

# Control Systems

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## CONTENTS

<b>1</b>	<b>Signal Flow Graph</b>	1
1.1	Mason's Gain Formula . . . .	1
1.2	Matrix Formula . . . . .	1
<b>2</b>	<b>Bode Plot</b>	1
2.1	Introduction . . . . .	1
2.2	Example . . . . .	1
<b>3</b>	<b>Second order System</b>	1
3.1	Damping . . . . .	1
3.2	Example . . . . .	1
<b>4</b>	<b>Routh Hurwitz Criterion</b>	1
4.1	Routh Array . . . . .	1
4.2	Marginal Stability . . . . .	1
4.3	Stability . . . . .	1
4.4	Example . . . . .	1
<b>5</b>	<b>State-Space Model</b>	1
5.1	Controllability and Observability . . . . .	1
5.2	Second Order System . . . .	1
5.3	Example . . . . .	1
5.4	Example . . . . .	1
<b>6</b>	<b>Nyquist Plot</b>	1
<b>7</b>	<b>Compensators</b>	2
7.1	Phase Lead . . . . .	2
7.2	Example . . . . .	2
<b>8</b>	<b>Gain Margin</b>	2
8.1	Introduction . . . . .	2
8.2	Example . . . . .	2
<b>9</b>	<b>Phase Margin</b>	2

## 10 Oscillator

2

**Abstract**—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/control/codes
```

### 1 SIGNAL FLOW GRAPH

1.1 Mason's Gain Formula

1.2 Matrix Formula

### 2 BODE PLOT

2.1 Introduction

2.2 Example

### 3 SECOND ORDER SYSTEM

3.1 Damping

3.2 Example

### 4 ROUTH HURWITZ CRITERION

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

4.4 Example

### 5 STATE-SPACE MODEL

5.1 Controllability and Observability

5.2 Second Order System

5.3 Example

5.4 Example

### 6 NYQUIST PLOT

6.1. The number of directions and encirclements around the point  $-1+j0$  in the complex plane by the Nyquist plot of  $G(s) = \frac{1-s}{4+2s}$

**Solution:** Substitute  $s = j\omega$  evaluate magnitude and phase at  $\omega = 0$  and  $\omega = \infty$

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## 7 COMPENSATORS

$$G(j\omega) = \frac{1 - j\omega}{4 + 2j\omega} \quad (6.1.1) \quad \begin{array}{l} 7.1 \text{ Phase Lead} \\ 7.2 \text{ Example} \end{array}$$

$$\angle G(j\omega) = \tan^{-1}\left(\frac{-\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) \quad (6.1.2) \quad \begin{array}{l} 8 \text{ GAIN MARGIN} \\ 8.1 \text{ Introduction} \\ 8.2 \text{ Example} \end{array}$$

so from this at  $\omega = 0$   $\angle G(j\omega) = 0$   
and at  $\omega = \infty$   $\angle G(j\omega) = -180$

## 9 PHASE MARGIN

## 10 OSCILLATOR

$$|G(j\omega)| = \frac{\sqrt{1 + \omega^2}}{\sqrt{16 + 4\omega^2}} \quad (6.1.3)$$

when  $\omega = 0$   $|G(j\omega)| = \frac{1}{4}$

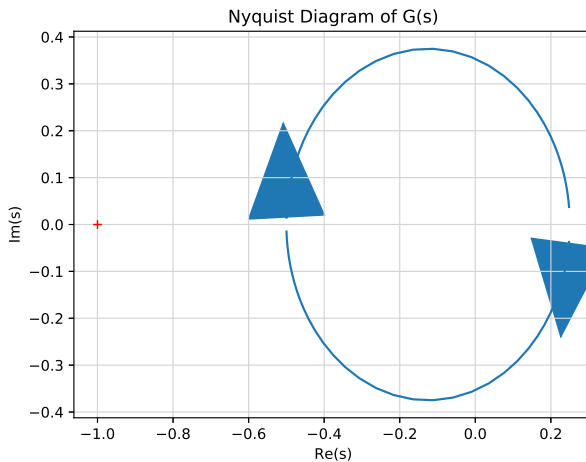
and at  $\omega = \infty$   $|G(j\omega)| = \frac{1}{2}$

Plot first 0.25 on positive x-axis then turn -180 degrees from that point i.e 180 degrees clockwise(in this case).

Substitute  $s = Re^{j\theta}$

$$\lim_{R \rightarrow \infty} G(Re^{j\theta}) = \frac{1 - Re^{j\theta}}{4 + 2Re^{j\theta}} = \frac{-1}{2} \quad (6.1.4)$$

As there are no  $e^{j\theta}$  terms. There will be no enclosed Nyquist path here. So, for this Transfer function  $G(s)$ , the Nyquist plot is the the Polar plot and its mirror image with respect to real axis.



As from the observed plot the co-ordinate  $-1 + j0$  is outside the contour.

Hence, the number of encirclements around the the given co-ordinate is zero.