ECE 111 : Digital Circuits

Week 2

17/08/23

Today:

operations on "binary" NUMBERS

(add, ...)

$$(10)^{11} = 1000$$

011.010 | 3.45

000 + 001

001

+ 001

010

011 + 001

100 + 2

+	00	01
00	00	01
01	01	10

Carry!

Addition table

+	00	01
00	00	01
01	01	10

Carry!

Addition table

Multiplication

10 x 10

---

10 x 11

1 start at 10

000 001 010 011 100 101 110

111

---

10 x 10

00 01 10 11 100

~~0100~~

~~× 0011~~

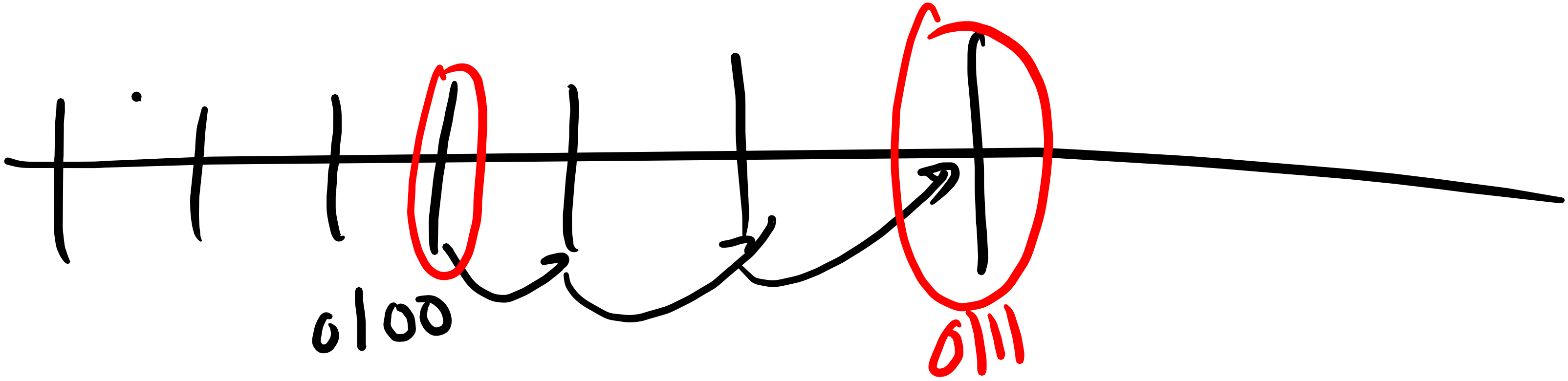
0100

Addition

0100

+ 0011

0001 + 0001 + 0001



0100
0111

addition:

$$
\begin{array}{r}
0100 \\
+\ 0111 \\
\hline
1011
\end{array}
$$

$$
\begin{array}{r}
0100 \\
\times\ 0011 \\
\hline
\end{array}
$$

34 x 16 =

$$\begin{array}{r} {}^{6}\cancel{3}4 \\ \times\ 16 \\ \hline 544 \end{array}$$

$$\begin{array}{r} 34 \\ \times 16 \\ \hline 204 \end{array}$$

X

$$
\begin{array}{r}
0100 \\
\times\ 011 \\
\hline
0100 \\
0100\text{X} \\
\hline
1100
\end{array}
$$

$$(6.436)_{10}$$

$$6\times10^{0} + 4\times10^{-1} + 3\times10^{-2} + 6\times10^{-3}$$

$$(6.436)_{16}$$

$$= \boxed{6\times16^{0} + 4\times16^{-1} + 3\times16^{-2} + 6\times16^{-3}}$$

Decimal

$(010.011)_2$

Convert it into decimal

$$1 \times 2^1 + 0 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}$$

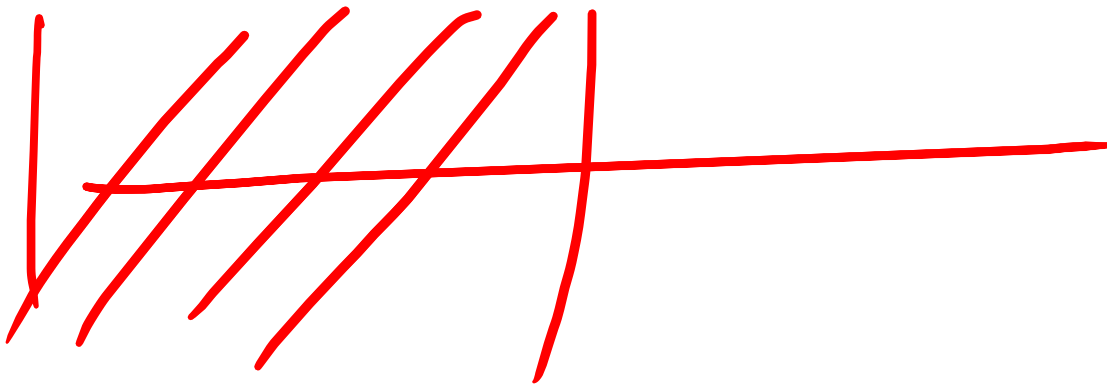
$$= 2 + 0 + 0 + 0.25 + 0.125$$

$$= 2.375$$

What is the "precision" of a 4 bit ~~b~~ binary number.

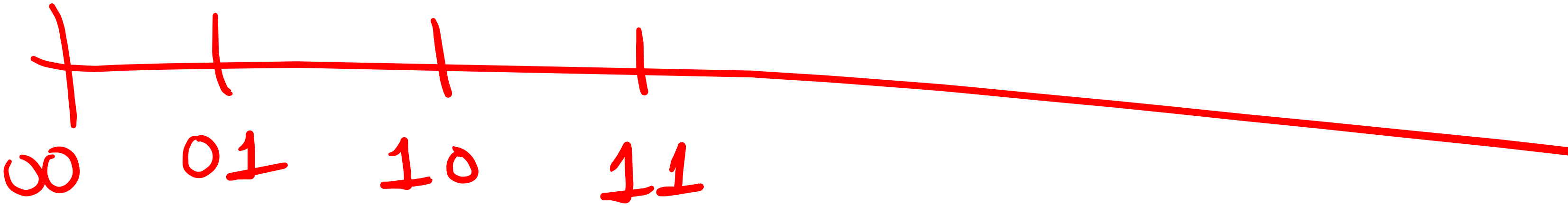
<u>Ans</u>: $1 \times 2^{-4} = \frac{1}{16} = 0.0625$

$(0.500)_{10} =$



$$34 \times 16$$

$$= 34 \times (10 + 6)$$


00
01
10
11

Class on 18/08:

1. Recap of multiplication of ~~b~~ binary numbers.

2. Substraction ~~of~~ in ~~#~~ any number system

   - r's complement
   - $(r-1)$'s "

Consider two numbers:

$$\begin{array}{rr|rr}
 & (1011)_2 \rightarrow & (?)_{10} & \\
\times & (0011)_2 \rightarrow & (?)_{10} & \times \\
\hline
 & (?) & & (?)
\end{array}$$

$$1011 \rightarrow \left(2^3 \times 1 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0\right)$$

$$011 \rightarrow \left(0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0\right)$$

$0 \times 2^3 +$

$$\left(\quad\right) \times \left(\quad\right)$$

$$= \left(1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0\right)$$

$+ 1 \times 2^1$

$0 - 011$

Radix Complement

We have two #'s M, N

$M - N$

$$M - N + r^m - r^n$$

$r \rightarrow$ (radix)

$\rightarrow$ 10, 2, 8, 16, - -

$$= M + (r^n - N) - r^n$$

$$= -(r^n - (M + \bar{N}))$$

$(r^n - N) \rightarrow$

if $r = 10$ (decimal)

$n = 3$

$r^n \rightarrow 1000$

if $r = 2, \quad n = 3$

$2^3 \rightarrow (1000)_2$

Define:

1. radix r's complement of number 'N'

$$(r^n - N)$$

2. radix (r-1)'s " " N

$$(r^n - N - 1)$$

$075 \rightarrow$

$n = 3$

$r^n = 1000 - 075$

$= 925$

$(r-1) = 924$

r's

c.

$$0075 \rightarrow \begin{array}{r} 9924 \\ +\ 1 \\ \hline 9925 \end{array} \quad (r-1\text{'s } c)$$

$$\frac{O\left(X \quad Y \quad Z\right)_{k} \longrightarrow \alpha}{(k-1)\text{s Compl. of } \alpha}$$

$$(k-1)\,\overline{X}\;\overline{Y}\;\overline{Z} \quad \Bigg| \quad \overline{X} + X \doteq k-1$$

$018 \longrightarrow 981$

$(r^n - 1) - N$

$(018)_{H,16} \longrightarrow \left(\dfrac{FE7}{r-1}\right)_{16}$

to find r's complement, we add 1 to (r-1)'s ...

$(124)_{10} \triangleq (876)_{10}$

Binary numbers

$$1011 \xrightarrow{1^s} 0100$$

$$\xrightarrow{2^s} 1^s + 0001$$

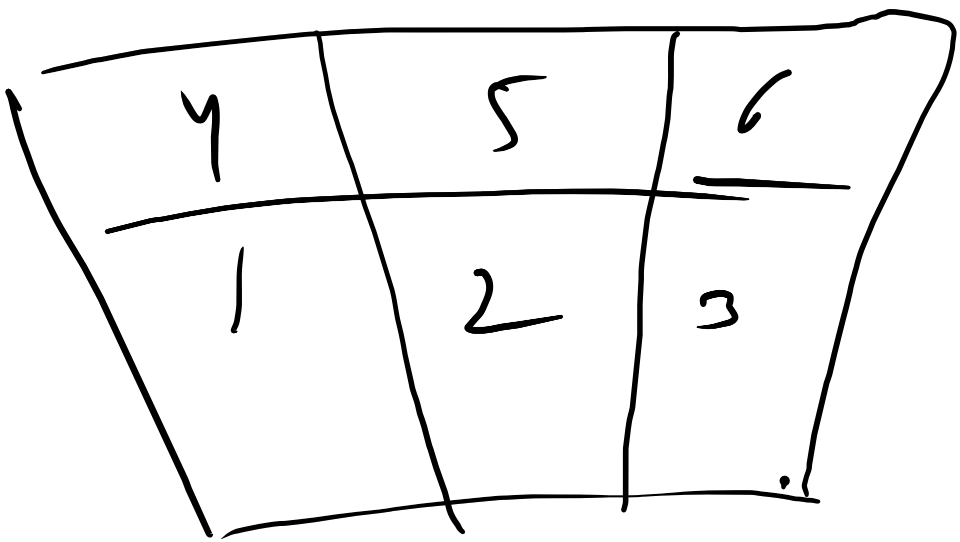
$$= 0101$$

$$1\times 2^4 + 1\times 2^3 + 1\times 2^2 + 1\times 2^1 + 1\times 2^0$$

$$+ 1\times 2^1$$

_ _ 1 1 1 1 1

Week 3



X3FF in decimal

X000
X001
⋮
X3FF

1010 0000 1111 1001 ~~00~~

1 Hexa-digit can be represented by 4 'binary bits'

x A 0 F 9

X3FF

0 1 1 1 1 1 1 1 1 1 1

XFF

[1 1 _ _]

$M - N$

if $M > N$

$212, \quad 124 \Big\} \quad n = 3$

$$X = M - N$$

$$= M + \underbrace{(2^n - N)}_{\bar{N}} - 2^n$$

$$= M + \bar{N} - 2^n$$

$$= \boxed{-(2^n - (M + \bar{N}))}$$

① Case: $M > N$

$M = 212$ ~~$\cancel{00}$~~

$N = 124$

$\bar{N} = 876$

$M + \bar{N} = 212 + 876$

$= 1086$

$A^n = 1000$

$$M + \bar{N} - 10^{n}$$

$$= 1088 - 1000$$

$$= 88$$

$\underline{\text{Case : 2}}$ $\qquad M < N$

$1\,2\,\uparrow\,4, \qquad 2\,\uparrow\,12$

$M = \qquad\qquad N$

$$= -\left({}^{n}C_{2} - (M + \bar{N}) \right)$$

$$= -($$

Binary numbers:

$M = 1010100$

$N = 1000011$

$\bar{N} = \text{flip}(N) + 1$

$0111100 + 1$

$= 0111101$

$M = 00$

$N = 01$