#### **Programming Challenges**

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#### Abstract

This is a collection of literate programs. If you are unfamiliar with the idea of literate programming please refer [1]. These programs are my solutions to the programming tasks from the "Programming Challenges" book[2] which in turn is a collection of problems from the UVa Online Judge hosted by University of Valladolid<sup>1</sup>.

### Contents

1	Cha	pter 1	1		
	1.1	The $3n+1$ Problem	1		
	1.2	Minesweeper	4		
	1.3	The Trip	6		
	1.4	LC Display	7		
	1.5	Graphical Editor	11		
	1.6	${\bf Interpreter}$	14		
	1.7	Check The Check	16		
	1.8	Australian Voting	20		
2	Chapter 2 22				
	2.1	Jolly Jumper	22		
	2.2	Crypt Kicker	23		
	Chapter 3 29				
3			29		
	3.1	Crypt Kicker II			
	3.2	File Fragmentation	31		
4	Cha	pter 4	33		
	4.1	Bridge	33		
	4.2	$ShellSort \dots \dots$	34		
5	Chapter 5 36				
9	5.1	Primary Arithmetic			
	5.2	Reverse And Add			
	5.2	The Archeologists' Dilemma			
	5.4	Ones			
	9.4	Ones	41		
6		pter 7	<b>43</b>		
	6.1	Light, More Light	43		
7	Lice	nse	46		

<sup>&</sup>lt;sup>1</sup>If you are going to submit any of these programs to the UVa Online Judge make sure the class name is Main and that it's not in any package; For the class names I use problem names, and I put everything into my package com.rvprg.pc)

## 1 Chapter 1

#### 1.1 The 3n+1 Problem

This task is not difficult if you notice that all the lengths of the sequences can easily be calculated up front. Then all that is needed is to lookup the pre-calculated table to find out the maximum lengths for the given input numbers.

(I noticed though that I could have simply calculated the values on the file without any tricks. The reason why I have done a more sophisticated algorithm is that at first I though the input number may go up to 1M, but in reality, according to the problem statement, they won't exceed 10000. So I solved a more tricky problem.)

So let's start with the definitions of the array that will hold all the lengths and the reader that will be used to read the input data.

```
\langle 3n+1 \text{ 1a} \rangle \equiv
1a
          package com.rvprg.pc;
          (1.1 Imports 1b)
          class Collatz {
              private static int MAX = 1000000;
              private int[] lengths = new int[MAX];
               private static final BufferedReader reader =
                         new BufferedReader(new InputStreamReader(System.in));
               (1.1 Helpers 2b)
               (1.1 Constructor 3a)
               ⟨1.1 Input/Output 3c⟩
          }
           We need the necessary imports:
1b
       \langle 1.1 \; Imports \; 1b \rangle \equiv
          import java.io.BufferedReader;
          import java.io.InputStreamReader;
```

The idea is to hold the lengths of the sequences in the lengths, but because the sequence member can sometimes go over 1M we will need to store them somewhere temporarily. For that a surplus hash map will be used. Its contents will be thrown away once the sequence lengths were computed.

So we write two helper methods: set and get. Both take an index and surplus hash map and depending on the index value either use the array or the hash map to set or get a value.

```
2a \langle 1.1 \; Imports \; 1b \rangle + \equiv import java.util.HashMap;
```

Now we can easily pre-calculate all the lengths using the helper methods set and get, but we must not re-calculate the lengths for the indexes that we have calculated already.

We calculate a member of the sequence at each step using the definition. Each time we calculate a new member of the sequence we push it onto the stack. We stop if we notice that we already have the length calculated for that specific value or when we reach 1. Now all the values that are on the stack are potential inputs, that is they are all potential initial ns. We use this knowledge to update elements in the lengths:

```
\langle 1.1 \; Imports \; 1b \rangle + \equiv
2c
         import java.util.ArrayDeque;
         import java.util.Deque;
       ⟨1.1 Constructor 3a⟩≡
За
         Collatz() {
             final HashMap<Long, Integer> surplus = new HashMap<Long, Integer>();
             lengths[1] = 1;
             for (long i = 2; i < MAX; ++i) {
                 final Deque<Long> stack = new ArrayDeque<Long>();
                 long n = i;
                 int len = 2;
                 while (n != 1) {
                      stack.push(n);
                      int prev = get(n, surplus);
                      if (prev > 0) {
                          len = prev;
                          break;
                      }
                      n = n \% 2 == 0 ? n / 2 : n * 3 + 1;
                 }
                 while (!stack.isEmpty()) {
                      set(stack.pop(), len++, surplus);
             }
         }
```

Processing the input is easy but cumbersome<sup>2</sup>:

```
\langle 1.1 \; Imports \; 1b \rangle + \equiv
3b
                                      import java.util.stream.IntStream;
                             \langle 1.1 \; Input/Output \; 3c \rangle \equiv
3c
                                     public static void main(String[] args) {
                                                        Collatz s = new Collatz();
                                                           String input;
                                                            while ((input = reader.readLine()) != null &&
                                                                                                 !input.trim().equalsIgnoreCase("")) {
                                                                             List<String> str = Arrays.stream(input.trim().split(" "))
                                                                                            .filter(x -> !x.equals("")).collect(Collectors.toList());
                                                                               int x[] = new int[] { Integer.parseInt(str.get(0)),
                                                                                                                  Integer.parseInt(str.get(1)) };
                                                                             System.out.println(x[0] + " " + x[1] + " " +
                                                                                                                  IntStream.rangeClosed(Math.min(x[0], x[1]), Math.max(x[0], x[1])), Math.max(x[0], x[1]), Math.max(x[0], x[1]
                                                                                                                                                     x[1])).map(v -> s.lengths[v]).max().getAsInt());
                                                           }
                                     }
```

#### 1.2 Minesweeper

This task is trivial: We simply count the number of mines around each cell. There are eight cells around each cell that we need to inspect. If our cell is (x, y), then we check (x - 1, y - 1), (x, y - 1) and so on, and count the number of cells that have '\*' in them.

Our program structure is simple as usual:

Of course, we need a reader, so we define it next. Then we need to define the constants. We are going to split the lines by spaces, so let's have it as a constant. We also define an array of the offsets p to determine the cells around a given cell.

```
4b ⟨1.2 Imports 4b⟩≡
import java.io.BufferedReader;
import java.io.IOException;
import java.io.InputStreamReader;
```

<sup>&</sup>lt;sup>2</sup>It turns out that the UVa Judge tends to give some extra spaces here and there in the input, so we need to make sure we account for some sporadic spaces in the input. This was my first submission and it took me seven attempts before I got past that super annoying "Runtime Error", because the judge was giving some extra spaces between the values which my program was not taking into account.

Now let's write the main method. I'll delibirately use one-dimentional array instead of the two-dimensional, and I will use a couple of helper lambdas. One, count, to count the mines around a cell, and another, mine, which returns a cell value for the given coordinates.

```
4d  \( \lambda 1.2 \) Imports 4b\rangle +\( \equiv \)
    import static java.util.Arrays.stream;
    import static java.util.stream.Collectors.joining;
    import static java.util.stream.Collectors.toList;
    import static java.util.stream.IntStream.range;
    import java.util.List;
    import java.util.function.IntBinaryOperator;
    import java.util.function.IntUnaryOperator;
```

```
5
     \langle 1.2 \ Main \ 5 \rangle \equiv
        public static void main(String[] args) throws IOException {
            int lineNum = 0;
            String currentLine = INPUT_END;
            while ((currentLine = reader.readLine()) != null) {
                if (currentLine.equalsIgnoreCase("")) {
                     continue;
                }
                List<Integer> nm = stream(currentLine.split(SPACE))
                         .filter(x -> !x.equals("")).map(Integer::parseInt).collect(toList());;
                int n = nm.get(0);
                int m = nm.get(1);
                if (n == 0 \&\& m == 0) {
                    break;
                final int[] field = reader.lines().limit(n)
                     .collect(joining()).chars().map(x \rightarrow x == '*' ? -1 : 0).toArray();
                final IntBinaryOperator mine =
                    (x, y) \rightarrow (x < 0 \mid | x > (n - 1) \mid | y < 0 \mid | y > (m - 1)) ? 0 : field[x * m + y];
                final IntUnaryOperator count = (i) -> range(0, p.length)
                     .map(j -> Math.abs(mine.applyAsInt(i / m + p[j][0], i % m + p[j][1]))).sum();
                int[] result = range(0, field.length)
                     .map(x -> field[x] >= 0 ? count.applyAsInt(x) : field[x]).toArray();
                if (lineNum > 0) {
                    System.out.println();
                System.out.println("Field #" + (++lineNum) + ":");
                for (int i = 0; i < n; ++i) {
                    for (int j = 0; j < m; ++j) {
                         System.out.print(result[i * m + j] == -1 ? "*" : result[i * m + j]);
                    System.out.println();
                }
            }
       }
```

#### 1.3 The Trip

This task is much more fun that the previous two. The important thing that we should note for ourselves is that we are not going to use the floating point types to do the calculations.

```
6a ⟨The Trip 6a⟩≡
package com.rvprg.pc;
⟨1.3 Imports 6b⟩

class TheTrip {
⟨1.3 Calculation 6c⟩
⟨1.3 Input/Output 7c⟩
}
```

First thing we need to do is to calculate the average spend, don't we? Because we know that the input is a list of how much each of n students spent, let's define a function that takes this list of values and returns the minimum amount of money asked in the problem. Of course, the types will be long. And we can immediately cover the degenerate case of a input consisting of one element:

```
6b ⟨1.3 Imports 6b⟩≡
import static java.util.Arrays.stream;
6c ⟨1.3 Calculation 6c⟩≡
static long calculate(long[] values) {
if (values.length == 1)
return 0;
long total = stream(values).sum();
⟨1.3 Finding the minimum 6e⟩
}
```

Now we need to partition the students into two groups: One group of students that will be giving money (those that spent less than group average) and the ones who will be receiving the money (those that spent more than the group average). But the total won't always divide without a reminder. So we divide the total by the number of students to get the quotient and the reminder, and we partition only using the quotient; that is group 1 will contain spends x such that  $x-quotient \leq 0$ , and group 2 will have the others.

So what do we do with the reminder? These are those cents that we need to finally re-distribute among the members of the two groups. Note that the reminder will always be less than n. We choose the following strategy: We distribute these cents to the group that spent less than or equal to the quotient, the remaining cents are finally distributed to group 2. This is captured in the following code:

```
\langle 1.3 \text{ Finding the minimum } 6e \rangle + \equiv
7a.
         long sum = abs(diff.get(false).stream().reduce(Long::sum).get());
         long len = diff.get(true).size();
         reminder = len <= reminder ? reminder - len : 0;</pre>
         return sum + reminder;
          All we need to do now is to write input reading, which is trivial:
7b
       \langle 1.3 \; Imports \; 6b \rangle + \equiv
         import java.io.BufferedReader;
         import java.io.IOException;
         import java.io.InputStreamReader;
         import java.math.BigDecimal;
       \langle 1.3 \; Input/Output \; 7c \rangle \equiv
7c
         public static void main(String[] args) throws IOException {
              BufferedReader r = new BufferedReader(new InputStreamReader(System.in));
              int n = 0;
              while ((n = Integer.parseInt(r.readLine().trim())) > 0) {
                  long[] values = r.lines().limit(n).map(x -> x.replaceAll("\\.",
                            "").trim()).mapToLong(Long::parseLong).toArray();
                  System.out.println("$" + BigDecimal.valueOf(calculate(values), 2));
              }
         }
```

## 1.4 LC Display

This task may seem quite involved at first sight, because you may start thinking about twodimensional patterns and scaling functions. But in reality this task is much easier if you notice that the digits can be constructed not in a top to bottom (or bottom to up) row-byrow manner, but in a columnar manner; at the same time scaling becomes very easy.

```
7d  ⟨LC Display 7d⟩≡
    package com.rvprg.pc;

⟨1.4 Imports 8a⟩

class LCDisplay {
    ⟨1.4 Constants 8b⟩
    ⟨1.4 Convertion 9a⟩
    ⟨1.4 Input/Output 8c⟩
}
```

Each LCD digit has 7 segments: Two in the first and the third columns and three in the second column. Let's encode our digits:

Digit	Binary	Hex
0	11 101 11	77
1	00 000 11	03
2	10 111 01	5D
3	00 111 11	1F
4	01 010 11	2B
5	01 111 10	3E
6	11 111 10	7E
7	00 001 11	07
8	11 111 11	7F
9	01 111 11	3F

Since we know that the input ends in two zeros we define this string constant plus a couple of other string constants.

The array pattern for the given digit i returns bits that correspond to the segments, so for example digits[5] would return segments for digit 5. We will be using masks to discover which bits are set and not set.

But let's write input/output first as this is very easy. At the same time, let's assume our method that converts a string into LCD style digits is called **segments**. This method takes two arguments, the digits string and **scale**. Let's assume it returns list of strings which we can simply output to the console.

Now all that's left is to implement segments.

```
9a \langle 1.4\ Convertion\ 9a \rangle \equiv private static List<String> segments(final String digits, final int scale) { \langle 1.4\ Helpers\ 9c \rangle \langle 1.4\ Process\ 10e \rangle \langle 1.4\ Return\ 10g \rangle }
```

The idea is simple: We check bit 6 and bit 5 of the pattern and construct ASCII representation of the first column, then we check bit 4, 3, and 2 and construct the middle column, finally we check bit 1 and 0 to construct the last column. Of course, we need to take into account the scaling.

So let's have a look at an example. Let's say we need to construct digit 2 with scale 3. First, we get the pattern value pattern[2]=5D, or 1011101. Then, we start with the masks 40 and 20 to see which segments are on in the first column (bit 6 and bit 5); so, 40 & 5D = 1 and 20 & 5D = 0, which means that the first segment is on and the second is off, so we output \_\_\_\_\_\_\_. Because our scale is 3, we output | and \_\_\_\_\_\_ three times, so we end up with \_\_\_\_\_\_\_; Similarly we construct the third column, but we use different masks: 02 and 01.

OK, let's write some helpers already before we get back to producing the middle column. We will need some function that replicates a specified string n times. There's a Java function nCopies that does that, so we will use it. However, it returns a list of strings, therefore we use join function to join that into a single string using EMPTY as a delimiter. Let's write that:

```
9b  ⟨1.4 Imports 8a⟩+≡
import static java.lang.String.join;
import static java.util.Collections.nCopies;
import java.util.function.Function;

9c  ⟨1.4 Helpers 9c⟩≡
Function<String, String> g = x -> join(EMPTY, nCopies(scale, x));
```

Note that we use the fact that the scale is captured in the closure.

OK, we also need a mapping function that checks which bits are on and off in the given value using the list of masks. Depending on whether bits are on or off it returns ASCII character | or ... It will be a stream of such characters:

```
9d \langle 1.4 \; Helpers \; 9c \rangle + \equiv
BiFunction<Stream<Integer>, Byte, Stream<String>> h =
(m, x) -> m.map(mask -> (x & mask) > 0 ? "|" : SPACE);
```

Here m is a stream of masks, and x is the value pattern[i] for some i. Note the function returns a stream as well.

Now we can write a function that constructs a column of our LCD digit. Let's call it k:

Note SPACEs around (as per requirement) and that the segments within a column are joined by a space.

So far so good. Basically we can now write function f that takes a digit pattern pattern[i] and returns a stream of strings (in fact, the columns of our LCD digits).

Also note the last line which adds spaces between consecutive digits.

Finally, let's get back to the middle of the digit. To contruct it, we will re-use exactly the same functions we've already defined. We use h to obtain the segments that are on and off. Note though that h returns | symbols, not the dashes, which are used to indicate horizontal LCD segments. So we will need to replace all occurances of vertical bars with dashes. It's easy to see that the number of spaces between the horizontal segments will be exactly scale, which is already captured in the g function implementation. Finally, all we need to do, is to replicate the middle column scale times. All this can be very easily implemented like so:

But remember, this gives us a list of columns of the LCD digits, not rows, so before returning it we need a little post-processing: For each column we take the last characters, concatenate these last characters into a string, and add to a list; then we take the next to the last characters and do the same, and so on:

And that completes the program.

### 1.5 Graphical Editor

Very straightforward task. The only difficult part being the fill operation, but I leave it without comments either as the code is self-explanatory.

```
11
      \langle Graphical\ Editor\ 11 \rangle \equiv
        package com.rvprg.pc;
         import static java.util.Arrays.stream;
        import static java.util.stream.Collectors.toList;
        import java.io.BufferedReader;
        import java.io.IOException;
         import java.io.InputStreamReader;
         import java.util.ArrayDeque;
         import java.util.Deque;
         import java.util.List;
        public class GraphicalEditor {
             private static final BufferedReader reader = new BufferedReader(
                     new InputStreamReader(System.in));
             private int[][] canvas;
             private int m = 0, n = 0;
             private void clear() {
                 for (int i = 0; i < n; ++i) {
                     for (int j = 0; j < m; ++j) {
                         canvas[j][i] = '0';
                 }
             }
             private void execute(List<String> command) {
                 int x, y1, y2, y, x1, x2, c;
                 switch (command.get(0)) {
                 case "I":
                     m = Integer.parseInt(command.get(1));
                     n = Integer.parseInt(command.get(2));
                     canvas = new int[m][n];
                     clear();
                     break;
                 case "C":
                     clear();
                     break;
                 case "L":
                     x = Integer.parseInt(command.get(1)) - 1;
                     y = Integer.parseInt(command.get(2)) - 1;
                     canvas[x][y] = command.get(3).charAt(0);
                     break;
                 case "V":
```

x = Integer.parseInt(command.get(1)) - 1;

```
y1 = Integer.parseInt(command.get(2)) - 1;
        y2 = Integer.parseInt(command.get(3)) - 1;
        c = command.get(4).charAt(0);
        for (y = Math.min(y1, y2); y \le Math.max(y1, y2); ++y) {
            canvas[x][y] = c;
        break;
    case "H":
        x1 = Integer.parseInt(command.get(1)) - 1;
        x2 = Integer.parseInt(command.get(2)) - 1;
        y = Integer.parseInt(command.get(3)) - 1;
        c = command.get(4).charAt(0);
        for (x = Math.min(x1, x2); x \le Math.max(x1, x2); ++x) {
            canvas[x][y] = c;
        }
        break;
    case "K":
        x1 = Integer.parseInt(command.get(1)) - 1;
        y1 = Integer.parseInt(command.get(2)) - 1;
        x2 = Integer.parseInt(command.get(3)) - 1;
        y2 = Integer.parseInt(command.get(4)) - 1;
        c = command.get(5).charAt(0);
        for (x = x1; x \le x2; ++x) {
            for (y = y1; y \le y2; ++y) {
                canvas[x][y] = c;
        }
        break;
    case "F":
        x = Integer.parseInt(command.get(1)) - 1;
        y = Integer.parseInt(command.get(2)) - 1;
        int newColor = command.get(3).charAt(0);
        int oldColor = canvas[x][y];
        fill(new Point(x, y), oldColor, newColor);
        break;
    case "S":
        String name = command.get(1);
        System.out.println(name);
        for (int i = 0; i < n; ++i) {
            for (int j = 0; j < m; ++j) {
                System.out.print((char) canvas[j][i]);
            System.out.println();
        }
        break;
    default:
        break;
    }
}
private static class Point {
    final int x, y;
```

```
public Point(int x, int y) {
            this.x = x;
            this.y = y;
        }
    }
    private void fill(Point pt, int oldColor, int newColor) {
        if (canvas[pt.x][pt.y] != oldColor || oldColor == newColor) {
            return;
        Deque<Point> q = new ArrayDeque<>();
        q.addLast(pt);
        canvas[pt.x][pt.y] = newColor;
        while (!q.isEmpty()) {
            pt = q.pop();
            if (pt.x + 1 < m \&\& canvas[pt.x + 1][pt.y] == oldColor) {
                canvas[pt.x + 1][pt.y] = newColor;
                q.addLast(new Point(pt.x + 1, pt.y));
            }
            if (pt.x - 1 \ge 0 \&\& canvas[pt.x - 1][pt.y] == oldColor) {
                canvas[pt.x - 1][pt.y] = newColor;
                q.addLast(new Point(pt.x - 1, pt.y));
            }
            if (pt.y + 1 < n \&\& canvas[pt.x][pt.y + 1] == oldColor) {
                canvas[pt.x][pt.y + 1] = newColor;
                q.addLast(new Point(pt.x, pt.y + 1));
            }
            if (pt.y - 1 \ge 0 \&\& canvas[pt.x][pt.y - 1] == oldColor) {
                canvas[pt.x][pt.y - 1] = newColor;
                q.addLast(new Point(pt.x, pt.y - 1));
            }
        }
    }
    public static void main(String[] args) throws IOException {
        GraphicalEditor editor = new GraphicalEditor();
        String currentLine;
        while ((currentLine = reader.readLine()) != null) {
            List<String> command = stream(currentLine.trim().split(" "))
                    .filter(x -> !x.equals("")).collect(toList());
            if (command.get(0).equalsIgnoreCase("X")) {
                break;
            }
            editor.execute(command);
        }
   }
}
```

#### 1.6 Interpreter

This task is disappointingly straightforward.

```
14
      \langle Interpreter \ 14 \rangle \equiv
        package com.rvprg.pc;
        import java.io.BufferedReader;
         import java.io.IOException;
         import java.io.InputStreamReader;
        import java.util.ArrayList;
        import java.util.List;
        class Interpreter {
             private static final BufferedReader reader =
                 new BufferedReader(new InputStreamReader(System.in));
             private static int interpret(List<Integer> input) {
                 int[] reg = new int[10];
                 int[] ram = new int[1000];
                 for (int i = 0; i < input.size(); ++i) {</pre>
                     ram[i] = input.get(i);
                 int pc = 0;
                 int r = 0;
                 while (ram[pc] != 100) {
                     int op = ram[pc];
                     int c = (op / 100) \% 10;
                     pc = (pc + 1) \% 1000;
                     r++;
                     switch (c) {
                     case 2:
                          reg[(op / 10) % 10] = op % 10;
                         break;
                     case 3:
                         reg[(op / 10) \% 10] = (reg[(op / 10) \% 10] + (op % 10)) \% 1000;
                     case 4:
                         reg[(op / 10) \% 10] = (reg[(op / 10) \% 10] * (op % 10)) % 1000;
                         break;
                     case 5:
                         reg[(op / 10) % 10] = reg[op % 10];
                     case 6:
                         reg[(op / 10) % 10] = (reg[(op / 10) % 10] + reg[op % 10]) % 1000;
                         break;
                     case 7:
                          reg[(op / 10) \% 10] = (reg[(op / 10) \% 10] * reg[op \% 10]) \% 1000;
                     case 8:
                         reg[(op / 10) % 10] = ram[reg[op % 10]];
                         break;
```

ram[reg[op % 10]] = reg[(op / 10) % 10];

```
break;
            case 0:
                if (reg[op % 10] != 0) {
                    pc = reg[(op / 10) \% 10];
                }
                break;
            }
        }
        return r + 1;
    }
    public static void main(String[] args) throws IOException {
        int n = Integer.valueOf(reader.readLine().trim());
        reader.readLine();
        String currentLine;
        for (int i = 0; i < n; ++i) {
            List<Integer> input = new ArrayList<Integer>();
            while ((currentLine = reader.readLine()) != null &&
                    !currentLine.trim().equalsIgnoreCase("")) {
                input.add(Integer.parseInt(currentLine.trim()));
            }
            System.out.println(interpret(input));
            if (i < n - 1) {
                System.out.println();
        }
   }
}
```

#### 1.7 Check The Check

```
This task is trivial.
```

16

```
⟨Check The Check 16⟩≡
 package com.rvprg.pc;
  import java.io.BufferedReader;
  import java.io.IOException;
  import java.io.InputStreamReader;
 public class CheckTheCheck {
     private static final BufferedReader reader = new BufferedReader(
              new InputStreamReader(System.in));
     private static final int BOARD_SIZE = 8;
      private static final int[][] king = new int[][] {
              { -1, -1 }, { 0, -1 }, { 1, -1 }, { -1, 0 }, { 1, 0 }, { -1, 1 },
              { 0, 1 }, { 1, 1 }
     };
      private static final int[][] knight = new int[][] {
              \{-2, -1\}, \{-1, -2\}, \{1, -2\}, \{2, -1\}, \{2, 1\}, \{1, 2\},
              { -1, 2 }, { -2, 1 }
     };
      private static final int[][] bishop = new int[][] {
              { 1, 1 }, { -1, -1 }, { -1, 1 }, { 1, -1 }
     };
      private static final int[][] rook = new int[][] {
              \{1, 0\}, \{-1, 0\}, \{0, 1\}, \{0, -1\}
     private static final int[][] queen = new int[][] {
              \{1, 0\}, \{-1, 0\}, \{0, 1\}, \{0, -1\}, \{1, 1\}, \{-1, -1\},
              { -1, 1 }, { 1, -1 }
      private static final int[][] white_pawn = new int[][] {
              { -1, -1 }, { -1, 1 }
      private static final int[][] black_pawn = new int[][] {
              { 1, -1 }, { 1, 1 }
     };
      private static boolean isWithinBounds(int d, int v) {
          if (d == 0) {
              return true;
          return d > 0 ? (v < BOARD_SIZE) : (v >= 0);
      private static void check(int di, int dj, int i, int j, int[][] board,
              int[][] attackBoard) {
          int c = j + dj;
          int r = i + di;
          while (isWithinBounds(dj, c) && isWithinBounds(di, r)) {
              attackBoard[r][c] = 1;
```

```
if (board[r][c] != '.')
            break;
        r += di;
        c += dj;
   }
}
private static void check(int[][] d, int i, int j, int[][] board,
        int[][] attackBoard) {
    if (d == king || d == knight || d == black_pawn || d == white_pawn) {
        for (int k = 0; k < d.length; ++k) {
            if ((i + d[k][0] >= 0 \&\& i + d[k][0] < BOARD_SIZE) \&\&
                    (j + d[k][1] >= 0 && j + d[k][1] < BOARD_SIZE)) {
                attackBoard[i + d[k][0]][j + d[k][1]] = 1;
            }
        }
        return;
    }
    for (int k = 0; k < d.length; ++k) {
        check(d[k][0], d[k][1], i, j, board, attackBoard);
    }
}
private static int[] locate(int v, int[][] board) {
    for (int i = 0; i < BOARD_SIZE; ++i) {</pre>
        for (int j = 0; j < BOARD_SIZE; ++j) {
            if (board[i][j] == v) {
                return new int[] { i, j };
            }
        }
    }
    return null;
}
private static String checkTheCheck(int[][] board) {
    int[][] attackBoardWhites = new int[BOARD_SIZE][BOARD_SIZE];
    int[][] attackBoardBlacks = new int[BOARD_SIZE][BOARD_SIZE];
    for (int i = 0; i < BOARD_SIZE; ++i) {</pre>
        for (int j = 0; j < BOARD_SIZE; ++j) {
            if (board[i][j] == 'R' || board[i][j] == 'r') {
                check(rook, i, j, board, board[i][j] == 'R'
                        ? attackBoardWhites : attackBoardBlacks);
            if (board[i][j] == 'B' || board[i][j] == 'b') {
                check(bishop, i, j, board, board[i][j] == 'B'
                        ? attackBoardWhites : attackBoardBlacks);
            }
            if (board[i][j] == 'K' || board[i][j] == 'k') {
                check(king, i, j, board, board[i][j] == 'K'
                        ? attackBoardWhites : attackBoardBlacks);
            }
```

```
check(knight, i, j, board, board[i][j] == 'N'
                            ? attackBoardWhites : attackBoardBlacks);
                }
                if (board[i][j] == 'Q' || board[i][j] == 'q') {
                    check(queen, i, j, board, board[i][j] == 'Q'
                            ? attackBoardWhites : attackBoardBlacks);
                }
                if (board[i][j] == 'P' || board[i][j] == 'p') {
                    boolean isWhite = board[i][j] == 'P';
                    check(isWhite ? white_pawn : black_pawn, i, j, board,
                            isWhite ? attackBoardWhites : attackBoardBlacks);
                }
            }
        }
        int[] wk = locate('K', board);
        int[] bk = locate('k', board);
        boolean bkCheck = (attackBoardWhites[bk[0]][bk[1]] == 1);
        boolean wkCheck = (attackBoardBlacks[wk[0]][wk[1]] == 1);
        if (wkCheck) {
            return "white king is in check.";
        if (bkCheck) {
            return "black king is in check.";
        return "no king is in check.";
    }
    public static void main(String[] args) throws IOException {
        boolean empty = true;
        int game = 1;
        do {
            int[][] board = new int[BOARD_SIZE][BOARD_SIZE];
            empty = true;
            for (int i = 0; i < BOARD_SIZE; ++i) {</pre>
                String currentLine = reader.readLine();
                for (int j = 0; j < BOARD_SIZE; ++j) {
                    board[i][j] = currentLine.charAt(j);
                    empty = empty && board[i][j] == '.';
                }
            }
            if (empty) {
                break;
            System.out.println("Game #" + game + ": " + checkTheCheck(board));
        } while (reader.readLine().trim().equals(""));
   }
}
```

if (board[i][j] == 'N' || board[i][j] == 'n') {

}

20

#### 1.8 Australian Voting

This task is very straightforward.

Let's sort out input/output first as usual. We will assume our function that does the election is called elect, and that it takes two arguments, a list of candidates and a list of ballots, and returns a list of those who win the election. Note the ballots is a list of deques, that's because we will be checking the next candidate in the ranking, and note that we subtract one from each index in the ballots, this is for easier access to the arrays, as they are indexed from 0.

```
\langle Australian \ Voting \ 20 \rangle \equiv
  package com.rvprg.pc;
  import static java.util.Arrays.stream;
  import static java.util.stream.Collectors.toList;
  import java.io.BufferedReader;
  import java.io.IOException;
  import java.io.InputStreamReader;
  import java.util.ArrayDeque;
  import java.util.ArrayList;
  import java.util.Deque;
  import java.util.List;
  \langle 1.8\ Imports\ 21e \rangle
  class AustralianVoting {
      private static final String EMPTY = "";
      private static final BufferedReader reader =
          new BufferedReader(new InputStreamReader(System.in));
      private static List<String> elect(List<String> candidates, List<Deque<Integer>> ballots) {
          (1.8 Implementation 21a)
      public static void main(String[] args) throws IOException {
          int n = Integer.valueOf(reader.readLine().trim());
          reader.readLine();
          for (int i = 0; i < n; ++i) {
              int count = Integer.valueOf(reader.readLine().trim());
              List<String> candidates = reader.lines().limit(count).collect(toList());
              List<Deque<Integer>> ballots = new ArrayList<Deque<Integer>>();
              String currentLine = EMPTY;
              while ((currentLine = reader.readLine()) != null && !currentLine.equalsIgnoreCase(EMPTY)) {
                   ballots.add(new ArrayDeque<Integer>(stream(currentLine.trim().split(" "))
                           .filter(x -> !x.equals(EMPTY))
                           .map(Integer::parseInt).map(x -> x - 1).collect(toList())));
              elect(candidates, ballots).forEach(System.out::println);
               if (i < n - 1) {
                  System.out.println();
              }
          }
```

```
}
```

Now let's implement elect function. First we need to figure out the majority. That's easy as that's simply the half of the ballots plus one.

```
21a \langle 1.8 \; Implementation \; 21a \rangle \equiv final int majority = ballots.size() / 2 + 1;
```

Because candidates in the ballots are numbered by their indexes in the table, let's have an array of ints, which will hold the number of votes.

```
21b \langle 1.8 \; Implementation \; 21a \rangle + \equiv final int[] counter = new int[candidates.size()];
```

Now let's count votes for the candidates specified as first in the ballots:

```
21c \langle 1.8 \ Implementation \ 21a \rangle + \equiv ballots.stream().map(Deque::peek).forEach(x -> counter[x]++);
```

After this point two things may happen: We will have somebody who got the majority of the votes, in which case we know the winner (or winners), or not, in which case we repeat the procedure described in the problem statement.

```
21d \langle 1.8 \ Implementation \ 21a \rangle + \equiv while (true) { \langle 1.8 \ Election \ loop \ 21f \rangle }
```

 $\langle 1.8 \; Imports \; 21e \rangle \equiv$ 

21e

OK, because some candidates may get equal number of votes we need to group them by votes. This is pretty easy:

Pay attention to the candidates who got zeros votes, because those will need to go through the elimination process too.

Now we need to find out who got the most votes and who got the least:

```
21g  \( \langle 1.8 \) Election \( loop \) 21f\rangle +\( \equiv \)
  int \( \text{max} = \text{result.keySet().stream().max(Integer::compareTo).get();} \)
  int \( \text{min} = \text{result.keySet().stream().min(Integer::compareTo).get();} \)
  It's easy to see that if \( \text{max} \geq majority \) or \( \text{max} = min, \) then we know the winner:

21h \( \langle 1.8 \) Election \( \loop \) 21f\rangle +\( \equiv \)
  if \( \text{max} >= \text{majority} \) | \( \text{max} == \text{min} \) \( \langle \)
  return \( \text{result.get(max).stream().map(x -> candidates.get(x)).collect(toList());} \)
```

Otherwise we need to re-distribute the votes. We get the indexes of the candidates who got the least votes and mark them as having -1 votes in the counter array so that we never consider them again in our filters.

```
21i ⟨1.8 Election loop 21f⟩+≡
List<Integer> eliminated = result.get(min);
eliminated.forEach(x -> counter[x] = -1);
```

Now we need to remove the eliminated candidates from the ballots. However it needs to be done carefully. We make note of who is currently the first in the ballot. If after the elimination process the first in the rank has changed, we need to take that into account. This is captured in the following chunk:

# 2 Chapter 2

}

### 2.1 Jolly Jumper

```
This task must be a joke.
22b
       \langle Jolly\ Jumpers\ 22b\rangle \equiv
         package com.rvprg.pc;
          import static java.lang.Math.abs;
          import static java.util.Arrays.stream;
          import static java.util.stream.Collectors.toList;
          import static java.util.stream.IntStream.range;
          import java.io.BufferedReader;
          import java.io.IOException;
          import java.io.InputStreamReader;
          import java.util.List;
          class JollyJumpers {
              private static final BufferedReader reader =
                  new BufferedReader(new InputStreamReader(System.in));
              public static void main(String[] args) throws IOException {
                  String currentLine;
                  while ((currentLine = reader.readLine()) != null) {
                      List<Integer> nums = stream(currentLine.trim().split(" ")).filter(x -> !x.equals(""))
                           .skip(1).map(Integer::parseInt).collect(toList());
                      int[] diffs = range(0, nums.size() - 1)
                           .map(i -> abs(nums.get(i) - nums.get(i + 1))).distinct().sorted().toArray();
                      boolean isJolly = range(0, diffs.length).boxed()
                           .map(i \rightarrow diffs[i] == i + 1).reduce(true, (x, y) \rightarrow x && y);
                      System.out.println(diffs.length == nums.size() - 1 && isJolly ? "Jolly" : "Not jolly");
                  }
              }
```

### 2.2 Crypt Kicker

This task is a lot of fun! To solve it we are going to need a very good bookkeeping discipline. Let's outline the general strategy. First thing to do is to to group the words by length. Then we need to come up with a method to compare a dictionary word and an encrypted word by looking at their patterns. So we somehow need to tell if the words "abbc" has a similar pattern as "xyyz". But this is very easy: we scan a word from left to right and output an index of the first occurrence of the character, or current index if it's the first occurrence. So for example "abbc" and "xyyz" would both have a pattern 1 2 2 3. Using a pattern and the word length we can find words from the dictionary that could be the potential matches for an encrypted word.

We start with the longest word (if multiple words of the same length, then any words of such length) and we find all the words from the dictionary that have the same length, the same pattern, and agree with the mapping found so far. By the mapping found so far we mean the following: if some previous word has been matched with a candidate, we note the mapping. So if we matched "abbc" with "xyyz" we now know that a maps to x, b maps to y, and c maps to z. This means that if we are now trying to match another word, say "zy", we can eliminate candidates such as "bc", because we now assume that z maps to c, not b. Once we filtered all the potential candidates we try to match the first candidate from the list and move on to the next word. If at any step we fail to find any candidate word, we return one step back, and try another word in the list, if the list is exhausted, we move one step back again. If we exhausted all the lists, then the decryption is impossible. For simplicity of implementation we will implement it as a recursion.

OK, now we just need to write code.

23

First input/output. The main class will be initialized by a dictionary and will have just one method decrypt that will take a string and return either a decrypted text or stars, as per problem statement.

```
\langle Crypt \ Kicker \ 23 \rangle \equiv
  package com.rvprg.pc;
  import java.io.BufferedReader;
  import java.io.IOException;
  import java.io.InputStreamReader;
  (2.4 Imports 24a)
  class CryptKicker {
      private static final BufferedReader reader =
          new BufferedReader(new InputStreamReader(System.in));
      \langle 2.4 \ Variables \ 24f \rangle
      (2.4 Constructor 24g)
      (2.4 Methods 24b)
      public static void main(String[] args) throws IOException {
          String currentLine;
          final int size = Integer.parseInt(reader.readLine().trim());
          final List<String> dictionary = reader.lines().limit(size).collect(toList());
           CryptKicker cryptKicker = new CryptKicker(dictionary);
```

```
while ((currentLine = reader.readLine()) != null &&
                             !currentLine.trim().equals("")) {
                        System.out.println(cryptKicker.decrypt(currentLine));
                    }
               }
          }
            Let's write the method that gives us the pattern of a given word. This method does
        exactly the thing we've described above.
        \langle 2.4 | Imports | 24a \rangle \equiv
24a
           import static java.util.stream.Collectors.toList;
           import static java.util.stream.IntStream.range;
24b
        \langle 2.4 \text{ Methods } 24b \rangle \equiv
          private static List<Integer> getPattern(String word) {
               return range(0, word.length()).map(i -> word.indexOf(word.charAt(i)))
                    .boxed().collect(toList());
          }
            And let's add a helper method that for a given list of words gives a map. (Note that we
        take distinct words as the words in the input dictionary aren't necessarily unique.)
24c
        \langle 2.4 | Imports | 24a \rangle + \equiv
           import static java.util.function.Function.identity;
           import static java.util.stream.Collectors.toMap;
           import java.util.List;
           import java.util.Map;
           import java.util.Deque;
24d
        \langle 2.4 \text{ Methods } 24b \rangle + \equiv
          private static Map<String, List<Integer>> getPatterns(Deque<String> words) {
               return words.stream().distinct().collect(
                    toMap(identity(), CryptKicker::getPattern));
          }
            Now we can do the constructor. In the constructor we will group the words by length and
        get their patterns.
        \langle 2.4 \; Imports \; 24a \rangle + \equiv
24e
          import static java.util.stream.Collectors.groupingBy;
           import java.util.ArrayDeque;
24f
        \langle 2.4 \ Variables \ 24f \rangle \equiv
          private final Map<Integer, List<String>> dictionary;
          private final Map<String, List<Integer>> patterns;
24g
        \langle 2.4 \ Constructor \ 24g \rangle \equiv
          public CryptKicker(List<String> inputDictionary) {
               dictionary = inputDictionary.stream()
                    .collect(groupingBy(String::length));
               patterns = getPatterns(new ArrayDeque<>(inputDictionary));
          }
```

We will also need a function to compare two given patterns. This is easy:

Let's sort out the variables that we are going to need. We will need a map that will hold patterns of the encrypted words for the given input string.

```
25c \langle 2.4 \ Variables \ 24f \rangle + \equiv private Map<String, List<Integer>> encryptedPatterns;
```

This variable could have been passed around via argument to the methods, because this variable's contents depend on each encrypted input line. But I have chosen to just have this as a private member.

We will also need to keep track of the words that have been mapped.

And we will need the mappings themselves. We will keep both the direct mapping and the reversed mappings in the arrays, where an index is the ASCII character code and the value is another ASCII character code. Because ASCII characters for the lower case letters go from 97 to 122 it should be enough to just create an array of no more than 128 bytes. We could have created a smaller array, but in that case we would need to adjust the indexes which would clutter the code unnecessarily.

The counter array will keep track of how many words have used this character mapping so far. This is needed because we will be mapping and unmapping the words multiple times during the search. An empty array will denote unsuccessful mapping.

26a

26b

Let's implement mapWord and unmapWord methods. Note that they keep track (with help of counter) of how many words have used a specific character mapping.

```
\langle 2.4 \text{ Methods } 24b \rangle + \equiv
  private void mapWord(String e, String c) {
      mappedWords.add(c);
      for (int i = 0; i < e.length(); ++i) {
          dirMapping[e.charAt(i)] = c.charAt(i);
          counter[e.charAt(i)]++;
          revMapping[c.charAt(i)] = e.charAt(i);
      }
  }
  private void unmapWord(String e, String c) {
      mappedWords.remove(c);
      for (int i = 0; i < e.length(); ++i) {
          counter[e.charAt(i)]--;
          if (counter[e.charAt(i)] == 0) {
               revMapping[dirMapping[e.charAt(i)]] = 0;
               dirMapping[e.charAt(i)] = 0;
          }
      }
  }
```

We need to keep track of these mapping and counter because of the filtering. For example, if we mapped the word "abc" to "xyz" and the word "ab" to "xy", we now know that a maps to x, b maps to y, and c maps to z. So we can find that the word's "ab" mapping to "xy" is a valid mapping and so we can map that too. If for some reason we unmap the word "abc", our mapping arrays should still keep the mapping of a to x, and b to y, because we haven't unmapped the word "ab".

Now let's implement the filtering. This function take an encrypted word and does the following filtering. First, it gets all the words from the dictionary of the same length. Then it filters out the words that have been mapped already and the words that don't have the same pattern. Next, it checks if this word agrees with the mapping (may be partial) of the mappings found so far. If the word passes all this filtering, it is add to the list, which then returned as the result.

```
\langle 2.4 \; Imports \; 24a \rangle + \equiv import java.util.ArrayList;
```

```
27a
        \langle 2.4 \text{ Methods } 24b \rangle + \equiv
          private List<String> filter(String encrypted) {
              List<String> matchedWords = new ArrayList<String>();
              for (String word : dictionary.get(encrypted.length())) {
                  if (mappedWords.contains(word) ||
                       !compare(encryptedPatterns.get(encrypted), patterns.get(word))) {
                       continue;
                  }
                  boolean matched = true;
                  for (int i = 0; i < word.length() && matched; ++i) {</pre>
                       boolean unmapped = dirMapping[encrypted.charAt(i)] == 0;
                      boolean mapped = dirMapping[encrypted.charAt(i)] == word.charAt(i);
                      boolean unused = revMapping[word.charAt(i)] == 0;
                      matched = (unmapped && unused) || mapped;
                  }
                  if (matched) {
                      matchedWords.add(word);
              }
              return matchedWords;
          }
```

We can now implement the recursive search method. It takes a deque of encrypted words and then tries to map them to the dictionary. (A deque because it has convenient methods such as pop and push.) This method assumes that the words in the deque are sorted by length in descending order.

```
27b
        \langle 2.4 \text{ Methods } 24b \rangle + \equiv
          private boolean map(Deque<String> encryptedWords) {
              if (encryptedWords.isEmpty()) {
                   return true;
              String encryptedWord = encryptedWords.pop();
              List<String> words = filter(encryptedWord);
              for (String candidate : words) {
                   mapWord(encryptedWord, candidate);
                   if (map(encryptedWords)) {
                       return true;
                   unmapWord(encryptedWord, candidate);
              }
              encryptedWords.push(encryptedWord);
              return false;
          }
```

Let's add another helper method that will do the clearing up and initialization of the data structures:

```
27c \langle 2.4 | Imports | 24a \rangle + \equiv import java.util.Arrays;
```

```
\langle 2.4 \ Methods \ 24b \rangle + \equiv
28a
          private int[] findMapping(Deque<String> encryptedWords) {
               encryptedPatterns = getPatterns(encryptedWords);
              mappedWords.clear();
              Arrays.fill(dirMapping, 0);
               Arrays.fill(revMapping, 0);
              Arrays.fill(counter, 0);
              return map(encryptedWords) ? dirMapping : NOT_FOUND;
          }
           Finally, we can now implement decrypt method:
28b
        \langle 2.4 | Imports | 24a \rangle + \equiv
          import static java.util.Comparator.comparing;
        \langle 2.4 \; Methods \; 24b \rangle + \equiv
28c
          public String decrypt(String input) {
              StringBuilder result = new StringBuilder();
               int[] mapping = findMapping(
                       new ArrayDeque<>(Arrays.stream(input.trim().split(" "))
                            .filter(x -> !x.equals("")).distinct()
                            .sorted(comparing(String::length).reversed()).collect(toList())));
               input.chars().map(c \rightarrow c != ' ' ? (mapping != NOT_FOUND ? mapping[c] : '*') : c)
                   .forEachOrdered(x -> result.append((char) x));
              return result.toString();
          }
```

## 3 Chapter 3

29

## 3.1 Crypt Kicker II

This task is much easier than Crypt Kicker. Here we have a very well known pangram "the quick brown fox jumps over the lazy dog". A pangram is a sentence that uses every letter of the alphabet at least once. So all we need to do is to locate the pangram in the input lines. We will use exactly the same technique as we used while solving the original Crypt Kicker problem.

```
\langle Crypt \ Kicker \ II \ 29 \rangle \equiv
  package com.rvprg.pc;
  import static java.lang.Math.abs;
  import static java.util.stream.Collectors.toList;
  import static java.util.stream.IntStream.range;
  import java.io.BufferedReader;
  import java.io.IOException;
  import java.io.InputStreamReader;
  import java.util.ArrayList;
  import java.util.Arrays;
  import java.util.List;
  class CryptKickerII {
      private static final BufferedReader reader =
          new BufferedReader(new InputStreamReader(System.in));
      private final int[] mapping = new int[128];
      private static final String pangram = "the quick brown fox jumps over the lazy dog";
      private static final String pangramSpaces = pangram.replaceAll("[^ ]", ".");
      private static final List<Integer> pangramPattern = getPattern(pangram);
      private static List<Integer> getPattern(String word) {
          return range(0, word.length()).map(i -> word.indexOf(word.charAt(i)))
                   .boxed().collect(toList());
      }
      private static boolean compare(List<Integer> a, List<Integer> b) {
          return a.size() == b.size() && range(0, a.size())
                  .map(i \rightarrow abs(a.get(i) - b.get(i))).sum() == 0;
      }
      private boolean isPangram(String input) {
          String line = String.join(" ", Arrays.stream(input.trim()
                   .split(" ")).filter(x -> !x.equals("")).collect(toList()));
          return compare(pangramPattern, getPattern(line.toString())) &&
                  line.replaceAll("[^ ]", ".").equalsIgnoreCase(pangramSpaces);
      public List<String> decrypt(List<String> input) {
          Arrays.fill(mapping, 0);
          List<String> output = new ArrayList<String>();
          String encryptedPangram = input.stream()
```

```
.filter(x -> isPangram(x)).findFirst().orElse("");
        if (encryptedPangram.equalsIgnoreCase("")) {
            output.add("No solution.");
            return output;
       }
        for (int i = 0; i < encryptedPangram.length(); ++i) {</pre>
            mapping[encryptedPangram.charAt(i)] = pangram.charAt(i);
        }
        return input.stream().map(x -> {
            StringBuilder result = new StringBuilder();
            x.chars().map(c -> c != ' ' ? mapping[c] : c)
                    .forEachOrdered(c -> result.append((char) c));
            return result.toString();
       }).collect(toList());
    }
    public static void main(String[] args) throws IOException {
        String currentLine;
        final int n = Integer.parseInt(reader.readLine().trim());
        reader.readLine();
        CryptKickerII cryptKicker = new CryptKickerII();
        for (int i = 0; i < n; ++i) {
            List<String> input = new ArrayList<String>();
            while ((currentLine = reader.readLine()) != null &&
                    !currentLine.trim().equalsIgnoreCase("")) {
                input.add(currentLine);
            }
            cryptKicker.decrypt(input).forEach(System.out::println);
            if (i < n - 1) {
                System.out.println();
       }
   }
}
```

## 3.2 File Fragmentation

Let's sort out input/output assuming that our function restore takes a list of strings (i.e. shards) and returns the restored string (i.e. original file). Input is rather straightforward and, unfortunately, due to the format of the input data, isn't very concise.

```
31
       \langle File\ Fragmentation\ 31 \rangle \equiv
         package com.rvprg.pc;
         import java.io.BufferedReader;
         import java.io.IOException;
         import java.io.InputStreamReader;
         import java.util.ArrayList;
         import java.util.List;
         \langle 3.6 \; Imports \; 32a \rangle
         class FileFragmentation {
             private static final BufferedReader reader =
                  new BufferedReader(new InputStreamReader(System.in));
             ⟨3.6 Helpers 32e⟩
             private static String restore(List<String> fragments) {
                  \langle 3.6 \; Implementation \; 32b \rangle
             public static void main(String[] args) throws IOException {
                  int n = Integer.parseInt(reader.readLine());
                  reader.readLine();
                  for (int i = 0; i < n; ++i) {
                      List<String> fragments = new ArrayList<String>();
                      do {
                           String s = reader.readLine();
                           if (s == null || s.equalsIgnoreCase("")) {
                               break;
                           }
                           fragments.add(s);
                      } while (true);
                      System.out.println(restore(fragments));
                      if (i < n - 1) {
                           System.out.println();
                      }
                  }
             }
```

So how do we restore the files? It's easy to see that if we sort the shards by length and then take the largest shard and the shortest one we will end up with a potential original file. But there may be numerous smallest shards and numerous largest shards, so we will need to try them one by one. This is not that bad as it seems at first sight. This is because we only need to try one largest shard with n shortest shards in the worst case, having only two cases: The long shard goes first and the short goes after it or vice versa. Once we got a candidate original file we simply try to fit the rest of the shards. This can be done very easily. We simply partition our candidate file at every point and then check if the list contains these shards, and if it does, we mark that. Once we found every shard in the list in this way we know that the original file was the same as our candidate file. Otherwise we try the next smallest shard. We continue until we fit every shard. This algorithm will always find the original file because of how the problem is formulated.

OK, so first thing we need to do is to sort the shards by length:

```
32a
        \langle 3.6 \; Imports \; 32a \rangle \equiv
           import static java.util.Comparator.comparing;
32b
        \langle 3.6 \; Implementation \; 32b \rangle \equiv
           fragments.sort(comparing(String::length));
            Then we find the largest (any will do) and get the list of the smallest shards:
        \langle 3.6 \text{ Imports } 32a \rangle + \equiv
32c
           import static java.util.stream.Collectors.toList;
32d
        \langle 3.6 \; Implementation \; 32b \rangle + \equiv
           String large = fragments.get(fragments.size() - 1);
           List<String> smallest = fragments.stream().filter(
               x -> x.length() == fragments.get(0).length()).collect(toList());
            Let's write fit function that takes a list of shards and a candidate and returns true or
        false depending on whether those shards could be fit with this candidate file or not. This is
        implemented in accordance to the algorithm described earlier.
        \langle 3.6 \text{ Helpers } 32e \rangle \equiv
32e
           private static boolean fit(List<String> fragments, String candidate) {
               List<String> temp = new ArrayList<String>(fragments);
               for (int i = 1; i < candidate.length() && !temp.isEmpty(); ++i) {</pre>
                    final int j = i;
                    temp.removeIf(x -> x.equalsIgnoreCase(candidate.substring(0, j)));
                    temp.removeIf(x -> x.equalsIgnoreCase(candidate.substring(j)));
               return temp.isEmpty();
           }
            For the largest and every smallest shard we try to fit the rest of the shards using fit
        function trying both cases: large + small, and small + large.
        \langle 3.6 \; Implementation \; 32b \rangle + \equiv
32f
           for (String small : smallest) {
               if (fit(fragments, large + small)) {
```

return large + small;

return small + large;

}

return "Impossible";

} else if (fit(fragments, small + large)) {

In accordance to the problem statement "Impossible" should never be returned, unless the input is malformed for any reason.

## 4 Chapter 4

#### 4.1 Bridge

This task is quite tricky. But before trying to solve it, let's just sort out input/output to get it out of the way.

We will assume that the we have a method getStrategy that takes a list of integers (crossing times) and returns two lists. The first list holds crossing times going from left to right, and the second list holds crossing times from right to left. (We assume the group of people starts on the left side of the bridge.) Let's assume there's a printResult method that takes that output of getStrategy and prints it out in the format specified in the problem statement.

```
33
       \langle Bridge \ 33 \rangle \equiv
         package com.rvprg.pc;
         \langle 4.3 \; Imports \; (never defined) \rangle
         import static java.util.stream.Collectors.toList;
         import java.io.BufferedReader;
         import java.io.IOException;
         import java.io.InputStreamReader;
         import java.util.List;
         class Bridge {
             private static final BufferedReader reader =
                 new BufferedReader(new InputStreamReader(System.in));
             private static final int LEFT_RIGHT = 0;
             private static final int RIGHT_LEFT = 1;
             (4.3 Variables (never defined))
             (4.3 Methods (never defined))
             public static void main(String[] args) throws IOException {
                  int n = Integer.valueOf(reader.readLine().trim());
                 reader.readLine();
                 for (int i = 0; i < n; ++i) {
                      int count = Integer.valueOf(reader.readLine().trim());
                      List<Integer> input = reader.lines().map(String::trim)
                          .limit(count).map(Integer::parseInt).collect(toList());
                      printResult(getStrategy(input));
                      if (i < n - 1) {
                          reader.readLine();
                          System.out.println();
                      }
                 }
          TBD.
```

#### 4.2 ShellSort

34

The key to this problem answer is to note that all the items in the stack above the one that is about to be moved will move down. Therefore we just need to find all such elements, and everything else will need to be moved using the operation described in the problem statement.

Let's start with the input/output assuming that we have getStrategy method which takes input array and the target array and returns an answer, i.e. a list of items that need to be moved to the top:

```
\langle ShellSort 34 \rangle \equiv
 package com.rvprg.pc;
 import static java.util.stream.Collectors.toList;
  import java.io.BufferedReader;
  import java.io.IOException;
  import java.io.InputStreamReader;
  import java.util.Collections;
  import java.util.List;
  class ShellSort {
      private static final BufferedReader reader =
          new BufferedReader(new InputStreamReader(System.in));
      private static List<String> getStrategy(List<String> input, List<String> target) {
          (4.7 Implementation 35)
      public static void main(String[] args) throws IOException {
          int n = Integer.valueOf(reader.readLine().trim());
          for (int i = 0; i < n; ++i) {
              int count = Integer.valueOf(reader.readLine().trim());
              List<String> input = reader.lines().limit(count).collect(toList());
              List<String> target = reader.lines().limit(count).collect(toList());
              getStrategy(input, target).forEach(System.out::println);
              System.out.println();
      }
 }
```

35

OK, let's get to the implementation of the method that finds the optimal strategy. We start from the bottom of the lists and work towards the top, comparing the items. The idea is that we move sequentially in the target array and move towards the top in the input array potentially skipping some elemnts until we hit the start of the array. The index in the target array, at which we broke the loop, will be the point that will divide the target array into two parts: Elements above it are the elements that will need to be moved, elemnts below do not need to be moved.

```
(4.7 Implementation 35)\(\equiv \)
int i = input.size() - 1;
int j = target.size() - 1;
while (i >= 0 && j >= 0) {
    while (j >= 0 && !target.get(i).equals(input.get(j))) {
        j--;
}
if (j < 0) {
        break;
}
i--;
j--;
}
List<String> output = target.subList(0, i + 1);
Collections.reverse(output);
return output;
```

# 5 Chapter 5

### 5.1 Primary Arithmetic

```
This task is trivial.
```

```
36
      \langle Primary\ Arithmetic\ 36 \rangle \equiv
        package com.rvprg.pc;
        import static java.util.stream.Collectors.toList;
        import java.io.BufferedReader;
        import java.io.IOException;
        import java.io.InputStreamReader;
        import java.util.Arrays;
         import java.util.Comparator;
         import java.util.List;
        public class PrimaryArithmetic {
             private static final BufferedReader reader = new BufferedReader(
                     new InputStreamReader(System.in));
             private static int[] asArray(String input, int pad) {
                 int[] a = new int[input.length() + pad];
                 for (int i = 0; i < input.length(); ++i) {</pre>
                     a[i] = input.charAt(input.length() - i - 1) - '0';
                 return a;
             }
             public static int count(List<String> input) {
                 int[] a = asArray(input.get(0), 0);
                 int[] b = asArray(input.get(1),
                         input.get(0).length() - input.get(1).length());
                 int carry = 0;
                 int count = 0;
                 for (int i = 0; i < a.length; ++i) {
                     int c = a[i] + b[i] + carry;
                     if (c >= 10) {
                         carry = 1;
                         count++;
                     } else {
                         carry = 0;
                     }
                 return count;
             }
             public static String toMessage(int count) {
                 if (count == 0) {
                     return "No carry operation.";
                 } else if (count == 1) {
                     return "1 carry operation.";
                 } else {
```

```
return count + " carry operations.";
        }
    }
    public static void main(String[] args) throws IOException {
        String currentLine;
        while ((currentLine = reader.readLine()) != null) {
            List<String> input = Arrays
                    .stream(currentLine.trim().split(" "))
                    .filter(x -> !x.equals(" "))
                    .sorted(Comparator.comparing(String::length).reversed())
                    .collect(toList());
            if (input.get(0).equals("0") && input.get(1).equals("0")) {
                break;
            }
            System.out.println(toMessage(count(input)));
        }
   }
}
```

#### 5.2 Reverse And Add

```
This task is trivial.
```

38

```
⟨Reverse And Add 38⟩≡
 package com.rvprg.pc;
 import java.io.BufferedReader;
  import java.io.IOException;
 import java.io.InputStreamReader;
 class ReverseAndAdd {
     private static final BufferedReader reader = new BufferedReader(
             new InputStreamReader(System.in));
     private static long reverse(long value) {
         long reversed = 0;
          while (value > 9) {
              reversed = reversed * 10 + (value % 10);
              value /= 10;
          reversed = reversed * 10 + value;
          return reversed;
     }
     private static boolean isPalindrome(long value) {
          return value == reverse(value);
     public static long[] calculate(long value) {
         int count = 0;
          do {
              value = value + reverse(value);
              count++;
          } while (!isPalindrome(value));
          return new long[] { count, value };
      public static void main(String[] args) throws IOException {
          int n = Integer.parseInt(reader.readLine().trim());
          for (int i = 0; i < n; ++i) {
              long v = Integer.parseInt(reader.readLine().trim());
              long[] res = calculate(v);
              System.out.println(res[0] + " " + res[1]);
         }
     }
 }
```

## 5.3 The Archeologists' Dilemma

Unlike the previous two tasks, this tasks is quite challenging.

Let's paraphrase this task in mathematical terms. For a given number v find positive integers m and n such that

$$v \cdot 10^n \le 2^m < (v+1) \cdot 10^n$$

where  $n \ge l(v) + 1$ , and l(v) is the number of digits in v. Let's take common logarithms on that inequality

$$log(v \cdot 10^n) \le log(2^m) < log((v+1) \cdot 10^n)$$

which is the same as

$$log(v) + log(10^n) \le log(2^m) < log(v+1) + log(10^n)$$

and

$$log(v) + n \cdot log(10) \le m \cdot log(2) < log(v+1) + n \cdot log(10)$$

which is the same as

$$log(v) + n \le m \cdot log(2) < log(v+1) + n.$$

This solves the task, because all we need to do now is to iterate on n starting with n = l(v) + 1. For a given n we find an initial m by using the left part of the inequality, so

$$m = \lfloor \frac{\log(v) + n}{\log(2)} \rfloor$$

Then we increment m while  $log(v) + n \ge m \cdot log(2)$ . Once this loop stops, we check if  $m \cdot log(2) < log(v+1) + n$ , and if so, m is the answer. Otherwise, we increment n and start everything all over again.

```
\langle The \ Archeologists \ Dilemma \ 39 \rangle \equiv
39
        package com.rvprg.pc;
         import java.io.BufferedReader;
         import java.io.IOException;
         import java.io.InputStreamReader;
         import java.math.BigDecimal;
        public class TheArcheologistsDilemma {
             private static final BufferedReader reader = new BufferedReader(
                     new InputStreamReader(System.in));
             private static long calculate(long v) {
                 long n = BigDecimal.valueOf(v).precision() + 1;
                 final double left = Math.log10(v);
                 final double right = Math.log10(v + 1);
                 final double log10_2 = Math.log10(2);
                 while (true) {
                     long m = (long) Math.floor((left / log10_2) + n / log10_2);
                     while (left + n > (log10_2 * m)) {
```

```
m++;
}
if (right + n > (log10_2 * m)) {
    return m;
}
n++;
}

public static void main(String[] args) throws IOException {
    String currentLine;
    while ((currentLine = reader.readLine()) != null) {
        System.out.println(calculate(Long.parseLong(currentLine.trim())));
}
}
}
```

#### 5.4 Ones

This is a little nice problem but it may take some time to come up with a proper solution. Obviously these "minimum multiples" of n can quickly become too large, and so we can't use the standard types of the language to do the calculations. The next natural idea would be to try to use BigInteger and repeatedly do  $x = x \times 10 + 1$  and then checking x % n == 0 until it becomes true. But this is not a solution, it's too slow.

Another idea would be too come up with some clever "divisibility rules" to see if a given n divides a number that has only 1s in it. But this a dead end too.

Of course, the general idea is to simply test if x % n == 0 for a given n where x is a number consisting of 1s only.

To do that we can simply do long division and keep appending 1s to the reminder until it doesn't divide without a reminder.

Before we implement the long division, let's write input/output:

```
⟨Ones 41a⟩≡
41a
         package com.rvprg.pc;
         import java.io.BufferedReader;
         import java.io.IOException;
         import java.io.InputStreamReader;
         class Ones {
             private static final BufferedReader reader =
                  new BufferedReader(new InputStreamReader(System.in));
              private static int calculate(int n) {
                  ⟨5.4 Calculation 41b⟩
              public static void main(String[] args) throws IOException {
                  reader.lines().map(Integer::parseInt)
                                 .map(Ones::calculate)
                                 .forEach(System.out::println);
             }
         }
          We implement the case when n is 1 first:
41b
       ⟨5.4 Calculation 41b⟩≡
         if (n == 1) {
              return 1;
         }
```

Any other number can be calculated using the long division.

Let's workout a small example. Let's say we want to find the minimum multiple for n=91. We start with s=11 and r=11. But clearly because s < n we need to append one more  $1, \ s=r\times 10+1, \ \text{so now} \ s=111, \ \text{and} \ r=s-(n*\lfloor s/n\rfloor), \ \text{so} \ r=20; \ \text{and since} \ r\neq 0$  we continue by extending  $s=r\times 10+1$  and then repeat the steps until r=0. But note though that  $r=s-(n*\lfloor s/n\rfloor)$  is equivalent to  $\mathbf{r}=\mathbf{s}\%$  n.

OK, now we can capture that in code:

```
42  \langle 5.4 Calculation 41b \rangle +\equiv \text{int 1 = 0;} \\
int r = 0; \\
do \{ r = (r * 10 + 1) % n; \\
1++; \rangle \text{while (r > 0);} \\
return 1;
```

Brilliant.

43

# 6 Chapter 7

#### 6.1 Light, More Light

With this tasks we are basically asked to find the number of divisors of the given number. Once we know the number of divisors, we can figure out the last bulb state by checking if the number of the divisors is even or odd.

Let's sort out the input/output first as usual. We assume that we have calculate method that returns the number of divisors for a given number.

```
\langle Light, More \ Light \ 43 \rangle \equiv
 package com.rvprg.pc;
  import java.io.BufferedReader;
  import java.io.IOException;
  import java.io.InputStreamReader;
  \langle 7.1 \; Imports \; 44a \rangle
  class LightMoreLight {
      private static final BufferedReader reader = new BufferedReader(
               new InputStreamReader(System.in));
      ⟨7.1 Variables 44b⟩
      LightMoreLight() {
           ⟨7.1 Constructor 44c⟩
      public long calculate(long value) {
           \langle 7.1 \ Implementation \ 45 \rangle
      public static void main(String[] args) throws IOException {
           LightMoreLight 1 = new LightMoreLight();
           String currentLine;
           while ((currentLine = reader.readLine()) != null &&
                    !currentLine.trim().equals("0")) {
               long value = Long.parseLong(currentLine.trim());
               System.out.println(1.calculate(value) % 2 == 0 ? "no" : "yes");
      }
  }
```

OK, to figure out the number of divisors we will use the fundamental theorem of arithmetic. This theorem states that: Every integer greater than one either is prime itself or is the product of prime numbers, and that this product is unique, up to the order of the factors. To find the prime factorization we can use a straightforward algorithm: simply by dividing a number by the primes less that the number itself, trying them one by one.

Since we need to know the prime numbers, let's pre-calculate them first in the constructor. We won't need primes larger than  $\sqrt{2^{32}-1}$ , but we'll define a constant MAX\_PRIMES a bit larger than that. We will use a classic algorithm for finding prime numbers, the sieve of Eratosthenes algorithm.

```
⟨7.1 Imports 44a⟩≡
44a
          import java.util.ArrayList;
          import java.util.BitSet;
          import java.util.List;
44b
        ⟨7.1 Variables 44b⟩≡
          private final List<Long> primes;
          private final static int MAX_PRIMES = 70000;
        \langle 7.1 \ Constructor \ 44c \rangle \equiv
44c
          BitSet bits = new BitSet(MAX_PRIMES);
          for (int i = 2; i < Math.sqrt(MAX_PRIMES); ++i) {</pre>
               if (!bits.get(i)) {
                   int k = 0;
                   int ii = i * i;
                   int j = ii + k * i;
                   while (j < MAX_PRIMES) {</pre>
                       bits.set(j);
                       k++;
                        j = ii + k * i;
                   }
              }
          primes = new ArrayList<Long>();
          for (int i = 2; i < bits.length(); ++i) {
              if (!bits.get(i)) {
                   primes.add((long) i);
          }
```

Now the interesting part: In fact we don't need the prime numbers of the factorization, we only need their exponents to find out the number of divisors.

To see why, consider a number of the form  $v = p_1^n$ . The divisors of this number are

```
1, p_1, p_1^2, p_1^3, \ldots, p_1^n; therefore the number of the divisors is n+1.
             Consider a number of the form v = p_1^n p_2^m, its divisors are:
                              \begin{array}{cccc} p_1^2 & \dots & p_1^n \\ p_1^2 p_2 & \dots & p_1^n p_2 \\ p_1^2 p_2^2 & \dots & p_1^n p_2^2 \end{array}
              1
              p_2
                     p_1p_2
              p_2^2
                   p_1p_2^m ... p_1^np_2^m
            Therefore the number of its divisors is (n+1)(m+1).
            Generally the number of the divisors for a number v = p_1^n p_2^m \cdots p_k^l is (n+1)(m+1) \cdots (l+1).
45
        \langle 7.1 \ Implementation \ 45 \rangle \equiv
           List<Long> factors = new ArrayList<Long>();
           for (int i = 0; i < primes.size() && value > 1 &&
                      (primes.get(i) * primes.get(i)) <= value; ++i) {</pre>
                long p = 0;
                while (value % primes.get(i) == 0) {
                      value /= primes.get(i);
                      p++;
                }
                if (p > 0) {
                      factors.add(p);
           }
           if (value > 1) {
                factors.add(1L);
```

return factors.stream().map(x  $\rightarrow$  x + 1).reduce(1L, (a, b)  $\rightarrow$  a \* b);

This concludes the program.

### 7 License

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### References

- [1] Donald E. Knuth, Literate Programming, The Computer Journal, 1984, pp 97–111
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#### **Definitions**

```
\begin{array}{lll} \langle 1.3 \; Calculation \; 6c \rangle & 6a, \; \underline{6c} \\ \langle 5.4 \; Calculation \; 41b \rangle & 41a, \; \underline{41b}, \; \underline{42} \\ \langle 1.2 \; Constants \; 4c \rangle & 4a, \; \underline{4c} \\ \langle 1.4 \; Constants \; 8b \rangle & 7d, \; \underline{8b} \\ \langle 1.1 \; Constructor \; 3a \rangle & 1a, \; \underline{3a} \\ \rangle & \langle 1.2 \; Constructor \; 3a \rangle & \langle 1.3 \; \underline{3a} \rangle \end{array}
```

- $\langle 2.4 \ Constructor \ 24g \rangle \ 23, \ \underline{24g}$  $\langle 7.1 \ Constructor \ 44c \rangle \ 43, \ \overline{44c}$
- $\langle 1.1 \ Constructor \ 44c \rangle \ 45, \underline{44}$  $\langle 1.4 \ Convertion \ 9a \rangle \ 7d, \underline{9a}$
- (1.8 Election loop 21f) 21d, 21f, 21g, 21h, 21i, 22a
- $\langle 1.3 \text{ Finding the minimum } 6e \rangle 6\overline{c, 6e, 7a}$
- $\langle 1.1 \; Helpers \; 2b \rangle \; 1a, \; \underline{2b}$
- (1.4 Helpers 9c) 9a, 9c, 9d, 9f, 10b
- $\langle 3.6 \text{ Helpers } 32e \rangle$  31, 32e
- (1.8 Implementation 21a) 20, 21a, 21b, 21c, 21d
- $\langle 3.6 \; Implementation \; 32b \rangle \; 31, \; \underline{32b}, \; \underline{32d}, \; \underline{32f}$
- $\langle 4.7 | Implementation 35 \rangle 34, \underline{35}$
- $\langle 7.1 \ Implementation \ 45 \rangle \ 43, \ \underline{45}$
- $\langle 1.1 \; Imports \; 1b \rangle \; 1a, \underline{1b}, \underline{2a}, \underline{2c}, \underline{3b}$
- $\langle 1.2 \; Imports \; 4b \rangle \; 4a, \; \underline{4b}, \; \underline{4d}$
- $\langle 1.3 \; Imports \; 6b \rangle \; 6a, \underline{6b}, \underline{6d}, \underline{7b}$
- (1.4 Imports 8a) 7d, 8a, 9b, 9e, 10a, 10d, 10f
- $\langle 1.8 \; Imports \; 21e \rangle \; 20, \; 21e$
- (2.4 Imports 24a) 23, <u>24a</u>, <u>24c</u>, <u>24e</u>, <u>25a</u>, <u>25d</u>, <u>26b</u>, <u>27c</u>, <u>28b</u>
- $\langle 3.6 \text{ Imports } 32a \rangle$  31, 32a, 32c
- $\langle 4.3 \; Imports \; (never defined) \rangle \; 33$
- $\langle 7.1 \; Imports \; 44a \rangle \; 43, \, \underline{44a}$
- $\langle 1.1 \; Input/Output \; 3c \rangle \; 1a, \; \underline{3c}$
- ⟨1.3 Input/Output 7c⟩ 6a, 7c
- $\langle 1.4 \; Input/Output \; 8c \rangle \; 7d, \; \underline{8c}$
- $\langle 1.2 \; Main \; 5 \rangle \; 4a, \underline{5}$
- (2.4 Methods 24b) 23, 24b, 24d, 25b, 26a, 27a, 27b, 28a, 28c
- $\langle 4.3 \; Methods \; (never defined) \rangle \; 33$
- (1.4 Middle Column Construction 10c) 10b, 10c
- $\langle 1.4 \ Process \ 10e \rangle \ 9a, \ \underline{10e}$
- (1.4 Return 10g) 9a, 10g
- (2.4 Variables 24f) 23, <u>24f</u>, <u>25c</u>, <u>25e</u>, <u>25f</u>
- $\langle 4.3 \ Variables \ (never defined) \rangle$  33
- $\langle 7.1 \ Variables \ 44b \rangle \ 43, \underline{44b}$
- $\langle Australian \ Voting \ 20 \rangle \ \ \underline{20}$
- $\langle Bridge \ 33 \rangle \ \underline{33}$

```
\langle \mathit{Check\ The\ Check\ 16} \rangle 16
\langle Crypt \ Kicker \ 23 \rangle \ \ \underline{23}
\langle Crypt \ Kicker \ II \ 29 \rangle \ \ \underline{29}
\langle File\ Fragmentation\ 31 \rangle\ \ \underline{31}
\langle Graphical\ Editor\ 11 \rangle \ \ \underline{11}
\langle Interpreter 14 \rangle \underline{14}
\langle Jolly\ Jumpers\ 22b \rangle\ \underline{22b}
\langle LC \ Display \ 7d \rangle \ \ \underline{7d}
\langle Light, More \ Light \ 43 \rangle \ \underline{43}
\langle Minesweeper 4a \rangle \underline{4a}
\langle Ones 41a \rangle \underline{41a}
\langle Primary \ Arithmetic \ 36 \rangle \ \ \underline{36}
\langle Reverse \ And \ Add \ 38 \rangle \ \ 38
\langle ShellSort 34 \rangle = 34
\langle The \ Archeologists \ Dilemma \ 39 \rangle \ \ \underline{39}
\langle \mathit{The Trip 6a} \rangle \ \underline{6a}
\langle 3n+1 \text{ 1a} \rangle \text{ } \underline{1a}
```

#### Index