STT 810

Homework 5

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Contents

Question 1

1. In the car_multi.csv file, there is a field called acceleration. Calculate

a. The mean value

```
# setwd("C:\\Users\\Adi\\Desktop\\Fall-22 Study Material\\STT 810\\HW\\HW5")
cars_multi <- read.csv('cars_multi.csv', header = TRUE)
mean_acc <- mean(cars_multi$acceleration)
paste(mean_acc)
```

```
## [1] "15.5680904522613"
```

b. The standard deviation

```
std_acc <- sd(cars_multi$acceleration)
paste(std_acc)
```

```
## [1] "2.75768892981268"
```

c. The 5th and 95th quantile

```
quantile_range <- quantile(cars_multi$acceleration, c(0.05,0.95))
paste(quantile_range)
```

```
## [1] "11.285" "20.415"
```

Question 2

2. For this acceleration field, assuming it is normally distributed, conduct a hypothesis test at the 95% confidence level to determine whether you can say the mean value is greater than 15.2. State the null hypothesis. Conduct the hypothesis test by

The null hypothesis is - the mean acceleration of the car is less than 15.2

a. Constructing an appropriate confidence interval

```
a1 <- mean_acc + (std_acc/sqrt(length(cars_multi$acceleration)))*
  qt(0.05,length(cars_multi$acceleration)-1)
paste(a1)
```

```
## [1] "15.3401897806772"
```

At 95 % Confidence interval we can say the null hypothesis is invalid and the mean value is greater than 15.2

b. Calculating a p-value, both by

```
# i. Analytically
a2 <- pt((15.2-mean_acc)/(std_acc/sqrt(length(cars_multi$acceleration)))
         ,length(cars_multi$acceleration)-1)
paste(a2)
```

```
## [1] "0.00403136148288605"
```

```
# ii. Running an appropriate simulation
trials <- 10000
sampmean <- mean_acc
n <- length(cars_multi$acceleration)
H0mean <- 15.2
simmean <- rep(0,trials)
for(i in 1:trials){
  simsamp <- H0mean + std_acc*rt(n, n - 1)
  simmean[i] <- mean(simsamp)
}
p_value <- sum(simmean >= sampmean)/trials
paste(p_value)
```

```
## [1] "0.0043"
```

As p value is less than 0.05, we can say that the null hypothesis is falsified.

Question 3

3. A Poisson process generates the following data: {3, 5, 6, 7, 10, 4, 5, 6, 4, 3}, covering 2 second intervals. Run a Monte Carlo simulation to test the hypothesis at the 95% confidence level that the rate parameter is greater than 2.

The null hypothesis is - the rate parameter is less than or equal to 2

```
c <- c(3,5,6,7,10,4,5,6,4,3)
trials <- 10000
samprate <- (sum(c)/length(c))
n <- 10
H0rate <- 2
simrate <- rep(0,trials)
for(i in 1:trials){
  simsamp <- rpois(n, H0rate*2) # rate times time (i.e 2 seconds)
  simrate[i] <- sum(simsamp)/n
}
p_value <- sum(simrate >= samprate)/trials
paste(p_value)
```

```
## [1] "0.0283"
```

As p-value is less than 0.05, we can say that the null hypothesis is falsified and the rate parameter is greater than or equal to 2 for the given poisson distribution.

Question 4

4. For the 4 values you calculated in #1, construct 95% confidence intervals using

a. Regular bootstrapping

```
# Regular Bootstrapping
mn0 <- rep(0,10000)
sd0 <- rep(0,10000)
five0 <- rep(0,10000)
ninefive0 <-  rep(0,10000)
for(i in 1:10000){
  samp0 <- sample(cars_multi$acceleration, length(cars_multi$acceleration),
                  replace = TRUE)
  mn0[i] <- mean(samp0)
  sd0[i] <- sd(samp0)
  five0[i] <- quantile(samp0, 0.05)
  ninefive0[i] <- quantile(samp0, 0.95)
}
paste(quantile(mn0, c(0.025, 0.975)))
```

```
## [1] "15.2979899497487" "15.835182160804"
```

```
paste(quantile(sd0, c(0.025, 0.975)))
```

```
## [1] "2.54541198204302" "2.96372472121071"
```

```
paste(quantile(five0, c(0.025, 0.975)))
```

```
## [1] "11"     "11.585"
```

```
paste(quantile(ninefive0, c(0.025, 0.975)))
```

```
## [1] "19.5" "21"
```

b. Bayesian bootstrapping

```
# Bayesian Bootstrapping
library(DirichletReg)
```

```
## Warning: package 'DirichletReg' was built under R version 4.2.2
```

```
## Loading required package: Formula
```

```
weight <- rep(0,length(cars_multi$acceleration))
mn1 <- rep(0,10000)
sd1 <- rep(0,10000)
five1 <- rep(0,10000)
ninefive1 <-  rep(0,10000)
```

```r
for(i in 1:10000){
  weight <- rdirichlet(1, rep(1,length(cars_multi$acceleration)))
  samp1 <- sample(cars_multi$acceleration,length(cars_multi$acceleration),weight,
                  replace = TRUE)
  mn1[i] <- mean(samp1)
  sd1[i] <- sd(samp1)
  five1[i] <- quantile(samp1, 0.05)
  ninefive1[i] <- quantile(samp1, 0.95)
}
paste(quantile(mn1, c(0.025, 0.975)))
```

```
## [1] "15.1813944723618" "15.9454773869347"
```

```r
paste(quantile(sd1, c(0.025, 0.975)))
```

```
## [1] "2.46368613439917" "3.05172889993441"
```

```r
paste(quantile(five1, c(0.025, 0.975)))
```

```
## [1] "10.425" "12"
```

```r
paste(quantile(ninefive1, c(0.025, 0.975)))
```

```
## [1] "19.245" "21.53"
```

Question 5

5. Import the data in the dataset called "longtail.csv." Calculate the standard deviation and the 0.99 quantile. Then create 95% confidence intervals for these two quantities, using both regular and Bayesian bootstrapping. Describe what you see with these results

```
lt <- read.csv("longtail.csv", stringsAsFactors = FALSE)
lt <- lt$x
ltsd <- sd(lt)
paste(ltsd)
```

```
## [1] "78.9768058514152"
```

```
lt99 <- quantile(lt,0.99)
paste(lt99)
```

```
## [1] "93.6482862423871"
```

```
# regular bootstrapping
ltsd0 <- rep(0,10000)
lt990 <-  rep(0,10000)
for(i in 1:10000){
  samp0 <- sample(lt, length(lt), replace = TRUE)
  ltsd0[i] <- sd(samp0)
  lt990[i] <- quantile(samp0, 0.99)
}
paste(quantile(ltsd0, c(0.025, 0.975)))
```

```
## [1] "50.9533245760065" "104.819443523271"
```

```
paste(quantile(lt990, c(0.025, 0.975)))
```

```
## [1] "74.6138513589416" "122.977316854457"
```

```
# Bayesian Bootstrapping
weight <- rep(0,length(lt))
ltsd1 <- rep(0,10000)
lt991 <-  rep(0,10000)
for(i in 1:10000){
  weight <- rdirichlet(1, rep(1,length(lt)))
  samp1 <- sample(lt,length(lt),weight, replace = TRUE)
  ltsd1[i] <- sd(samp1)
  lt991[i] <- quantile(samp1, 0.99)
}
paste(quantile(ltsd1, c(0.025, 0.975)))
```

```
## [1] "43.915543220358" "116.96638502209"
```

```r
paste(quantile(lt991, c(0.025, 0.975)))
```

```
## [1] "69.0952961242343" "130.213957312874"
```

Question 6

(a) Find the eigenvectors and eigenvalues for the matrix

$$A = \begin{bmatrix} 1 & 7 & 3 \\ 7 & 4 & 5 \\ 3 & 5 & 0 \end{bmatrix}$$

```
A = as.matrix(cbind(c(1,7,3),c(7,4,5),c(3,5,0)))
eigenA = eigen(A)
eigenA
```

```
## eigen() decomposition
## $values
## [1] 12.364127 -2.454628 -4.909498
##
## $vectors
##            [,1]       [,2]       [,3]
## [1,] -0.5534805 -0.5016948  0.6648020
## [2,] -0.7167664 -0.1195787 -0.6869839
## [3,] -0.4241524  0.8567399  0.2934135
```

(b) Express the vectors x = <1, 3, 1> and y = <-1, 4, 9> in terms of the eigenvectors basis for the above matrix.

```
x = as.matrix(c(1,3,1))
y = as.matrix(c(-1,4,9))
xa = solve(eigenA$vectors)%*%x
ya = solve(eigenA$vectors)%*%y
paste(xa)
```

```
## [1] "-3.12793216336437"    "-0.00369094288744376" "-1.10273603293411"
```

```
paste(ya)
```

```
## [1] "-6.13095663154334"  "7.73403921917673"   "-0.772015633501637"
```

(c) Find the inner product of x and y in the original coordinates. Then find the inner product of x and y in terms of the eigenvector basis. Do you get the same value?

```
inner_original = t(x)%*%y
inner_basis = t(xa)%*%ya
paste(inner_original)
```

```
## [1] "20"
```

```
paste(inner_basis)
```

```
## [1] "20"
```

```r
paste("We get the same value.")
```

```
## [1] "We get the same value."
```

Question 7

For this problem we will use the nndb dataset available in the sample data. There are 45 columns, 38 of which are numerical. a. Calculate the covariance matrix for the numerical data.

```
nndb = read.csv("nndb_flat.csv", stringsAsFactors = FALSE)
nndb_num = nndb[8:ncol(nndb)]
cov_nndb = cov(nndb[8:ncol(nndb)])
cov_nndb
```

```
##                    Energy_kcal     Protein_g         Fat_g        Carb_g
## Energy_kcal       2.869260e+04 197.126585658  2.168329e+03  2.247455e+03
## Protein_g         1.971266e+02 111.314944832  9.159321e+00 -8.678354e+01
## Fat_g             2.168329e+03   9.159321342  2.517411e+02 -2.338810e+01
## Carb_g            2.247455e+03 -86.783538935 -2.338810e+01  7.419631e+02
## Sugar_g           7.215148e+02 -38.198033251 -4.234431e-01  2.279897e+02
## Fiber_g           1.441923e+02  -3.312228183 -1.974011e+00  5.383758e+01
## VitA_mcg          3.475814e+03 217.760904248  3.014578e+02  3.971645e+01
## VitB6_mg          9.880400e+00   1.153773367 -3.565283e-01  2.549656e+00
## VitB12_mcg       -8.993246e+00  11.183925768 -1.424434e+00 -1.113389e+01
## VitC_mg          -3.267044e+02 -40.373894767 -5.481173e+01  1.203750e+02
## VitE_mg           1.980078e+02  -1.197227827  2.064218e+01  7.370322e+00
## Folate_mcg        4.611778e+03  17.595392168 -1.718278e+02  1.652423e+03
## Niacin_mg         1.433043e+02  19.226762927 -1.739030e+00  2.281969e+01
## Riboflavin_mg     1.187501e+01   0.961069860 -2.590076e-01  2.847988e+00
## Thiamin_mg        1.645824e+01   0.537939021 -4.758314e-02  3.979099e+00
## Calcium_mg        4.214856e+03  99.614417728  4.804869e+01  1.037238e+03
## Copper_mcg        9.322461e+00   0.927183833  3.752596e-01  9.355972e-01
## Iron_mg           1.897139e+02   7.635961231 -3.671758e+00  5.467115e+01
## Magnesium_mg      2.430184e+03 129.320244096  6.296670e+01  4.386367e+02
## Manganese_mg      4.067899e+01   2.676615002 -5.993033e-02  9.662447e+00
## Phosphorus_mg     6.714554e+03 952.684456473  2.129536e+02  4.048385e+02
## Selenium_mcg      2.715990e+02 112.943556962  1.686016e+01 -8.831674e+01
## Zinc_mg           6.414646e+01  14.750522644  1.505954e+00 -1.114788e+00
## VitA_USRDA        3.862015e+00   0.241956560  3.349531e-01  4.412939e-02
## VitB6_USRDA       5.812000e+00   0.678690216 -2.097225e-01  1.499798e+00
## VitB12_USRDA     -3.747186e+00   4.659969070 -5.935141e-01 -4.639122e+00
## VitC_USRDA       -3.630049e+00  -0.448598831 -6.090192e-01  1.337500e+00
## VitE_USRDA        1.320052e+01  -0.079815188  1.376145e+00  4.913548e-01
## Folate_USRDA      1.152945e+01   0.043988480 -4.295696e-01  4.131057e+00
## Niacin_USRDA      8.956518e+00   1.201672683 -1.086894e-01  1.426231e+00
## Riboflavin_USRDA  9.134624e+00   0.739284508 -1.992366e-01  2.190760e+00
## Thiamin_USRDA     1.371520e+01   0.448282517 -3.965262e-02  3.315916e+00
## Calcium_USRDA     3.512380e+00   0.083012015  4.004058e-02  8.643651e-01
## Copper_USRDA      1.035829e-02   0.001030204  4.169551e-04  1.039552e-03
## Magnesium_USRDA   5.786153e+00   0.307905343  1.499207e-01  1.044373e+00
## Phosphorus_USRDA  9.592220e+00   1.360977795  3.042194e-01  5.783407e-01
## Selenium_USRDA    4.938164e+00   2.053519217  3.065484e-01 -1.605759e+00
## Zinc_USRDA        5.831497e+00   1.340956604  1.369049e-01 -1.013443e-01
##                        Sugar_g       Fiber_g     VitA_mcg      VitB6_mg
## Energy_kcal       7.215148e+02  1.441923e+02 3.475814e+03  9.880400e+00
## Protein_g        -3.819803e+01 -3.312228e+00 2.177609e+02  1.153773e+00
## Fat_g            -4.234431e-01 -1.974011e+00 3.014578e+02 -3.565283e-01
## Carb_g            2.279897e+02  5.383758e+01 3.971645e+01  2.549656e+00
```

```
## Sugar_g           1.850171e+02  6.801384e+00 1.075286e+02  5.778901e-01
## Fiber_g           6.801384e+00  1.860775e+01 1.986968e+01  5.012894e-01
## VitA_mcg          1.075286e+02  1.986968e+01 6.074054e+05  5.095539e+01
## VitB6_mg          5.778901e-01  5.012894e-01 5.095539e+01  2.290714e-01
## VitB12_mcg       -2.963953e+00 -9.696786e-01 1.955779e+03  5.464182e-01
## VitC_mg           5.024715e+01  2.033140e+01 4.993958e+03  7.477142e+00
## VitE_mg           3.458409e+00  2.659107e+00 1.323772e+02  5.269891e-01
## Folate_mcg        4.092077e+02  2.004462e+02 1.577821e+04  4.911759e+01
## Niacin_mg         2.260734e+00  2.820760e+00 5.715085e+02  1.652450e+00
## Riboflavin_mg     7.662900e-01  3.060437e-01 1.142001e+02  1.252462e-01
## Thiamin_mg        6.892329e-01  4.588663e-01 2.137979e+01  1.034600e-01
## Calcium_mg        3.548058e+02  2.017672e+02 5.464847e+03  1.708429e+01
## Copper_mcg       -4.390633e-02  4.414566e-01 2.456622e+02  3.079147e-02
## Iron_mg           9.173768e+00  9.553387e+00 4.618463e+02  1.309941e+00
## Magnesium_mg      3.150365e+01  1.310976e+02 1.073741e+03  7.867222e+00
## Manganese_mg      9.439116e-01  2.799346e+00 1.840719e+03  2.043186e-01
## Phosphorus_mg    -1.375281e+02  1.387903e+02 8.834385e+03  2.074273e+01
## Selenium_mcg     -4.760707e+01 -4.381037e+00 1.059354e+03  1.476032e+00
## Zinc_mg          -1.004507e+00  1.233557e+00 1.846253e+02  7.186878e-01
## VitA_USRDA        1.194762e-01  2.207742e-02 6.748949e+02  5.661710e-02
## VitB6_USRDA       3.399353e-01  2.948761e-01 2.997376e+01  1.347479e-01
## VitB12_USRDA     -1.234981e+00 -4.040328e-01 8.149081e+02  2.276742e-01
## VitC_USRDA        5.583017e-01  2.259044e-01 5.548842e+01  8.307935e-02
## VitE_USRDA        2.305606e-01  1.772738e-01 8.825148e+00  3.513261e-02
## Folate_USRDA      1.023019e+00  5.011156e-01 3.944553e+01  1.227940e-01
## Niacin_USRDA      1.412959e-01  1.762975e-01 3.571928e+01  1.032781e-01
## Riboflavin_USRDA  5.894539e-01  2.354182e-01 8.784621e+01  9.634321e-02
## Thiamin_USRDA     5.743608e-01  3.823886e-01 1.781649e+01  8.621667e-02
## Calcium_USRDA     2.956715e-01  1.681394e-01 4.554040e+00  1.423691e-02
## Copper_USRDA     -4.878482e-05  4.905073e-04 2.729580e-01  3.421274e-05
## Magnesium_USRDA   7.500868e-02  3.121373e-01 2.556527e+00  1.873148e-02
## Phosphorus_USRDA -1.964687e-01  1.982718e-01 1.262055e+01  2.963247e-02
## Selenium_USRDA   -8.655830e-01 -7.965521e-02 1.926098e+01  2.683695e-02
## Zinc_USRDA       -9.131886e-02  1.121416e-01 1.678412e+01  6.533526e-02
##                    VitB12_mcg       VitC_mg       VitE_mg    Folate_mcg
## Energy_kcal      -8.993246e+00 -3.267044e+02  1.980078e+02  4.611778e+03
## Protein_g         1.118393e+01 -4.037389e+01 -1.197228e+00  1.759539e+01
## Fat_g            -1.424434e+00 -5.481173e+01  2.064218e+01 -1.718278e+02
## Carb_g           -1.113389e+01  1.203750e+02  7.370322e+00  1.652423e+03
## Sugar_g          -2.963953e+00  5.024715e+01  3.458409e+00  4.092077e+02
## Fiber_g          -9.696786e-01  2.033140e+01  2.659107e+00  2.004462e+02
## VitA_mcg          1.955779e+03  4.993958e+03  1.323772e+02  1.577821e+04
## VitB6_mg          5.464182e-01  7.477142e+00  5.269891e-01  4.911759e+01
## VitB12_mcg        1.865534e+01  1.806737e+00  1.007749e+00  1.277037e+02
## VitC_mg           1.806737e+00  3.315774e+03  1.482434e+01  7.820334e+02
## VitE_mg           1.007749e+00  1.482434e+01  1.481502e+01  1.321031e+02
## Folate_mcg        1.277037e+02  7.820334e+02  1.321031e+02  3.480488e+04
## Niacin_mg         5.826592e+00  5.821277e+01  3.665325e+00  4.826579e+02
## Riboflavin_mg     8.475519e-01  5.852727e+00  2.721440e-01  4.790286e+01
## Thiamin_mg        2.230002e-01  1.701012e+00  2.380549e-01  5.210870e+01
## Calcium_mg        1.018873e+01  1.296620e+03  8.500071e+01  5.613864e+03
## Copper_mcg        1.363112e+00  8.108057e-01  1.585982e-01  1.249629e+01
## Iron_mg           4.386490e+00  2.454079e+01  3.446064e+00  5.067375e+02
## Magnesium_mg      9.987496e-01  3.720016e+02  4.212704e+01  2.539697e+03
```

```
## Manganese_mg      6.675315e+00  6.594993e+00  2.538949e+00  6.943299e+01
## Phosphorus_mg     1.305041e+02  1.403372e+02  5.335232e+01  4.619803e+03
## Selenium_mcg      2.631060e+01 -5.737511e+01  1.701512e+00  1.838881e+02
## Zinc_mg           4.324385e+00  3.572704e+00  2.159825e+00  2.002202e+02
## VitA_USRDA        2.173088e+00  5.548842e+00  1.470858e-01  1.753135e+01
## VitB6_USRDA       3.214225e-01  4.398319e+00  3.099936e-01  2.889270e+01
## VitB12_USRDA      7.773058e+00  7.528069e-01  4.198953e-01  5.320988e+01
## VitC_USRDA        2.007485e-02  3.684193e+01  1.647149e-01  8.689260e+00
## VitE_USRDA        6.718324e-02  9.882893e-01  9.876679e-01  8.806876e+00
## Folate_USRDA      3.192593e-01  1.955083e+00  3.302578e-01  8.701219e+01
## Niacin_USRDA      3.641620e-01  3.638298e+00  2.290828e-01  3.016612e+01
## Riboflavin_USRDA  6.519630e-01  4.502098e+00  2.093415e-01  3.684835e+01
## Thiamin_USRDA     1.858335e-01  1.417510e+00  1.983791e-01  4.342392e+01
## Calcium_USRDA     8.490607e-03  1.080517e+00  7.083392e-02  4.678220e+00
## Copper_USRDA      1.514569e-03  9.008952e-04  1.762202e-04  1.388477e-02
## Magnesium_USRDA   2.377975e-03  8.857181e-01  1.003025e-01  6.046898e+00
## Phosphorus_USRDA  1.864344e-01  2.004817e-01  7.621760e-02  6.599719e+00
## Selenium_USRDA    4.783746e-01 -1.043184e+00  3.093659e-02  3.343420e+00
## Zinc_USRDA        3.931259e-01  3.247913e-01  1.963477e-01  1.820184e+01
##                      Niacin_mg Riboflavin_mg    Thiamin_mg   Calcium_mg
## Energy_kcal       1.433043e+02  1.187501e+01  1.645824e+01 4.214856e+03
## Protein_g         1.922676e+01  9.610699e-01  5.379390e-01 9.961442e+01
## Fat_g            -1.739030e+00 -2.590076e-01 -4.758314e-02 4.804869e+01
## Carb_g            2.281969e+01  2.847988e+00  3.979099e+00 1.037238e+03
## Sugar_g           2.260734e+00  7.662900e-01  6.892329e-01 3.548058e+02
## Fiber_g           2.820760e+00  3.060437e-01  4.588663e-01 2.017672e+02
## VitA_mcg          5.715085e+02  1.142001e+02  2.137979e+01 5.464847e+03
## VitB6_mg          1.652450e+00  1.252462e-01  1.034600e-01 1.708429e+01
## VitB12_mcg        5.826592e+00  8.475519e-01  2.230002e-01 1.018873e+01
## VitC_mg           5.821277e+01  5.852727e+00  1.701012e+00 1.296620e+03
## VitE_mg           3.665325e+00  2.721440e-01  2.380549e-01 8.500071e+01
## Folate_mcg        4.826579e+02  4.790286e+01  5.210870e+01 5.613864e+03
## Niacin_mg         2.337308e+01  1.624134e+00  1.508239e+00 1.297562e+02
## Riboflavin_mg     1.624134e+00  2.023920e-01  1.477858e-01 2.012578e+01
## Thiamin_mg        1.508239e+00  1.477858e-01  2.687065e-01 1.417969e+01
## Calcium_mg        1.297562e+02  2.012578e+01  1.417969e+01 4.054732e+04
## Copper_mcg        3.702548e-01  7.483199e-02  2.857073e-02 9.276577e+00
## Iron_mg           1.383697e+01  1.304294e+00  1.198336e+00 3.985656e+02
## Magnesium_mg      7.367333e+01  5.781764e+00  7.092435e+00 3.531488e+03
## Manganese_mg      3.084916e+00  3.067682e-01  1.597647e-01 9.117719e+01
## Phosphorus_mg     2.522220e+02  2.046336e+01  1.838020e+01 2.326078e+04
## Selenium_mcg      2.446795e+01  1.791385e+00  8.168598e-01 5.368660e+01
## Zinc_mg           7.657354e+00  5.553840e-01  4.341580e-01 9.516145e+01
## VitA_USRDA        6.350094e-01  1.268890e-01  2.375533e-02 6.072053e+00
## VitB6_USRDA       9.720296e-01  7.367422e-02  6.085882e-02 1.004958e+01
## VitB12_USRDA      2.427746e+00  3.531466e-01  9.291674e-02 4.245304e+00
## VitC_USRDA        6.468085e-01  6.503030e-02  1.890014e-02 1.440689e+01
## VitE_USRDA        2.443550e-01  1.814293e-02  1.587033e-02 5.666714e+00
## Folate_USRDA      1.206645e+00  1.197571e-01  1.302718e-01 1.403466e+01
## Niacin_USRDA      1.460818e+00  1.015084e-01  9.426496e-02 8.109765e+00
## Riboflavin_USRDA  1.249334e+00  1.556861e-01  1.136814e-01 1.548137e+01
## Thiamin_USRDA     1.256866e+00  1.231548e-01  2.239221e-01 1.181641e+01
## Calcium_USRDA     1.081302e-01  1.677148e-02  1.181641e-02 3.378944e+01
## Copper_USRDA      4.113942e-04  8.314666e-05  3.174525e-05 1.030731e-02
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## Magnesium_USRDA   1.754127e-01  1.376611e-02  1.688675e-02 8.408304e+00
## Phosphorus_USRDA  3.603171e-01  2.923337e-02  2.625743e-02 3.322969e+01
## Selenium_USRDA    4.448719e-01  3.257064e-02  1.485200e-02 9.761200e-01
## Zinc_USRDA        6.961231e-01  5.048945e-02  3.946891e-02 8.651041e+00
##                     Copper_mcg       Iron_mg Magnesium_mg  Manganese_mg
## Energy_kcal       9.322461e+00  1.897139e+02 2.430184e+03  4.067899e+01
## Protein_g         9.271838e-01  7.635961e+00 1.293202e+02  2.676615e+00
## Fat_g             3.752596e-01 -3.671758e+00 6.296670e+01 -5.993033e-02
## Carb_g            9.355972e-01  5.467115e+01 4.386367e+02  9.662447e+00
## Sugar_g          -4.390633e-02  9.173768e+00 3.150365e+01  9.439116e-01
## Fiber_g           4.414566e-01  9.553387e+00 1.310976e+02  2.799346e+00
## VitA_mcg          2.456622e+02  4.618463e+02 1.073741e+03  1.840719e+03
## VitB6_mg          3.079147e-02  1.309941e+00 7.867222e+00  2.043186e-01
## VitB12_mcg        1.363112e+00  4.386490e+00 9.987496e-01  6.675315e+00
## VitC_mg           8.108057e-01  2.454079e+01 3.720016e+02  6.594993e+00
## VitE_mg           1.585982e-01  3.446064e+00 4.212704e+01  2.538949e+00
## Folate_mcg        1.249629e+01  5.067375e+02 2.539697e+03  6.943299e+01
## Niacin_mg         3.702548e-01  1.383697e+01 7.367333e+01  3.084916e+00
## Riboflavin_mg     7.483199e-02  1.304294e+00 5.781764e+00  3.067682e-01
## Thiamin_mg        2.857073e-02  1.198336e+00 7.092435e+00  1.597647e-01
## Calcium_mg        9.276577e+00  3.985656e+02 3.531488e+03  9.117719e+01
## Copper_mcg        3.058670e-01  5.684484e-01 1.056994e+01  5.918735e-01
## Iron_mg           5.684484e-01  3.279815e+01 1.291584e+02  3.414680e+00
## Magnesium_mg      1.056994e+01  1.291584e+02 3.143672e+03  6.381415e+01
## Manganese_mg      5.918735e-01  3.414680e+00 6.381415e+01  4.074828e+01
## Phosphorus_mg     2.425184e+01  2.566585e+02 4.882429e+03  1.011897e+02
## Selenium_mcg      1.983955e+00  1.034686e+01 2.016020e+02  4.048977e+00
## Zinc_mg           4.800148e-01  7.217161e+00 4.823707e+01  1.398135e+00
## VitA_USRDA        2.729580e-01  5.131625e-01 1.193046e+00  2.045243e+00
## VitB6_USRDA       1.811263e-02  7.705537e-01 4.627777e+00  1.201874e-01
## VitB12_USRDA      5.679633e-01  1.827704e+00 4.161457e-01  2.781381e+00
## VitC_USRDA        9.008952e-03  2.726754e-01 4.133351e+00  7.327770e-02
## VitE_USRDA        1.057321e-02  2.297376e-01 2.808469e+00  1.692632e-01
## Folate_USRDA      3.124073e-02  1.266844e+00 6.349242e+00  1.735825e-01
## Niacin_USRDA      2.314093e-02  8.648109e-01 4.604583e+00  1.928072e-01
## Riboflavin_USRDA  5.756307e-02  1.003303e+00 4.447511e+00  2.359755e-01
## Thiamin_USRDA     2.380894e-02  9.986132e-01 5.910363e+00  1.331372e-01
## Calcium_USRDA     7.730481e-03  3.321380e-01 2.942906e+00  7.598099e-02
## Copper_USRDA      3.398522e-04  6.316094e-04 1.174437e-02  6.576372e-04
## Magnesium_USRDA   2.516651e-02  3.075199e-01 7.484932e+00  1.519385e-01
## Phosphorus_USRDA  3.464548e-02  3.666550e-01 6.974899e+00  1.445568e-01
## Selenium_USRDA    3.607191e-02  1.881247e-01 3.665491e+00  7.361776e-02
## Zinc_USRDA        4.363771e-02  6.561056e-01 4.385189e+00  1.271032e-01
##                   Phosphorus_mg  Selenium_mcg       Zinc_mg   VitA_USRDA
## Energy_kcal       6.714554e+03  2.715990e+02  6.414646e+01 3.862015e+00
## Protein_g         9.526845e+02  1.129436e+02  1.475052e+01 2.419566e-01
## Fat_g             2.129536e+02  1.686016e+01  1.505954e+00 3.349531e-01
## Carb_g            4.048385e+02 -8.831674e+01 -1.114788e+00 4.412939e-02
## Sugar_g          -1.375281e+02 -4.760707e+01 -1.004507e+00 1.194762e-01
## Fiber_g           1.387903e+02 -4.381037e+00  1.233557e+00 2.207742e-02
## VitA_mcg          8.834385e+03  1.059354e+03  1.846253e+02 6.748949e+02
## VitB6_mg          2.074273e+01  1.476032e+00  7.186878e-01 5.661710e-02
## VitB12_mcg        1.305041e+02  2.631060e+01  4.324385e+00 2.173088e+00
## VitC_mg           1.403372e+02 -5.737511e+01  3.572704e+00 5.548842e+00
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## VitE_mg           5.335232e+01  1.701512e+00  2.159825e+00 1.470858e-01
## Folate_mcg        4.619803e+03  1.838881e+02  2.002202e+02 1.753135e+01
## Niacin_mg         2.522220e+02  2.446795e+01  7.657354e+00 6.350094e-01
## Riboflavin_mg     2.046336e+01  1.791385e+00  5.553840e-01 1.268890e-01
## Thiamin_mg        1.838020e+01  8.168598e-01  4.341580e-01 2.375533e-02
## Calcium_mg        2.326078e+04  5.368660e+01  9.516145e+01 6.072053e+00
## Copper_mcg        2.425184e+01  1.983955e+00  4.800148e-01 2.729580e-01
## Iron_mg           2.566585e+02  1.034686e+01  7.217161e+00 5.131625e-01
## Magnesium_mg      4.882429e+03  2.016020e+02  4.823707e+01 1.193046e+00
## Manganese_mg      1.011897e+02  4.048977e+00  1.398135e+00 2.045243e+00
## Phosphorus_mg     4.124686e+04  1.285294e+03  1.818136e+02 9.815984e+00
## Selenium_mcg      1.285294e+03  8.004569e+02  1.950577e+01 1.177060e+00
## Zinc_mg           1.818136e+02  1.950577e+01  1.127759e+01 2.051392e-01
## VitA_USRDA        9.815984e+00  1.177060e+00  2.051392e-01 7.498833e-01
## VitB6_USRDA       1.220160e+01  8.682542e-01  4.227575e-01 3.330418e-02
## VitB12_USRDA      5.437671e+01  1.096275e+01  1.801827e+00 9.054535e-01
## VitC_USRDA        1.559302e+00 -6.375013e-01  3.969671e-02 6.165380e-02
## VitE_USRDA        3.556821e+00  1.134341e-01  1.439883e-01 9.805721e-03
## Folate_USRDA      1.154951e+01  4.597202e-01  5.005506e-01 4.382837e-02
## Niacin_USRDA      1.576387e+01  1.529247e+00  4.785846e-01 3.968809e-02
## Riboflavin_USRDA  1.574105e+01  1.377988e+00  4.272184e-01 9.760690e-02
## Thiamin_USRDA     1.531684e+01  6.807165e-01  3.617984e-01 1.979611e-02
## Calcium_USRDA     1.938398e+01  4.473883e-02  7.930121e-02 5.060044e-03
## Copper_USRDA      2.694649e-02  2.204395e-03  5.333497e-04 3.032867e-04
## Magnesium_USRDA   1.162483e+01  4.800048e-01  1.148502e-01 2.840585e-03
## Phosphorus_USRDA  5.892409e+01  1.836134e+00  2.597336e-01 1.402283e-02
## Selenium_USRDA    2.336898e+01  1.455376e+01  3.546504e-01 2.140109e-02
## Zinc_USRDA        1.652850e+01  1.773252e+00  1.025236e+00 1.864902e-02
##                    VitB6_USRDA  VitB12_USRDA    VitC_USRDA    VitE_USRDA
## Energy_kcal       5.812000e+00 -3.747186e+00 -3.630049e+00  1.320052e+01
## Protein_g         6.786902e-01  4.659969e+00 -4.485988e-01 -7.981519e-02
## Fat_g            -2.097225e-01 -5.935141e-01 -6.090192e-01  1.376145e+00
## Carb_g            1.499798e+00 -4.639122e+00  1.337500e+00  4.913548e-01
## Sugar_g           3.399353e-01 -1.234981e+00  5.583017e-01  2.305606e-01
## Fiber_g           2.948761e-01 -4.040328e-01  2.259044e-01  1.772738e-01
## VitA_mcg          2.997376e+01  8.149081e+02  5.548842e+01  8.825148e+00
## VitB6_mg          1.347479e-01  2.276742e-01  8.307935e-02  3.513261e-02
## VitB12_mcg        3.214225e-01  7.773058e+00  2.007485e-02  6.718324e-02
## VitC_mg           4.398319e+00  7.528069e-01  3.684193e+01  9.882893e-01
## VitE_mg           3.099936e-01  4.198953e-01  1.647149e-01  9.876679e-01
## Folate_mcg        2.889270e+01  5.320988e+01  8.689260e+00  8.806876e+00
## Niacin_mg         9.720296e-01  2.427746e+00  6.468085e-01  2.443550e-01
## Riboflavin_mg     7.367422e-02  3.531466e-01  6.503030e-02  1.814293e-02
## Thiamin_mg        6.085882e-02  9.291674e-02  1.890014e-02  1.587033e-02
## Calcium_mg        1.004958e+01  4.245304e+00  1.440689e+01  5.666714e+00
## Copper_mcg        1.811263e-02  5.679633e-01  9.008952e-03  1.057321e-02
## Iron_mg           7.705537e-01  1.827704e+00  2.726754e-01  2.297376e-01
## Magnesium_mg      4.627777e+00  4.161457e-01  4.133351e+00  2.808469e+00
## Manganese_mg      1.201874e-01  2.781381e+00  7.327770e-02  1.692632e-01
## Phosphorus_mg     1.220160e+01  5.437671e+01  1.559302e+00  3.556821e+00
## Selenium_mcg      8.682542e-01  1.096275e+01 -6.375013e-01  1.134341e-01
## Zinc_mg           4.227575e-01  1.801827e+00  3.969671e-02  1.439883e-01
## VitA_USRDA        3.330418e-02  9.054535e-01  6.165380e-02  9.805721e-03
## VitB6_USRDA       7.926347e-02  1.339260e-01  4.887021e-02  2.066624e-02
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## VitB12_USRDA      1.339260e-01  3.238774e+00  8.364521e-03  2.799302e-02
## VitC_USRDA        4.887021e-02  8.364521e-03  4.093548e-01  1.098099e-02
## VitE_USRDA        2.066624e-02  2.799302e-02  1.098099e-02  6.584452e-02
## Folate_USRDA      7.223175e-02  1.330247e-01  2.172315e-02  2.201719e-02
## Niacin_USRDA      6.075185e-02  1.517342e-01  4.042553e-02  1.527219e-02
## Riboflavin_USRDA  5.667248e-02  2.716512e-01  5.002331e-02  1.395610e-02
## Thiamin_USRDA     5.071569e-02  7.743062e-02  1.575012e-02  1.322527e-02
## Calcium_USRDA     8.374654e-03  3.537753e-03  1.200575e-02  4.722262e-03
## Copper_USRDA      2.012514e-05  6.310703e-04  1.000995e-05  1.174802e-05
## Magnesium_USRDA   1.101852e-02  9.908230e-04  9.841312e-03  6.686832e-03
## Phosphorus_USRDA  1.743086e-02  7.768101e-02  2.227574e-03  5.081173e-03
## Selenium_USRDA    1.578644e-02  1.993228e-01 -1.159093e-02  2.062439e-03
## Zinc_USRDA        3.843250e-02  1.638025e-01  3.608792e-03  1.308985e-02
##                   Folate_USRDA  Niacin_USRDA Riboflavin_USRDA Thiamin_USRDA
## Energy_kcal       1.152945e+01  8.956518e+00     9.134624e+00  1.371520e+01
## Protein_g         4.398848e-02  1.201673e+00     7.392845e-01  4.482825e-01
## Fat_g            -4.295696e-01 -1.086894e-01    -1.992366e-01 -3.965262e-02
## Carb_g            4.131057e+00  1.426231e+00     2.190760e+00  3.315916e+00
## Sugar_g           1.023019e+00  1.412959e-01     5.894539e-01  5.743608e-01
## Fiber_g           5.011156e-01  1.762975e-01     2.354182e-01  3.823886e-01
## VitA_mcg          3.944553e+01  3.571928e+01     8.784621e+01  1.781649e+01
## VitB6_mg          1.227940e-01  1.032781e-01     9.634321e-02  8.621667e-02
## VitB12_mcg        3.192593e-01  3.641620e-01     6.519630e-01  1.858335e-01
## VitC_mg           1.955083e+00  3.638298e+00     4.502098e+00  1.417510e+00
## VitE_mg           3.302578e-01  2.290828e-01     2.093415e-01  1.983791e-01
## Folate_mcg        8.701219e+01  3.016612e+01     3.684835e+01  4.342392e+01
## Niacin_mg         1.206645e+00  1.460818e+00     1.249334e+00  1.256866e+00
## Riboflavin_mg     1.197571e-01  1.015084e-01     1.556861e-01  1.231548e-01
## Thiamin_mg        1.302718e-01  9.426496e-02     1.136814e-01  2.239221e-01
## Calcium_mg        1.403466e+01  8.109765e+00     1.548137e+01  1.181641e+01
## Copper_mcg        3.124073e-02  2.314093e-02     5.756307e-02  2.380894e-02
## Iron_mg           1.266844e+00  8.648109e-01     1.003303e+00  9.986132e-01
## Magnesium_mg      6.349242e+00  4.604583e+00     4.447511e+00  5.910363e+00
## Manganese_mg      1.735825e-01  1.928072e-01     2.359755e-01  1.331372e-01
## Phosphorus_mg     1.154951e+01  1.576387e+01     1.574105e+01  1.531684e+01
## Selenium_mcg      4.597202e-01  1.529247e+00     1.377988e+00  6.807165e-01
## Zinc_mg           5.005506e-01  4.785846e-01     4.272184e-01  3.617984e-01
## VitA_USRDA        4.382837e-02  3.968809e-02     9.760690e-02  1.979611e-02
## VitB6_USRDA       7.223175e-02  6.075185e-02     5.667248e-02  5.071569e-02
## VitB12_USRDA      1.330247e-01  1.517342e-01     2.716512e-01  7.743062e-02
## VitC_USRDA        2.172315e-02  4.042553e-02     5.002331e-02  1.575012e-02
## VitE_USRDA        2.201719e-02  1.527219e-02     1.395610e-02  1.322527e-02
## Folate_USRDA      2.175305e-01  7.541529e-02     9.212088e-02  1.085598e-01
## Niacin_USRDA      7.541529e-02  9.130111e-02     7.808337e-02  7.855414e-02
## Riboflavin_USRDA  9.212088e-02  7.808337e-02     1.197586e-01  9.473448e-02
## Thiamin_USRDA     1.085598e-01  7.855414e-02     9.473448e-02  1.866017e-01
## Calcium_USRDA     1.169555e-02  6.758138e-03     1.290114e-02  9.847008e-03
## Copper_USRDA      3.471192e-05  2.571214e-05     6.395897e-05  2.645438e-05
## Magnesium_USRDA   1.511724e-02  1.096329e-02     1.058931e-02  1.407229e-02
## Phosphorus_USRDA  1.649930e-02  2.251982e-02     2.248721e-02  2.188119e-02
## Selenium_USRDA    8.358550e-03  2.780449e-02     2.505434e-02  1.237666e-02
## Zinc_USRDA        4.550460e-02  4.350769e-02     3.883804e-02  3.289076e-02
##                  Calcium_USRDA  Copper_USRDA Magnesium_USRDA Phosphorus_USRDA
## Energy_kcal       3.512380e+00  1.035829e-02    5.786153e+00     9.592220e+00
```

```
## Protein_g          8.301201e-02  1.030204e-03    3.079053e-01     1.360978e+00
## Fat_g              4.004058e-02  4.169551e-04    1.499207e-01     3.042194e-01
## Carb_g             8.643651e-01  1.039552e-03    1.044373e+00     5.783407e-01
## Sugar_g            2.956715e-01 -4.878482e-05    7.500868e-02    -1.964687e-01
## Fiber_g            1.681394e-01  4.905073e-04    3.121373e-01     1.982718e-01
## VitA_mcg           4.554040e+00  2.729580e-01    2.556527e+00     1.262055e+01
## VitB6_mg           1.423691e-02  3.421274e-05    1.873148e-02     2.963247e-02
## VitB12_mcg         8.490607e-03  1.514569e-03    2.377975e-03     1.864344e-01
## VitC_mg            1.080517e+00  9.008952e-04    8.857181e-01     2.004817e-01
## VitE_mg            7.083392e-02  1.762202e-04    1.003025e-01     7.621760e-02
## Folate_mcg         4.678220e+00  1.388477e-02    6.046898e+00     6.599719e+00
## Niacin_mg          1.081302e-01  4.113942e-04    1.754127e-01     3.603171e-01
## Riboflavin_mg      1.677148e-02  8.314666e-05    1.376611e-02     2.923337e-02
## Thiamin_mg         1.181641e-02  3.174525e-05    1.688675e-02     2.625743e-02
## Calcium_mg         3.378944e+01  1.030731e-02    8.408304e+00     3.322969e+01
## Copper_mcg         7.730481e-03  3.398522e-04    2.516651e-02     3.464548e-02
## Iron_mg            3.321380e-01  6.316094e-04    3.075199e-01     3.666550e-01
## Magnesium_mg       2.942906e+00  1.174437e-02    7.484932e+00     6.974899e+00
## Manganese_mg       7.598099e-02  6.576372e-04    1.519385e-01     1.445568e-01
## Phosphorus_mg      1.938398e+01  2.694649e-02    1.162483e+01     5.892409e+01
## Selenium_mcg       4.473883e-02  2.204395e-03    4.800048e-01     1.836134e+00
## Zinc_mg            7.930121e-02  5.333497e-04    1.148502e-01     2.597336e-01
## VitA_USRDA         5.060044e-03  3.032867e-04    2.840585e-03     1.402283e-02
## VitB6_USRDA        8.374654e-03  2.012514e-05    1.101852e-02     1.743086e-02
## VitB12_USRDA       3.537753e-03  6.310703e-04    9.908230e-04     7.768101e-02
## VitC_USRDA         1.200575e-02  1.000995e-05    9.841312e-03     2.227574e-03
## VitE_USRDA         4.722262e-03  1.174802e-05    6.686832e-03     5.081173e-03
## Folate_USRDA       1.169555e-02  3.471192e-05    1.511724e-02     1.649930e-02
## Niacin_USRDA       6.758138e-03  2.571214e-05    1.096329e-02     2.251982e-02
## Riboflavin_USRDA   1.290114e-02  6.395897e-05    1.058931e-02     2.248721e-02
## Thiamin_USRDA      9.847008e-03  2.645438e-05    1.407229e-02     2.188119e-02
## Calcium_USRDA      2.815786e-02  8.589423e-06    7.006920e-03     2.769141e-02
## Copper_USRDA       8.589423e-06  3.776135e-07    2.796279e-05     3.849498e-05
## Magnesium_USRDA    7.006920e-03  2.796279e-05    1.782127e-02     1.660690e-02
## Phosphorus_USRDA   2.769141e-02  3.849498e-05    1.660690e-02     8.417727e-02
## Selenium_USRDA     8.134333e-04  4.007990e-05    8.727360e-03     3.338425e-02
## Zinc_USRDA         7.209201e-03  4.848634e-05    1.044093e-02     2.361215e-02
##                  Selenium_USRDA    Zinc_USRDA
## Energy_kcal        4.9381638962  5.831497e+00
## Protein_g          2.0535192175  1.340957e+00
## Fat_g              0.3065484240  1.369049e-01
## Carb_g            -1.6057588219 -1.013443e-01
## Sugar_g           -0.8655830263 -9.131886e-02
## Fiber_g           -0.0796552134  1.121416e-01
## VitA_mcg          19.2609841441  1.678412e+01
## VitB6_mg           0.0268369471  6.533526e-02
## VitB12_mcg         0.4783746115  3.931259e-01
## VitC_mg           -1.0431838724  3.247913e-01
## VitE_mg            0.0309365863  1.963477e-01
## Folate_mcg         3.3434198231  1.820184e+01
## Niacin_mg          0.4448718533  6.961231e-01
## Riboflavin_mg      0.0325706372  5.048945e-02
## Thiamin_mg         0.0148519964  3.946891e-02
## Calcium_mg         0.9761199707  8.651041e+00
```

```
## Copper_mcg          0.0360719142  4.363771e-02
## Iron_mg             0.1881247132  6.561056e-01
## Magnesium_mg        3.6654912705  4.385189e+00
## Manganese_mg        0.0736177609  1.271032e-01
## Phosphorus_mg      23.3689773145  1.652850e+01
## Selenium_mg        14.5537619189  1.773252e+00
## Zinc_mg             0.3546504187  1.025236e+00
## VitA_USRDA          0.0214010935  1.864902e-02
## VitB6_USRDA         0.0157864395  3.843250e-02
## VitB12_USRDA        0.1993227548  1.638025e-01
## VitC_USRDA         -0.0115909319  3.608792e-03
## VitE_USRDA          0.0020624391  1.308985e-02
## Folate_USRDA        0.0083585496  4.550460e-02
## Niacin_USRDA        0.0278044908  4.350769e-02
## Riboflavin_USRDA    0.0250543363  3.883804e-02
## Thiamin_USRDA       0.0123766637  3.289076e-02
## Calcium_USRDA       0.0008134333  7.209201e-03
## Copper_USRDA        0.0000400799  4.848634e-05
## Magnesium_USRDA     0.0087273602  1.044093e-02
## Phosphorus_USRDA    0.0333842533  2.361215e-02
## Selenium_USRDA      0.2646138531  3.224095e-02
## Zinc_USRDA          0.0322409472  9.320325e-02
```

b. The eigenvalues of the covariance matrix are called the principal component values. How many of the 38 principal components are within 0.1% of the largest component?

```
eigen_nndb = eigen(cov_nndb)
eigen_nndb$values
```

```
##  [1]  6.081308e+05  6.818525e+04  3.403074e+04  2.647470e+04  1.739410e+04
##  [6]  3.307536e+03  2.245686e+03  8.021377e+02  5.516910e+02  9.999359e+01
## [11]  8.100209e+01  3.402110e+01  2.278368e+01  1.294877e+01  1.179346e+01
## [16]  1.114752e+01  8.411638e+00  6.624400e+00  2.029089e+00  2.661836e-01
## [21]  1.421729e-01  1.047091e-01  8.001511e-02  2.902667e-12  1.978165e-12
## [26]  2.119974e-13  6.569654e-15  2.431439e-15  4.177439e-16  3.362896e-16
## [31] -4.824302e-19 -6.562843e-18 -1.022111e-17 -3.601724e-17 -8.956535e-17
## [36] -1.409050e-16 -9.955419e-14 -2.053233e-13
```

```
paste(sum(eigen_nndb$values[2:length(eigen_nndb$values)]
          < 0.001*eigen_nndb$values[1]))
```

```
## [1] "30"
```

The number of principal components that are less than 0.1% of the largest component is equal to - 30

c. Transform the data into the eigenvector basis, also called the principal component basis. Calculate the covariance matrix in the new basis. What structure can you see from this matrix

```
nndb_num = as.matrix(nndb_num)
transform_nndb_num = nndb_num%*%solve(eigen_nndb$vectors)
transform_cov = cov(transform_nndb_num)
transform_cov
```

```
##               [,1]          [,2]          [,3]          [,4]          [,5]
##  [1,]   2995.133164  -542.7271211    878.892886  -2974.394629   489.9374345
##  [2,]   -542.727121   722.3943924    661.884045    832.827248   -75.5263585
##  [3,]    878.892886   661.8840452   2719.785274  -1037.648258   906.6519507
##  [4,]  -2974.394629   832.8272477  -1037.648258   4328.988108 -1604.0146095
##  [5,]    489.937435   -75.5263585    906.651951  -1604.014610  3002.3684993
##  [6,]   -827.064857  1588.0250845  -1795.001784   1581.220966  -842.5852644
##  [7,]   2873.522656  -466.7067744  -1656.566577   -306.569391   474.8773997
##  [8,]     39.489749  -122.5902554   1439.540801    501.050564  -246.0654261
##  [9,]    641.202700  1345.9318103   -268.349445   -357.201914   -91.9617094
## [10,]  11544.528098 -1970.9910774   6811.235241 -15075.838106  3174.2353706
## [11,]   -193.626129  -363.6808659   -218.500024    261.390907    18.4723769
## [12,]   1603.464039  -247.1977430   1084.332003  -2285.262934   586.9432017
## [13,]   -416.935235 -1428.6358076     73.679208    176.147977    34.2067849
## [14,]     50.733236   -26.5201764     53.923368    -69.693182    18.1680133
## [15,]     15.937485  -134.5048372    467.819231     95.587080   -60.9979458
## [16,]    199.901838    40.5860291    205.001126    -89.220303   -21.6395295
## [17,]     49.190910   444.9202972  -2382.373232   -725.360794   318.9288650
## [18,]   -943.357779   153.0781152   -690.021247   1316.216699  -527.1965294
## [19,] -33703.703107  5736.9085827 -20331.672790  44191.038745 -9278.4293419
## [20,]    -69.981751  -877.8616472  -2325.599784   1651.342531  -798.1622163
## [21,]   3639.299080  -622.8228390   2244.800453  -4798.877557  1024.1930493
## [22,]  -1322.284496   196.6547662   -793.929496   1721.607266  -341.8070181
## [23,]   -156.570161    16.3191737   -269.985412    262.577923   -54.0737954
## [24,]    -42.845517     6.8083922    -27.971541     60.705237   -18.8768379
## [25,]     41.224599   -73.7299146    862.504285    269.439179  -134.4091824
## [26,]    309.295606   555.7200773    -87.244507   -199.058144   -35.3062406
## [27,]    130.779802   -24.4897272     74.826228   -168.284131    32.6316939
## [28,]    -27.001002   -23.1285402    -16.267189     27.945252    20.4223368
## [29,]      5.832186    -0.4073801      5.984492     -4.573911    -1.2136920
## [30,]    -24.448271   -89.1328245      4.477266     13.899903   -15.8089083
## [31,]     14.470483   -14.8828367     26.897437    -19.548689     4.7183951
## [32,]     26.476166  -111.4515518    397.681233     70.497713   -57.0988246
## [33,]     -4.498176     2.5462755      4.569118      1.619719     4.3956685
## [34,]     -2.500398     0.3163589     -9.147396      6.265839    -4.9580496
## [35,]    -86.035603    14.7883989    -45.400042    108.483678   -12.8030988
## [36,]      6.395909    -2.3424651      1.496359     -6.548377    -0.3051263
## [37,]    -23.025937     1.7346919    -12.584164     30.948175    -8.5363683
## [38,]    -14.226196    -0.7161804    -32.913401     26.098848   -10.4594098
##               [,6]          [,7]          [,8]          [,9]         [,10]
##  [1,]   -827.064857  2873.5226561     39.489749  6.412027e+02  1.154453e+04
##  [2,]   1588.025085  -466.7067744   -122.590255  1.345932e+03 -1.970991e+03
##  [3,]  -1795.001784 -1656.5665773   1439.540801 -2.683494e+02  6.811235e+03
##  [4,]   1581.220966  -306.5693909    501.050564 -3.572019e+02 -1.507584e+04
##  [5,]   -842.585264   474.8773997   -246.065426 -9.196171e+01  3.174235e+03
##  [6,]  15985.151069  3118.8862100  -7060.226281  1.297599e+04 -8.395772e+03
##  [7,]   3118.886210 28599.0874626  -3240.320167  2.325722e+03 -1.430236e+03
##  [8,]  -7060.226281 -3240.3201670   9733.526448 -5.508401e+03  1.140164e+03
##  [9,]  12975.987930  2325.7220858  -5508.400668  1.157414e+04  1.165197e+02
## [10,]  -8395.772149 -1430.2364281   1140.163928  1.165197e+02  6.105022e+04
## [11,]  -2096.859816    81.0570698    776.010308 -1.935247e+03 -1.170085e+03
## [12,]  -1446.785101  1431.7166490    132.452439 -1.360997e+02  9.257684e+03
## [13,] -12008.663790 -1564.7351640   4597.386652 -1.064625e+04  2.575307e+02
## [14,]   -295.207241   -51.1690543     47.576983 -2.330126e+02  3.258752e+02
```

```
## [15,]  -2895.221477 -1068.4401702   3112.037180 -2.302564e+03  6.380230e+02
## [16,]   -156.803014  -336.6069451    436.782667 -1.287369e+01  6.836132e+02
## [17,]  13333.937637  5574.0322148 -16174.507120  1.047416e+04 -2.313356e+03
## [18,]    955.230869   307.3129973   -328.041967  1.660713e+02 -5.143248e+03
## [19,]  25526.811837  5460.4459593  -3708.689780  3.387581e+02 -1.795542e+05
## [20,]   5228.311603  4775.5924530  -2403.148976  4.421719e+03 -8.898006e+03
## [21,]  -2955.837864 -1074.2751260    672.389357 -1.833859e+02  1.951646e+04
## [22,]    839.575306   204.6543725   -119.364332 -1.190421e+02 -6.997860e+03
## [23,]    727.904443   372.5453328   -210.719538  4.915081e+02 -1.328577e+03
## [24,]     35.378587    28.5868976    -21.797087  2.640735e+00 -2.257101e+02
## [25,]  -4176.151503 -1914.4833823   5738.865087 -3.247898e+03  7.711914e+02
## [26,]   5374.588962   962.8170154  -2274.764030  4.819158e+03  2.571078e+02
## [27,]   -105.470237   -15.3461690     30.829204 -7.749097e+00  6.838331e+02
## [28,]   -139.391100     2.2414250     46.652413 -1.338608e+02 -1.412912e+02
## [29,]     -6.137103     0.2049089     10.196799  1.484527e-01  2.711251e+01
## [30,]   -752.622473  -112.8824632    292.804446 -6.679566e+02  2.843588e+01
## [31,]   -209.599927   -34.6849225     44.630245 -1.814748e+02  1.106745e+02
## [32,]  -2418.902286  -895.9759819   2626.699249 -1.917889e+03  5.811001e+02
## [33,]    -11.775569   -15.7323750     10.624782 -1.090481e+01 -3.313411e+00
## [34,]     33.345034    18.2334180    -28.315301  2.486549e+01 -3.134330e+01
## [35,]     44.832304     4.3937043      2.195329 -1.484822e+01 -4.446255e+02
## [36,]     -2.823011    -1.0299288      1.294033  1.541945e+00  2.607686e+01
## [37,]     -3.093125     1.5814194      7.044250 -1.709017e+01 -1.192897e+02
## [38,]     81.773270    44.0973476    -35.422757  5.572118e+01 -1.350063e+02
##                [,11]         [,12]         [,13]         [,14]         [,15]
##  [1,]  -193.6261290  1.603464e+03 -4.169352e+02   50.73323552    15.9374853
##  [2,]  -363.6808659 -2.471977e+02 -1.428636e+03  -26.52017644  -134.5048372
##  [3,]  -218.5000238  1.084332e+03  7.367921e+01   53.92336810   467.8192310
##  [4,]   261.3909066 -2.285263e+03  1.761480e+02  -69.69318202    95.5870798
##  [5,]    18.4723769  5.869432e+02  3.420678e+01   18.16801330   -60.9979458
##  [6,] -2096.8598156 -1.446785e+03 -1.200866e+04 -295.20724126 -2895.2214773
##  [7,]    81.0570698  1.431717e+03 -1.564735e+03  -51.16905428 -1068.4401702
##  [8,]   776.0103082  1.324524e+02  4.597387e+03   47.57698272  3112.0371795
##  [9,] -1935.2471628 -1.360997e+02 -1.064625e+04 -233.01262162 -2302.5639724
## [10,] -1170.0850426  9.257684e+03  2.575307e+02  325.87521049   638.0230092
## [11,]   639.5421736 -2.124945e+02  2.024583e+03   19.21674138   354.9412476
## [12,]  -212.4944690  1.808768e+03  1.518355e+02   55.19523405   100.1005281
## [13,]  2024.5827246  1.518355e+02  1.004311e+04  205.46330032  2004.1389458
## [14,]    19.2167414  5.519523e+01  2.054633e+02   18.63966968    23.3274231
## [15,]   354.9412476  1.001005e+02  2.004139e+03   23.32742306  1081.2932922
## [16,]   -27.6821920  5.080123e+01 -2.318376e+01    5.87946938   122.5704069
## [17,] -1583.6867722 -3.357439e+02 -8.991611e+03  -19.55212575 -5517.4659208
## [18,]    74.5510731 -7.832222e+02 -1.530037e+02  -31.65051569  -128.9676779
## [19,]  3350.1537866 -2.722067e+04 -1.367014e+03 -972.48711604 -2028.0905560
## [20,]  2487.4609483 -2.116220e+03 -1.541634e+03 -277.76308207  -850.1689633
## [21,]  -348.9827611  2.944192e+03  2.592656e+02  111.95019248   300.1295173
## [22,]   173.3524616 -1.061666e+03  8.788674e+01  -36.88266661   -60.6679904
## [23,]    23.1215311 -2.333440e+02 -3.978053e+02  -27.64541341   -87.4933917
## [24,]     5.5285186 -3.270532e+01 -1.909597e-01   -1.49023094    -6.6851514
## [25,]   452.9011118  9.411491e+01  2.708499e+03   29.07984564  1834.7375183
## [26,]  -813.0237567 -2.493490e+01 -4.435382e+03  -95.06100330  -952.9678860
## [27,]    -9.6874824  1.031431e+02  1.232389e+01    3.44450046    13.2260342
## [28,]    46.8649188 -2.431090e+01  1.401466e+02    1.07561568    22.1555459
## [29,]     2.2378602  4.079779e+00  1.861650e+00   -0.02757171     3.2782882
```

```
## [30,]   123.9031452  1.009348e+01  6.281465e+02   13.05578449   127.0906679
## [31,]    15.4716432  2.158480e+01  1.564088e+02   14.08289266    19.1858096
## [32,]   292.8854377  8.995851e+01  1.666027e+03   20.80272889   907.2785122
## [33,]    -2.9820105  9.285319e-01  5.380881e+00    0.42300534     3.4699763
## [34,]     0.4437372 -6.183720e+00 -1.858881e+01   -0.70076545    -9.6167292
## [35,]     8.1622323 -6.640807e+01  7.908521e+00   -1.92691285    -0.9876763
## [36,]     2.1096117  2.992274e+00  9.655205e-01    0.03787283     0.5352070
## [37,]     6.3439463 -1.813152e+01  1.618857e+01   -0.42332128     2.6292645
## [38,]     3.9514619 -2.448303e+01 -4.272324e+01   -2.85564423   -13.2261930
##               [,16]        [,17]        [,18]         [,19]        [,20]
##  [1,]   199.9018377     49.19091  -943.357779  -33703.70311   -69.98175
##  [2,]    40.5860291    444.92030   153.078115    5736.90858  -877.86165
##  [3,]   205.0011261  -2382.37323  -690.021247  -20331.67279 -2325.59978
##  [4,]   -89.2203029   -725.36079  1316.216699   44191.03875  1651.34253
##  [5,]   -21.6395295    318.92887  -527.196529   -9278.42934  -798.16222
##  [6,]  -156.8030141  13333.93764   955.230869   25526.81184  5228.31160
##  [7,]  -336.6069451   5574.03221   307.312997    5460.44596  4775.59245
##  [8,]   436.7826666 -16174.50712  -328.041967   -3708.68978 -2403.14898
##  [9,]   -12.8736863  10474.15708   166.071296     338.75812  4421.71851
## [10,]   683.6132492  -2313.35568 -5143.247561 -179554.21653 -8898.00641
## [11,]   -27.6821920  -1583.68677    74.551073    3350.15379  2487.46095
## [12,]    50.8012269   -335.74394  -783.222167  -27220.66958 -2116.21968
## [13,]   -23.1837617  -8991.61055  -153.003669   -1367.01360 -1541.63430
## [14,]     5.8794694    -19.55213   -31.650516    -972.48712  -277.76308
## [15,]   122.5704069  -5517.46592  -128.967678   -2028.09056  -850.16896
## [16,]   120.9825234   -655.07583   -70.395724   -2022.18520  -177.97070
## [17,]  -655.0758276  29294.17032   588.884203    7505.76713  4016.84393
## [18,]   -70.3957239    588.88420   474.880932   15139.09236   903.78761
## [19,] -2022.1852020   7505.76713 15139.092358  528323.21119 26616.02932
## [20,]  -177.9706994   4016.84393   903.787612   26616.02932 35434.74261
## [21,]   205.8468851  -1231.63943 -1661.007207  -57428.02119 -3004.02750
## [22,]   -86.8094063    217.04374   588.764713   20580.44764  1194.91384
## [23,]   -26.6034188    350.17896   125.321619    3965.64503  1222.55305
## [24,]    -4.1236117     31.08540    20.824123     663.07861    53.67554
## [25,]   258.9773001  -9533.69448  -202.818118   -2477.68210 -1462.12484
## [26,]    -1.1051880   4332.49723    51.257215    -471.32206  1779.30498
## [27,]     8.2078931    -58.53478   -57.527899   -2011.22330   -81.15630
## [28,]    -3.4639636    -97.71033     8.477039     408.66552   189.14612
## [29,]     0.9682070    -17.37209    -2.487922     -80.06395    28.41204
## [30,]    -0.7558275   -570.87877    -9.239170    -122.84081  -122.70847
## [31,]     4.1644233    -25.06652   -13.175174    -335.33469  -217.22366
## [32,]   105.4791958  -4621.49643  -111.828804   -1834.34689  -724.87096
## [33,]     0.7526629    -18.00173    -1.095204       8.52524   -53.55693
## [34,]    -1.6033407     49.45402     4.368187      93.81212    54.15866
## [35,]    -4.5933466     -1.47320    35.862439    1307.25415    29.23846
## [36,]     0.5283899     -2.42714    -2.372635     -76.40488    28.68243
## [37,]    -1.4231928    -13.37873    10.220385     349.70764    24.55832
## [38,]    -3.3166544     59.35792    13.600127     403.54278   153.34383
##               [,21]         [,22]         [,23]         [,24]         [,25]
##  [1,]  3.639299e+03 -1322.2844957 -1.565702e+02 -4.284552e+01    41.2245988
##  [2,] -6.228228e+02   196.6547662  1.631917e+01  6.808392e+00   -73.7299146
##  [3,]  2.244800e+03  -793.9294960 -2.699854e+02 -2.797154e+01   862.5042848
##  [4,] -4.798878e+03  1721.6072662  2.625779e+02  6.070524e+01   269.4391795
##  [5,]  1.024193e+03  -341.8070181 -5.407380e+01 -1.887684e+01  -134.4091824
```

```
##  [6,] -2.955838e+03   839.5753061  7.279044e+02  3.537859e+01 -4176.1515026
##  [7,] -1.074275e+03   204.6543725  3.725453e+02  2.858690e+01 -1914.4833823
##  [8,]  6.723894e+02  -119.3643323 -2.107195e+02 -2.179709e+01  5738.8650875
##  [9,] -1.833859e+02  -119.0420997  4.915081e+02  2.640735e+00 -3247.8976957
## [10,]  1.951646e+04 -6997.8601124 -1.328577e+03 -2.257101e+02   771.1914207
## [11,] -3.489828e+02   173.3524616  2.312153e+01  5.528519e+00   452.9011118
## [12,]  2.944192e+03 -1061.6658005 -2.333440e+02 -3.270532e+01    94.1149088
## [13,]  2.592656e+02    87.8867352 -3.978053e+02 -1.909597e-01  2708.4987569
## [14,]  1.119502e+02   -36.8826666 -2.764541e+01 -1.490231e+00    29.0798456
## [15,]  3.001295e+02   -60.6679904 -8.749339e+01 -6.685151e+00  1834.7375183
## [16,]  2.058469e+02   -86.8094063 -2.660342e+01 -4.123612e+00   258.9773001
## [17,] -1.231639e+03   217.0437394  3.501790e+02  3.108540e+01 -9533.6944788
## [18,] -1.661007e+03   588.7647128  1.253216e+02  2.082412e+01  -202.8181179
## [19,] -5.742802e+04 20580.4476441  3.965645e+03  6.630786e+02 -2477.6821022
## [20,] -3.004027e+03  1194.9138368  1.222553e+03  5.367554e+01 -1462.1248410
## [21,]  6.330963e+03 -2235.2276175 -4.395043e+02 -7.402736e+01   428.6609868
## [22,] -2.235228e+03   808.0394251  1.557068e+02  2.763291e+01   -82.2781005
## [23,] -4.395043e+02   155.7068190  1.012307e+02  5.969161e+00  -127.7937826
## [24,] -7.402736e+01    27.6329096  5.969161e+00  1.994872e+00   -13.4555290
## [25,]  4.286610e+02   -82.2781005 -1.277938e+02 -1.345553e+01  3383.9284147
## [26,] -8.206671e+00   -74.9969066  1.986037e+02 -4.391277e-01 -1340.7414424
## [27,]  2.189584e+02   -78.2556295 -1.425284e+01 -2.562326e+00    19.2473708
## [28,] -4.357302e+01    19.7257106  3.376200e+00  9.062873e-01    27.0556038
## [29,]  9.002135e+00    -3.0396109 -4.109822e-02 -1.350101e-01     6.0378651
## [30,]  2.038780e+01     3.7135763 -2.616147e+01 -1.145685e-01   172.5350786
## [31,]  4.237837e+01   -13.4306626 -1.933902e+01 -1.228791e+00    27.0338905
## [32,]  2.674181e+02   -57.1509691 -7.490042e+01 -6.203983e+00  1548.7750917
## [33,] -4.864645e-01     0.1794995 -2.050224e+00  3.406642e-03     6.3207827
## [34,] -1.138112e+01     3.8367999  3.104814e+00  2.699666e-01   -16.8185233
## [35,] -1.414742e+02    51.1321323  8.278041e+00  1.707604e+00     0.6149157
## [36,]  8.579512e+00    -2.8478769  6.213984e-02 -1.006752e-01     0.7916730
## [37,] -3.790346e+01    13.8866623  2.327462e+00  4.359509e-01     3.9340619
## [38,] -4.516938e+01    15.9930979  1.093385e+01  6.774245e-01   -21.2761627
##               [,26]         [,27]         [,28]         [,29]         [,30]
##  [1,]   309.2956058  1.307798e+02  -27.00100170   5.832186408  -24.44827116
##  [2,]   555.7200773 -2.448973e+01  -23.12854023  -0.407380070  -89.13282447
##  [3,]   -87.2445069  7.482623e+01  -16.26718930   5.984491679    4.47726592
##  [4,]  -199.0581444 -1.682841e+02   27.94525221  -4.573910727   13.89990309
##  [5,]   -35.3062406  3.263169e+01   20.42233684  -1.213692049  -15.80890830
##  [6,]  5374.5889622 -1.054702e+02 -139.39109987  -6.137102543 -752.62247319
##  [7,]   962.8170154 -1.534617e+01    2.24142503   0.204908861 -112.88246325
##  [8,] -2274.7640295  3.082920e+01   46.65241263  10.196799462  292.80444569
##  [9,]  4819.1584395 -7.749097e+00 -133.86075757   0.148452668 -667.95663358
## [10,]   257.1077722  6.838331e+02 -141.29121548  27.112512042   28.43588155
## [11,]  -813.0237567 -9.687482e+00   46.86491876   2.237860189  123.90314518
## [12,]   -24.9349049  1.031431e+02  -24.31090393   4.079778806   10.09347927
## [13,] -4435.3824730  1.232389e+01  140.14658125   1.861649871  628.14648163
## [14,]   -95.0610033  3.444500e+00    1.07561568  -0.027571706   13.05578449
## [15,]  -952.9678860  1.322603e+01   22.15554593   3.278288204  127.09066792
## [16,]    -1.1051880  8.207893e+00   -3.46396360   0.968206985   -0.75582748
## [17,]  4332.4972253 -5.853478e+01  -97.71033481 -17.372094176 -570.87876800
## [18,]    51.2572146 -5.752790e+01    8.47703928  -2.487922343   -9.23916973
## [19,]  -471.3220621 -2.011223e+03  408.66551759 -80.063951954 -122.84081125
## [20,]  1779.3049753 -8.115630e+01  189.14612348  28.412039676 -122.70847074
```

```
## [21,]    -8.2066714  2.189584e+02  -43.57302306   9.002135446   20.38780490
## [22,]   -74.9969066 -7.825563e+01   19.72571063  -3.039610908    3.71357631
## [23,]   198.6036859 -1.425284e+01    3.37620031  -0.041098218  -26.16146906
## [24,]    -0.4391277 -2.562326e+00    0.90628733  -0.135010072   -0.11456846
## [25,] -1340.7414424  1.924737e+01   27.05560382   6.037865077  172.53507865
## [26,]  2007.9716303 -8.638095e-01  -56.76196082   0.183831321 -278.11220405
## [27,]    -0.8638095  7.723826e+00   -1.38196794   0.338325513    0.91152526
## [28,]   -56.7619608 -1.381968e+00    3.81087382   0.101088515    8.41185838
## [29,]     0.1838313  3.383255e-01    0.10108851   0.073011123    0.12292123
## [30,]  -278.1122041  9.115253e-01    8.41185838   0.122921230   39.47382548
## [31,]   -74.0387435  1.097367e+00    0.79678908  -0.064928413    9.98574215
## [32,]  -793.2205522  1.161488e+01   18.02411186   2.798782946  105.74879549
## [33,]    -4.5283865 -7.578686e-02   -0.17775754  -0.034005994    0.34703977
## [34,]    10.1163991 -3.603997e-01    0.09360205  -0.010970818   -1.19345270
## [35,]    -7.7620514 -4.998046e+00    1.11255375  -0.227972613    0.36027986
## [36,]     0.7310557  3.087386e-01    0.11067918   0.054324656    0.05952023
## [37,]    -7.5246343 -1.306055e+00    0.53799153  -0.037838312    0.99690780
## [38,]    22.5426995 -1.453213e+00    0.43676331   0.008109045   -2.79321381
##               [,31]         [,32]         [,33]         [,34]         [,35]
##  [1,]   14.47048281    26.4761664  -4.498175791  -2.500398467  -86.03560310
##  [2,]  -14.88283674  -111.4515518   2.546275508   0.316358949   14.78839894
##  [3,]   26.89743722   397.6812335   4.569118072  -9.147396269  -45.40004184
##  [4,]  -19.54868881    70.4977133   1.619718504   6.265839163  108.48367750
##  [5,]    4.71839507   -57.0988246   4.395668530  -4.958049641  -12.80309883
##  [6,] -209.59992728 -2418.9022865 -11.775569332  33.345033567   44.83230427
##  [7,]  -34.68492245  -895.9759819 -15.732374978  18.233418006    4.39370430
##  [8,]   44.63024547  2626.6992486  10.624781650 -28.315301382    2.19532897
##  [9,] -181.47480384 -1917.8892204 -10.904813474  24.865493152  -14.84822104
## [10,]  110.67446687   581.1000605  -3.313411288 -31.343301822 -444.62545590
## [11,]   15.47164320   292.8854377  -2.982010503   0.443737206    8.16223231
## [12,]   21.58480135    89.9585066   0.928531881  -6.183719926  -66.40806775
## [13,]  156.40883022  1666.0269825   5.380880659 -18.588813843    7.90852081
## [14,]   14.08289266    20.8027289   0.423005344  -0.700765453   -1.92691285
## [15,]   19.18580955   907.2785122   3.469976255  -9.616729153   -0.98767630
## [16,]    4.16442328   105.4791958   0.752662943  -1.603340673   -4.59334659
## [17,]  -25.06651753 -4621.4964253 -18.001732397  49.454019359   -1.47320010
## [18,]  -13.17517403  -111.8288035  -1.095203747   4.368186612   35.86243900
## [19,] -335.33469047 -1834.3468892   8.525239544  93.812117837 1307.25414616
## [20,] -217.22366465  -724.8709556 -53.556934924  54.158656678   29.23845565
## [21,]   42.37837335   267.4180949  -0.486464534 -11.381118745 -141.47420938
## [22,]  -13.43066262   -57.1509691   0.179499459   3.836799918   51.13213230
## [23,]  -19.33902464   -74.9004162  -2.050223550   3.104813506    8.27804101
## [24,]   -1.22879061    -6.2039828   0.003406642   0.269966574    1.70760377
## [25,]   27.03389053  1548.7750917   6.320782742 -16.818523279    0.61491569
## [26,]  -74.03874347  -793.2205522  -4.528386545  10.116399110   -7.76205139
## [27,]    1.09736738    11.6148769  -0.075786855  -0.360399686   -4.99804639
## [28,]    0.79678908    18.0241119  -0.177757545   0.093602050    1.11255375
## [29,]   -0.06492841     2.7987829  -0.034005994  -0.010970818   -0.22797261
## [30,]    9.98574215   105.7487955   0.347039772  -1.193452705    0.36027986
## [31,]   11.32626526    17.1810970   0.356185535  -0.571703251   -0.49398986
## [32,]   17.18109703   761.9297556   2.897605575  -8.121725337   -1.21863872
## [33,]    0.35618553     2.8976056   0.212545562  -0.157314656    0.09467838
## [34,]   -0.57170325    -8.1217253  -0.157314656   0.242047886    0.14551122
## [35,]   -0.49398986    -1.2186387   0.094678378   0.145511222    3.36493673
```

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## [36,]   -0.03163275     0.4778501  -0.027114338   0.003463122   -0.21678968
## [37,]   -0.05434835     2.0975606  -0.005669372   0.052722834    0.85494104
## [38,]   -2.03936597   -11.2577887  -0.273481024   0.395764980    0.79167564
##               [,36]         [,37]         [,38]
##  [1,]   6.395908983 -2.302594e+01 -1.422620e+01
##  [2,]  -2.342465108  1.734692e+00 -7.161804e-01
##  [3,]   1.496358969 -1.258416e+01 -3.291340e+01
##  [4,]  -6.548376967  3.094818e+01  2.609885e+01
##  [5,]  -0.305126255 -8.536368e+00 -1.045941e+01
##  [6,]  -2.823010938 -3.093125e+00  8.177327e+01
##  [7,]  -1.029928799  1.581419e+00  4.409735e+01
##  [8,]   1.294033258  7.044250e+00 -3.542276e+01
##  [9,]   1.541945130 -1.709017e+01  5.572118e+01
## [10,]  26.076862364 -1.192897e+02 -1.350063e+02
## [11,]   2.109611716  6.343946e+00  3.951462e+00
## [12,]   2.992273652 -1.813152e+01 -2.448303e+01
## [13,]   0.965520495  1.618857e+01 -4.272324e+01
## [14,]   0.037872833 -4.233213e-01 -2.855644e+00
## [15,]   0.535207014  2.629264e+00 -1.322619e+01
## [16,]   0.528389898 -1.423193e+00 -3.316654e+00
## [17,]  -2.427140140 -1.337873e+01  5.935792e+01
## [18,]  -2.372634781  1.022039e+01  1.360013e+01
## [19,] -76.404879016  3.497076e+02  4.035428e+02
## [20,]  28.682429975  2.455832e+01  1.533438e+02
## [21,]   8.579512291 -3.790346e+01 -4.516938e+01
## [22,]  -2.847876855  1.388666e+01  1.599310e+01
## [23,]   0.062139844  2.327462e+00  1.093385e+01
## [24,]  -0.100675170  4.359509e-01  6.774245e-01
## [25,]   0.791672960  3.934062e+00 -2.127616e+01
## [26,]   0.731055693 -7.524634e+00  2.254270e+01
## [27,]   0.308738638 -1.306055e+00 -1.453213e+00
## [28,]   0.110679179  5.379915e-01  4.367633e-01
## [29,]   0.054324656 -3.783831e-02  8.109045e-03
## [30,]   0.059520229  9.969078e-01 -2.793214e+00
## [31,]  -0.031632746 -5.434835e-02 -2.039366e+00
## [32,]   0.477850081  2.097561e+00 -1.125779e+01
## [33,]  -0.027114338 -5.669372e-03 -2.734810e-01
## [34,]   0.003463122  5.272283e-02  3.957650e-01
## [35,]  -0.216789680  8.549410e-01  7.916756e-01
## [36,]   0.058218572 -4.607850e-02  3.534342e-02
## [37,]  -0.046078499  2.736932e-01  2.380360e-01
## [38,]   0.035343419  2.380360e-01  1.234573e+00
```

```
dim(transform_cov)
```

```
## [1] 38 38
```