MovieLens Report

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Overview:

The objective of the project comprised of creating a movie rating prediction system with the provided dataset. The "edx" and "validation" datasets were created with code provided in the project introduction on the course website. The "edx" dataset was used for data-exploration, analysis, and machine learning algorithm training purposes. With due consideration for computing capacity and computing time, a random sampling approach was employed. The predictions generated were tested against the "validation" set to obtain an **RMSE** (root mean square error) of **0.69**.

The following linear model was fit to the data:

```
y = mu + b\_i + b\_u + b\_y + b\_g + epsilon
where,
y = \text{predicted rating},
mu = \text{average rating},
b\_i = \text{movie bias},
b\_u = \text{user bias},
b\_y = \text{movie release year bias},
b\_g = \text{genre bias},
epsilon = \text{independent errors from sampling from the same dataset}.
```

The above variables required for the linear model represent sources of variation in rating prediction, and hence are considered as "biases". They were either directly obtained from the data on variables present in the dataset, or were derived by wrangling and processing the available data. A regularization parameter, lambda = 3.25, was obtained using a representative sample of 10000 observations. This was used to regularize the predictions generated by an averaging approach. Readings for the "biases" and the predictions obtained by applying them one after the other, their regularized equivalents, and the average ratings for the movies were used together as a set of 17 predictors to train the machine learning models. Once again, with due consideration for computing time and computing capacity, six different sets of 1000 samples each were used. Each set was trained on nine different individual machine learning models, and one ensemble model. Rating prediction results of each of those were checked against actual ratings for the same movies in the validation dataset. The RMSEs showed a **standard deviation** of **0.022**, and all results (individual RMSEs) were within two standard deviations of the overall mean RMSE, thus indicating good reproducibility of results. The **minimum RMSE** obtained among all runs of the machine learning models was **0.65**, and the mean of all RMSEs, yielded a **final RMSE** of **0.69**.

Methodology and Analysis:

Definition of the RMSE(root mean square error) function:

The RMSE(root mean square error) function calculates the difference between the predicted value and the true (observed) value of a variable in a dataset which, in this case, is the rating of a movie by a specific user. The function involves taking a square of the difference to obtain a positive number and then taking the square root, thus giving the absolute value of the difference. The average of these values is taken across the dataset to give the RMSE of the dataset. If the difference between the predicted values and the actual observed values is minimal, then the RMSE function will output a minimum value, thus indicating the accuracy of the predictions. An RMSE of less than 0.86490 was expected for full points in the final result section of the project.

```
RMSE <- function(true_ratings, predicted_ratings) {
    sqrt(mean((true_ratings - predicted_ratings)^2))
}
```

Dataset Structures:

The "edx" and the "validation" datasets are both created from the MovieLens dataset using code provided on the course website. These are the datasets to be used for analysis and algorithm training, and for validating the generated predictions respectively. Examining the structure of the given datasets reveals each to be comprising of observations on six variables, namely, userId, movieId, rating, timestamp, title, and genres.

Structure of edx dataset for analysis and training of algorithms to generate predictions:

```
## Classes 'data.table' and 'data.frame':
                                            6 obs. of 6 variables:
   $ userId
              : int 1 1 1 1 1 1
##
   $ movieId
              : num
                     122 185 292 316 329 355
##
              : num 5 5 5 5 5 5
   $ rating
                     838985046 838983525 838983421 838983392 838983392 838984474
   $ timestamp: int
                      "Boomerang (1992)" "Net, The (1995)" "Outbreak (1995)" "Stargate (1994)" ...
   $ title
              : chr
                      "Comedy|Romance" "Action|Crime|Thriller" "Action|Drama|Sci-Fi|Thriller" "Action|A
              : chr
   - attr(*, ".internal.selfref")=<externalptr>
```

Structure of validation dataset for final RMSE evaluation:

```
## Classes 'data.table' and 'data.frame': 6 obs. of 6 variables:
## $ userId : int 1 1 1 2 2 2
## $ movieId : num 231 480 586 151 858 ...
## $ rating : num 5 5 5 3 2 3
## $ timestamp: int 838983392 838983653 838984068 868246450 868245645 868245920
## $ title : chr "Dumb & Dumber (1994)" "Jurassic Park (1993)" "Home Alone (1990)" "Rob Roy (1995)
## $ genres : chr "Comedy" "Action|Adventure|Sci-Fi|Thriller" "Children|Comedy" "Action|Drama|Roman
## - attr(*, ".internal.selfref")=<externalptr>
```

First six rows of the edx dataset for analysis and training of algorithms to generate predictions:

```
##
     userId movieId rating timestamp
                                                                   title
## 1
           1
                 122
                           5 838985046
                                                       Boomerang (1992)
## 2
           1
                 185
                           5 838983525
                                                        Net, The (1995)
## 3
                 292
                           5 838983421
           1
                                                        Outbreak (1995)
## 4
                           5 838983392
           1
                 316
                                                        Stargate (1994)
## 5
                 329
                           5 838983392 Star Trek: Generations (1994)
           1
## 6
           1
                 355
                           5 838984474
                                               Flintstones, The (1994)
                               genres
##
## 1
                      Comedy | Romance
## 2
              Action | Crime | Thriller
      Action|Drama|Sci-Fi|Thriller
## 3
## 4
            Action | Adventure | Sci-Fi
## 5 Action | Adventure | Drama | Sci-Fi
            Children | Comedy | Fantasy
```

First six rows of the validation dataset for final RMSE evaluation:

```
##
     userId movieId rating timestamp
## 1
                 231
                           5 838983392
          1
## 2
                 480
                           5 838983653
          1
## 3
          1
                 586
                           5 838984068
          2
## 4
                 151
                           3 868246450
## 5
          2
                 858
                           2 868245645
## 6
          2
                1544
                           3 868245920
##
                                                           title
## 1
                                           Dumb & Dumber (1994)
## 2
                                           Jurassic Park (1993)
## 3
                                              Home Alone (1990)
## 4
                                                 Rob Roy (1995)
## 5
                                          Godfather, The (1972)
## 6 Lost World: Jurassic Park, The (Jurassic Park 2) (1997)
                                         genres
##
## 1
                                         Comedy
## 2
             Action | Adventure | Sci-Fi | Thriller
## 3
                               Children | Comedy
## 4
                     Action|Drama|Romance|War
## 5
                                   Crime | Drama
## 6 Action|Adventure|Horror|Sci-Fi|Thriller
```

Rating being the variable of interest for which the predictions are to be generated, the other data present in the dataset was examined in terms of its distribution for having potential contribution in the variation in rating.

Distribution of the count of ratings received by each movie, and the count of ratings provided by each user were examined directly by grouping the data based on the userId and movieId variable. The timestamp variable was converted to rating_year by means of the lubridate package. Additionally, every entry in the title column has the movie title along with the year of it's release written in brackets after it. This regular pattern of entries was put to use for string processing using regex (regular expressions). Finally, to quantify the effect based on genres, the following method was used:

- 1. Each movie was observed to have either "(no genres listed)", or one or more genre entered in the *genres* column. Movies having more than one genre have all of them entered in the genre column separated using the vertical bar sign, "|", as a delimiter.
- 2. A character vector was made by grouping entries in the genres column.
- 3. The unnest_tokens command was used to separate individual genre names, applying necessary corrections wherever required, such as in cases of hyphenated genre names. Thus, individual genres occuring across the entire dataframe were put as single entries in one character vector.
- 4. An arbitrary score was assigned to each individual category based on the order of occurrence in the created character vector, thus yielding the following dataframe:

##				genres	score
##	1	(no	genres	listed)	1
##	2			Action	2
##	3		Adventure		3
##	4		Animation		4
##	5		Children		5
##	6		Comedy		6
##	7			${\tt Fantasy}$	7
##	8			IMAX	8
##	9			Sci-Fi	9
##	10			Drama	10
##	11			Horror	11
##	12			${\tt Mystery}$	12
##	13			${\tt Romance}$	13
##	14		7	Thriller	14
##	15			Crime	15
##	16			War	16
##	17			${\tt Western}$	17
##	18			${\tt Musical}$	18
##	19		Docı	umentary	19
##	20		F	ilm-Noir	20

- 5. This score, while being nominal, served as a quantitative representation of the occurrence of a specific genre in the entries of the *genres* variable.
- 6. Adding up individual scores for a particular entry in the *genres* column gave a total genre score which was a quantitative representation of all the genres that a particular movie belonged to. This was calculated using the following function:

Function to define genre scores:

```
 \begin{array}{l} genre\_score\_fun <- \ function(g) \\ \{g\_dat <- g \%>\% \ str\_split(pattern = "\|") \%>\% \ .[[1]] \%>\% \ data.frame(genre = .) \\ genre\_score <- \ sapply(g\_dat, \ function(genre)\{ \\ ind <- \ match(genre\_ \ score\_ \ chart\$ \ genres) \\ genre\_score\_chart\$score[ind]\}) \\ sum(genre\_score)\} \\ \end{array}
```

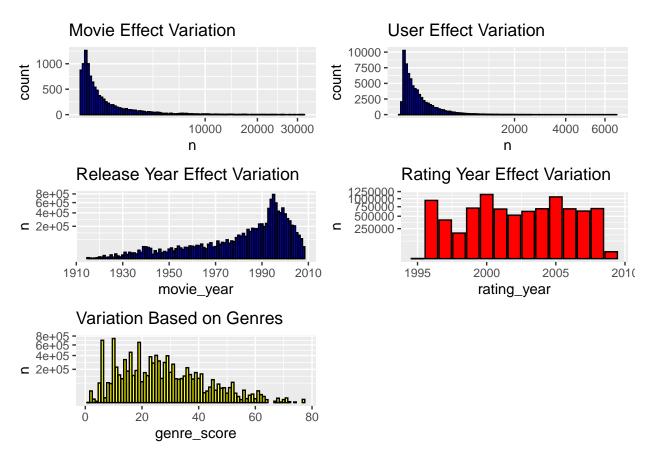
Distributions of selected variables:

The variables available in the original dataset, and those obtained by the wrangling and processing of data as described were examined for their distribution in the dataset to check if they have any kind of a skewed distribution, thus, making them potential sources of variability in rating prediction.

The variables considered were:

- * movieId
- * userId
- * rating_year
- * movie_year (movie release year)
- * genre score

Variability in rating observed based on selected potential sources of variation:



The graphs obtained show that the variables *movieId*, *userId* and *movie_year* are significantly skewed in their distribution, thus indicating a strong effect on the number of ratings. The *genre_score* variable shows some variability in it's effect on the number of ratings, while the *rating_year* variable does not show any significant variability.

Hence, movie Id, user Id, movie year, and genre score were the variables chosen for further analysis.

The linear model, $y = mu + b_i + b_u + b_y + b_g + epsilon$, was applied using an averaging based approach. This was done as follows:

- The average of all ratings in the dataset was calculated mu = 3.51
- The movie bias is then calculated as: b_i = mean(rating mu)
- The predictions using movie bias are then calculated as: pred b i = mu + b i
- This is repeated for the remaining variables as follows:

```
- b_u = mean(rating - mu - b_i)
- pred_b_u = mu + b_i + b_u
- b_y = mean(rating - mu - b_i - b_u)
- pred_b_y = mu + b_i + b_u + b_y
- b_g = mean(rating - mu - b_i - b_u - b_y)
- pred_b_g = mu + b_i + b_u + b_y + b_g
```

Thus, we now construct a dataset which has the variables under consideration for being sources of variation in rating prediction, the effects of the biases induced by each of them, and the predictions generated by applying each individual bias to the average rating.

Dataset with variables chosen as sources of variation in rating prediction:

```
## Classes 'data.table' and 'data.frame':
                                             6 obs. of 13 variables:
##
                 : num
                        5 5 5 5 5 5
    $ rating
                        122 185 292 316 329 355
    $ movieId
                 : num
##
    $ userId
                 : num
                        1 1 1 1 1 1
##
    $ movie_year : num
                        1992 1995 1995 1994 1994 ...
##
    $ genre_score: num
                        19 31 35 14 24 18
##
    $ b i
                        -0.6539 -0.3831 -0.0945 -0.1628 -0.175 ...
                 : num
##
    $ pred b i
                 : num
                        2.86 3.13 3.42 3.35 3.34 ...
    $ b_u
##
                 : num
                        1.68 1.68 1.68 1.68 1.68 ...
##
    $ pred_b_u
                        4.54 4.81 5.1 5.03 5.02 ...
                 : num
   $ b_y
                        0.00698 -0.03792 -0.03792 -0.032 -0.032 ...
##
                 : num
##
    $ pred_b_y
                        4.54 4.77 5.06 5 4.98 ...
                 : num
                        -0.0054 -0.00312 0.00151 -0.01969 -0.00395 ...
##
    $ b_g
                 : num
    $ pred_b_g
                 : num 4.54 4.77 5.06 4.98 4.98 ...
##
    - attr(*, ".internal.selfref")=<externalptr>
##
     rating movieId userId movie_year genre_score
                                                            b_i pred_b_i
                                                                              b_u
## 1
          5
                122
                                  1992
                                                19 -0.65387934 2.858586 1.679235
                         1
                185
## 2
          5
                         1
                                  1995
                                                31 -0.38313118 3.129334 1.679235
## 3
          5
                292
                         1
                                  1995
                                                35 -0.09445454 3.418011 1.679235
## A
          5
                316
                         1
                                  1994
                                                14 -0.16278816 3.349677 1.679235
## 5
          5
                329
                         1
                                  1994
                                                24 -0.17500816 3.337457 1.679235
## 6
                355
                         1
                                  1994
                                                18 -1.02467799 2.487787 1.679235
          5
##
     pred_b_u
                       b_y pred_b_y
                                              b_g pred_b_g
## 1 4.537821 0.006983816 4.544804 -0.005395510 4.539409
## 2 4.808569 -0.037918647 4.770650 -0.003120829 4.767529
## 3 5.097245 -0.037918647 5.059327 0.001505932 5.060833
## 4 5.028912 -0.032003181 4.996909 -0.019691473 4.977217
## 5 5.016692 -0.032003181 4.984689 -0.003945963 4.980743
## 6 4.167022 -0.032003181 4.135019 -0.014654944 4.120364
```

Lambda Optimization:

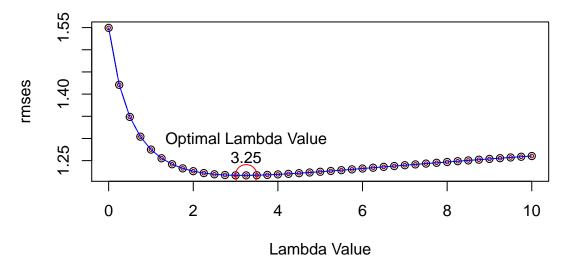
The effects of the biases and the predictions generated by them as shown above can be subject to regularization for better accuracy of prediction using a regularization parameter, *lambda*. For calculating the optimal value of *lambda*, the following procedure was used:

- A representative sample of 10000 ratings was chosen such that there were 1000 movies for each unique rating.
- This was divided into a training and test set.
- A range of *lambdas* from 0 to 10 with an interval of 0.25 was created as a numeric vector for each value to be tested individually for the RMSE value that it would yield.
- For n() being the number of ratings and the letter "1" being the value of lambda, the subsequent steps were followed.
- The average of all ratings in the dataset was calculated mu = 3.51
- The regularized movie bias is then calculated as: $b_i = sum(rating mu)/(n() + l)$
- The regularized predictions using movie bias are then calculated as: pred b i l = mu + b i l
- This is repeated for the remaining variables as follows:

```
- b_u_l = sum(rating - mu - b_i_l)/(n() + l)
- pred_b_u_l = mu + b_i_l + b_u_l
- b_y_l = sum(rating - mu - b_i_l - b_u_l)/(n() + l)
- pred_b_y_l = mu + b_i_l + b_u_l + b_y_l
- b_g_l = sum(rating - mu - b_i_l - b_u_l - b_y_l)/(n() + l)
- pred_b_g_l = mu + b_i_l + b_u_l + b_y_l + b_g_l
```

The following graph was obtained showing the RMSE for different values of lambda across the selected range:

Lambda Optimization Curve



It is seen that the RMSE was minimum at lambda = 3.25. This was the value used to generate regularized effect bias and prediction data.

Regularized Prediction Data:

Using all the above results, the following dataset was generated:

```
## Classes 'data.table' and 'data.frame':
                                          6 obs. of 21 variables:
   $ rating
                : num 555555
## $ movieId
                : num 122 185 292 316 329 355
## $ userId
                : num 1 1 1 1 1 1
   $ movie_year : num
                      1992 1995 1995 1994 1994 ...
## $ genre_score: num
                       19 31 35 14 24 18
## $ b_i
                : num
                       -0.6539 -0.3831 -0.0945 -0.1628 -0.175 ...
## $ pred_b_i
                       2.86 3.13 3.42 3.35 3.34 ...
                : num
## $ b_u
                : num
                       1.68 1.68 1.68 1.68 1.68 ...
## $ pred_b_u
                : num
                      4.54 4.81 5.1 5.03 5.02 ...
## $ b_y
                       0.00698 -0.03792 -0.03792 -0.032 -0.032 ...
                : num
## $ pred_b_y
                       4.54 4.77 5.06 5 4.98 ...
                : num
## $ b_g
                       -0.0054 -0.00312 0.00151 -0.01969 -0.00395 ...
                : num
## $ pred_b_g
                : num 4.54 4.77 5.06 4.98 4.98 ...
## $ b_i_1
                      -3.00e-04 -2.84e-05 -6.54e-06 -9.56e-06 -1.20e-05 ...
                : num
## $ pred_b_i_1 : num 3.51 3.51 3.51 3.51 3.51 ...
                       0.0669 0.0669 0.0669 0.0669 ...
## $ b u 1
              : num
## $ pred_b_u_1 : num
                       3.58 3.58 3.58 3.58 ...
## $ b_y_1
              : num
                       -3.43e-07 -8.94e-08 -8.94e-08 -6.12e-08 -6.12e-08 ...
                       3.58 3.58 3.58 3.58 ...
## $ pred_b_y_1 : num
## $ b_g_1
              : num -3.48e-08 4.25e-07 -2.92e-07 -8.73e-08 -1.91e-07 ...
## $ pred_b_g_1 : num 3.58 3.58 3.58 3.58 3.58 ...
## - attr(*, ".internal.selfref")=<externalptr>
##
    rating movieId userId movie_year genre_score
                                                        b_i pred_b_i
## 1
         5
               122
                        1
                                1992
                                             19 -0.65387934 2.858586 1.679235
## 2
         5
               185
                        1
                                1995
                                             31 -0.38313118 3.129334 1.679235
## 3
               292
                                             35 -0.09445454 3.418011 1.679235
         5
                        1
                                1995
## 4
         5
               316
                        1
                                1994
                                             14 -0.16278816 3.349677 1.679235
## 5
         5
               329
                                1994
                                             24 -0.17500816 3.337457 1.679235
                        1
## 6
         5
               355
                                1994
                                             18 -1.02467799 2.487787 1.679235
                        1
                                                                b_i_l pred_b_i_l
    pred_b_u
                      b_y pred_b_y
                                            b_g pred_b_g
## 1 4.537821 0.006983816 4.544804 -0.005395510 4.539409 -2.997728e-04
                                                                        3.512165
## 2 4.808569 -0.037918647 4.770650 -0.003120829 4.767529 -2.843854e-05
                                                                        3.512437
## 3 5.097245 -0.037918647 5.059327 0.001505932 5.060833 -6.536533e-06
                                                                        3.512459
## 4 5.028912 -0.032003181 4.996909 -0.019691473 4.977217 -9.557082e-06
                                                                        3.512456
## 5 5.016692 -0.032003181 4.984689 -0.003945963 4.980743 -1.202537e-05
                                                                        3.512453
## 6 4.167022 -0.032003181 4.135019 -0.014654944 4.120364 -2.119621e-04
                                                                        3.512253
         b_u_l pred_b_u_l
                                  b_y_l pred_b_y_l
                                                          b_g_l pred_b_g_l
## 1 0.06685757
                 3.579023 -3.431769e-07
                                          3.579023 -3.484433e-08
                                                                  3.579023
## 2 0.06685757
                 3.579294 -8.942414e-08
                                          3.579294 4.245029e-07
                                                                  3.579295
## 3 0.06685757
                 3.579316 -8.942414e-08
                                          3.579316 -2.917229e-07
                                                                  3.579316
## 4 0.06685757
                 3.579313 -6.116911e-08
                                          3.579313 -8.728339e-08
                                                                  3.579313
## 5 0.06685757
                 3.579311 -6.116911e-08
                                          3.579311 -1.910742e-07
                                                                  3.579310
## 6 0.06685757
                 3.579111 -6.116911e-08
                                          3.579111 -6.621431e-07
                                                                  3.579110
```

Machine Learning Algorithm Training:

In the above dataset, the values for the variables b_i , b_u , b_y , b_g , $pred_b_i$, $pred_b_u$, and $pred_b_u$, are all obtained by applying a set of uniform operations on the data previously present in the dataset. These variables would hence serve as a good predictor space for rating predictions, and thus were used as such.

Sample set creation:

Running machine learning algorithms on the full dataset being beyond the limitations of computing time and computing capacity, a random sampling approach was used. Random samples of 1000 movies were chosen for one run of each set of machine learning algorithms. Six such samples were taken by setting the seed at different points to obtain six different sample sets of 1000 movies each from the entire dataset. These were used to demonstrate consistency in reproducibility of results.

One particular sample set thus has a total of 1000 movies, with the average rating for each movie and the variables mentioned above as predictor space used for training the machine learning algorithms.

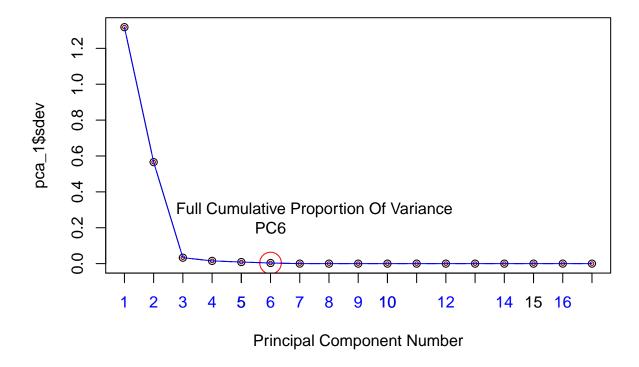
This sample set was then divided into a training set and test set comprising of 500 observations each.

The principal component analysis (for the first sample) showed the following results:

Principal Component Analysis of machine learning training data:

```
## Importance of components:
                             PC1
                                    PC2
                                            PC3
                                                    PC4
                                                              PC5
                                                                      PC6
                                                                                PC7
##
                          1.3180 0.5658 0.03304 0.01524 0.008822 0.00340 2.374e-05
## Standard deviation
## Proportion of Variance 0.8438 0.1555 0.00053 0.00011 0.000040 0.00001 0.000e+00
## Cumulative Proportion 0.8438 0.9993 0.99984 0.99996 0.999990 1.00000 1.000e+00
##
                                PC8
                                          PC9
                                                    PC10
                                                              PC11
                                                                        PC12
## Standard deviation
                          1.001e-05 1.041e-15 3.473e-16 1.812e-16 1.369e-16
## Proportion of Variance 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00
## Cumulative Proportion 1.000e+00 1.000e+00 1.000e+00 1.000e+00 1.000e+00
##
                               PC13
                                         PC14
                                                   PC15
                                                             PC16
                                                                        PC17
## Standard deviation
                          1.097e-16 9.599e-17 8.744e-17 6.466e-17 6.358e-17
## Proportion of Variance 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00
## Cumulative Proportion 1.000e+00 1.000e+00 1.000e+00 1.000e+00 1.000e+00
```

Principal Component Analysis



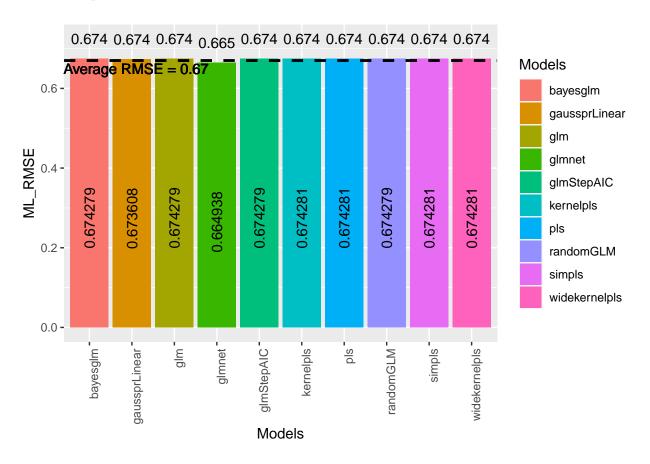
The data and the graph show that the first six principal components account for all the variability of the data. Thus, PC1 to PC6 were selected to train the machine learning algorithms.

Nine different linear classification models were chosen for machine learning training. These were "bayesglm", "gaussprLinear", "glm", "glmStepAIC", "glmnet", "pls", "simpls", "kernelpls", and "widekernelpls". Additionally one ensemble model, "randomGLM", was chosen. These models each had *ncomp* (number of components) as the only tuning parameter. With this parameter already being chosen optimally by principal component analysis, there was no further need for optimization, thus reducing computing effort, and making the choice of these models aptly suited for training and prediction purposes here.

Results:

The first run all all models yielded an average RMSE of 0.67 and a minimum RMSE of 0.66 as can be seen from following graph:

RMSE Graph:



```
#Minimum RMSE obtained:
min(final_rmses_1$ML_RMSE)

## [1] 0.6649379

#Model yielding the minimal RMSE:
min_ind <- which.min(final_rmses_1$ML_RMSE)
final_rmses_1[min_ind,]

## Models ML_RMSE rmse_rounded
## 5 glmnet 0.6649379     0.665

#Average RMSE from all models:
result <- mean(final_rmses_1$ML_RMSE)
result</pre>
```

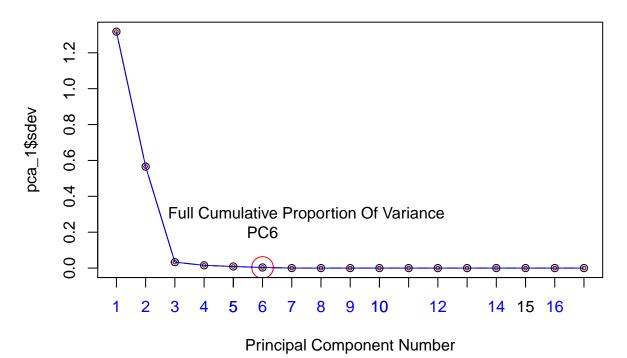
[1] 0.6732785

Reproduced results:

To demonstrate consistency in reproducibility of results obtained by this method all models were run on six different samples. All samples consistently showed the first six components to account for all variability in the data as can be seen from the PCA data and graphs below:

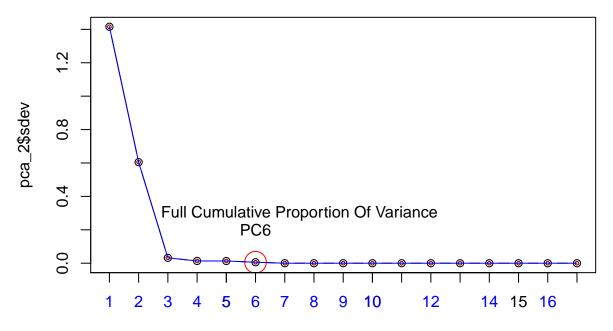
PCA_1:

```
## Importance of components:
##
                             PC1
                                    PC2
                                            PC3
                                                    PC4
                                                              PC5
                                                                      PC6
                                                                                PC7
## Standard deviation
                          1.3180 0.5658 0.03304 0.01524 0.008822 0.00340 2.374e-05
## Proportion of Variance 0.8438 0.1555 0.00053 0.00011 0.000040 0.00001 0.000e+00
                          0.8438 0.9993 0.99984 0.99996 0.999990 1.00000 1.000e+00
##
  Cumulative Proportion
##
                                PC8
                                          PC9
                                                    PC10
                                                              PC11
## Standard deviation
                          1.001e-05 1.041e-15 3.473e-16 1.812e-16 1.369e-16
## Proportion of Variance 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00
## Cumulative Proportion 1.000e+00 1.000e+00 1.000e+00 1.000e+00 1.000e+00
##
                               PC13
                                         PC14
                                                    PC15
                                                              PC16
## Standard deviation
                          1.097e-16 9.599e-17 8.744e-17 6.466e-17 6.358e-17
## Proportion of Variance 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00
## Cumulative Proportion 1.000e+00 1.000e+00 1.000e+00 1.000e+00 1.000e+00
```



PCA_2:

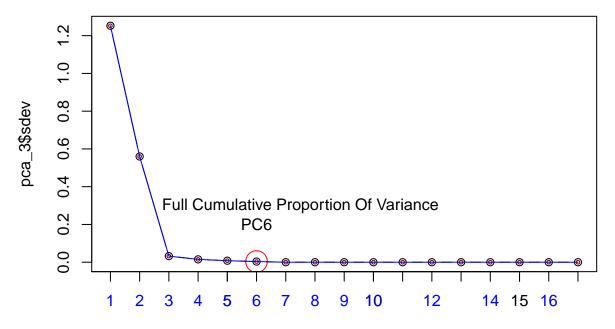
```
## Importance of components:
                                    PC2
                                            PC3
                                                    PC4
                                                             PC5
                                                                     PC6
                                                                               PC7
##
                             PC1
## Standard deviation
                          1.4155 0.6048 0.03218 0.01348 0.01315 0.00586 1.779e-05
## Proportion of Variance 0.8451 0.1543 0.00044 0.00008 0.00007 0.00001 0.000e+00
## Cumulative Proportion 0.8451 0.9994 0.99984 0.99991 0.99999 1.00000 1.000e+00
##
                                PC8
                                          PC9
                                                    PC10
                                                              PC11
                                                                        PC12
## Standard deviation
                          2.616e-06 7.855e-16 3.842e-16 3.012e-16 1.252e-16
## Proportion of Variance 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00
## Cumulative Proportion 1.000e+00 1.000e+00 1.000e+00 1.000e+00 1.000e+00
                               PC13
                                         PC14
                                                   PC15
                                                             PC16
##
                                                                        PC17
## Standard deviation
                          1.164e-16 9.126e-17 8.019e-17 6.968e-17 6.647e-17
## Proportion of Variance 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00
## Cumulative Proportion 1.000e+00 1.000e+00 1.000e+00 1.000e+00 1.000e+00
```



Principal Component Number

PCA_3:

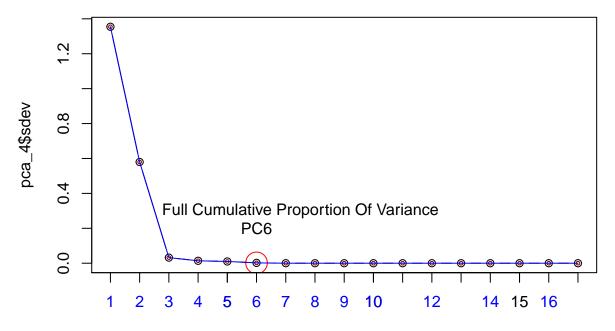
```
## Importance of components:
                                    PC2
                                            PC3
                                                    PC4
                                                             PC5
##
                             PC1
                                                                       PC6
## Standard deviation
                          1.2524 0.5600 0.03234 0.01509 0.008014 0.003466
## Proportion of Variance 0.8328 0.1665 0.00056 0.00012 0.000030 0.000010
## Cumulative Proportion 0.8328 0.9993 0.99984 0.99996 0.999990 1.000000
##
                                PC7
                                          PC8
                                                    PC9
                                                              PC10
## Standard deviation
                          1.588e-05 1.121e-05 5.045e-16 4.614e-16 1.983e-16
## Proportion of Variance 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00
## Cumulative Proportion 1.000e+00 1.000e+00 1.000e+00 1.000e+00 1.000e+00
                              PC12
                                        PC13
                                                  PC14
                                                            PC15
                                                                    PC16
## Standard deviation
                          1.13e-16 9.809e-17 8.485e-17 7.634e-17 7.1e-17 5.955e-17
## Proportion of Variance 0.00e+00 0.000e+00 0.000e+00 0.000e+00 0.0e+00 0.000e+00
## Cumulative Proportion 1.00e+00 1.000e+00 1.000e+00 1.000e+00 1.0e+00 1.000e+00
```



Principal Component Number

PCA_4:

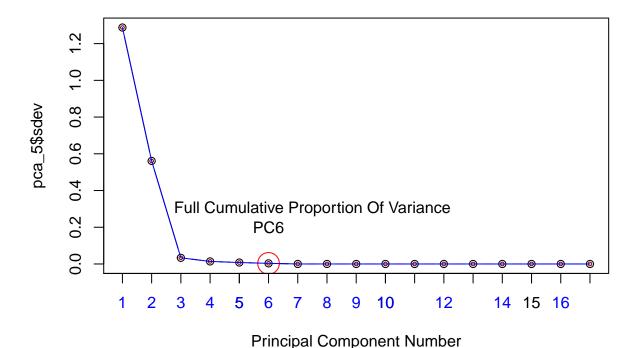
```
## Importance of components:
                                    PC2
                                            PC3
                                                    PC4
                                                             PC5
                                                                      PC6
                                                                                PC7
##
                             PC1
## Standard deviation
                          1.3545 0.5797 0.03198 0.01386 0.00996 0.001909 2.262e-05
## Proportion of Variance 0.8447 0.1547 0.00047 0.00009 0.00005 0.000000 0.000e+00
## Cumulative Proportion 0.8447 0.9994 0.99986 0.99995 1.00000 1.000000 1.000e+00
##
                                PC8
                                          PC9
                                                   PC10
                                                              PC11
## Standard deviation
                          7.996e-06 4.539e-16 3.448e-16 2.413e-16 1.597e-16
## Proportion of Variance 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00
## Cumulative Proportion 1.000e+00 1.000e+00 1.000e+00 1.000e+00 1.000e+00
                               PC13
                                         PC14
                                                   PC15
                                                             PC16
                                                                        PC17
## Standard deviation
                          1.227e-16 1.018e-16 8.801e-17 7.003e-17 6.225e-17
## Proportion of Variance 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00
## Cumulative Proportion 1.000e+00 1.000e+00 1.000e+00 1.000e+00 1.000e+00
```



Principal Component Number

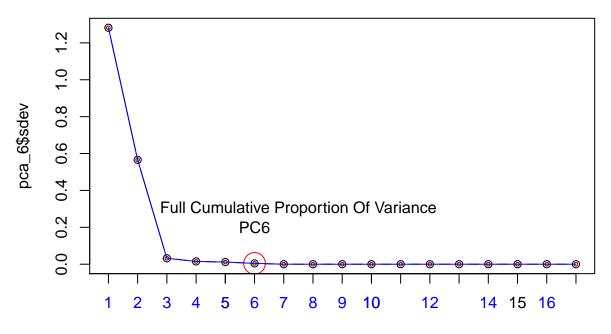
PCA_5:

```
## Importance of components:
                                    PC2
                                             PC3
                                                     PC4
                                                              PC5
                             PC1
                                                                       PC6
##
## Standard deviation
                          1.2876 0.5612 0.03354 0.01385 0.007882 0.003315
## Proportion of Variance 0.8398 0.1595 0.00057 0.00010 0.000030 0.000010
## Cumulative Proportion 0.8398 0.9993 0.99987 0.99996 0.999990 1.000000
##
                                PC7
                                          PC8
                                                     PC9
                                                              PC10
## Standard deviation
                          1.979e-05 1.215e-05 5.026e-16 2.612e-16 1.5e-16
## Proportion of Variance 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.0e+00
## Cumulative Proportion 1.000e+00 1.000e+00 1.000e+00 1.000e+00 1.0e+00
                               PC12
                                         PC13
                                                   PC14
                                                             PC15
##
                                                                      PC16
## Standard deviation
                          1.187e-16 1.122e-16 1.01e-16 8.038e-17 6.82e-17
## Proportion of Variance 0.000e+00 0.000e+00 0.00e+00 0.000e+00 0.00e+00
## Cumulative Proportion 1.000e+00 1.000e+00 1.000e+00 1.000e+00 1.00e+00
##
                               PC17
## Standard deviation
                          5.708e-17
## Proportion of Variance 0.000e+00
## Cumulative Proportion 1.000e+00
```



PCA_6:

```
## Importance of components:
                                    PC2
                                            PC3
                                                    PC4
                                                             PC5
                                                                      PC6
                                                                                PC7
##
                             PC1
## Standard deviation
                          1.2819 0.5659 0.03194 0.01519 0.01160 0.004495 7.941e-06
## Proportion of Variance 0.8363 0.1630 0.00052 0.00012 0.00007 0.000010 0.000e+00
## Cumulative Proportion 0.8363 0.9993 0.99980 0.99992 0.99999 1.000000 1.000e+00
##
                                PC8
                                          PC9
                                                   PC10
                                                              PC11
## Standard deviation
                          4.737e-06 3.577e-16 2.651e-16 1.709e-16 1.327e-16
## Proportion of Variance 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00
## Cumulative Proportion 1.000e+00 1.000e+00 1.000e+00 1.000e+00 1.000e+00
                               PC13
                                         PC14
                                                   PC15
                                                             PC16
                                                                        PC17
## Standard deviation
                          1.152e-16 8.267e-17 7.534e-17 6.061e-17 5.622e-17
## Proportion of Variance 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00
## Cumulative Proportion 1.000e+00 1.000e+00 1.000e+00 1.000e+00 1.000e+00
```

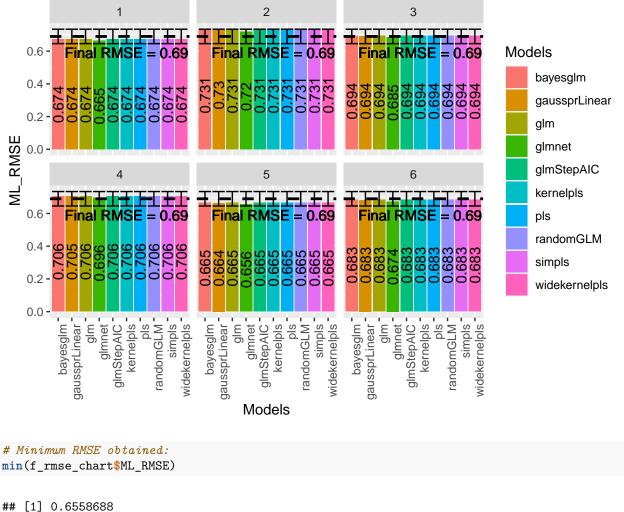


Principal Component Number

Final Results:

All runs of all models on each sample set show RMSEs being within two standard deviations of the average RMSE, 0.69, and a standard deviation as low as 0.022, thus indicating good accuracy of models and consistent reproducibility of results, as can be seen from the graph below:

Graph showing results of all sample analyses combined:



[1] 0.02206774

```
# Average RMSE from all models:
final_result <- mean(f_rmse_chart$ML_RMSE)
final_result</pre>
```

[1] 0.691203

Conclusion:

Among the variables present in the provided dataset, the *movieId* and *userId* variables were clear sources of variability in ratings of movies. The genre(s) and release year of the movies contributed to additional variability. With these variables in consideration, the linear model as described previously was fit to the data:

```
y = mu + b\_i + b\_u + b\_y + b\_g + epsilon
where,
y = \text{predicted rating},
mu = \text{average rating},
b\_i = \text{movie bias},
b\_u = \text{user bias},
b\_y = \text{movie release year bias},
b\_g = \text{genre bias},
epsilon = \text{independent errors from sampling from the same dataset}.
```

The various "biases" and the predictions obtained by applying them across the dataset, along with their equivalents regularized with the optimized regularization parameter, lambda = 3.25, served as a proper predictor space for rating prediction. Principal component analysis of this predictor space showed the first six principal components to account for all variability in the data, hence these six principal components were chosen to train machine learning algorithms on.

The results were reproduced six times with different samples, yielding an average **final RMSE** of **0.69**, and showing a **standard deviation** as low as **0.022**, thus indicating good reproducibility and accuracy of this method.

The limitations of this analysis mainly comprised of limited sample sizes (10000 for *lambda* optimization and 1000 for each set of machine learning models). While the method's consistent reproducibility was demonstrated by repeated random sampling and a low standard deviation (0.022) in the results obtained, there is no guaranteed indicator of scalability for larger datasets. Limited computing capacity imposed these restrictions on sample sizes and computing time. Future goals would involve testing this method on larger datasets, but this can only be done with an increase in computing capacity beyond what is available with regular commercial laptops, and would require more powerful workstations.