4th sem
CSE
Done

INDIAN INSTITUTE OF INFORMATION TECHNOLOGY, NAGPUR
Department of Computer Science & Engineering
CSL 205: Design and Analysis of Algorithms

Date: 8th May 2024 (Wed)

Exam: End Sem Exam
Duration: 5 hour | Semester – IV (CSE, DSA, AI&ML, HCI&GT) | Max. Marks: 55

Important Instructions:

- This is a closed book, closed notes examination.
- This question paper comprises total 6 questions.
- All the questions are compulsory.

Q1 a) A tree is called a ternary tree if every node has at most 3 children. Let us call a ternary tree KGP tree if, at every node x, the difference between the heights of all the subtrees rooted at the children of x differ by at most 5. Find the asymptotic behavior of the maximum height of any KGP tree with n nodes. (3M) CO1

b) Derive the asymptotic complexity of T(n) in terms of Θ for the following recurrence.

$$T(n) = \begin{cases} T\left(2^{\log^{\frac{1}{11}} n}\right) + T\left(2^{\log^{\frac{9}{17}} n}\right) + 13 \log \log n, & if\ n \geq 10^{20} \\ 1, & otherwise \end{cases}$$

(3M) CO1

c) Consider the recurrence relation

$$t_n = t_{n-1} + t_{n-2} \quad ; \quad n > 1$$
$$t_n = n \quad ; \quad \text{if } n = 0 \text{ or } n = 1$$

(4M) CO1

Apply Generating function method to solve the above recurrence relation.

Q.2 a) A cable connection operator has n cable wires of various lengths and wants to give the cable connection to a customer from his office. He wants to connect these wires into a single wire. The cost to join two cable wires is equal to the sum of their lengths. Write an algorithm based on priority queue to joint these cable wires with minimum cost. (5M) CO1, CO3, CO4

Example:
Suppose the cable operator has four cable wires of lengths {5, 2, 3, 9}. He first joins the cables of lengths 2 and 3. The cost of joining is {2 + 3 = 5}. Now he has three wires of lengths {5, 5, 9}. Further, he joins the wire of lengths 5 and 5, and the cost is {5 + 5 = 10}. The total cost till now becomes {5 + 10 = 15}. Now, he has two wires of lengths
10 and 9. Finally, he connects the last two wires to have all the wires connected. The total cost becomes {15 + 19 = 34}.

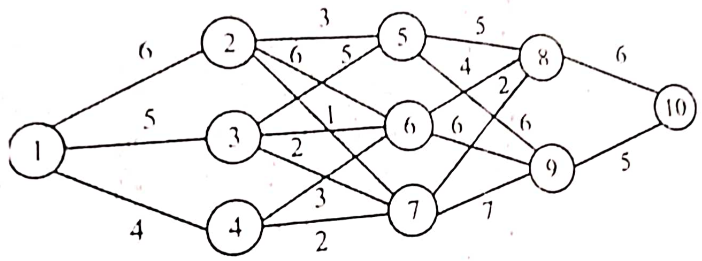
b) Assume that a task dependency scenario is given in the ordered pair form as (a_i, b_i) = {(1, 2), (2, 3), (2, 4), (2, 5), (3, 6), (4, 6), (5, 7)}, which represents that task b can be completed after the completion of task a. If each task takes one unit of time, then write an efficient approach to calculate the minimum time in which tasks can complete their execution. (5M) CO1, CO3, CO4
Please note that independent tasks can be executed in parallel. Also, discuss your approach's space and time complexity if there are T tasks and P task dependencies.

Q.3 a) A traveling salesman has to cover a set of 5 cities (his own included) periodically (say, once per week) and return home. The distances between the cities are given in Table 1, as could have been read on a map. Write an algorithm to find the minimum cost of the tour if he starts his journey from city A and visits every city exactly once, and returns to the same city. Also, write the minimum cost of the tour obtained using the table given below. (5M) CO5

Table 1: Cost matrix (distances in km) from one city to another

City	A	B	C	D	E
A	0	132	217	164	58
B	132	0	290	201	79
C	217	290	0	113	303
D	164	201	113	0	196
E	58	79	303	196	0

b) Apply multistage graph approach to solve the given graph by using backward approach. (5M) CO1, CO3, CO4



Q.4 For the following statements, indicate that the statement is True or False. Also, give the suitable justification. (10M) CO1, CO3, CO4, CO5

a) Negating all the edge weights in a weighted undirected graph G and then finding the minimum spanning tree gives us the maximum-weight spanning tree of the original graph G.

b) In the recursion of the Floyd-Warshall algorithm

$$d_{uv}^{(k)} = min\left\{d_{uv}^{(k-1)}, d_{uk}^{(k-1)} + d_{kv}^{(k-1)}\right\},$$

$d_{uv}^{(k)}$ represents the length of the shortest path from vertex u to vertex v that contains at most k edges.

c) If a dynamic-programming problem satisfies the optimal-substructure property, then a locally optimal solution is globally optimal.

d) Let $G = (V, E)$ be a directed graph with negative-weight edges, but no negative-weight cycles. Then, one can compute all shortest paths from a source $s \in V$ to all $v \in V$ faster than Bellman-Ford using the technique of reweighting.

e) Ram and Shyam have been asked to show that a certain problem L is NP-complete. Ram shows a polynomial reduction from the 3-satisfiability problem to L and Shyam shows a polynomial reduction from L to the 3-satisfiability problem. State with justification which of the following can be inferred from these reductions.

(i) L is NP-hard but not NP-complete (ii) L is in NP but not NP-complete
(iii) L is NP-complete (iv) L is neither NP-hard nor in NP

Q.5 a) Write a dynamic programming-based solution to find the longest triangular subsequence from the given sequence. Here, triangular subsequence refers to a subsequence where numbers are first strictly increasing then strictly decreasing. Also, discuss the time complexity and space complexity of your solution. (5M) CO1, CO3, CO4

Example: If given sequence is [6 16 7 15 9 10 7 6] then its longest triangular subsequence will be [6, 7, 15, 9, 7, 6] Or [6, 16, 15, 10, 7, 6] Or [6, 7, 9, 10, 7, 6].

b) Imagine you have a chessboard of size N×N and you want to solve the N-Queens problem for a given value of N. Each row and each column of the chessboard should contain exactly one queen, and no two queens should be able to attack each other (i.e., no two queens should be in the same row, column, or diagonal). (5M) CO1, CO3, CO4

i) Devise an algorithm to determine all possible solutions to the N-Queens problem?

ii) How would you optimize your algorithm to efficiently find solutions, especially for larger values of N?

Q.6 a) Consider the following instance for 3-SAT: (2½M) CO5

$$\emptyset = (\overline{x_1} \vee x_3 \vee \overline{x_4}) \wedge (x_1 \vee x_2 \vee x_4)$$

Give the corresponding instance for the independent set problem Independent_Set, according to the reduction algorithm for 3-SAT $\leq_P$ Independent_Set. You need to find (G, k), where G is a graph and k is a target size for the independent set.

b) Design the Randomized Quick Sort algorithm and also analyze the time complexity for the same. (2½M) CO5