MTH100 – Section A **Math 1 (Linear Algebra)** **Monsoon Semester – 2023-24**

Tutorial exercises for the week commencing Monday 21st August 2023

1. Find the solution set in vector form for the homogeneous system A**x** = **0** given A below. NB: A must be row-reduced to an RREF matrix in order to give the solution in standard form.

$$A = \begin{bmatrix} 1 & -2 & 3 & -1 \\ 2 & -1 & 2 & 2 \\ 3 & 1 & 2 & 3 \end{bmatrix}$$

2. a) Row reduce the augmented matrix of the system given below to an RREF matrix:

$$\begin{aligned} 3x + 2y + 7z + 9w &= 7 \\ 6x + 14y + 22z + 15w &= 13 \\ x + 4y + 5z + 2w &= 2 \end{aligned}$$

b) Is the system consistent or inconsistent ? If consistent, express the solution in the form of a vector **u** which is a solution of the non-homogeneous system plus scalar multiples of vector(s) which are solutions of the associated homogeneous system.

3. Repeat Q2, both parts a) and b), for the non-homogeneous system A**x** = **b**, where A and **b** are given below.

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 2 & -1 \\ 0 & 2 & -2 \end{bmatrix}$$

b = (3, –3, 1) taken as a column vector

4. Row reduce the augmented matrix of the system given below to an RREF matrix:

$$\begin{aligned} x + 5y - 3z &= -4 \\ -x - 4y + z &= 3 \\ -2x - 7y &= a \end{aligned}$$

b) For what values of a is the above system consistent and for what values of a is it inconsistent ? Justify your answer.

5. Is it possible for a non-homogeneous system A**x** = **b**, **b** ≠ **0**, to be inconsistent when the associated homogeneous system A**x** = **0** has a unique solution (i.e. only the trivial solution) ? Answer YES or NO, and justify your answer. If YES, construct an example and verify. If NO, explain with reference to suitable propositions. (***Note: See Observation 5 in Friday's lecture.***).

6. a) Find the values of x for which the following matrix is an augmented matrix corresponding to a consistent system.

$$A = \begin{bmatrix} 1 & -2 & 1 & x \\ 0 & 5 & -2 & x^2 \\ 4 & -23 & 10 & x^3 \end{bmatrix}$$

b) Find the RREF of the matrix formed by replacing x in A by π.

7. Recall the following from the lecture on Monday (slightly abbreviated): **Observation 1:** If we obtain a row equivalent matrix to the coefficient matrix, then the solution sets of the two linear systems are the same. ***In fact, that is why we defined the elementary row operations in the way we did !***

a) Make the above observation rigorous by proving the following: If the matrix B has been obtained from the matrix A by an elementary row operation, then the vector $\mathbf{v}$ is a solution of the homogeneous system $A\mathbf{x} = \mathbf{0}$ if and only if $\mathbf{v}$ is a solution of the homogeneous system $B\mathbf{x} = \mathbf{0}$.

b) Formulate the nonhomogeneous version of the above statement, and then prove it.