

Problem 1:

Apart from solving forward problems, PINN can also be used for solving inverse problems. In an inverse problem, we start with a set of observations and then use those observations to calculate the causal factors that produced them. To illustrate how to solve inverse problems with a neural network solver, we consider inverting out the source term $f(x)$ from the following equation

$$\frac{d^2u}{dx^2} = f(x); \quad u(0) = u(1) = 0$$

With this setup, do the followings:

- (A) Generate the solution $u_{true}(x)$ at 100 random points between 0 and 1. You can use the true solution $u_{true}(x) = \frac{1}{48}(8x(-1 + x^2) - (3 \sin(4\pi x))/\pi^2)$ corresponding to $f(x) = x + \sin(4\pi x)$.
- (B) Use the generated data and the governing physics described above to setup a physics-informed loss-function.
- (C) Write computer code to identify $f(x)$ by using PINN