Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

1 Polar Plot

1.1 Introduction

1.2 Example

1.3 Example

1.4 Example

1.5 Example

1.6 Example

1.7 Example

1.7.1. Consider the system shown in the figure below.Sketch the nyquist plot of the system and determine the maximum value of K for stability. Take

$$G(s) = \frac{K}{s(1+s)(1+4s)} \tag{1.7.1.1}$$

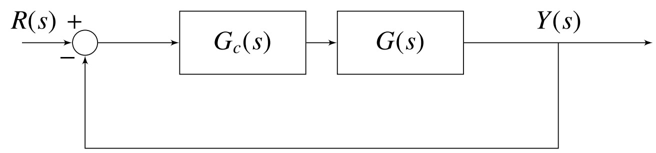

Fig. 1.7.1

Solution:

1.7.2. Part 1 of the question when $G_c(s) = 1$
The open loop transfer function is

$$G_c(s)G(s) = \frac{K}{s(1+s)(1+4s)} \tag{1.7.2.1}$$

$$G_c(\jmath\omega)G(\jmath\omega) = \frac{K}{\jmath\omega(1+\jmath\omega)(1+4\jmath\omega)} \tag{1.7.2.2}$$

$$= \frac{K}{\jmath\omega(1-4\omega^2+5\jmath\omega)} \tag{1.7.2.3}$$

$$= \frac{K\left(-5\omega-\jmath\left(1-4\omega^2\right)\right)}{\omega\left(\left(1-4\omega^2\right)^2+25\omega^2\right)} \tag{1.7.2.4}$$

The maximum K for stability is obtained where the nyquist plot of open loop transfer function cuts the coordinate $(-1, \jmath 0)$
This is done through

$$\implies \text{Re}\{G(\jmath\omega)G_c(\jmath\omega)\} = -1 \tag{1.7.2.5}$$

$$\implies \text{Im}\{G(\jmath\omega)G_c(\jmath\omega)\} = 0 \tag{1.7.2.6}$$

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$$\implies \mathrm{Re}\left\{G(\jmath\omega)\, G_c(\jmath\omega)\right\} = \frac{-5K\omega}{\omega\left(\left(1-4\omega^2\right)^2 + 25\omega^2\right)} \tag{1.7.2.7}$$

$$\implies \mathrm{Im}\left\{G(\jmath\omega)\, G_c(\jmath\omega)\right\} = \frac{-K\left(1-4\omega^2\right)}{\omega\left(\left(1-4\omega^2\right)^2 + 25\omega^2\right)} \tag{1.7.2.8}$$

From (1.7.2.8) and (1.7.2.6)

$$1 - 4\omega^2 = 0 \implies \omega = \frac{1}{2} \tag{1.7.2.9}$$

From (1.7.2.7),(1.7.2.5) and substituting $\omega = \frac{1}{2}$

$$\frac{-5K\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)\left(\frac{25}{4}\right)} = -1 \implies K = \frac{5}{4} = 1.25 \tag{1.7.2.10}$$

From this the maximum value of K for stability is $K_{max} = \frac{5}{4}$ and the system is stable for

$$K < \frac{5}{4} \tag{1.7.2.11}$$

Pratically for $K < 0$ the system with negative feedback is unstable the range of K is

$$0 < K < \frac{5}{4} \tag{1.7.2.12}$$

1.7.3. Sketching the Nyquist plot for $G(s)G_c(s)$ in Fig. 1.7.3 The following code gives the nyquist plot

codes/ee18btech11034_1.py

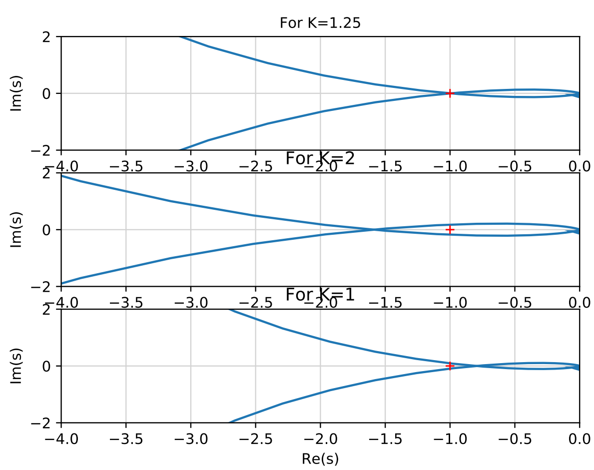

Fig. 1.7.3

1.7.4. Stability Criterian for K

$$N + P = Z \tag{1.7.4.1}$$

K	P	N	Z	Description
1.25	0	0	0	System is not unstable
2	0	1	1	System is unstable
1	0	0	0	System is not unstable

TABLE 1.7.4

And from the Fig.1.7.3 $G_c(s)\, G(s)$ cuts $(-1, \jmath 0)$ at $K_{max} = \frac{5}{4}$

Solution:

1.7.5. Part 2 of the question when $G_c(s) = \frac{1+s}{s}$ The open loop transfer function is

$$G_c(s)\, G(s) = \frac{K(s+1)}{s^2(1+s)(1+4s)} \tag{1.7.5.1}$$

$$G_c(s)\, G(s) = \frac{K}{s^2(1+4s)} \tag{1.7.5.2}$$

$$G_c(\jmath\omega)\, G(\jmath\omega) = \frac{K}{(\jmath\omega)^2(1+4\jmath\omega)} \tag{1.7.5.3}$$

$$= \frac{\frac{-K}{\omega^2}(1-4\jmath\omega)}{1+16\jmath\omega^2} \tag{1.7.5.4}$$

From (1.7.2.6)

$$\implies \mathrm{Im}\left\{G(\jmath\omega)\, G_c(\jmath\omega)\right\} = \frac{4K}{\omega\left(1+16\omega^2\right)} = 0 \tag{1.7.5.5}$$

This is possible when

$$K = 0 \tag{1.7.5.6}$$

Practically the system is unstable for both $K < 0$ and $K > 0$ and theoretically the system is marginally stable for $K = 0$ which is not possible.

1.7.6. Sketching the Nyquist plot for $G(s)G_c(s)$ in Fig. 1.7.6 The following code gives the nyquist plot codes/ee18btech11034_2.py

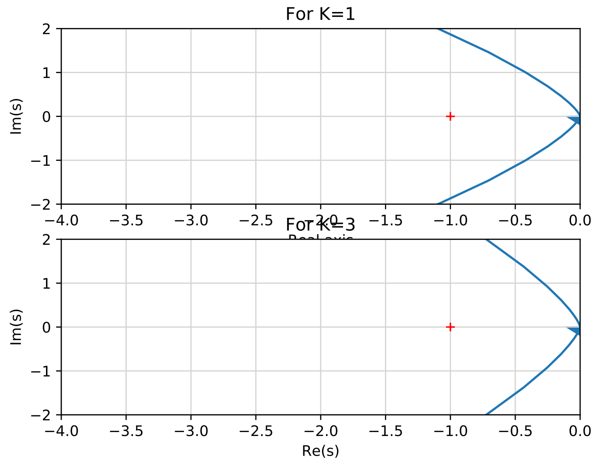

Fig. 1.7.6

From (1.7.4.1)

K	P	N	Z	Description
1	0	1	1	System is unstable
3	0	1	1	System is unstable

TABLE 1.7.6

From (1.7.5.6) K_{max} must be 0 which is not possible. Hence the system is unstable for all real K

2 Bode Plot

2.1 Gain and Phase Margin

2.2 Example

3 PID Controller

3.1 Introduction