Reg/DFA/NFA (1)

- (**DFA**) $M = (Q, \Sigma, \delta, q_0, F)$, $\delta : Q \times \Sigma \to Q$
- (**NFA**) $M = (Q, \Sigma, \delta, q_0, F)$, $\delta : Q \times \Sigma_\varepsilon \to \mathcal{P}(Q)$
- (GNFA) $(Q, \Sigma, \delta, q_0, q_\mathrm{a})$, $\delta : (Q \setminus \{q_\mathrm{a}\}) \times (Q \setminus \{q_\mathrm{start}\} \longrightarrow \mathcal{R}$ (where $\mathcal{R} = \{\text{all regex over } \Sigma\}$)
- GNFA accepts $w \in \Sigma^*$ if $w = w_1 \cdots w_k$, where $w_i \in \Sigma^*$ and there exists a sequence of states $q_0, q_1, \dots, q_k$ s.t. $q_0 = q_\mathrm{start}$, $q_k = q_\mathrm{a}$ and for each i, we have $w_i \in L(R_i)$, where $R_i = \delta(q_{i-1}, q_i)$.
- (**DFA-to-GNFA**) $G = (Q', \Sigma, \delta', s, a)$, $\quad Q' = Q \cup \{s, a\}$, $\delta'(s, \varepsilon) = q_0$, $\quad$ For each $q \in F$, $\delta'(q, \varepsilon) = a$,
- (**P.L.**) If A is a regular lang., then $\exists p$ s.t. every string $s \in A$, $|s| \geq p$, can be written as $s = xyz$, satisfying the following:
  - $\forall i \geq 0, xy^iz \in A$
  - $|y| > 0$
  - $|xy| \leq p$
- Every NFA can be converted to an equivalent one that has a single accept state.
- (**regular grammar**) $G = (V, \Sigma, R, S)$. Rules: $A \to aB$, $A \to a$ or $S \to \varepsilon$. ($A, B, S \in V$ and $a \in \Sigma$).

	NReg	**Reg**	**CFL**	**TD**	**TR**
$L_1 \cup L_2$		✓	✓	✓	✓
$L_1 \cap L_2$		✓	✗	✓	✓
$\overline{L}$	✓	✓	✗	✓	✗
$L_1 \cdot L_2$		✓	✓	✓	✓
L^*		✓	✓	✓	✓
$L^\mathcal{R}$		✓	✓	✓	✓
$L \cap R$		✓	✓	✓	✓

CFG (2)

- (**CFG**) $G = (\underset{\text{n.t.}}{V}, \underset{\text{ter.}}{\Sigma}, R, S)$. Rules: $A \to w$. (where $A \in V$ and $w \in (V \cup \Sigma)^*$).
  - A derivation of w is a **leftmost derivation** if at every step the leftmost remaining variable is the one replaced.
  - w is derived **ambiguously** in G if it has at least two different l.m. derivations.
  - G is **ambiguous** if it generates at least one string ambiguously.
  - A CFG is ambiguous iff it generates some string with two different parse trees.
- (**P.L.**) If L is a CFL, then $\exists p$ s.t. any string $s \in L$ with $|s| \geq p$ can be written as $s = uvxyz$, where:
  - $\forall i \geq 0, uv^ixy^iz \in L$
  - $|vxy| \leq p$
  - $|vy| > 0$
- (**CNF**) $A \to BC$, $A \to a$, or $S \to \varepsilon$, (where $A, B, C \in V$, $a \in \Sigma$, and $B, C \neq S$).
  - If G is a CFG in CNF, and $w \in L(G)$, then $|w| \leq 2^{|h|} - 1$, where h is the height of the parse tree for w.
  - Every CFL is generated by a CFG in CNF.
- L is **CFL** if it is generated by some CFG.
  - A CFL is **inherently ambiguous** if all CFGs that generate it are ambiguous.
  - Every CFL is generated by a CFG in CNF.
  - Every regular lang. is CFL.
- (**derivation**) $S \Rightarrow u_1 \Rightarrow u_2 \Rightarrow \cdots \Rightarrow u_n = w$, where each u_i is in $(V \cup \Sigma)^*$. (in this case, G **generates** w (or S **derives** w), $S \overset{*}{\Rightarrow} w$)
- (**PDA**) $M = (Q, \underset{\text{input}}{\Sigma}, \underset{\text{stack}}{\Gamma}, \delta, q_0 \in Q, \underset{\text{accepts}}{F} \subseteq Q)$. (where Q, Σ, Γ, F finite). $\delta : Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \longrightarrow \mathcal{P}(Q \times \Gamma_\varepsilon)$.
- M **accepts** $w \in \Sigma^*$ if there is a seq. $r_0, r_1, \dots, r_m \in Q$ and $s_0, , s_1, \dots, s_m \in \Gamma^*$ s.t.:
  - $r_0 = q_0$ and $s_0 = \varepsilon$
  - For $i = 0, 1, \dots, m-1$, we have $(r_i, b) \in \delta(r_i, w_{i+1}, a)$, where $s_i = at$ and $s_{i+1} = bt$ for some $a, b \in \Gamma_\varepsilon$ and $t \in \Gamma^*$.
  - $r_m \in F$
- A PDA can be represented using a state diagram, where each transition is labeled by the notation "$a, b \to c$" to denote that the PDA:
  - Reads a from the input (or read nothing if $a = \varepsilon$)
  - Pops b from the stack (or pops nothing if $b = \varepsilon$)
  - Pushes c onto the stack (or pushes nothing if $c = \varepsilon$)
- (**CSG**) $G = (V, \Sigma, R, S)$. Rules: $S \to \varepsilon$ or $\alpha A \beta \to \alpha \gamma \beta$ where:
  - $\alpha, \beta \in (V \cup \Sigma \setminus \{S\})^*$
  - $\gamma \in (V \cup \Sigma \setminus \{S\})^+$
  - $A \in V$