# Control Systems

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***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

## 1 Polar Plot

### *1.1 Introduction*

### *1.2 Example*

### *1.3 Example*

### *1.4 Example*

### *1.5 Example*

### *1.6 Example*

### *1.7 Example*

1.7.1. Consider the system shown in the figure below.Sketch the nyquist plot of the system and determine the maximum value of K for stability. Take

$$G(s) = \frac{K}{s(1+s)(1+4s)} \tag{1.7.1.1}$$

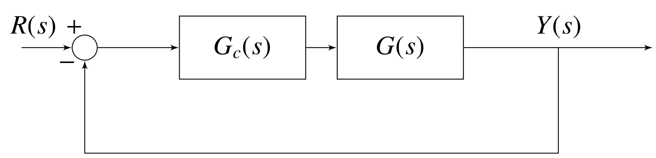


Fig. 1.7.1

**Solution:**

1.7.2. Part 1 of the question when $G_c(s) = 1$
The open loop transfer function is

$$G_c(s) G(s) = \frac{K}{s(1+s)(1+4s)} \tag{1.7.2.1}$$

$$G_c(\jmath\omega) G(\jmath\omega) = \frac{K}{\jmath\omega(1+\jmath\omega)(1+4\jmath\omega)} \tag{1.7.2.2}$$

$$= \frac{K}{\jmath\omega\left(1-4\omega^2+5\jmath\omega\right)} \tag{1.7.2.3}$$

$$= \frac{K\left(-5\omega - \jmath\left(1-4\omega^2\right)\right)}{\omega\left(\left(1-4\omega^2\right)^2+25\omega^2\right)} \tag{1.7.2.4}$$

The maximum K for stability is where the nyquist plot of open loop transfer function cuts the coordinate $(-1, \jmath 0)$

$$\implies \text{Re}\left\{G(\jmath\omega) G_c(\jmath\omega)\right\} = -1 \tag{1.7.2.5}$$

$$\implies \text{Im}\left\{G(\jmath\omega) G_c(\jmath\omega)\right\} = 0 \tag{1.7.2.6}$$

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$$\implies \text{Re}\left\{G(\jmath\omega)G_c(\jmath\omega)\right\} = \frac{-5K\omega}{\omega\left(\left(1-4\omega^2\right)^2+25\omega^2\right)} \tag{1.7.2.7}$$

$$\implies \text{Im}\left\{G(\jmath\omega)G_c(\jmath\omega)\right\} = \frac{-K\left(1-4\omega^2\right)}{\omega\left(\left(1-4\omega^2\right)^2+25\omega^2\right)} \tag{1.7.2.8}$$

From (1.7.2.8) and (1.7.2.6)

$$1-4\omega^2 = 0 \implies \omega = \frac{1}{2} \tag{1.7.2.9}$$

From (1.7.2.7),(1.7.2.5) and substituting $\omega = \frac{1}{2}$

$$\frac{-5K\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)\left(\frac{25}{4}\right)} = -1 \implies K = \frac{5}{4} = 1.25 \tag{1.7.2.10}$$

For $K < 0$ the system with negative feedback is unstable the range of K is

$$0 < K < \frac{5}{4} \tag{1.7.2.11}$$

1.7.3. Sketching the Nyquist plot for $G(s)G_c(s)$ in Fig. 1.7.3 The following code gives the nyquist plot

codes/ee18btech11034/ee18btech11034_1.py

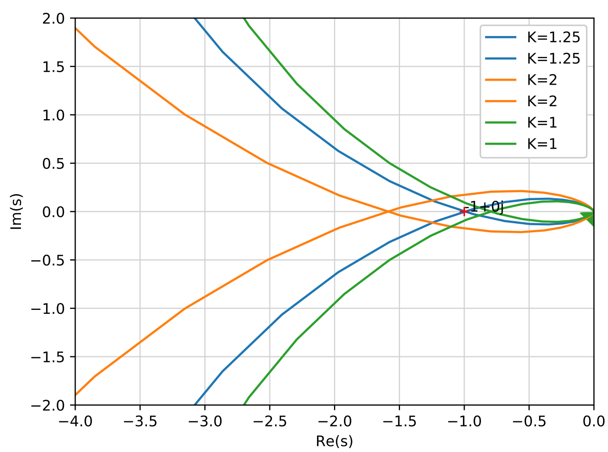


Fig. 1.7.3

1.7.4. Stability Criterian for K

$$N + P = Z \tag{1.7.4.1}$$

| K | P | N | Z | Descrip-tion |
|---|---|---|---|---|
| 1.25 | 0 | 0 | 0 | System is marginally stable |
| 2 | 0 | 1 | 1 | System is unstable |
| 1 | 0 | 0 | 0 | System is stable |

TABLE 1.7.4

From the Fig.1.7.3

$$K_{max} = \frac{5}{4} \tag{1.7.4.2}$$

**Solution:**

1.7.5. Part 2 of the question when $G_c(s) = \frac{1+s}{s}$ The open loop transfer function is

$$G_c(s)G(s) = \frac{K(s+1)}{s^2(1+s)(1+4s)} \tag{1.7.5.1}$$

$$G_c(s)G(s) = \frac{K}{s^2(1+4s)} \tag{1.7.5.2}$$

$$G_c(\jmath\omega)G(\jmath\omega) = \frac{K}{(\jmath\omega)^2(1+4\jmath\omega)} \tag{1.7.5.3}$$

$$= \frac{\frac{-K}{\omega^2}(1-4\jmath\omega)}{1+16\jmath\omega^2} \tag{1.7.5.4}$$

From (1.7.2.6)

$$\implies \text{Im}\left\{G(\jmath\omega)G_c(\jmath\omega)\right\} = \frac{4K}{\omega\left(1+16\omega^2\right)} = 0 \tag{1.7.5.5}$$

This is possible when

$$K = 0 \tag{1.7.5.6}$$

The system is unstable for both

$$K < 0 \tag{1.7.5.7}$$

$$K > 0 \tag{1.7.5.8}$$

1.7.6. Sketching the Nyquist plot for $G(s)G_c(s)$ in Fig. 1.7.6 The following code gives the nyquist plot

codes/ee18btech11034/ee18btech11034_2.py

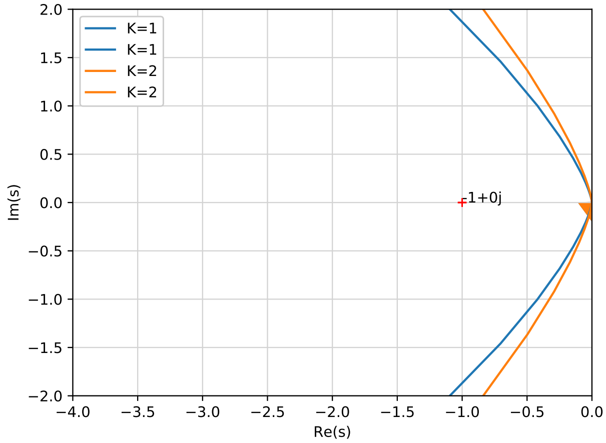


Fig. 1.7.6

From (1.7.4.1)

| K | P | N | Z | Description |
|---|---|---|---|---|
| 1 | 0 | 1 | 1 | System is unstable |
| 2 | 0 | 1 | 1 | System is unstable |

TABLE 1.7.6

From (1.7.5.6) $K_{max}$ must be 0 which is not possible. Hence the system is unstable for all real K

# 2 Bode Plot

## 2.1 Gain and Phase Margin

## 2.2 Example

# 3 PID Controller

## 3.1 Introduction