mathbf{y}$ for example）;
3) Perform Fourier transform to the k-th estimation $\mathbf{x}_k$, obtaining $\mathbf{X}_k(\omega)$;
4) Let $\mathbf{X}_k(\omega)$, $\mathbf{Y}(\omega)$ and $\mathbf{H}(\omega)$ be Fourier correspondence of $\mathbf{y}$ and $\mathbf{h}$ respectively, calculate the frequency error: $\mathbf{S}_k(\omega) = \mathbf{Y}(\omega) - \mathbf{X}_k(\omega)\mathbf{H}(\omega)$;
5) Update the $(k+1)$-th estimation through $\mathbf{X}_{k+1}(\omega) = \mathbf{X}_k(\omega) + \dfrac{\mathbf{H}^*(\omega)\mathbf{S}_k(\omega)}{|\mathbf{H}(\omega)|^2 + \gamma}$, where γ is a constant;
6) Inverse Fourier transform to $\mathbf{X}_{k+1}(\omega)$ to get $\mathbf{x}_{k+1}$;
7) Apply the non-negative constrains (Eq.(5)) to current solution $\mathbf{x}_{k+1}$ and get $\mathbf{x}_{k+1,p}$;
8) Apply sparseness representation constrain to $\mathbf{x}_{k+1,p}$ by solving Eq.(3) approximately using Orthogonal Matching Pursuit (OMP) for $\boldsymbol{\alpha}$, then get $\mathbf{x}_{k+1,p,s}$ through Eq.(6);
9) Set $\mathbf{x}_{k+1,p,s}$ as the current estimation.

Repeat 3) ~ 9) until certain goals are attained. As we observe that the algorithm converges in a few iterations, we take 5 iterations in our experiments. For sparse representation step we set $\lambda = 0.9\sigma$ where σ is the estimated noise variance.

4. EXPERIMENTS AND ANALYSIS

To verify the applicability of proposed algorithm, we construct several simulations, and compare the performance of different algorithms. To evaluate the result objectively, we adopt the blurred signal-to-noise ratio (BSNR) and improvement in signal-to-noise ratio (ISNR), signal-to-noise ratio (SNR) for restoration quality assessment. BSNR, SNR and ISNR are defined as follows:

$$\text{BSNR} = 10\log_{10}\left\{\frac{\sum\{\mathbf{y}-mean(\mathbf{y})\}^2}{\sigma^2}\right\} \tag{4}$$

$$\text{SNR} = 10\log_{10}\left\{\frac{var(\mathbf{x})}{\sum[\mathbf{x}-\mathbf{x}_{est}]^2}\right\} \tag{5}$$

$$\text{ISNR} = 10\log_{10}\left\{\frac{\sum[\mathbf{x}-\mathbf{y}]^2}{\sum[\mathbf{x}-\mathbf{x}_{est}]^2}\right\} \tag{6}$$

These measures are used widely in restoration filed. BSNR denotes the signal-to-noise ratio of the blurry and noisy input image. ISNR specifies the improvement in signal-to-noise ratio after restoration. SNR is also used as a criterion for image restoration due to the existence of noise.

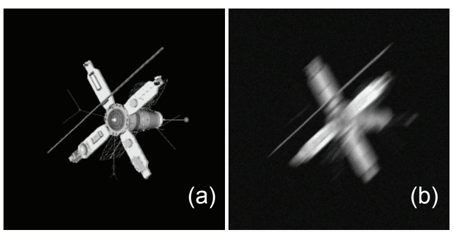

Figure 1 (a) Clear and sharp Satellite image (b) Blurry and noisy observation with BSNR=20dB, motion direction 45 degrees, length 20 pixels

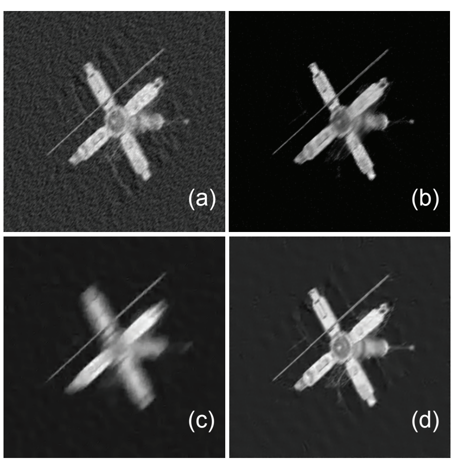

Figure 2 (a) Deblur Estimation via Wiener Filtering (b) Deblur Estimation using Lucy method [15] (c) Deblur Estimation using ForWaRD method[7] (d) The deblurred image through the algorithm proposed in this paper. As can be seen from comparison, our method has the best restoration output. ForWaRD is not so desirable in this case, indicating that it is not suitable for the image of morphological diversity. Lucy and ForWaRD method suffers from under-fitting while Wiener method over-fit and produces ring.

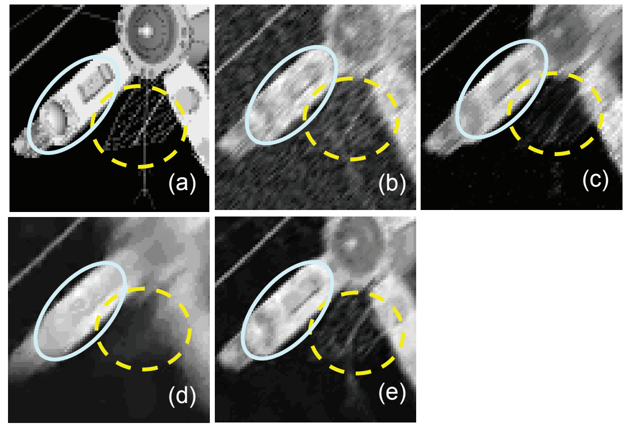

Figure 3 A close-up look into the restored images via different methods. (a) True image (b)Wiener (c)Lucy[15] (d) ForWaRD[7] (e) Proposed Method. Line and dash mark the smooth and fine areas respectively. Wiener method is apt to over-fit in smooth areas while ForWaRD method under-fitting in fine areas. The proposed method is free from over and under fitting in smooth and fine areas respectively.

First we describe the experiment setup. The ‘satellite’ image (Figure **1** (a)) is utilized on consideration of the morphological diversity. Experiment setups are:

- PSF: motion blur with direction of 45 degree and length of 20 pixels
- Noise variance: set proper variance making the BSNR=20dB

Figure **2** demonstrates the results of the proposed algorithm comparing with other methods (including both classical Wiener method as well as the state-of-art ForWaRD[7] method). We comparison the performance of different methods under different noise levels, and summarize the results in Table **1**.

As can be seen from comparisons between the close-lookups in Figure **3**, different methods perform differently. Wiener method can restore the fine structures (Figure **3** (b)), but suffered form over-fitting, which generates obvious ripples in (b); Lucy method [15] generates fewer ripples (Figure **3** (c)) but is not able to restore the fine structures. For example, the solar panel substrate (dashed area) is not well restored indicating the under-fitting problem; ForWaRD[7] (Figure **3** (d)) method suffers from the similar deficiency as Lucy method; the restored image using propose method (Figure.**3** (e)) is sufficiently smooth in smooth areas (lined) indicating no over-fitting. The fine structures (dashed), such as the solar panel substrate, are well restored demonstrating it has enough DOF and is free from under-fitting. Table **1** lists the comparisons between several method evaluated by ISNR and SNR. Conclusions can be drawn from the table that the proposed method performs better than others under low, moderate and high noise levels, indicating its robustness in the existence of noise; as the BSNR increases, the restoration quality with proposed method increases, which parallels our former theoretic analysis.

Table1. Algorithm evaluation ISNR (dB) and SNR (dB) with PSF: direction=45deg, length=20pixels

Method	High Noise BSNR= 5 dB		Moderate Noise BSNR= 10dB		Low Noise BSNR= 30 dB	
	ISNR	SNR	ISNR	SNR	ISNR	SNR
Wiener	6.59	3.74	5.61	5.76	-0.02	7.85
Lucy	5.62	2.78	4.41	4.56	2.87	10.74
ForWaRD	6.22	3.37	2.96	3.11	-3.4	4.47
Proposed	**10.39**	**7.55**	**11.54**	**11.69**	**9.6**	**17.48**

In addition, we do not assume a specific form of the blurring process, which means our methods are applicable to generic forms of blurry images. To verify this point, experiments on another blurry form (out-of-focus blur) images are performed, and results are compared in Figure **4**. It is obvious that the results through our method do not affected by under-fitting or over-fitting and the result is desirable.

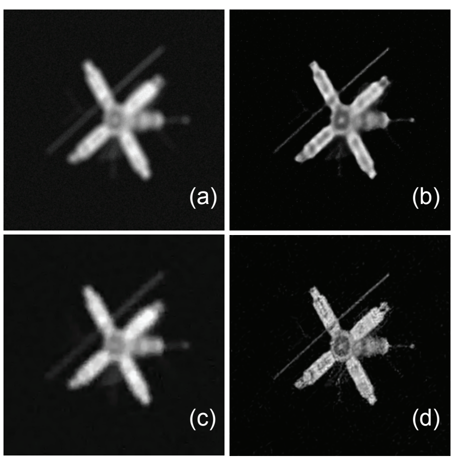

Figure 4 (a) Observed blurry and noisy image with BSNR=20 dB, out of focus blur with radius=5. (b) Deblur Estimation using Lucy method[15] (c) Deblur Estimation using ForWaRD method[7]. (d) The deblurred image through the algorithm proposed in this paper. As can be seen from comparison, our method has the best restoration output

5. CONCLUSIONS AND FUTURE WORK

This paper adopts an intermediate-language for image restoration from the angle of image representation exploring the recent advance in the sparse representation community. As this language exhibits small DOF than pixel level model but not so specific as the parametric ones, a better estimation can be expected without so severe over/under-fitting problems as the other methods. This can be explained further. As the 'words' in the constructed dictionary have large enough DOF and flexibility to represent the image to be estimated, under-fitting is avoided; with the limitation to represent only a group of images, the words cannot express the artifacts-like contents effectively, thus won't over-fit. So it is more competent for image restoration. Although desirable results are obtained through the proposed method, there is still room for improvement. We believe a proper representation or transformation for image will be fundamental for improving the result of restoration in the long run.

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