EE3025 INDEPENDENT PROJECT

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Assignment-1 Question 7.1

The command $output_{signal} = signal.lfilter(b, a, input_{signal})$ in Problem 2.3 is executed with the following difference equation

$$\sum_{m=0}^{M} a(m)y(n-m) = \sum_{k=0}^{N} b(k)x(n-k) \tag{1}$$

where input signal is $x(n)$ with initial values all 0.Replace **signal.filtfilt** with your own routine and verify

Solution

Computing $X(z)$ from $x(n)$ using inbuilt fft (fast fourier transfrom)command

$$X(z) = fft(x(n)) \tag{2}$$

Obtaining $H(z)$ from coefficients b,a as

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{N} b(k)z^{-k}}{\sum_{m=0}^{M} a(m)z^{-m}} \tag{3}$$

Obtaining $Y(z)$ as

$$Y(z) = X(z)H(z) \tag{4}$$

and computing $y(n)$ from $Y(z)$ using inbuit ifft(inverse fast fourier transform) command

$$y(n) = ifft(Y(z)) \tag{5}$$

where $y(n)$ is the output signal

Graphical results

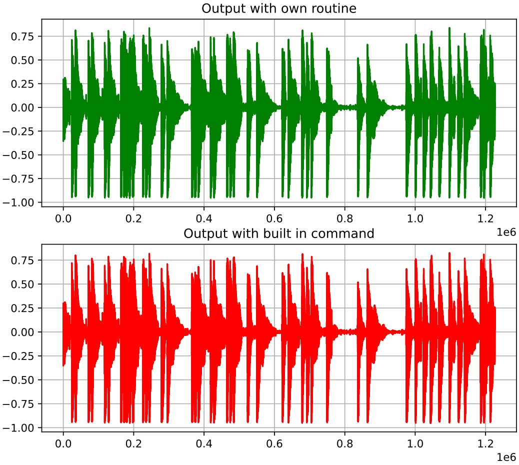

Figure: Time domain response of output signal

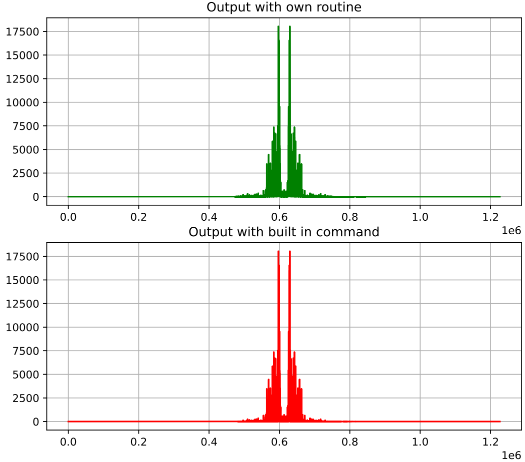

Figure: Frequency domain response of output signal

FFT Algorithm implementation in C for Assignment-1

- Implementing Radix-2 FFT alogorithm making the length of the signal as $N = 2^k$ by appending zeros to the original signal.
- Spliting the original sequence $x(n)$ into two sequences (even indexing and odd indexing) as $s_1(m) = x(2n)$ and $s_2(m) = x(2n+1)$
- Now the main DFT expression

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \tag{6}$$

where $k = 0, 1..N-1$ and $W_N = \exp(\frac{-j2\pi}{N})$

- Reduces to the form

$$X(k) = S_1(k) + W_N^k S_2(k) \tag{7}$$

$$X(k + \frac{N}{2}) = S_1(k) - W_N^k S_2(k) \tag{8}$$

where $k = 0, 1..\frac{N}{2} - 1$, $S_1(k)$ and $S_2(k)$ are $\frac{N}{2}$ point DFT's of the sequences $s_1(m)$ and $s_2(m)$ respectively.
- Again the $\frac{N}{2}$ point DFT is computed using the same expression above and this is done in a recursive process.

Graphical results

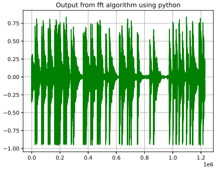

Figure: Time domain response in python of output signal

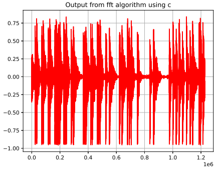

Figure: Time domain response in C of output signal

We are supposed to design FIR and IIR realizations for the filter number 114.Observing magnitude response of the filters.Chebyshev filter is taken in the case of IIR filter
Chebyshev filter have a steeper roll-off than butterworth filter and has stopband monotonic and passband equiripple.

After choosing required specifications setting $N = 4$ (4th order Chebyshev filter) and $0.3184 < \epsilon < 0.6197$ (parameters of filter), plotting the magnitude response of Lowpass Chebyshev filter for the above values
The transfer function is given as

$$|H_{a,LP}(j\Omega_L)|^2 = \frac{1}{1 + \epsilon^2 c_N^2(\Omega_L)} \tag{9}$$

where $c_N(x) = \cosh(N \cosh^{-1} x)$

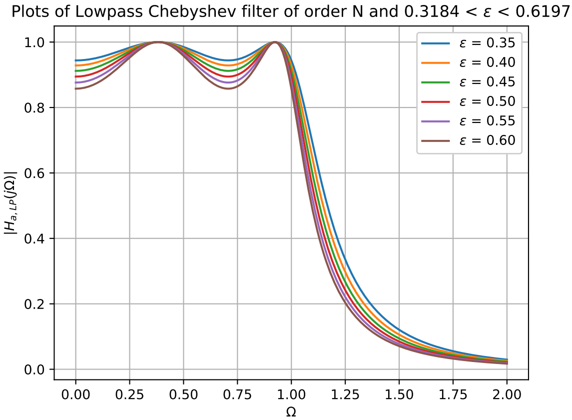

Figure: Lowpass Chebyshev filter magnitude response for different values of ϵ

We choose $\epsilon = 0.4$ for our IIR design

Verifying the design with magnitude response of Lowpass stable Chebyshev filter ($H_{a,LP}(s_L)$) for even order (i.e using Chebyshev polynomial) and obtaining gain as $G_{lp} = 0.3125$ and poles as $p = [1, 1.1068, 1.6125, 0.9140, 0.3366]$

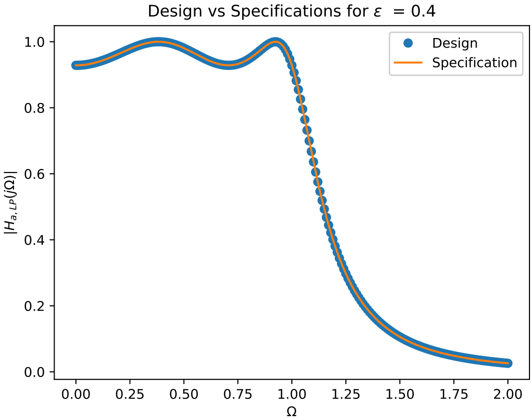

Figure: Design vs Specifications

From the above graph it is verified the design meets the specifications for Lowpass IIR filter.
Obtaining analog band pass Chebyshev filter as

$$H_{a,BP}(s) = G_{BP} H_{a,LP}(s_L) \tag{10}$$

where G_{BP} is gain of analog band pass filter
Obtaining bilinear transformation by substituting $s = \frac{1-z^{-1}}{1+z^{-1}}$ which converts analog filter transfer function to digital filter transfer function and using below equation

$$H_{d,BP}(z) = G_{BP} H_{a,LP}(s) \tag{11}$$

Plots of Analog Lowpass,Analog Bandpass and Digital Bandpass IIR filters

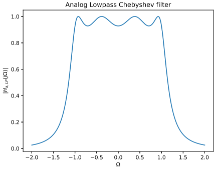

Figure: Analog Lowpass IIR filter magnitude response

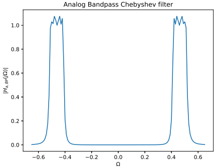

Figure: Analog Bandpass IIR filter magnitude response

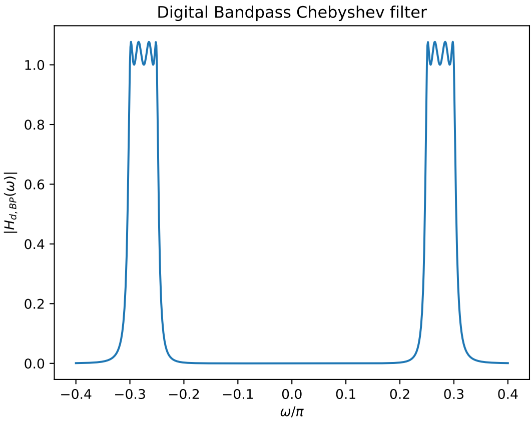

Figure: Digital Bandpass IIR filter magnitude response vs Normalized Frequency

FIR Filter Design

The general impulse response $h_{lp}(n)$ for the desired lowpass FIR filter with cutoff frequency ω_l is given by

$$h_l(n) = \frac{\sin(n\omega_l)}{n\pi} w(n) \tag{12}$$

where $w(n)$ is the Kaiser window obtained from the design specifications.
We obtain the desired low pass filter impulse as

$$\begin{aligned} h_{lp}(n) &= \frac{\sin(\frac{n\pi}{40})}{n\pi} - 100 \leq n \leq 100 \\ &= 0, \qquad\qquad \text{otherwise} \end{aligned} \tag{13}$$

The Kaiser window in the interval [-100,100] is chosen to be 1

The desired band pass filter impulse response is obtained as

$$h_{bp}(n) = 2h_{lp}(n)cos(n\omega_c) \tag{14}$$

Thus now we obtain

$$\begin{aligned} h_{bp}(n) &= \frac{2\sin(\frac{n\pi}{40})\cos(\frac{11n\pi}{40})}{n\pi} \quad -100 \leq n \leq 100 \\ &= 0, \qquad \text{otherwise} \end{aligned} \tag{15}$$

Multiplication of $\cos(n\omega_c)$ with sinc function in time domain gives convolution of rectangular window function centered around 0 with two delta functions at ω_c which finally gives two rectangular window functions centered around ω_c (genral bandpass FIR function).

Plots of magnitude response of lowpass and bandpass FIR filters

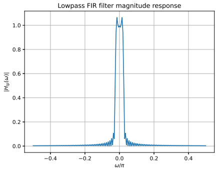

Figure: Lowpass FIR filter magnitude response

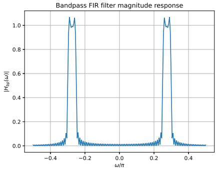

Figure: Bandpass FIR filter magnitude response