Problem 1 →

A.

(a) factor analysis model differs the prob. PCA except for the fact that cond. var on $\sigma^2 I$ is replaced by a matrix (diag.) Ψ. hence one free parameter σ^2 is replaced by 'D' new parameters in

$\Psi \rightarrow \{\Psi_{11}\ \Psi_{22}\ \cdots\ \Psi_{DD}\}$

hence a total of :

$$DM + D - \frac{M(M-1)}{2}$$

$$D(M+1) - \frac{M(M-1)}{2}$$

↳ total no. of indepandent parameters.

(b) $\tilde{z} = R z$ rotation matrix. (MXM orthog. matrix)

latent space

rotated latent space

define a modified factor loading matrix

$\tilde{w} = w R^T$ original loading matrix.

rotation matrix

$\tilde{z}^T \tilde{z} = (Rz)^T Rz = z^T R^T R z$

$= z^T z \quad (R^T R = 1)$

hence we can use this and comment that $p(z)$ is independent of rotation.

$P(x|z)$ defined by only (wz) hence we need to prove $wz = \tilde{w}\tilde{z}$, which can be done easily $\tilde{w}\tilde{z} = (wR^T)(Rz)$

$= wz$

hence $P(x,z) = P(z)\, P(x|z)$ remains the same even on rotation of the latent space.

This is also seen in the predictive distr. $p(x)$ which depends on w only through $ww^T = \tilde{w}\tilde{w}^T$ and is invariant of R.