

Phase Margin

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Consider an op amp having a single pole open loop response $G_0 = 10^5$ and $f_p = 10$ Hz. Let the OPAMP be ideal connected in non-inverting terminal with a nominal low frequency of closed loop gain of 100

- 1) A manufacturing error introducing a second pole at 10 kHz. Find the frequency at which $|GH| = 1$ and the corresponding phase margin.
- 2) For what values of H is the phase margin greater than 45° ?

1. Find the transfer function of the two pole OPAMP.

Solution: For a two-pole amplifier open loop transfer function is

$$G(s) = \frac{G_0}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)} \quad (1.1)$$

Poles are at $f_1 = 10$ and $f_2 = 10^4$

$$G(f) = \frac{G_0}{\left(1 + j\frac{f}{f_1}\right)\left(1 + j\frac{f}{f_2}\right)} \quad (1.2)$$

$$= \frac{10^5}{\left(1 + j\frac{f}{10}\right)\left(1 + j\frac{f}{10^4}\right)} \quad (1.3)$$

2. Find the feedback H .

Solution: Since the closed loop gain

$$|T| = 100 \quad (2.1)$$

and for nominal low frequency $|GH| \gg 1$,

$$H \approx \frac{1}{|T|} = 0.01 \quad (2.2)$$

3. Find the PM and the crossover frequency.

Solution: From (1.3) and (2.2)

$$|GH| = 1 \quad (3.1)$$

$$\Rightarrow \frac{10^3}{\left(\sqrt{1 + \frac{f^2}{100}}\right)\left(\sqrt{1 + \frac{f^2}{10^8}}\right)} = 1 \quad (3.2)$$

$$\text{or } f_{180} = 7.8615 \text{ kHz.} \quad (3.3)$$

using the following python code.

```
codes/ee18btech11034/ee18btech11034.py
```

From (1.3), $\therefore \angle H = 0^\circ$,

$$\angle G(f)H(f) = \angle G(f) \quad (3.4)$$

$$- \tan^{-1}\left(\frac{f}{10}\right) - \tan^{-1}\left(\frac{f}{10^4}\right) \quad (3.5)$$

$$\Rightarrow PM = 180^\circ + \angle G(f_{180}) \quad (3.6)$$

$$= 180^\circ - 128.1^\circ = 51.9^\circ \quad (3.7)$$

4. Verify your result using a Bode plot.

Solution: The following code generates Fig. 4

```
codes/ee18btech11034/ee18btech11034_1.py
```

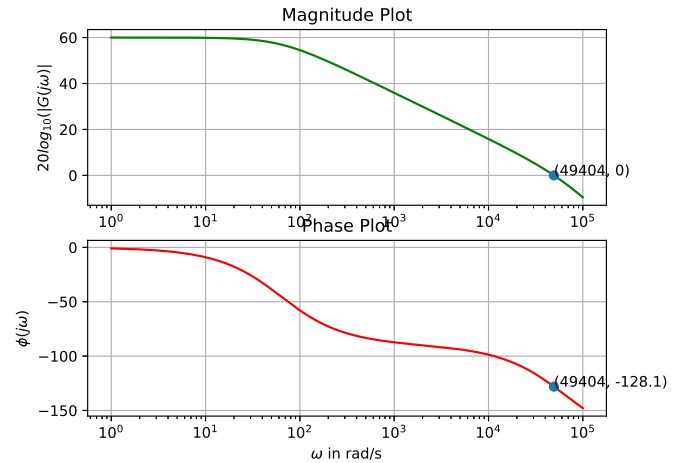


Fig. 4

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5. Realise the above system with $PM = 51.9^\circ$ using a feedback circuit.

Solution:

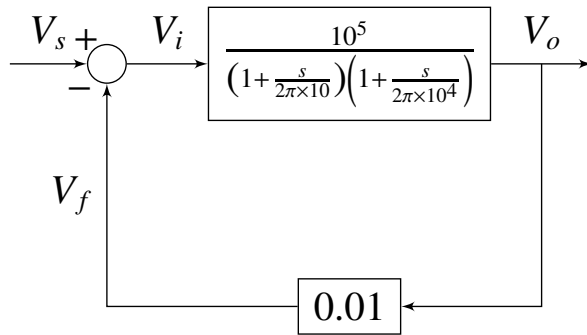


Fig. 5

The transfer function of OPAMP is

$$G(s) = \frac{10^5}{\left(1 + \frac{s}{2\pi \times 10}\right) \left(1 + \frac{s}{2\pi \times 10^4}\right)} \quad (5.1)$$

6. For the feedback gain H

Solution:

Choose a resistance network such that

$$H = \frac{V_f}{V_o} = \frac{R_1}{R_1 + R_2} \approx 0.01 \quad (6.1)$$

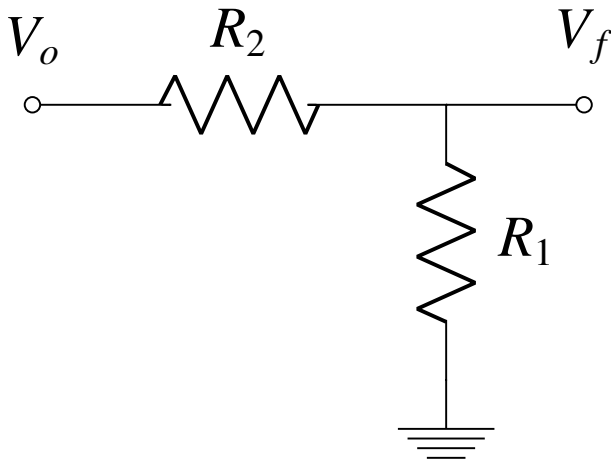


Fig. 6

Choose R_1 and R_2 as

$$R_1 = 10\Omega \quad (6.2)$$

$$R_2 = 990\Omega \quad (6.3)$$

$$H = \frac{R_1}{R_1 + R_2} = \frac{10}{10 + 990} = 0.01 \quad (6.4)$$

7. Feedback Circuit for $PM = 51.9^\circ$

Solution:

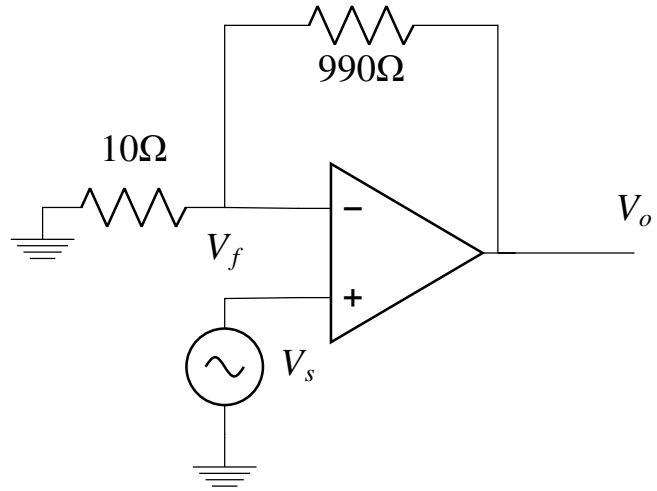


Fig. 7

8. Verification using spice simulation

Solution: For $H = 0.01$ the closed loop response is

$$|T| \approx \frac{1}{H} = 100 \quad (8.1)$$

The following is the netlist file for spice

```
spice/ee18btech11034/ee18btech11034_1.net
```

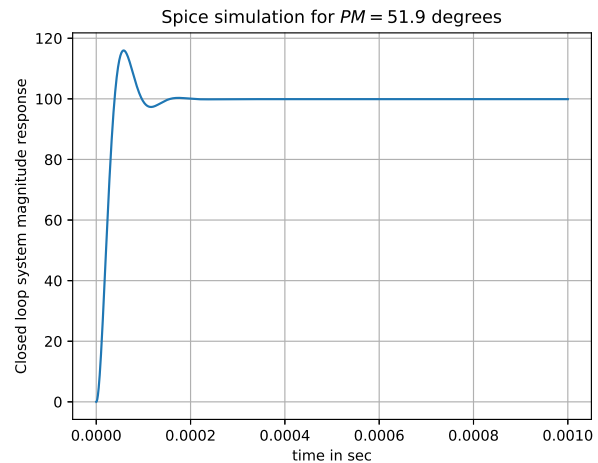


Fig. 8

The following python code plots the closed loop response verses time

```
spice/ee18btech11034/
ee18btech11034_spice_result1.py
```

9. Find H such that $PM = 45^\circ$.

Solution: From (3.4), assuming constant H ,

$$\angle G(f_{180}) = 45^\circ - 180^\circ = -135^\circ \quad (9.1)$$

$$\Rightarrow -\tan^{-1}\left(\frac{f}{10}\right) - \tan^{-1}\left(\frac{f}{10^4}\right) = -135^\circ \quad (9.2)$$

$$\Rightarrow \frac{\frac{f}{10} + \frac{f}{10^4}}{1 - \frac{f^2}{10^5}} = -1 \quad (9.3)$$

$$\text{or, } f_{180} \approx 10 \text{ kHz} \quad (9.4)$$

From (1.3),

$$\because |G(f_{180})H| = 1, \quad (9.5)$$

$$\frac{(10^5)H}{\left(\sqrt{1 + \frac{10^8}{100}}\right)\left(\sqrt{1 + \frac{10^8}{10^8}}\right)} = 1 \quad (9.6)$$

$$\Rightarrow H = 1.414 \times 10^{-2} \quad (9.7)$$

$$\text{or, } H_{\max} = 1.414 \times 10^{-2} \quad (9.8)$$

which is the value of H for which $PM > 45^\circ$.

10. Verify the above using a Bode plot.

Solution: The following code plots Fig. 10.

codes/ee18btech11034/ee18btech11034_2.py

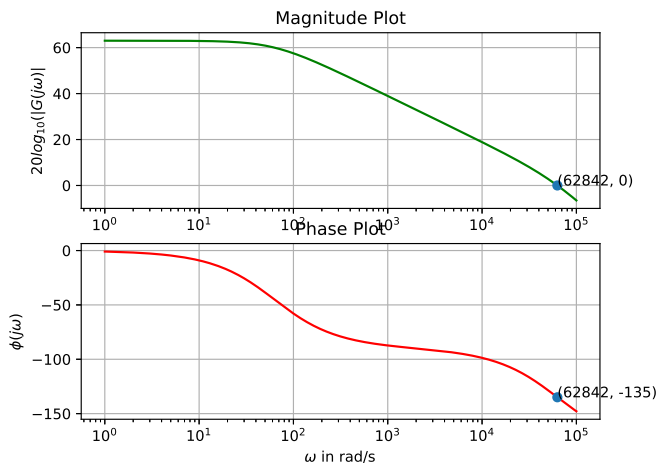


Fig. 10

The transfer function of OPAMP will be unchanged. For the required feedback gain H the feedback circuit changes

11. Realise the above system with $PM = 45^\circ$ using a feedback circuit.

Solution:

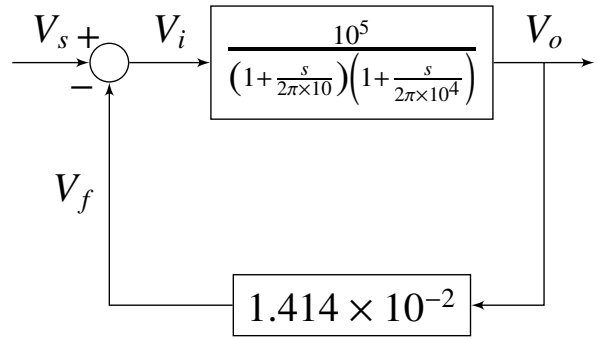


Fig. 11

12. For the feedback gain H

Solution:

$$R_1 = 10\Omega \quad (12.1)$$

$$R_2 = 700\Omega \quad (12.2)$$

$$H = \frac{R_1}{R_1 + R_2} \Rightarrow \frac{10}{10 + 700} \approx 1.41 \times 10^{-2} \quad (12.3)$$

13. Feedback Circuit for $PM = 45^\circ$

Solution:

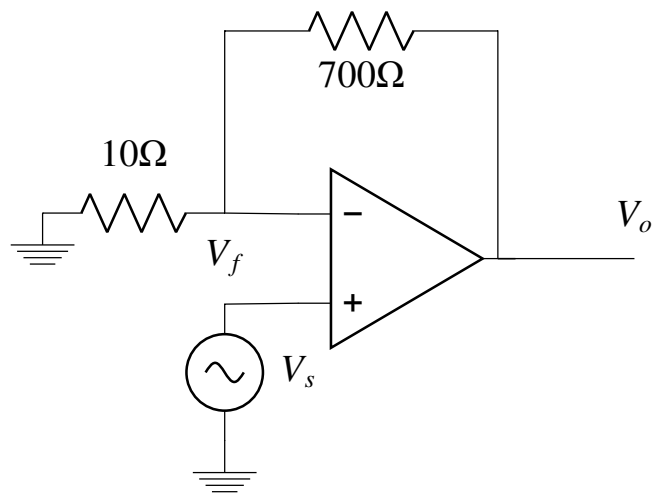


Fig. 13

14. Verification using spice simulation

Solution: For $H = 0.014$ the closed loop response is

$$|T| \approx \frac{1}{H} = 70.72 \quad (14.1)$$

The following is the netlist file for spice

```
spice/ee18btech11034/ee18btech11034_2.net
```

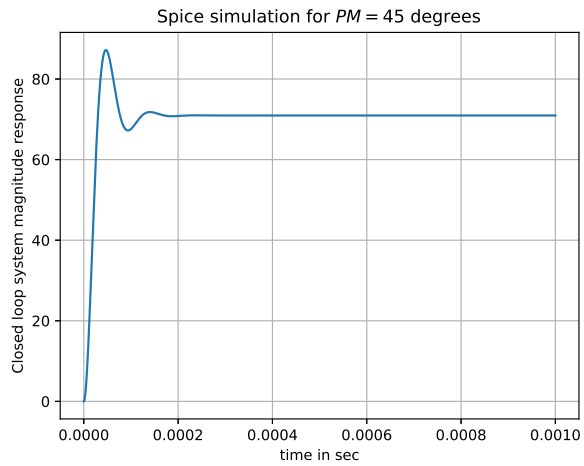


Fig. 14

The following python code plots the closed loop response verses time

```
spice/ee18btech11034/  
ee18btech11034_spice_result2.py
```