Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

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6.1. The number of directions and encirclements around the point -1+j0 in the complex plane by the Nyquist plot of $G(s) = \frac{1-s}{4+2s}$

Solution: Substitute s = $\jmath\omega$ evaluate magnitude and phase from $\omega = -\infty$ to $\omega = \infty$

$$G(\jmath\omega) = \frac{1-\jmath\omega}{4+2\jmath\omega} \tag{6.1.1}$$

6.2. Phase of G(ȷω)

$$\angle G(\jmath\omega) = \angle G(\jmath\omega)_{num} - \angle G(\jmath\omega)_{den} \quad (6.2.1)$$

$$\angle G(\jmath\omega) = \tan^{-1}\left(\frac{-\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) \quad (6.2.2)$$

At $\omega = 0$

$$\angle G(\jmath\omega) = \tan^{-1}(0) - \tan^{-1}(0) = 0 \quad (6.2.3)$$

At $\omega = \infty$

$$\angle G(\jmath\omega) = \tan^{-1}(-\infty) - \tan^{-1}(\infty) = -180 \quad (6.2.4)$$

At $\omega = -\infty$

$$\angle G(\jmath\omega) = \tan^{-1}(\infty) - \tan^{-1}(-\infty) = 180 \quad (6.2.5)$$

6.3. Magnitude of G(ȷω)

$$|G(\jmath\omega)| = \frac{\sqrt{1+\omega^2}}{\sqrt{16+4\omega^2}} \quad (6.3.1)$$

At $\omega = 0$

$$|G(\jmath\omega)| = \frac{1}{4} \quad (6.3.2)$$

At $\omega = \infty$

$$|G(\jmath\omega)| = \frac{1}{2} \quad (6.3.3)$$

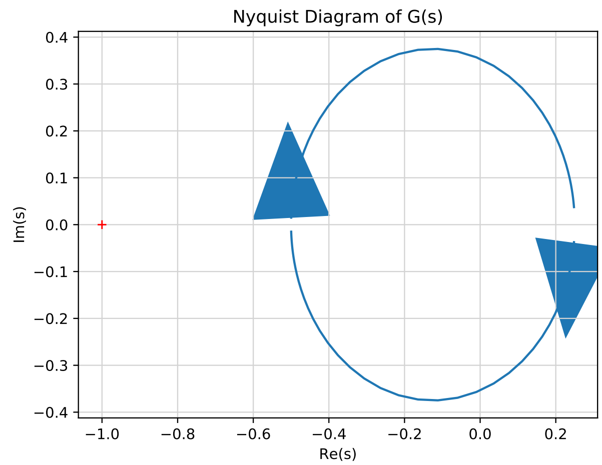
At $\omega = -\infty$

$$|G(\jmath\omega)| = \frac{1}{2} \quad (6.3.4)$$

6.4. Nyquist plotting
Solution:
From $\omega = 0$ to $\omega = \infty$ the path is $\frac{1}{4}\angle 0$ to $\frac{1}{2}\angle -180$
From $\omega = -\infty$ to $\omega = 0$ the path is $\frac{1}{2}\angle 180$ to $\frac{1}{4}\angle 0$
Substitute $s = Re^{\jmath\theta}$

$$\lim_{R\to\infty} G\left(Re^{\jmath\theta}\right) = \frac{1-Re^{\jmath\theta}}{4+2Re^{\jmath\theta}} = \frac{-1}{2} \quad (6.4.1)$$

As there are no $e^{\jmath\theta}$ terms. There will be no enclosed Nyquist path here. The Nyquist plot is the the Polar plot drawn by varying ω from $-\infty$ to ∞.



Hence,the number of encirclements of -1+j0 is zero.

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