1) **show that students' T is a mixture of infinite gaussians** ?

$$S_t(x\,|\,\mu,\sigma,\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\cdot\sqrt{\pi\nu\sigma^2}}\left(1+\frac{1}{\nu}\left(\frac{x-\mu}{\sigma}\right)^2\right)^{-\frac{\nu+1}{2}}$$

, is the univariate student's T distribution with a given 'v' degrees of freedom. Now all that we have to do is to prove that :

$$S_t(x\,|\,\mu,\sigma,\nu) = \int_0^{\infty} N(x\,|\,\mu,\frac{\sigma^2}{z}) \;\cdot\; Ga(z\,|\,\frac{\nu}{2},\frac{\nu}{2})\;dz$$

This can be done using normal integration of the integrand under the limits, so let us first manipulate the integrand into a much more easy to handle form :

$$N(x\,|\,\mu,\frac{\sigma^2}{z}) = \frac{1}{\sqrt{2\pi}\cdot\frac{\sigma}{\sqrt{z}}}\,e^{-\frac{z(x-\mu)^2}{2\sigma^2}}$$

$$Ga(z\,|\,\frac{\nu}{2},\frac{\nu}{2}) = \frac{\left(\frac{\nu}{2}\right)^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)} z^{\frac{\nu}{2}-1}\cdot e^{-z\frac{\nu}{2}}$$

Using these and a little manipulation the integrand becomes something like this :

$$\int_0^{\infty}\left(\frac{\nu^{\frac{\nu}{2}}}{\Gamma(\frac{\nu}{2})\cdot 2^{\frac{\nu}{2}}\cdot\sqrt{2\pi\sigma^2}}\right)e^{-z\left(\frac{\nu}{2}+\frac{(x-\mu)^2}{2\sigma^2}\right)}z^{\frac{\nu-1}{2}}dz$$

(here we can use the formulae for the gamma function for integration over z after making the integrand in a standard form (i.e. $z \rightarrow z\cdot\left(\frac{\nu}{2}+\frac{(x-\mu)^2}{2\sigma^2}\right)^{\frac{-1}{2}}$) and the final result is a students' T distribution)

$$S_t(x\,|\,\mu,\sigma,\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\cdot\sqrt{\pi\nu\sigma^2}}\left(1+\frac{1}{\nu}\left(\frac{x-\mu}{\sigma}\right)^2\right)^{-\frac{\nu+1}{2}}$$