Phase Margin

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Consider an op amp having a single pole open loop response $G_0 = 10^5$ and $f_p = 10$ Hz. Let the OPAMP be ideal connected in non-inverting terminal with a nominal low frequency of closed loop gain of 100

1) A manufacturing error introducing a second pole at 10 kHz. Find the frequency at which $|GH| = 1$ and the corresponding phase margin.
2) For what values of H is the phase margin greater than 45°?

1. Find the transfer function of the two pole OPAMP.
Solution: For a two-pole amplifier open loop transfer function is

$$G(s) = \frac{G_0}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)} \tag{1.1}$$

Poles are at $f_1 = 10$ and $f_2 = 10^4$

$$G(f) = \frac{G_0}{\left(1+\jmath\frac{f}{f_1}\right)\left(1+\jmath\frac{f}{f_2}\right)} \tag{1.2}$$

$$= \frac{10^5}{\left(1+\jmath\frac{f}{10}\right)\left(1+\jmath\frac{f}{10^4}\right)} \tag{1.3}$$

2. Find the feedback H.
Solution: Since the closed loop gain

$$|T| = 100 \tag{2.1}$$

and for nominal low frequency $|GH| \gg 1$,

$$H \approx \frac{1}{|T|} = 0.01 \tag{2.2}$$

3. Find the PM and the crossover frequency.

Solution: From (1.3) and (2.2)

$$|GH| = 1 \tag{3.1}$$

$$\implies \frac{10^3}{\left(\sqrt{1+\frac{f^2}{100}}\right)\left(\sqrt{1+\frac{f^2}{10^8}}\right)} = 1 \tag{3.2}$$

$$\text{or } f_{180} = 7.8615\,kHz. \tag{3.3}$$

using the following python code.

```
codes/ee18btech11034/ee18btech11034.py
```

From (1.3), $\because \angle H = 0°$,

$$\angle G(f)H(f) = \angle G(f) \tag{3.4}$$

$$- \tan^{-1}\left(\frac{f}{10}\right) - \tan^{-1}\left(\frac{f}{10^4}\right) \tag{3.5}$$

$$\implies PM = 180° + \angle G(f_{180}) \tag{3.6}$$

$$= 180° - 128.1° = 51.9° \tag{3.7}$$

4. Verify your result using a Bode plot.
Solution: The following code generates Fig. 4

```
codes/ee18btech11034/ee18btech11034_1.py
```

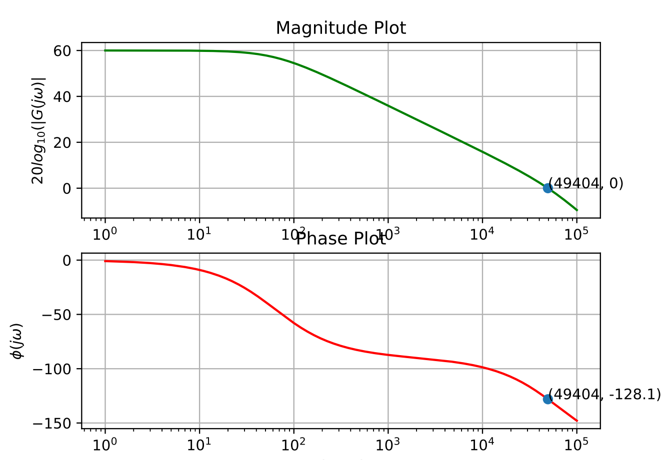

Fig. 4

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5. Realise the above system with $PM = 51.9°$ using a feedback circuit.
Solution:

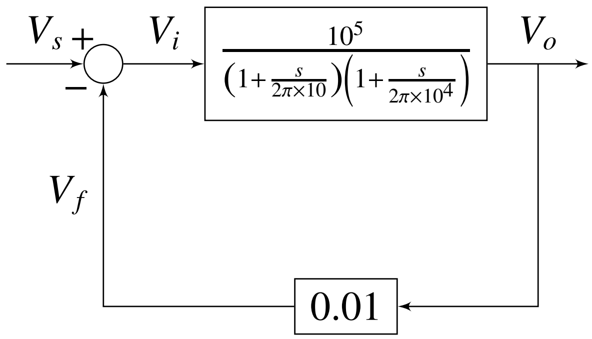

Fig. 5

The transfer function of OPAMP is

$$G(s) = \frac{10^5}{\left(1 + \frac{s}{2\pi\times10}\right)\left(1 + \frac{s}{2\pi\times10^4}\right)} \tag{5.1}$$

6. For the feedback gain H
Solution:
Choose a resistance network such that

$$H = \frac{V_f}{V_o} = \frac{R_1}{R_1 + R_2} \approx 0.01 \tag{6.1}$$

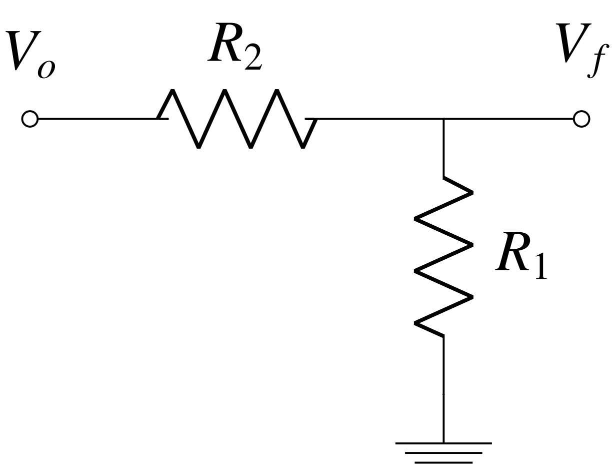

Fig. 6

Choose R_1 and R_2 as

$$R_1 = 10\Omega \tag{6.2}$$
$$R_2 = 990\Omega \tag{6.3}$$

$$H = \frac{R_1}{R_1 + R_2} = \frac{10}{10 + 990} = 0.01 \tag{6.4}$$

7. Feedback Circuit for $PM = 51.9°$
Solution:

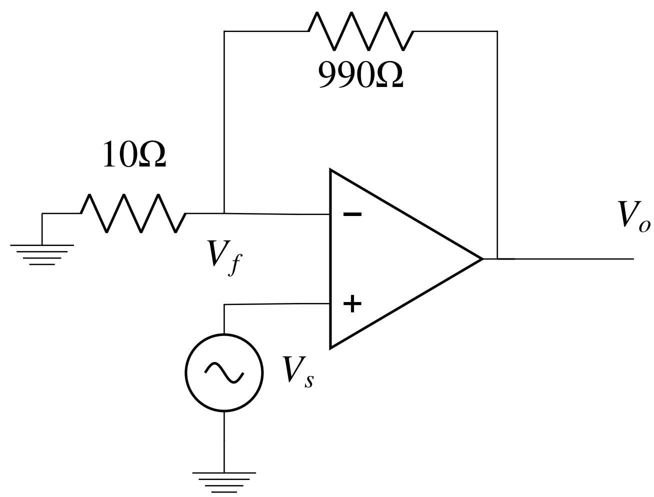

Fig. 7

8. Verification using spice simulation
Solution: For $H = 0.01$ the closed loop response is

$$|T| \approx \frac{1}{H} = 100 \tag{8.1}$$

The following is the netlist file for spice

spice/ee18btech11034/ee18btech11034_1.net

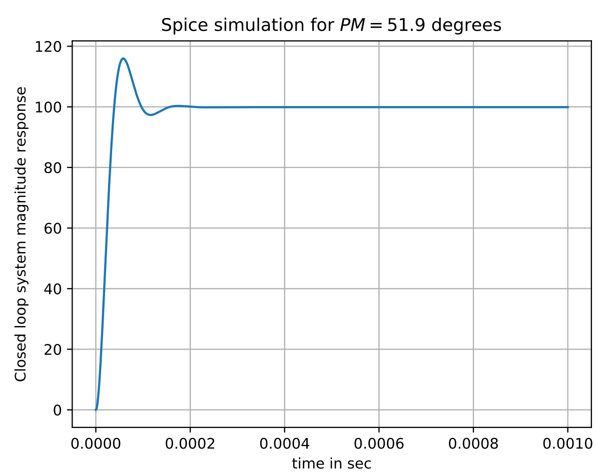

Fig. 8

The following python code plots the closed loop step response verses time

spice/ee18btech11034/ ee18btech11034_spice_result1.py

9. Find H such that $PM = 45°$.
Solution: From (3.4), assuming constant H,

$$\angle G(f_{180}) = 45° - 180° = -135° \tag{9.1}$$

$$\implies -\tan^{-1}\left(\frac{f}{10}\right) - \tan^{-1}\left(\frac{f}{10^4}\right) = -135° \tag{9.2}$$

$$\implies \frac{\frac{f}{10} + \frac{f}{10^4}}{1 - \frac{f^2}{10^5}} = -1 \tag{9.3}$$

$$\text{or, } f_{180} \approx 10\,kHz \tag{9.4}$$

From (1.3),

$$\because |G(f_{180})\,H| = 1, \tag{9.5}$$

$$\frac{\left(10^5\right)H}{\left(\sqrt{1+\frac{10^8}{100}}\right)\left(\sqrt{1+\frac{10^8}{10^8}}\right)} = 1 \tag{9.6}$$

$$\implies H = 1.414 \times 10^{-2} \tag{9.7}$$

$$\text{or, } H_{max} = 1.414 \times 10^{-2} \tag{9.8}$$

which is the value of H for which $PM > 45°$.

10. Verify the above using a Bode plot.
Solution: The following code plots Fig. 10.

```
codes/ee18btech11034/ee18btech11034_2.py
```

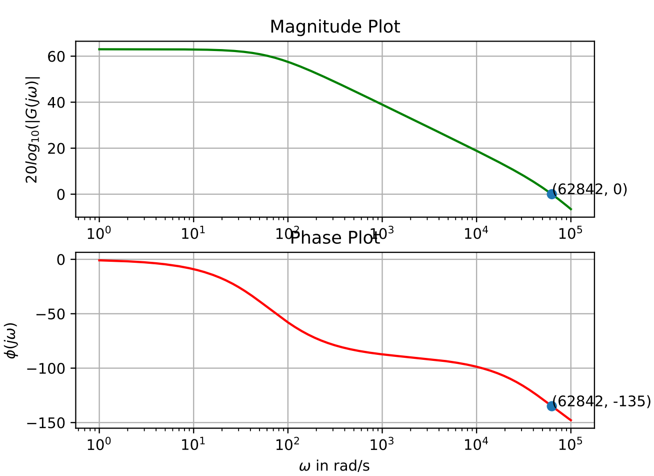

Fig. 10

The transfer function of OPAMP will be unchanged. For the required feedback gain H the feedback circuit changes

11. Realise the above system with $PM = 45°$ using a feedback circuit.
Solution:

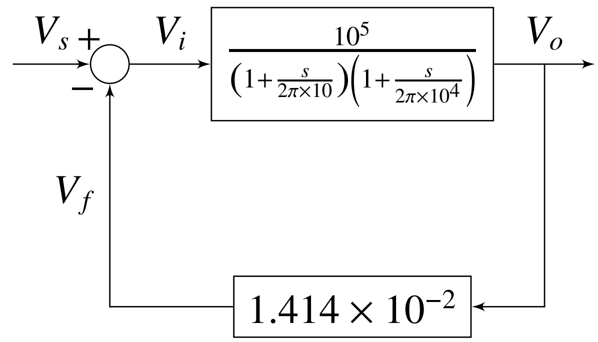

Fig. 11

12. For the feedback gain H
Solution:

$$R_1 = 10\Omega \tag{12.1}$$

$$R_2 = 700\Omega \tag{12.2}$$

$$H = \frac{R_1}{R_1 + R_2} \implies \frac{10}{10 + 700} \approx 1.41 \times 10^{-2} \tag{12.3}$$

13. Feedback Circuit for $PM = 45°$
Solution:

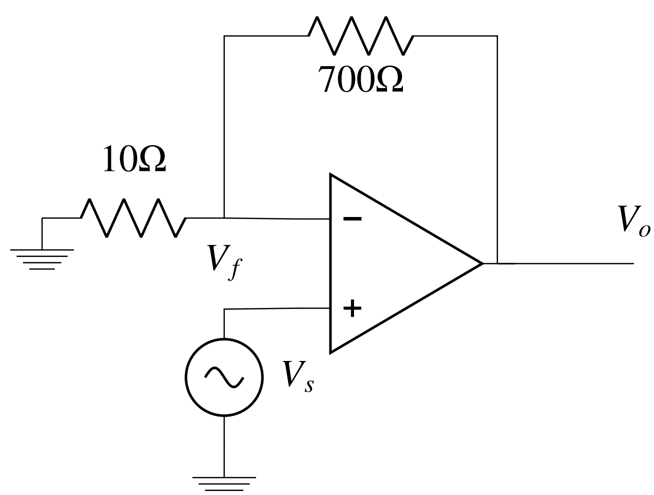

Fig. 13

14. Verification using spice simulation
Solution: For $H = 0.014$ the closed loop response is

$$|T| \approx \frac{1}{H} = 70.72 \tag{14.1}$$

The following is the netlist file for spice

spice/ee18btech11034/ee18btech11034_2.net

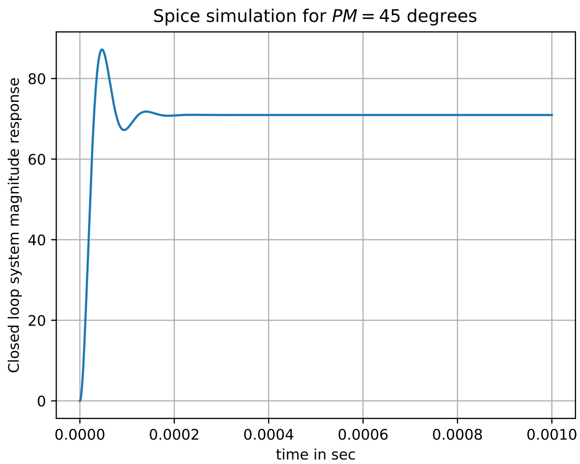

Fig. 14

The following python code plots the closed loop step response verses time

spice/ee18btech11034/ee18btech11034_spice_result2.py

Follow the instructions in below file for running spice files

spice/ee18btech11034/README.md

15. Check for unstability
Solution: For a closed loop system to be unstable PM of GH is negative

$$PM < 0^\circ \tag{15.1}$$

$$\implies \angle G(f)H(f) < -180^\circ \tag{15.2}$$

For the given GH

$$\angle G(f)H(f) = \angle G(f) \tag{15.3}$$

$$\implies -\tan^{-1}\left(\frac{f}{10}\right) - \tan^{-1}\left(\frac{f}{10^4}\right) \tag{15.4}$$

At $f = \infty$

$$\angle G(f) = -90^\circ - 90^\circ = -180^\circ \tag{15.5}$$

So there will be no positive f where $\angle G(f) < -180^\circ$
Hence,the system is stable for any constant feedback gain H