Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

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6.1. The number of directions and encirclements around the point -1+j0 in the complex plane by the Nyquist plot of $G(s) = \frac{1-s}{4+2s}$

Solution: First,we need to draw the polar plot of given G(S). In the polar plot,substitute s = $j\omega$

$$G(j\omega) = \frac{1 - j\omega}{4 + 2j\omega} \tag{6.1.1}$$

$$\lim_{\omega\to\infty} G(j\omega) = \frac{1 - j\omega}{4 + 2j\omega} \tag{6.1.2}$$

$$\lim_{\omega\to\infty} G(j\omega) = \frac{j\omega(\frac{1}{j\omega} - 1)}{j\omega(\frac{4}{j\omega} + 2)} \tag{6.1.3}$$

$$\lim_{\omega\to\infty} G(j\omega) = \frac{-1}{2}\angle 0 \tag{6.1.4}$$

which is equal to $\frac{1}{2}\angle{-180}$

Now substitute $\omega = 0$

$$\lim_{\omega\to\ 0} G(j\omega) = \frac{1 - j\omega}{4 + 2j\omega} = \frac{1}{4}\angle 0 \tag{6.1.5}$$

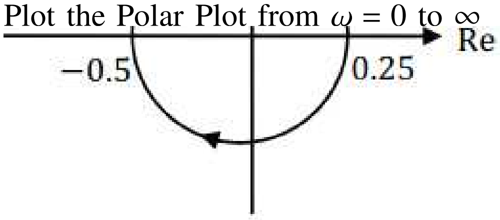
$$\angle(G(j\omega)) = \tan^{-1}(\frac{-\omega}{1}) - \tan^{-1}(\frac{\omega}{2}) \tag{6.1.6}$$

so from this at $\omega = 0$ $\angle G(j\omega) = 0$
and at $\omega = \infty$ $\angle G(j\omega) = -180$

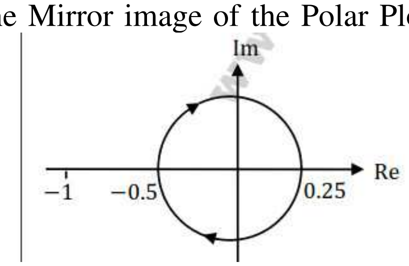
$$\mid (G(j\omega)) \mid = \frac{\sqrt{1 + \omega^2}}{\sqrt{16 + 4\omega^2}} \tag{6.1.7}$$

when $\omega = 0$ $\mid (G(j\omega)) \mid = \frac{1}{4}$
and at $\omega = \infty$ $\mid (G(j\omega)) \mid = \frac{1}{2}$
So,we have to plot first 0.25 on positive x-axis then we have to turn -180 degrees from that point i.e 180 degrees clockwise(in this case).

Plot the Polar Plot from $\omega = 0$ to ∞



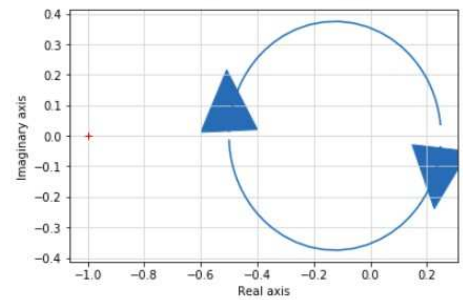
Draw the Mirror image of the Polar Plot.



Substitute $s = Re^{j\theta}$

$$\lim_{R\to\infty} G(Re^{j\theta}) = \frac{1 - Re^{j\theta}}{4 + 2Re^{j\theta}} = \frac{-1}{2} \tag{6.1.8}$$

As there are no $e^{j\theta}$ terms.
There will be no enclosed Nyquist path here.
So, for this Transfer function $G(s)$,the Nyquist plot is the the Polar plot and its mirror image with respect to real axis.



As from the observed plot the co-ordinate -1 + j0 is outside the contour.
Hence,the number of encirclements around the the given co-ordinate is zero.

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