Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

1 Polar Plot

1.1 Introduction

1.2 Example

1.3 Example

1.4 Example

1.5 Example

1.6 Example

1.7 Example

1.7.1. Consider the system shown in the figure below.Sketch the nyquist plot of the system and determine the maximum value of K for stability. Take

$$G(s) = \frac{K}{s(1+s)(1+4s)} \tag{1.7.1.1}$$

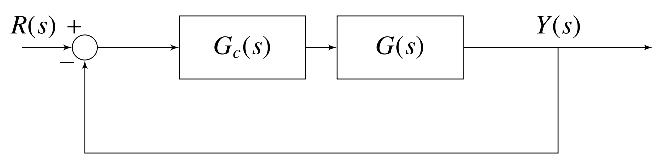

Fig. 1.7.1

Solution:

1.7.2. Part 1 of the question when $G_c(s) = 1$
The open loop transfer function is

$$G_c(s)G(s) = \frac{K}{s(1+s)(1+4s)} \tag{1.7.2.1}$$

$$G_c(\jmath\omega)G(\jmath\omega) = \frac{K}{\jmath\omega(1+\jmath\omega)(1+4\jmath\omega)} \tag{1.7.2.2}$$

$$= \frac{K}{\jmath\omega(1-4\omega^2+5\jmath\omega)} \tag{1.7.2.3}$$

$$= \frac{K\left(-5\omega-\jmath\left(1-4\omega^2\right)\right)}{\omega\left(\left(1-4\omega^2\right)^2+25\omega^2\right)} \tag{1.7.2.4}$$

The maximum K for stability is where the nyquist plot of open loop transfer function cuts the coordinate $(-1, \jmath 0)$

$$\implies \text{Re}\{G(\jmath\omega)G_c(\jmath\omega)\} = -1 \tag{1.7.2.5}$$

$$\implies \text{Im}\{G(\jmath\omega)G_c(\jmath\omega)\} = 0 \tag{1.7.2.6}$$

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$$\implies \mathrm{Re}\left\{G(\jmath\omega)\,G_c(\jmath\omega)\right\} = \frac{-5K\omega}{\omega\left(\left(1-4\omega^2\right)^2+25\omega^2\right)} \tag{1.7.2.7}$$

$$\implies \mathrm{Im}\left\{G(\jmath\omega)\,G_c(\jmath\omega)\right\} = \frac{-K\left(1-4\omega^2\right)}{\omega\left(\left(1-4\omega^2\right)^2+25\omega^2\right)} \tag{1.7.2.8}$$

From (1.7.2.8) and (1.7.2.6)

$$1-4\omega^2 = 0 \implies \omega = \frac{1}{2} \tag{1.7.2.9}$$

From (1.7.2.7),(1.7.2.5) and substituting $\omega = \frac{1}{2}$

$$\frac{-5K\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)\left(\frac{25}{4}\right)} = -1 \implies K = \frac{5}{4} = 1.25 \tag{1.7.2.10}$$

For $K < 0$ the system with negative feedback is unstable the range of K is

$$0 < K < \frac{5}{4} \tag{1.7.2.11}$$

1.7.3. Sketching the Nyquist plot for $G(s)G_c(s)$ in Fig. 1.7.3 The following code gives the nyquist plot

codes/ee18btech11034/ee18btech11034_1.py

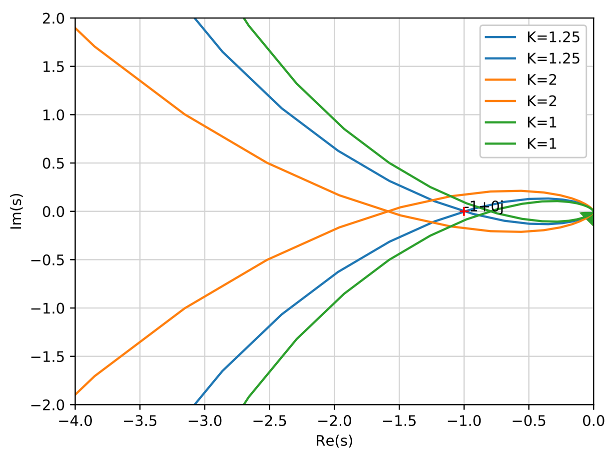

Fig. 1.7.3

1.7.4. Stability Criterian for K

$$N + P = Z \tag{1.7.4.1}$$

K	P	N	Z	Description
1.25	0	0	0	System is marginally stable
2	0	1	1	System is unstable
1	0	0	0	System is stable

TABLE 1.7.4

From the Fig.1.7.3

$$K_{max} = \frac{5}{4} \tag{1.7.4.2}$$

Solution:

1.7.5. Part 2 of the question when $G_c(s) = \frac{1+s}{s}$ The open loop transfer function is

$$G_c(s)\,G(s) = \frac{K(s+1)}{s^2(1+s)(1+4s)} \tag{1.7.5.1}$$

$$G_c(s)\,G(s) = \frac{K}{s^2(1+4s)} \tag{1.7.5.2}$$

$$G_c(\jmath\omega)\,G(\jmath\omega) = \frac{K}{(\jmath\omega)^2(1+4\jmath\omega)} \tag{1.7.5.3}$$

$$= \frac{\frac{-K}{\omega^2}(1-4\jmath\omega)}{1+16\omega^2} \tag{1.7.5.4}$$

From (1.7.2.6)

$$\implies \mathrm{Im}\left\{G(\jmath\omega)\,G_c(\jmath\omega)\right\} = \frac{4K}{\omega(1+16\omega^2)} = 0 \tag{1.7.5.5}$$

This is possible when

$$K = 0 \tag{1.7.5.6}$$

The system is unstable for both

$$K < 0 \tag{1.7.5.7}$$

$$K > 0 \tag{1.7.5.8}$$

1.7.6. Sketching the Nyquist plot for $G(s)G_c(s)$ in Fig. 1.7.6 The following code gives the nyquist plot

codes/ee18btech11034/ee18btech11034_2.py

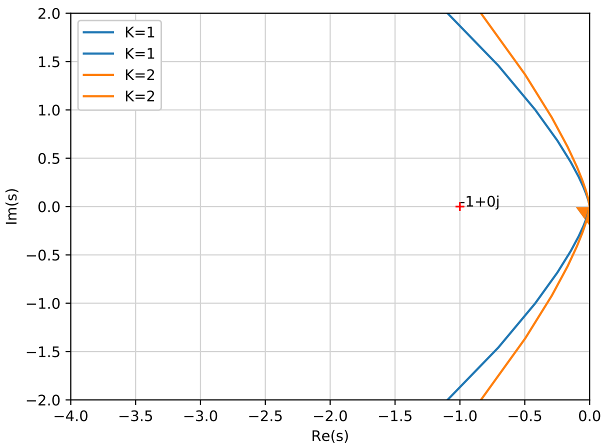

Fig. 1.7.6

From (1.7.4.1)

K	**P**	**N**	**Z**	**Description**
1	0	1	1	System is unstable
2	0	1	1	System is unstable

TABLE 1.7.6

From (1.7.5.6) K_{max} must be 0 which is not possible. Hence the system is unstable for all real K

2 Bode Plot

2.1 Gain and Phase Margin

2.2 Example

3 PID Controller

3.1 Introduction