MTH100 MATH 1 Section A Monsoon Semester – 2023-24

Tutorial exercise for the Week commencing 20230814

1. Reduce the following matrix to an RREF matrix using elementary row operations:

$$A = \begin{bmatrix} 1 & 2 & -3 & 0 \\ 2 & 4 & -2 & 2 \\ 3 & 6 & -4 & 1 \end{bmatrix}$$

2. Reduce the following matrix to an RREF matrix using elementary row operations:

$$A = \begin{bmatrix} 1 & -2 & 3 & -1 \\ 2 & -1 & 2 & 2 \\ 3 & 1 & 2 & 3 \end{bmatrix}$$

3. Explicitly describe all non-zero 2×2 RREF matrices. You may also try to do this for 2×3 and 3×3 RREF matrices.

4. Verify that row-equivalence is an equivalence relation on the set $\mathbb{R}^{m\times n}$ of all m by n matrices with real entries.

5. Show that if E is an equivalence relation on a set X, then any two distinct equivalence classes must be disjoint. Also, show that every element of X has to belong to an equivalence class. NB: the equivalence class of any element $a \in X$ is the set of all elements of X which are related to a; the formal definition is:

   $[a] = \{ x \in X$: x E a, i.e. x is related to a under the relation E$\}$

6. Show that if $\mathbb{P}$ is a partition of a set X, then there exists an equivalence relation E on X such that the equivalence classes correspond to the parts of the given partition $\mathbb{P}$. (*Q6 is the converse of Q5.*)

7. Define a relation T on the real number system $\mathbb{R}$ by xTy if $y - x \in \mathbb{Z}$, the set of integers. Is T an equivalence relation? Justify your answer. If yes, can you find a special representative in each equivalence class, just as we could do for row-equivalence of matrices?