Control Systems

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***Abstract*—This manual is an introduction to control systems in feedback circuits. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/feedback/codes
```

1 Feedback Voltage Amplifier: Series-Shunt

2 Feedback Current Amplifier: Shunt-Series

2.1 Ideal Case

2.2 Practical Case

2.2.1. Consider an op amp having a single pole open loop response $G_o = 10^5$ and $f_p = 10$ Hz.Let op amp be ideal connected in non-inverting terminal with a nominal low frequency of closed loop gain of 100

A manufacturing error introducing a second pole at 10^4 Hz.Find the frequency at which $|GH| = 1$ and phase margin

What values of H phase margin is greater than 45°

Solution: Part 1 of the question For a two-pole amplifier open loop transfer function is

$$G(s) = \frac{G_o}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)} \tag{2.2.1.1}$$

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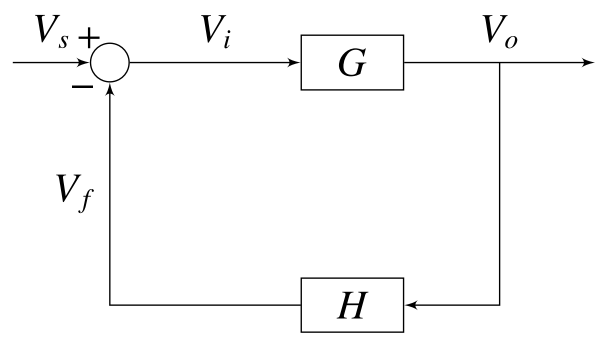

Fig. 2.2.1

Poles are at $f_1 = 10$ and $f_2 = 10^4$

$$G(f) = \frac{G_o}{\left(1 + \jmath\frac{f}{f_1}\right)\left(1 + \jmath\frac{f}{f_2}\right)} \tag{2.2.1.2}$$

$$\implies \frac{10^5}{\left(1 + \jmath\frac{f}{10}\right)\left(1 + \jmath\frac{f}{10^4}\right)} \tag{2.2.1.3}$$

As given closed loop gain is 100

$$|T| = 100 \tag{2.2.1.4}$$

For nominal low frequency $|GH| \gg 1$ and from Fig.2.2.1

$$T = \frac{G}{1 + GH} \tag{2.2.1.5}$$

$$\implies \approx \frac{1}{H} \tag{2.2.1.6}$$

So from this

$$H = 0.01 \tag{2.2.1.7}$$

For the $|GH| = 1$ and from (2.2.1.3) and (2.2.1.7)

$$\frac{10^3}{\left(\sqrt{1 + \frac{f^2}{100}}\right)\left(\sqrt{1 + \frac{f^2}{10^8}}\right)} = 1 \tag{2.2.1.8}$$

$$\left(1 + \frac{f^2}{100}\right)\left(1 + \frac{f^2}{10^8}\right) = 10^6 \tag{2.2.1.9}$$

Solving f using python code

$$f = 7861.5 \tag{2.2.1.10}$$

From definition of phase margin $\alpha = 180° + \phi$ where ϕ is the phase of GH

$$\phi = -\tan^{-1}\left(\frac{f}{10}\right) - \tan^{-1}\left(\frac{f}{10^4}\right) \tag{2.2.1.11}$$

At $f = 7861.5$

$$\phi = -128.1° \tag{2.2.1.12}$$

$$\implies \alpha = 180° + \phi \tag{2.2.1.13}$$

$$\implies \alpha = 51.9° \tag{2.2.1.14}$$

Hence for frequency $f = 7861.5$ Hz $|GH| = 1$ and phase margin is 51.9°

The following code for bode plot of part 1

codes/ee18btech11034/ee18btech11034_1.py

2.2.2. Verification using Bode plot of part 1

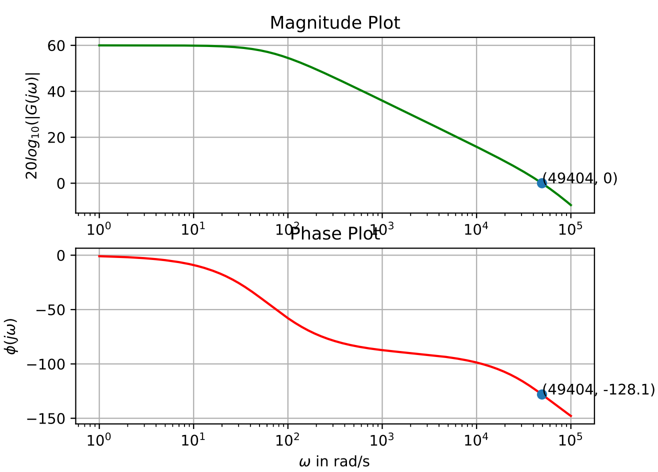

Fig. 2.2.2

Solution: Part 2 of the question For phase margin $\alpha = 45°$ and from (2.2.1.13)

$$\phi = -135° \tag{2.2.2.1}$$

From (2.2.1.11) and (2.2.2.1)

$$\phi = -\tan^{-1}\left(\frac{f}{10}\right) - \tan^{-1}\left(\frac{f}{10^4}\right) = -135° \tag{2.2.2.2}$$

$$\tan^{-1}\left(\frac{\frac{f}{10} + \frac{f}{10^4}}{1 - \frac{f^2}{10^5}}\right) = 135° \tag{2.2.2.3}$$

$$\frac{\frac{f}{10} + \frac{f}{10^4}}{1 - \frac{f^2}{10^5}} = -1 \tag{2.2.2.4}$$

Solving f using python

$$f \approx 10^4 \tag{2.2.2.5}$$

For the above f equating $|GH| = 1$ and from (2.2.1.3)

$$\frac{\left(10^5\right)H}{\left(\sqrt{1 + \frac{10^8}{100}}\right)\left(\sqrt{1 + \frac{10^8}{10^8}}\right)} = 1 \tag{2.2.2.6}$$

Solving H using python code

$$H = 1.414 \times 10^{-2} \tag{2.2.2.7}$$

$$\implies H_{max} = 1.414 \times 10^{-2} \tag{2.2.2.8}$$

In the part 1 of question for $H = 0.01$ which is less than H_{max} phase margin is greater than 45°

So for

$$H < H_{max} \tag{2.2.2.9}$$

$$\implies H < 1.414 \times 10^{-2} \tag{2.2.2.10}$$

the phase margin is greater than 45°

The following code for bode plot of part 2

codes/ee18btech11034/ee18btech11034_2.py

2.2.3. Verification using Bode plot of part 2

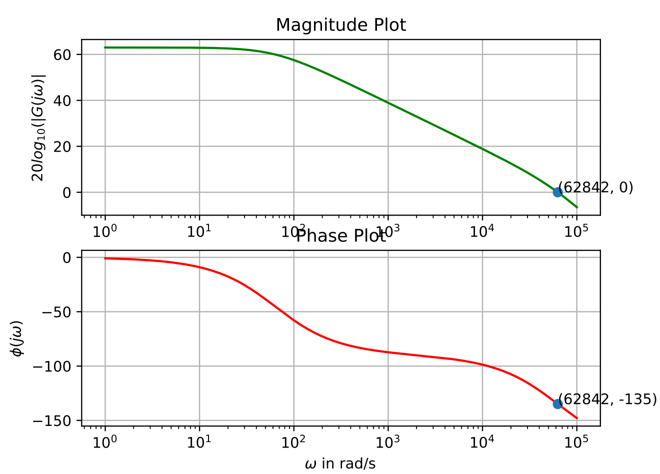

Fig. 2.2.3

Below is the code for computations

codes/ee18btech11034/ee18btech11034.py

2.2.4. Circuit design for part 1

As given op-amp is in non-inverting configuration and having a pole at $f_p = 10$ Hz
The transfer function of op-amp is

$$G_1(s) = \frac{10^5}{1 + \frac{s}{2\pi\times10}} \tag{2.2.4.1}$$

Required $G(s)$

$$G(s) = G_1(s)\frac{1}{1 + \frac{s}{2\pi\times10^4}} \tag{2.2.4.2}$$

So for the error second pole we cascade a RC circuit

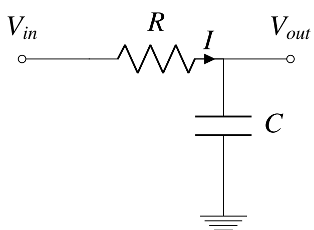

Fig. 2.2.4

$$\frac{V_{out}}{V_{in}} = \frac{I\times\frac{1}{Cs}}{I\times\left(R + \frac{1}{Cs}\right)} \implies \frac{1}{1 + sCR} \tag{2.2.4.3}$$

Choosing R and C such that

$$RC = \frac{1}{2\pi\times10^4} \tag{2.2.4.4}$$

$$\implies 159\times10^{-7} \tag{2.2.4.5}$$

Assuming $R = 5300\Omega$ and $C = 3\times10^{-9}F = 3nF$
Circuit for open loop transfer function $G(s)$

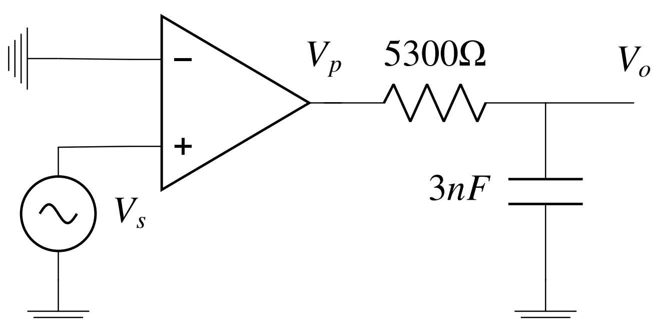

Fig. 2.2.4

At operational amplifier

$$\frac{V_p}{V_s} = G_1(s) \tag{2.2.4.6}$$

At RC circuit

$$\frac{V_o}{V_p} = \frac{1}{1 + \frac{s}{2\pi\times10^4}} \tag{2.2.4.7}$$

From (2.2.4.1),(2.2.4.6) and (2.2.4.7) Open loop gain

$$G = \frac{V_o}{V_s} = \frac{10^5}{\left(1 + \frac{s}{2\pi\times10}\right)\left(1 + \frac{s}{2\pi\times10^4}\right)} \tag{2.2.4.8}$$

For the feedback gain H Choose a resistance network such that

$$H = \frac{V_f}{V_o} = \frac{R_1}{R_1 + R_2} \approx 0.01 \tag{2.2.4.9}$$

Choosig R1 and R2 as

$$R1 = 10\Omega \tag{2.2.4.10}$$

$$R2 = 1000\Omega \tag{2.2.4.11}$$

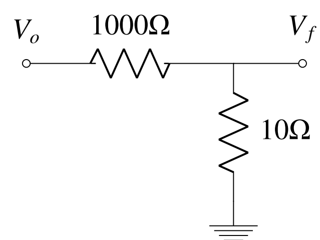

Fig. 2.2.4

From Fig.2.2.4

$$\frac{V_f}{V_o} = \frac{10}{10 + 1000} \tag{2.2.4.12}$$

$$\implies \approx 0.01 \tag{2.2.4.13}$$

Circuit for closed loop transfer function

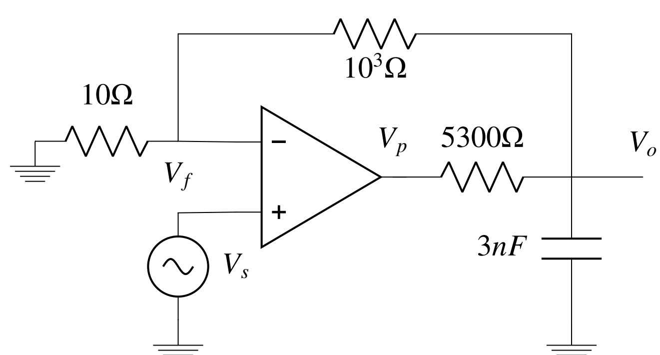

Fig. 2.2.4

The above Fig.2.2.4 is feed back circuit for part 1 of question

2.2.5. Circuit design for part 2

As open loop system is not changed for phase margin to be 45° we change feedback gain H

Required $H = 1.414 \times 10^{-2}$

Choosing R1 and R2 as

$$R1 = 14\Omega \tag{2.2.5.1}$$

$$R2 = 1000\Omega \tag{2.2.5.2}$$

$$H = \frac{R_1}{R_1 + R_2} \tag{2.2.5.3}$$

$$\implies \frac{14}{14 + 1000} \approx 1.4 \times 10^{-2} \tag{2.2.5.4}$$

Circuit for closed loop transfer function

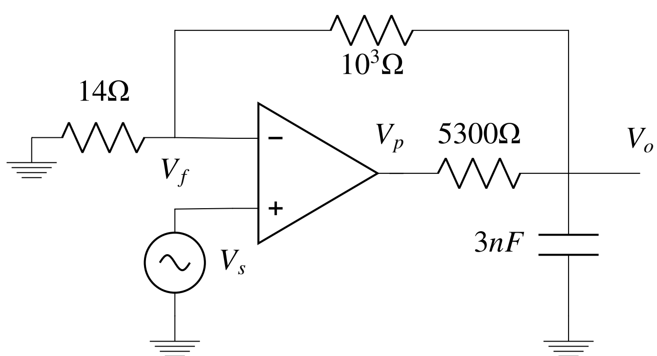

Fig. 2.2.5

The above Fig.2.2.5 is feed back circuit for part 2 of question