Example for Determinant

$$A_1 = \begin{bmatrix} 0 & 5 & 8 \\ 1 & 2 & 7 \\ 2 & 4 & 6 \end{bmatrix} \xrightarrow[(1)]{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 2 & 7 \\ 0 & 5 & 8 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 - 2R_1} \begin{bmatrix} 1 & 2 & 7 \\ 0 & 5 & 8 \\ 0 & 0 & -8 \end{bmatrix}$$ Echelon Form → RREF will be $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\det(A_1) = (-1)(5)(-8) = 40$

Using ~~Recursive Definition, we get:~~ Cofactor Expansion, we get:

$(-5)(6-14) + 8(4-4) = (-5)(-8) = 40$ ✓

Solving the Homogeneous System $A\bar{x} = \bar{0}$

where $A = \begin{bmatrix} 0 & 5 & 10 & 8 \\ 1 & 2 & 6 & 7 \\ 2 & 4 & 12 & 6 \end{bmatrix}$

Recall that $R = \begin{bmatrix} \textcircled{1} & 0 & 2 & 0 \\ 0 & \textcircled{1} & 2 & 0 \\ 0 & 0 & 0 & \textcircled{1} \end{bmatrix}$ x_1, x_2, x_4 basic, x_3 free

The system $R\bar{x} = \bar{0}$ is:

$$\left.\begin{aligned} x_1 + 2x_3 &= 0 \\ x_2 + 2x_3 &= 0 \\ x_4 &= 0 \end{aligned}\right\} \longrightarrow \begin{aligned} x_1 &= -2x_3 \\ x_2 &= -2x_3 \\ x_3 &= x_3 \\ x_4 &= 0 \end{aligned}$$

Example (continued)

i.e. $\bar{x} = x_3 \begin{bmatrix} -2 \\ -2 \\ 1 \\ 0 \end{bmatrix} = x_3 \bar{v}$

Check: $A\bar{v} = \begin{bmatrix} 0 & 5 & 10 & 8 \\ 1 & 2 & 6 & 7 \\ 2 & 4 & 12 & 6 \end{bmatrix} \begin{bmatrix} -2 \\ -2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ ✓

In short, $\bar{v}$ is a solution of $R\bar{x} = 0$, and also of $A\bar{x} = \bar{0}$.

But then, $A(x_3 \bar{v}) = x_3 A\bar{v} = x_3 \bar{0}$
$= \bar{0}$.

In other words, x_3 acts like a parameter; we can give it any real value. Infinitely many solutions.

Another Example: Solve $R_1\bar{x} = \bar{0}$ where

$R_1 = \begin{bmatrix} 1 & 0 & 2 & 1 & 0 & 2 \\ 0 & 1 & 4 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$ (Already an RREF matrix)