

Control Systems

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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

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by the Nyquist plot of $G(s) = \frac{1-s}{4+2s}$

Solution: Draw the polar plot of given $G(S)$. In the polar plot, substitute $s = j\omega$ evaluate magnitude and phase at $\omega = 0$ and $\omega = \infty$

$$G(j\omega) = \frac{1 - j\omega}{4 + 2j\omega} \quad (6.1.1)$$

$$\lim_{\omega \rightarrow \infty} G(j\omega) = \frac{1 - j\omega}{4 + 2j\omega} \quad (6.1.2)$$

$$\lim_{\omega \rightarrow \infty} G(j\omega) = \frac{j\omega(\frac{1}{j\omega} - 1)}{j\omega(\frac{4}{j\omega} + 2)} \quad (6.1.3)$$

$$\lim_{\omega \rightarrow \infty} G(j\omega) = \frac{-1}{2} \angle 0 \quad (6.1.4)$$

which is equal to $\frac{1}{2} \angle -180$

$$\lim_{\omega \rightarrow 0} G(j\omega) = \frac{1 - j\omega}{4 + 2j\omega} = \frac{1}{4} \angle 0 \quad (6.1.5)$$

$$\angle(G(j\omega)) = \tan^{-1}\left(\frac{-\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) \quad (6.1.6)$$

so from this at $\omega = 0$ $\angle(G(j\omega)) = 0$

and at $\omega = \infty$ $\angle(G(j\omega)) = -180$

$$|(G(j\omega))| = \frac{\sqrt{1 + \omega^2}}{\sqrt{16 + 4\omega^2}} \quad (6.1.7)$$

when $\omega = 0$ $|(G(j\omega))| = \frac{1}{4}$

and at $\omega = \infty$ $|(G(j\omega))| = \frac{1}{2}$

Plot first 0.25 on positive x-axis then turn -180 degrees from that point i.e 180 degrees clockwise(in this case).

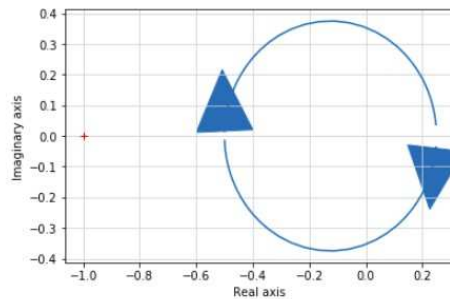
Substitute $s = Re^{j\theta}$

$$\lim_{R \rightarrow \infty} G(Re^{j\theta}) = \frac{1 - Re^{j\theta}}{4 + 2Re^{j\theta}} = \frac{-1}{2} \quad (6.1.8)$$

As there are no $e^{j\theta}$ terms.

There will be no enclosed Nyquist path here.

So, for this Transfer function $G(s)$, the Nyquist plot is the the Polar plot and its mirror image with respect to real axis.



As from the observed plot the co-ordinate $-1 + j0$ is outside the contour.

Hence, the number of encirclements around the the given co-ordinate is zero.

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