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Q3

$$P(\theta) = 0.5\, B(\theta \mid 100, 100) + 0.5\, B(\theta \mid 0.5, 0.5)$$

for the first dataset 2H,3T

$$P(D \mid z=k) = \binom{N}{N_H} \frac{\text{beta}(a_k + N_H, b_k + N - N_H)}{\text{beta}(a_k, b_k)}$$

$$\Rightarrow P(D \mid z=1) = \binom{5}{2} \frac{\beta(102, 103)}{\beta(100, 100)}$$

$$\Rightarrow P(D \mid z=2) = \binom{5}{2} \frac{\beta(2.5, 3.5)}{\beta(0.5, 0.5)}$$

$$P(D) = \int P(D, z)\, dz = \int P(D \mid z=k) \cdot P(z=k)\, dz$$

as the number of models is finite we can simply use the summation instead of integration (discrete)

$$P(D) = \sum_k P(D \mid z=k)\, \underbrace{P(z=k)} = \binom{5}{2} \frac{\beta(102,103)}{\beta(100,100)} \left(P(z=1)\right) + \binom{5}{2} \frac{\beta(2.5, 3.5)}{\beta(0.5, 0.5)} \left(P(z=2)\right)$$

$$\approx 0.310 \times 0.5 + 0.117 \times 0.5$$

$$\approx 0.2135$$

$$\cancel{\sum_{k'} \frac{P(z=k' \mid D)}{P(z=k')} = \frac{0.5 \times \binom{5}{2} \frac{\beta(102, 103)}{\beta(100, 100)} + 0.5 \times \binom{5}{2} \frac{\beta(2.5, 3.5)}{\beta(0.5, 0.5)}}{} = 0.2315}$$

$$P(z=1 \mid D) = \frac{P(z=1)\, P(D \mid z=1)}{\sum P(z=k')\, P(D \mid z=k')} = \frac{0.5 \times \binom{5}{2} \frac{\beta(102, 103)}{\beta(100, 100)}}{0.2315} = 0.7258$$

$$P(z=2 \mid D) = \frac{P(z=2)\, P(D \mid z=2)}{\sum P(z=k')\, P(D \mid z=k')} = \frac{0.5 \times \binom{5}{2} \frac{\beta(2.5, 3.5)}{\beta(0.5, 0.5)}}{0.2315} = 0.2742$$

for the second data set 5H, 0T

$$P(D|\chi=k) = \binom{5}{5} \frac{\beta(a_k+5, b_k)}{\beta(a_k, b_k)}$$

$$\to \frac{\beta(105, 100)}{\beta(100, 100)} = 0.03241$$

$$\to \frac{\beta(5.5, 0.5)}{\beta(0.5, 0.5)} = 0.24609$$

$$P(D) = 0.5 \times \left(\frac{\beta(105, 100)}{\beta(100, 100)} + \frac{\beta(5.5, 0.5)}{\beta(0.5, 0.5)} \right) \simeq 0.13925$$

$$P(\chi=1|D) = \frac{(0.03241) \times 0.5}{0.13925} \simeq 0.116378$$

$$P(\chi=2|D) = \frac{0.24609 \times 0.5}{0.13925} \simeq 0.883621$$

these results are pretty intuitive, for the first case we recieve an equitable distr. of heads, tails (2, 3) hence most likely the coin isn't biased while for the second case as (H, T) ≡ (5, 0) it is likely that the coin was biased.