Deep Learning for Mechanics (APL 745)

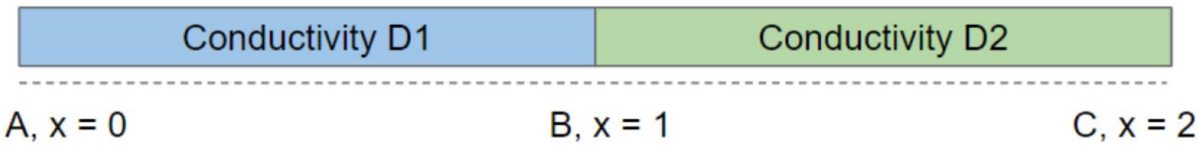
Homework – 2

Handed out: 16-03-2022 Submission due: 31-03-2022

Instructions:

- We highly encourage handwritten homework for derivation and other theoretical part. Compile as single report containing solutions, derivations, figures, etc
- Zipped folder should be turned in on Sakai with the following naming scheme: HW2_Enrl_No.zip
- Collaboration is encouraged however all submitted reports, programs, figures, etc. should be an individual student's writeup. Direct copying will be considered cheating.
- Discussion on methods used, results obtained for programming assignment and technical discussion is essential. Homework problems that simply provide computer outputs with no technical discussion, Algorithms, etc. will receive no credit.
- This homework will be counted as HW 2 and HW 3, and will have more weightage

Problem – 1: In this problem, our aim is to obtain the temperature distribution inside the bar that is made up of two materials with different thermal conductivity. The geometry and the problem specification of the problem can be seen below



The composite bar extends from $x = 0$ to $x = 2$. The bar has material of conductivity $D_1 = 10$ from $x = 0$ to $x = 1$ and $D_2 = 0.1$ from $x = 1$ to $x = 2$. Both end of the bars are at a constant temperature of 0 and 100 respectively. For simplicity of modeling, we will treat the composite bar as two separate bars, bar 1 and bar 2, whose ends are joined together. We will treat the temperatures in the bar 1 as U_1 and the temperature in bar 2 as U_2. The governing equation of the problem is given as

$$\frac{d}{dx}\left(D_1 \frac{dU_1}{dx}\right) = 0, \;\; when\; 0 < x < 1$$

$$\frac{d}{dx}\left(D_2 \frac{dU_2}{dx}\right) = 0, \;\; when\; 1 < x < 2$$

Flux and temperature continuity at interface is

$$D_1 \frac{dU_1}{dx} = D_2 \frac{dU_2}{dx} \quad when\; x = 1$$

$$U_1 = U_2 \quad when\; x = 1$$

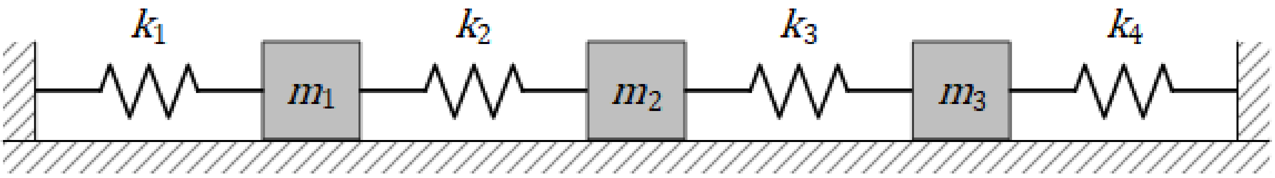
(A) Write an algorithm illustrating how you will solve it using PINN (strong form) with boundary loss term.
(B) Following the algorithm, write a computer code to solve it.

(C) Carry out case studies by varying the no. of collocation point (consider 3 different set). Report your findings with possible explanations.
(D) How you will modify the NN output such that BC is automatically satisfied. Rewrite the algorithm highlighting appropriate changes
(E) Write a computer code following the algorithm and solve the problem. Report on the results obtained with proper explanation., Compare with previously obtained results in (B).

Problem – 2: Let us revisit problem 1. Suppose we want to solve it using v-PINN. In this context, answer the followings:

(A) Consider a single layer neural network with cosine activation function. Also consider a test function of the form discussed in the lecture. Derive the loss function in weak form.
(B) Write an algorithm for v-PINN with deep neural network. Use Gauss quadrature for computing the integration.
(C) Write a computer code following the algorithm. Report the results and provide your observations.

Problem – 3: Consider a spring mass system as shown below



The model equations are given as follows:

$$m_1 x_1''(t) = -k_1 x_1(t) + k_2\big(x_2(t) - x_1(t)\big)$$

$$m_2 x_2''(t) = -k_2(x_2(t) - x_1(t)) + k_3\big(x_3(t) - x_2(t)\big)$$

$$m_3 x_3''(t) = -k_3(x_3(t) - x_2(t)) - k_4 x_3(t)$$

Carry out the following operations:

(A) Considering $m_1 = m_2 = m_3 = 1$ and $k_1 = k_4 = 2, k_2 = k_3 = 1$, generate solution in $t = [0,10]$. Use $x_1 = 1$ and everything else to be zero as the initial condition.
(B) Using the data generated in (A), consider that m_1 and k_4 are unknown. Write the PINN loss-function for estimating the unknown parameters.
(C) Write the algorithm for estimating m_1 and k_4.
(D) Following the algorithm, write a code to estimate the m_1 and k_4 using PINN
(E) Repeat (C) by adding 1% and 5% white Gaussian noise to the data.