AN EXAMPLE FOR ROW- REDUCTION
(GAUSS- JORDAN ELIMINATION)

$$\begin{bmatrix} 0 & 5 & 10 & 8 \\ 1 & 2 & 6 & 7 \\ 2 & 4 & 12 & 6 \end{bmatrix} = A$$

$$\Big\downarrow R_1 \longleftrightarrow R_2 \quad \text{(interchange)}$$

$$\begin{bmatrix} 1 & 2 & 6 & 7 \\ 0 & 5 & 10 & 8 \\ 2 & 4 & 12 & 6 \end{bmatrix}$$

$$\Big\downarrow R_3 \longrightarrow R_3 - 2R_1 \quad \text{(replacement)}$$

$$\begin{bmatrix} 1 & 2 & 6 & 7 \\ 0 & 5 & 10 & 8 \\ 0 & 0 & 0 & -8 \end{bmatrix}$$

Complete forward phase to get an echelon form matrix

$$R_2 \to \frac{1}{5} R_2 \qquad R_3 \to \left(-\frac{1}{8}\right) R_3 \quad \Big\downarrow$$

$$\begin{bmatrix} 1 & 2 & 6 & 7 \\ 0 & 1 & 2 & \frac{8}{5} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- scaling
- normalization

We had obtained the following matrix

$$\begin{bmatrix} 1 & 2 & 6 & 7 \\ 0 & 1 & 2 & \frac{8}{5} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, do the main backward phase, starting with the right-most lowest 1

$$\Big\downarrow \begin{array}{l} R_1 \to R_1 - 7R_3 \\ R_2 \to R_2 - \frac{8}{5}R_3 \end{array} \quad \Big| \text{ replacement}$$

$$\begin{bmatrix} 1 & 2 & 6 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Big\downarrow \; R_1 \to R_1 - 2R_2$$

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = R$$

$\longrightarrow$ RREF matrix

NB: all the matrices obtained while executing the algorithm are row-equivalent to A. However, there is <u>only one</u> RREF matrix equivalent to A, namely R.