Problem 3

1. **derive the expression for $s = 4$ backward differentiation method**

   to derive the scheme we need to find β which is given by the following expression :

$$\beta = \frac{1}{\sum \frac{1}{m}} = \frac{1}{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}} = \frac{12}{25}$$

   now we need to write the polynomial that will denote the left hand side of the scheme (also known as the characteristic polynomial).

$$\rho(w) = \frac{12}{25}\left(w^3(w-1) + \frac{1}{2}w^2(w-1)^2 + \frac{1}{3}w(w-1)^3 + \frac{1}{4}(w-1)^4\right)$$

$$= w^4 - \frac{48}{25}w^3 + \frac{36}{25}w^2 + \frac{16}{25}w + \frac{3}{25}$$

   hence the left hand side of the scheme can be written as the following explicit formulae:

$$y_{n+4} - \frac{48}{25}y_{n+3} + \frac{36}{25}y_{n+2} + \frac{16}{25}y_{n+1} + \frac{3}{25}y_n$$

   also the right hand side of the scheme can be written as the following explicit formulae:

$$\frac{12}{25}hf(t_{n+4}, y_{n+4})$$

   the total scheme is given by the following formulae :

$$y_{n+4} - \frac{48}{25}y_{n+3} + \frac{36}{25}y_{n+2} + \frac{16}{25}y_{n+1} + \frac{3}{25}y_n = \frac{12}{25}hf(t_{n+4}, y_{n+4})$$