

STT 810

Homework 4

Due Monday, October 31 at 11:59:59pm

1. Take the function $f(x, y, z) = x^2 - 3x + y^4 - 3y + z^2 + 10z + \cos(xyz)$. Find the location of the minimum value. Use $c(0, 0, 0)$ as the initialization location. Try different initialization locations. Can you find different locations for minima? If you see multiple ones, which one seems to be a global minimum?
2. On the interval $[1, \infty]$ a probability distribution is defined as $p(x) = \alpha/x^{(\alpha+1)}$. Given the sample $\{2, 3, 2.5, 3, 1.6, 1.4, 1.3, 1.8, 1.9, 2.4, 4.6\}$, find the log-likelihood, and then calculate the maximum likelihood estimate for the parameter α . (Hint: the likelihood function will look very similar to what we had in ICA 14, #2)

3. The beta distribution on the interval $0 < x < 1$ has the form

$$p(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$$

Where Γ is the gamma function. The two parameters α and β are positive constants. Fortunately, you do not need to use this messy expression, since R has a function to express the probability density: $p(x) = \text{dbeta}(x, \alpha, \beta)$

- a. Take the sample data in D2L called `beta_samp.csv`. Construct the log-likelihood function using `dbeta`.
 - b. Use the `optim` function in R to find the maximum likelihood estimate for the parameters.
 - c. Graph the beta distribution for the given parameters between $0 < x < 1$.
4. With the penguins dataset, construct a 99% two-sided confidence interval for the mean value for body mass with the t-distribution. Then do the same for each species individually.