

# Control Systems

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## 10 Oscillator

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**Abstract**—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/control/codes
```

### 1 SIGNAL FLOW GRAPH

- 1.1 Mason's Gain Formula
- 1.2 Matrix Formula

### 2 BODE PLOT

- 2.1 Introduction
- 2.2 Example

### 3 SECOND ORDER SYSTEM

- 3.1 Damping
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### 4 ROUTH HURWITZ CRITERION

- 4.1 Routh Array
- 4.2 Marginal Stability
- 4.3 Stability
- 4.4 Example

### 5 STATE-SPACE MODEL

- 5.1 Controllability and Observability
- 5.2 Second Order System
- 5.3 Example
- 5.4 Example

### 6 NYQUIST PLOT

- 6.1. The number of directions and encirclements around the point  $-1+j0$  in the complex plane by the Nyquist plot of  $G(s) = \frac{1-s}{4+2s}$

**Solution:** Substitute  $s = j\omega$  evaluate magnitude and phase from  $\omega = -\infty$  to  $\omega = \infty$

$$G(j\omega) = \frac{1-j\omega}{4+2j\omega} \quad (6.1.1)$$

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## 6.2. Phase of $G(j\omega)$

$$\angle G(j\omega) = \angle G(j\omega)_{num} - \angle G(j\omega)_{den} \quad (6.2.1)$$

$$\angle G(j\omega) = \tan^{-1}\left(\frac{-\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) \quad (6.2.2)$$

At  $\omega = 0$

$$\angle G(j\omega) = \tan^{-1}(0) - \tan^{-1}(0) = 0 \quad (6.2.3)$$

At  $\omega = \infty$

$$\angle G(j\omega) = \tan^{-1}(-\infty) - \tan^{-1}(\infty) = -180 \quad (6.2.4)$$

At  $\omega = -\infty$

$$\angle G(j\omega) = \tan^{-1}(\infty) - \tan^{-1}(-\infty) = 180 \quad (6.2.5)$$

## 6.3. Magnitude of $G(j\omega)$

$$|G(j\omega)| = \frac{\sqrt{1 + \omega^2}}{\sqrt{16 + 4\omega^2}} \quad (6.3.1)$$

At  $\omega = 0$

$$|G(j\omega)| = \frac{1}{4} \quad (6.3.2)$$

At  $\omega = \infty$

$$|G(j\omega)| = \frac{1}{2} \quad (6.3.3)$$

At  $\omega = -\infty$

$$|G(j\omega)| = \frac{1}{2} \quad (6.3.4)$$

## 6.4. Nyquist plotting

### Solution:

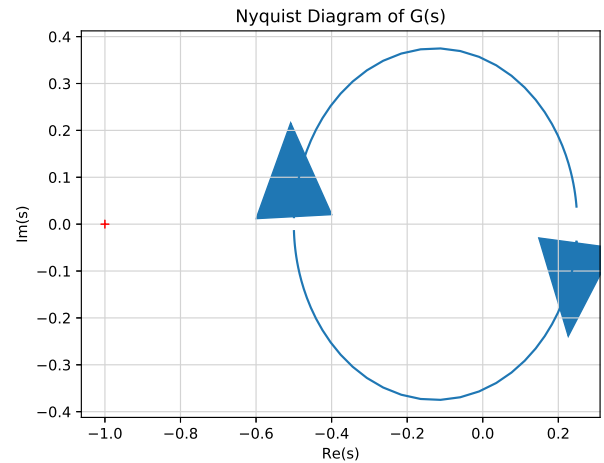
From  $\omega = 0$  to  $\omega = \infty$  the path is  $\frac{1}{4}\angle 0$  to  $\frac{1}{2}\angle -180$

From  $\omega = -\infty$  to  $\omega = 0$  the path is  $\frac{1}{2}\angle 180$  to  $\frac{1}{4}\angle 0$

Substitute  $s = Re^{j\theta}$

$$\lim_{R \rightarrow \infty} G(Re^{j\theta}) = \frac{1 - Re^{j\theta}}{4 + 2Re^{j\theta}} = \frac{-1}{2} \quad (6.4.1)$$

As there are no  $e^{j\theta}$  terms. There will be no enclosed Nyquist path here. The Nyquist plot is the the Polar plot drawn by varying  $\omega$  from  $-\infty$  to  $\infty$ .



Hence, the number of encirclements of  $-1+j0$  is zero.

## 7 COMPENSATORS

### 7.1 Phase Lead

### 7.2 Example

## 8 GAIN MARGIN

### 8.1 Introduction

### 8.2 Example

## 9 PHASE MARGIN

## 10 OSCILLATOR