

Q1) showing that the posterior can be converted into a Pareto Form:

as we have to derive the posterior, $p(\theta | D)$ and we have been given with the $p(\theta, D), p(D)$ we use Bayes Rule in the following form : $p(\theta | D) = \frac{p(\theta, D)}{p(D)}$ and consider the two cases $m \leq b, m > b$; as the value for prior for D changes with these conditions.

Case 1 : $m \leq b$

$$p(\theta | D) = \frac{Kb^k \mathbb{I}(\theta > \max(D, b))}{\theta^{N+K+1}} \frac{(N+K)b^N}{K} = \text{Par}(\theta | N+K, b)$$

Case 2 : $m > b$

$$p(\theta | D) = \frac{Kb^k \mathbb{I}(\theta > \max(D, b))}{\theta^{N+K+1}} \frac{(N+K)m^{N+K}}{Kb^K} = \frac{(N+K)m^{N+K} \mathbb{I}(\theta \geq m)}{\theta^{N+K+1}} = \text{Par}(\theta | N+K, m)$$

So if we designate the maximum of m, b by M ($\max(m, b) = M$), then we can write the combined posterior for the two cases as :

$$p(\theta | D) = \text{Par}(\theta | N+K, M)$$