$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} \quad \rho \neq 0$$

$$0 = \left| A - \lambda I \right| = \begin{vmatrix} \sigma_1^2 - \lambda & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 - \lambda \end{vmatrix}$$

$$\det\left| A - \lambda I \right| = (\sigma_1^2 - \lambda)(\sigma_2^2 - \lambda) - (\rho\sigma_1\sigma_2)^2 = 0$$

$$4\left(\sigma_1^2\sigma_2^2 - \lambda\sigma_2^2 - \lambda\sigma_1^2 + \lambda^2 - \rho^2\sigma_1^2\sigma_2^2 = 0\right)$$

$$4\sigma_1^2\sigma_2^2 - 4\lambda\sigma_2^2 - 4\lambda\sigma_1^2 + 4\lambda^2 - 4\rho^2\sigma_1^2\sigma_2^2 = 0$$

$$4\lambda^2 + 2\sigma_1^2\sigma^2 - 4\lambda\sigma_1^2 - 4\lambda\sigma_2^2 + \sigma_1^4 + \sigma_2^4 = 4\rho^2\sigma_1^2\sigma_2^2 - 2\sigma_1^2\sigma_2^2 + \sigma_1^4 + \sigma_2^4$$

$$\left[(2\lambda - \sigma_1^2 - \sigma_2^2)^2 = (\sigma_1^2 - \sigma_2^2)^2 + 4\sigma_1^2\sigma_2^2\rho^2\right]^{1/2}$$

$$2\lambda - \sigma_1^2 - \sigma_2^2 = \pm\sqrt{(\sigma_1^2 - \sigma_2^2)^2 + 4\sigma_1^2\sigma_2^2\rho^2}$$

$$2\lambda = \sigma_1^2 + \sigma_2^2 \pm \sqrt{\Delta} \quad \text{where } \Delta = (\sigma_1^2 - \sigma_2^2)^2 + 4\sigma_1^2\sigma_2^2\rho^2$$

$$\lambda = \frac{\sigma_1^2 + \sigma_2^2 \pm \sqrt{\Delta}}{2} \quad \text{eigenvalues}$$

$$\Sigma\lambda = \frac{1}{2}\left(\sigma_1^2 + \sigma_2^2 + \sqrt{\Delta} + \sigma_1^2 + \sigma_2^2 - \sqrt{\Delta}\right) = \sigma_1^2 + \sigma_2^2 = \text{variance}$$

$$A\gamma = \lambda\gamma$$

$$\begin{pmatrix} \sigma_2^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}\begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} = \frac{\sigma_1^2 + \sigma_2^2 + \sqrt{\Delta}}{2}\begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix}$$

$$\begin{vmatrix} \sigma_2^2\gamma_1 + \rho\sigma_1\sigma_2\gamma_2 \\ \rho\sigma_1\sigma_2\gamma_1 + \sigma_2^2\gamma_2 \end{vmatrix} = \begin{matrix} \left(\frac{\sigma_1^2 + \sigma_2^2 + \sqrt{\Delta}}{2}\right)\gamma_1 \\ \left(\frac{\sigma_1^2 + \sigma_2^2 + \sqrt{2}}{2}\right)\gamma_2 \end{matrix}$$

$$2(\sigma_2^2\gamma_1 + \rho\sigma_1\sigma_2\gamma_2) = (\sigma_1^2 + \sigma_2^2 + \sqrt{\Delta})\gamma_1$$

$$2\rho\sigma_1\sigma_2\gamma_2 = (\sigma_1^2 + \sigma_{\cancel{2}}^2 - 2\sigma_2^2 + \sqrt{2})\gamma_1$$

$$2\rho\sigma_1\sigma_2\gamma_2 = (\sigma_1^2 + \sigma_2^2 + \sqrt{\Delta})\gamma_1$$

$$2\rho\sigma_1\sigma_2\gamma_1 + \cancel{2\sigma_2^2\gamma_2} = \gamma_2(\sigma_1^2 + \sigma_2^2 + \sqrt{\Delta} - 2\sigma_2^2)$$

$$2\rho\sigma_1\sigma_2\gamma_1 = \gamma_2(\sigma_1^2 - \sigma_2^2 + \sqrt{\Delta})$$