Q4

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Butcher Tableu for the 4th order Explicit Runge Kutta Method :

$$y^1 = y_n$$

$$y^2 = y_n + \frac{h}{2} f(y^1)$$

$$y^3 = y_n + \frac{h}{2} f(y^2)$$

$$y^4 = y_n + h f(y^3)$$

$$y_{n+1} = y_n + \frac{h}{6}\left[f(y^1) + 2f(y^2) + 2f(y^3) + f(y^4)\right]$$

$$\begin{array}{c|c} c^T & A \\ \hline & b \end{array}$$

$$\left.\begin{array}{l} a_{11} = a_{12} = a_{13} = a_{14} = 0 \\ a_{21} = \frac{1}{2} \text{ and } a_{22} = a_{23} = a_{24} = 0 \\ a_{32} = \frac{1}{2} \text{ and } a_{31} = a_{33} = a_{34} = 0 \\ a_{43} = 1 \text{ and } a_{41} = a_{42} = a_{44} = 0 \end{array}\right\} A$$

$$\left. b_1 = b_4 = \frac{1}{6} \text{ and } b_2 = b_3 = \frac{1}{3} \right\} B$$

$$\left. c_1 = 0, \quad c_2 = \frac{1}{2} = c_3, \quad c_4 = 1 \right\} c^T$$

$$\begin{array}{c|cccc} 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ \hline & 1/6 & 1/3 & 1/3 & 1/6 \end{array}$$

→ ans Q.E.D

Final scheme is explicit as the tableau has a lower triangular matrix A.