

**Problem 1:**

Physics-informed neural network, unlike its data-driven counterpart is trained directly from the governing physics. Considering the following governing equation

$$\frac{d^2u}{dx^2} = f(x); \quad u(0) = u(1) = 0$$

We want to train a neural network based on the physics defined above. The network architecture for the neural network can be decided by the student.

- (A) The loss-function in PINN consists of physics-informed loss and boundary loss. Write the expression for the physics-informed loss function and data-driven loss function for this problem.
- (B) Select a suitable activation function and justify your selection
- (C) Write a computer code to solve this problem.
- (D) The true solution of the above problem is  $\frac{1}{2}x(x - 1)$ . Plot and compare the PINN predicted results with the true solution.

**Problem 2:**

One advantage of PINN resides in its ability to solve parameterized geometries. Suppose, we want to know how the solution to this equation changes as we move the position on the boundary condition  $u(l) = 0$ . We can parameterize this position with a variable  $l \in [1,2]$  and our equation now has the form

$$\frac{d^2u}{dx^2} = f(x); \quad u(0) = u(l) = 0$$

- (A) State how you will formulate a PINN network (what will be inputs, what will be output, what will be boundary loss and what will be physics informed loss)?
- (B) Solve the equation using PINN and plot results for different values of  $l$ .
- (C) Explain the utility of parametric solution from an industrial perspective.