MTH100A – 20230825

Supplementary Notes

ROW & COLUMN FORM OF A MATRIX.

Let A be an $m \times n$ matrix.

(i) Regard each row $\bar{r}_i$ as a row vector in $\mathbb{R}^n$, or equivalently, as a $1 \times n$ matrix.

Then, $A = \begin{bmatrix} \bar{r}_1 \\ \vdots \\ \bar{r}_m \end{bmatrix}$ is called the row form of the matrix A.

(ii) Regard each column $\bar{v}_j$ as a column vector in $\mathbb{R}^m$, or equivalently as an $m \times 1$ matrix.

Then, $A = [\bar{v}_1 \ \bar{v}_2 \ \ldots \ \bar{v}_n]$ is called the column form of A.

These alternative forms of the matrix are occasionally useful, particularly the column form.

Let A be an $m \times n$ matrix in row form as $A = \begin{bmatrix} \bar{r}_1 \\ \vdots \\ \bar{r}_m \end{bmatrix}$ and let B be an $n \times p$ matrix written in column form as $[\bar{v}_1, \bar{v}_2 \ldots \bar{v}_p]$.

Then, the product matrix $AB = C = [c_{ij}]$ is a well-defined $m \times p$ matrix.

Observation 1: A typical entry c_{ij} of C is given as $c_{ij} = \bar{r}_i \cdot \bar{c}_j$ (1)
(standard matrix multiplication)

Since $\bar{r}_i$ is a $1 \times n$ matrix and $\bar{v}_j$ is an $n \times 1$ matrix, $\bar{r}_i \cdot \bar{v}_j$ is a 1×1 matrix, i.e. a scalar.

The expression (1) is a compact version of the definition of matrix multiplication, and is sometimes useful.

Observation 2. The product matrix $C = AB$ in column form is:

$$AB = [A\bar{v}_1, A\bar{v}_2 \cdots A\bar{v}_p], \quad (2)$$

i.e. the columns of AB are the column vectors $A\bar{v}_1, \ldots, A\bar{v}_p$ ($m \times n$ multiplied by $n \times 1$ $= m \times 1$ matrix).

The expression (2) is often useful.

Exercise left to the Student :-

a) Illustrate Observation 2 with an example. The matrices A and B in your example should be at least 3×3 in size (the product AB should be defined).

b) Give a proof of Observation 2 in the general case.

Hint: Recall that if $A = [a_{ij}]$ and $B = [b_{ij}]$ and the product $AB = C = [c_{ij}]$ is defined, then the typical element

$$c_{ij} = a_{i1} b_{1j} + a_{i2} b_{2j} + \dots + a_{in} b_{nj}$$

(for $i = 1$ to m, $j = 1$ to p)

Remark: We will use Observation 2 in Monday's lecture, so you should be comfortable with it. a) above is helpful for this. b) is optional, but will improve your matrix algebra skills