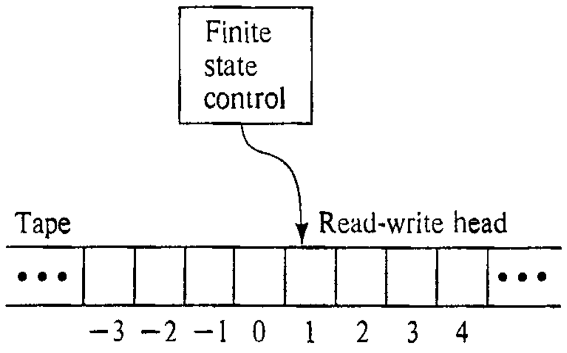

Figure 2.1 Schematic representation of a deterministic one-tape Turing machine (DTM).

A *program* for a DTM specifies the following information:

(1) A finite set Γ of tape *symbols*, including a subset $\Sigma \subset \Gamma$ of *input symbols* and a distinguished *blank symbol* $b \in \Gamma - \Sigma$;

(2) a finite set Q of *states*, including a distinguished *start-state* q_0 and two distinguished *halt-states* q_Y and q_N;

(3) a *transition function* $\delta\colon (Q - \{q_Y, q_N\}) \times \Gamma \rightarrow Q \times \Gamma \times \{-1, +1\}$.

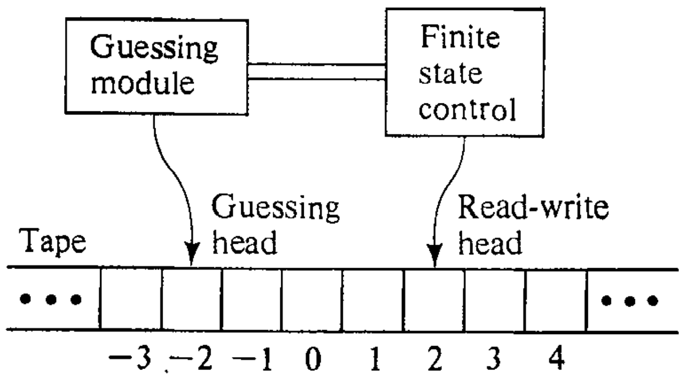

Figure 2.4 Schematic representation of a nondeterministic one-tape Turing machine (NDTM).

*Michael R. Garey and David S. Johnson. 1972/1990. Computers and Intractability; A Guide to the Theory of NP-Completeness. W. H. Freeman & Co., USA.

Proof of Cook's Theorem (Garey & Johnson, 1972*)

Variable	Range	Intended meaning
$Q[i,k]$	$0 \leqslant i \leqslant p(n)$ $0 \leqslant k \leqslant r$	At time i, M is in state q_k.
$H[i,j]$	$0 \leqslant i \leqslant p(n)$ $-p(n) \leqslant j \leqslant p(n)+1$	At time i, the read-write head is scanning tape square j.
$S[i,j,k]$	$0 \leqslant i \leqslant p(n)$ $-p(n) \leqslant j \leqslant p(n)+1$ $0 \leqslant k \leqslant v$	At time i, the contents of tape square j is symbol s_k.

Figure 2.7 Variables in $f_L(x)$ and their intended meanings.

*Michael R. Garey and David S. Johnson. 1972/1990. Computers and Intractability; A Guide to the Theory of NP-Completeness. W. H. Freeman & Co., USA.

Proof of Cook's Theorem (Garey & Johnson, 1972)

$x \in L \iff$ there is an accepting computation of M on x

$\iff$ there is an accepting computation of M on x with $p(n)$ or fewer steps in its checking stage and with a guessed string w of length exactly $p(n)$

$\iff$ there is a satisfying truth assignment for the collection of clauses in $f_L(x)$.

*Michael R. Garey and David S. Johnson. 1972/1990. Computers and Intractability; A Guide to the Theory of NP-Completeness. W. H. Freeman & Co., USA.

Proof of Cook’s Theorem

Clause group	Restriction imposed
G_1	At each time i, M is in exactly one state.
G_2	At each time i, the read-write head is scanning exactly one tape square.
G_3	At each time i, each tape square contains exactly one symbol from Γ.
G_4	At time 0, the computation is in the initial configuration of its checking stage for input x.
G_5	By time $p(n)$, M has entered state q_Y and hence has accepted x.
G_6	For each time i, $0 \leqslant i < p(n)$, the configuration of M at time $i+1$ follows by a single application of the transition function δ from the configuration at time i.

Figure 2.8 Clause groups in $f_L(x)$ and the restrictions they impose on satisf truth assignments.

Clause group	Clauses in group
G_1	$\{Q[i,0],Q[i,1],\ldots,Q[i,r]\},\ 0 \leqslant i \leqslant p(n)$ $\{\overline{Q[i,j]},\overline{Q[i,j']}\},\ 0 \leqslant i \leqslant p(n), 0 \leqslant j < j' \leqslant r$
G_2	$\{H[i,-p(n)],H[i,-p(n)+1],\ldots,H[i,p(n)+1]\},\ 0 \leqslant i \leqslant p(n)$ $\{\overline{H[i,j]},\overline{H[i,j']}\},\ 0 \leqslant i \leqslant p(n), -p(n) \leqslant j < j' \leqslant p(n)+1$
G_3	$\{S[i,j,0],S[i,j,1],\ldots,S[i,j,v]\},\ 0 \leqslant i \leqslant p(n), -p(n) \leqslant j \leqslant p(n)+1$ $\{\overline{S[i,j,k]},\overline{S[i,j,k']}\}, 0 \leqslant i \leqslant p(n), -p(n) \leqslant j \leqslant p(n)+1, 0 \leqslant k < k' \leqslant v$
G_4	$\{Q[0,0]\},\{H[0,1]\},\{S[0,0,0]\},$ $\{S[0,1,k_1]\},\{S[0,2,k_2]\},\cdots,\{S[0,n,k_n]\},$ $\{S[0,n+1,0]\},\{S[0,n+2,0]\},\ldots,\{S[0,p(n)+1,0]\},$ where $x = s_{k_1} s_{k_2} \cdots s_{k_n}$
G_5	$\{Q[p(n),1]\}$

Figure 2.9 The first five clause groups in $f_L(x)$.

*Michael R. Garey and David S. Johnson. 1972/1990. Computers and Intractability; A Guide to the Theory of NP-Completeness. W. H. Freeman & Co., USA.

Proof of Cook's Theorem

The remaining subgroup of G_6 guarantees that the *changes* from one configuration to the next are in accord with the transition function δ for M. For each quadruple (i,j,k,l), $0 \leqslant i < p(n)$, $-p(n) \leqslant j \leqslant p(n)+1$, $0 \leqslant k \leqslant r$, and $0 \leqslant l \leqslant v$, this subgroup contains the following three clauses:

$$\{\overline{H[i,j]}, \overline{Q[i,k]}, \overline{S[i,j,l]}, H[i+1,j+\Delta]\}$$
$$\{\overline{H[i,j]}, \overline{Q[i,k]}, \overline{S[i,j,l]}, Q[i+1,k']\}$$
$$\{\overline{H[i,j]}, \overline{Q[i,k]}, \overline{S[i,j,l]}, S[i+1,j,l']\}$$

where if $q_k \in Q-\{q_Y,q_N\}$, then the values of Δ, k', and l' are such that $\delta(q_k,s_l) = (q_{k'},s_{l'},\Delta)$, and if $q_k \in \{q_Y,q_N\}$, then $\Delta=0$, $k'=k$, and $l'=l$.

Length $[f_L(x)] = |U|\cdot|C| = O(p(n)^4)$, and is bounded by a polynomial function of n as desired.

*Michael R. Garey and David S. Johnson. 1990. Computers and Intractability; A Guide to the Theory of NP-Completeness. W. H. Freeman & Co., USA.