

Control Systems

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CONTENTS

Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/codes>

1 SIGNAL FLOW GRAPH

1.1 Mason's Gain Formula

1.2 Matrix Formula

2 BODE PLOT

2.1 Introduction

2.2 Example

3 SECOND ORDER SYSTEM

3.1 Damping

3.2 Example

4 ROUTH HURWITZ CRITERION

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

4.4 Example

5 STATE-SPACE MODEL

5.1 Controllability and Observability

5.2 Second Order System

5.3 Example

5.4 Example

6 NYQUIST PLOT

6.1. The number of directions and encirclements around the point $-1+j0$ in the complex plane by the Nyquist plot of

$$G(s) = \frac{1-s}{4+2s}$$

Solution: First, we need to draw the polar plot of given $G(s)$. In the polar plot, substitute $s = j\omega$

$$G(j\omega) = \frac{1-j\omega}{4+2j\omega} \quad (6.1.1)$$

$$\lim_{\omega \rightarrow \infty} G(j\omega) = \frac{1-j\omega}{4+2j\omega} \quad (6.1.2)$$

$$\lim_{\omega \rightarrow \infty} G(j\omega) = \frac{j\omega(\frac{1}{j\omega} - 1)}{j\omega(\frac{4}{j\omega} + 2)} \quad (6.1.3)$$

$$\lim_{\omega \rightarrow \infty} G(j\omega) = \frac{-1}{2} \angle 0 \quad (6.1.4)$$

which is equal to $\frac{1}{2} \angle -180$

As the Magnitude is taken positive in Nyquist Plot. Now substitute $\omega = 0$

$$\lim_{\omega \rightarrow 0} G(j\omega) = \frac{1-j\omega}{4+2j\omega} = \frac{1}{4} \angle 0 \quad (6.1.5)$$

$$\angle(G(j\omega)) = \tan^{-1}\left(\frac{-\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) \quad (6.1.6)$$

so from this at $\omega = 0$ $\angle G(j\omega) = 0$ and at $\omega = \infty$ $\angle G(j\omega) = -180$

$$|(G(j\omega))| = \frac{\sqrt{1+\omega^2}}{\sqrt{16+4\omega^2}} \quad (6.1.7)$$

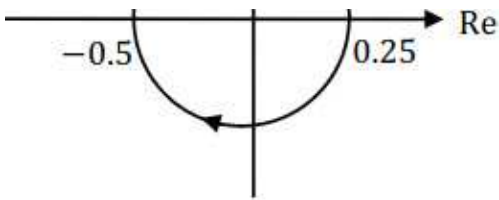
when $\omega = 0$ $|(G(j\omega))| = \frac{1}{4}$ and at when $\omega = \infty$ $|(G(j\omega))| = \frac{1}{2}$

Hence, magnitude should be every time positive.

So, we have to plot first 0.25 then we have to turn -180 degrees to that point i.e. 180 degrees clockwise (in this case).

Now Plot the Polar Plot 1 from $\omega = 0$ to ∞

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7 COMPENSATORS

7.1 Phase Lead

7.2 Example

8 GAIN MARGIN

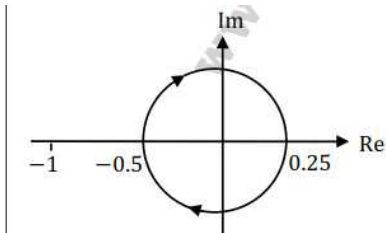
8.1 Introduction

8.2 Example

9 PHASE MARGIN

10 OSCILLATOR

Draw the Mirror image of the Polar Plot 1.



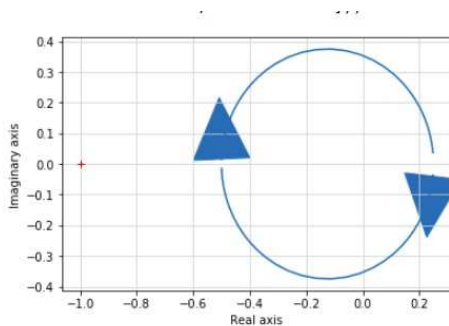
Put $s = Re^{j\theta}$

$$\lim_{R \rightarrow \infty} G(Re^{j\theta}) = \frac{1 - Re^{j\theta}}{4 + 2Re^{j\theta}} = \frac{-1}{2}$$

As there are no $e^{j\theta}$ terms,

There there will be no enclosed Nyquist path here.

So, for this Transfer function $G(s)$, the Nyquist plot is same as the Polar plot.



As from the observed plot the co-ordinate $-1 + j0$ is outside the contour.

Hence, the number of encirclements around the the given co-ordinate is zero.