Deliverables 3

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1) 6 independent fair dice

$$P(X \geq 1) = 1 - P(X=0)$$

$$1 - \left(\frac{5}{6}\right)^6 = 1 - \frac{5^6}{6^6} \approx 1 - .3349 = .6651$$

Probability of at least 1 6 appearing is .6651

12 dice w/ 2 success'

$$P(X \geq 2) = 1 - P(X=1) - P(X=0)$$

$$= 1 - \binom{12}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{11} - \left(\frac{5}{6}\right)^{12} \approx .6187$$

18 dice w/ 3 success'

$$P(X \geq 3) = 1 - P(X=2) - P(X=1) - P(X=0)$$

$$= 1 - \binom{18}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{16} - \binom{18}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{17} - \left(\frac{5}{6}\right)^{18} \approx .5973$$

Tossing 6 dice and trying to obtain 1 6 is the best choice.

2) $E[X] = \frac{1}{p}$

$20 = \frac{1}{p}$ $\quad p = \frac{1}{20}$

$p = \frac{R}{R+B}$ where R is total red balls, B is total black balls

$\frac{1}{20} = \frac{R}{R+B}$

$R + B = 100$

$\frac{1}{20} = \frac{R}{100}$

$$\frac{1}{20} = \frac{R}{100}$$

$$R = 5$$

A good estimate would be 5 red balls and 95 black balls.

3) $1 = P(X=3) + P(X=2) + P(X=1) + P(X=0)$

$$= \left(\frac{1}{6}\right)^3 + \binom{3}{2}\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)^2 + \binom{3}{1}\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3$$

$$E[X] = \sum_k p_k x_k$$

$$= \left(\frac{1}{6}\right)^3 3 + \binom{3}{2}\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)^2 2 + \binom{3}{1}\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right) - \left(\frac{5}{6}\right)^3$$

$$= \frac{1}{72} + \frac{10}{72} + \frac{25}{72} - \frac{125}{216}$$

$$= \frac{36}{72} - \frac{125}{216}$$

$$= \frac{108}{216} - \frac{125}{216} = \frac{17}{216} \approx -.08 \text{ galleons}$$

Hermione would lose about .08 galleons.