

Reg / DFA / NFA

	REG	REG	CFL	DEC.	REC.	P	NP	NPC
$L_1 \cup L_2$	no	✓	✓	✓	✓	✓	✓	no
$L_1 \cap L_2$	no	✓	no	✓	✓	✓	✓	no
\overline{L}	✓	✓	no	✓	no	✓	?	?
$L_1 \cdot L_2$	no	✓	✓	✓	✓	✓	✓	no
L^*	no	✓	✓	✓	✓	✓	✓	no
L^R	✓	✓	✓	✓	✓	✓		
$L_1 \setminus L_2$	no	✓	no	✓	no	✓	?	
$L \cap R$	no	✓	✓	✓	✓	✓		

- **(DFA)** $M = (Q, \Sigma, \delta, q_0, F)$, $\delta : Q \times \Sigma \rightarrow Q$
- **(NFA)** $M = (Q, \Sigma, \delta, q_0, F)$, $\delta : Q \times \Sigma_\varepsilon \rightarrow \mathcal{P}(Q)$

- **(GNFA)** $(Q, \Sigma, \delta, q_0, q_a)$, $\delta : (Q \setminus \{q_a\}) \times (Q \setminus \{q_{\text{start}}\}) \rightarrow \mathcal{R}$ (where $\mathcal{R} = \{\text{all regex over } \Sigma\}$)
- GNFA accepts $w \in \Sigma^*$ if $w = w_1 \cdots w_k$, where $w_i \in \Sigma^*$ and there exists a sequence of states q_0, q_1, \dots, q_k s.t. $q_0 = q_{\text{start}}, q_k = q_a$ and for each i , we have $w_i \in L(R_i)$, where $R_i = \delta(q_{i-1}, q_i)$.
- **(DFA \rightsquigarrow GNFA)** $G = (Q', \Sigma, \delta', s, a)$, $Q' = Q \cup \{s, a\}$, $\delta'(s, \varepsilon) = q_0$, For each $q \in F$, $\delta'(q, \varepsilon) = a$, ((TODO...))
- Every NFA can be converted to an equivalent one that has a single accept state.
- **(reg. grammar)** $G = (V, \Sigma, R, S)$. Rules: $A \rightarrow aB$, $A \rightarrow a$ or $S \rightarrow \varepsilon$. ($A, B, S \in V$; $a \in \Sigma$).
- **(NFA \rightsquigarrow DFA)**

- $N = (Q, \Sigma, \delta, q_0, F)$
- $D = (Q' = \mathcal{P}(Q), \Sigma, \delta', q'_0 = E(\{q_0\}), F')$
- $F' = \{q \in Q' \mid \exists p \in F : p \in q\}$
- $E(\{q\}) := \{q\} \cup \{\text{states reachable from } q \text{ via } \varepsilon\text{-arrows}\}$
- $\forall R \subseteq Q, \forall a \in \Sigma, \delta'(R, a) = E\left(\bigcup_{r \in R} \delta(r, a)\right)$
- $L(\varepsilon \cup 0\Sigma^*0 \cup 1\Sigma^*1) = \{w \mid \#_w(01) = \#_w(10)\}$,
- $L = \{a^nb^n : n \in \Sigma^*\} \equiv a(a \cup b)^*b$
- $L = \{w \in \Sigma^* : \#_w(0) \geq 2 \wedge \#_w(1) \leq 1\} \equiv ((0 \cup 1)^*0(0 \cup 1)^*)$

PL: $A \in \text{REG} \implies \exists p : \forall s \in A, |s| \geq p, s = uvxyz, \text{ (i) } \forall i \geq 0, uv^i xy^i z \in A, \text{ (ii) } |y| > 0 \text{ and (iii) } |xy| \leq p.$

- $\{w = a^{2^k}\}; \quad k = \lfloor \log_2 |w| \rfloor, s = a^{2^k} = xyz.$
 $2^k = |xyz| < |xy^2z| \leq |xyz| + |xy| \leq 2^k + p < 2^{k+1}.$

- $\{w = a^N b^N\}; \quad N = 0^p 1^p \notin L.$
- $\{a^n b^n\}; \quad s = a^p b^p = xyz, \text{ where } |y| > 0 \text{ and } |xy| \leq p.$

- Then $xy^2z = a^{p+|y|}b^p \notin L.$
- $L = \{a^p : p \text{ is prime}\}; \quad s = a^t = xyz \text{ for prime } t \geq p.$
 $r := |y| > 0$

CFL / CFG / PDA

- **(CFG)** $G = (V, \Sigma, R, S)$. Rules: $A \rightarrow w$. (where $A \in V$ and $w \in (V \cup \Sigma)^*$).
- A derivation of w is a **leftmost derivation** if at every step the leftmost remaining variable is the one replaced.
- w is derived **ambiguously** in G if it has at least two different l.m. derivations. G is **ambiguous** if it generates at least one string ambiguously. A CFG is ambiguous iff it generates some string with two different parse trees. A CFL is **inherently ambiguous** if all CFGs that generate it are ambiguous.
- **(CNF)** $A \rightarrow BC$, $A \rightarrow a$, or $S \rightarrow \varepsilon$, (where $A, B, C \in V$, $a \in \Sigma$, and $B, C \neq S$).
- **(CFG \rightsquigarrow CNF)** **(1.)** Add a new start variable S_0 and a rule $S_0 \rightarrow S$. **(2.)** Remove ε -rules of the form $A \rightarrow \varepsilon$ (except for $S_0 \rightarrow \varepsilon$). and remove A 's occurrences on the RH of a rule (e.g.: $R \rightarrow uAvAw$ becomes

- $R \rightarrow uAvAw \mid uAvw \mid uvAw \mid uvw$. where $u, v, w \in (V \cup \Sigma)^*$. **(3.)** Remove unit rules $A \rightarrow B$ then whenever $B \rightarrow u$ appears, add $A \rightarrow u$, unless this was a unit rule previously removed. ($u \in (V \cup \Sigma)^*$). **(4.)** Replace each rule $A \rightarrow u_1 u_2 \cdots u_k$ where $k \geq 3$ and $u_i \in (V \cup \Sigma)$, with the rules $A \rightarrow u_1 A_1, A_1 \rightarrow u_2 A_2, \dots, A_{k-2} \rightarrow u_{k-1} u_k$, where A_i are new variables. Replace terminals u_i with $U_i \rightarrow u_i$.
- If $G \in \text{CNF}$, and $w \in L(G)$, then $|w| \leq 2^{|h|} - 1$, where h is the height of the parse tree for w .
- $L \in \text{CFL} \Leftrightarrow \exists \frac{G}{\text{CFG}} : L = L(G) \Leftrightarrow \exists \frac{M}{\text{PDA}} : L = L(M)$
- $\forall L \in \text{CFL}, \exists G \in \text{CNF} : L = L(G)$.
- **(derivation)** $S \Rightarrow u_1 \Rightarrow u_2 \Rightarrow \cdots \Rightarrow u_n = w$, where each u_i is in $(V \cup \Sigma)^*$. (in this case, G **generates** w (or S **derives** w), $S \xRightarrow{*} w$)
- **(PDA)** $M = (Q, \Sigma, \Gamma, \delta, q_0 \in Q, \frac{F}{\text{accepts}} \subseteq Q)$. (where Q, Σ, Γ, F finite). $\delta : Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \rightarrow \mathcal{P}(Q \times \Sigma_\varepsilon)$.

- M **accepts** $w \in \Sigma^*$ if there is a seq. $r_0, r_1, \dots, r_m \in Q$ and $s_0, s_1, \dots, s_m \in \Gamma^*$ s.t.:
- $r_0 = q_0$ and $s_0 = \varepsilon$
- For $i = 0, 1, \dots, m-1$, we have $(r_i, b) \in \delta(r_i, w_{i+1}, a)$, where $s_i = at$ and $s_{i+1} = bt$ for some $a, b \in \Gamma_\varepsilon$ and $t \in \Gamma^*$.
- $r_m \in F$
- A PDA can be represented by a state diagram, where each transition is labeled by the notation " $a, b \rightarrow c$ " to denote that the PDA: **Reads** a from the input (or read nothing if $a = \varepsilon$). **Pops** b from the stack (or pops nothing if $b = \varepsilon$). **Pushes** c onto the stack (or pushes nothing if $c = \varepsilon$)
- **(CSG)** $G = (V, \Sigma, R, S)$. Rules: $S \rightarrow \varepsilon$ or $\alpha A \beta \rightarrow \alpha \gamma \beta$ where: $\alpha, \beta \in (V \cup \Sigma \setminus \{S\})^*$; $\gamma \in (V \cup \Sigma \setminus \{S\})^+$; $A \in V$.

PL: $L \in \text{CFL} \implies \exists p : \forall s \in L, |s| \geq p, s = uvxyz, \text{ (i) } \forall i \geq 0, uv^i xy^i z \in L, \text{ (ii) } |vxy| \leq p, \text{ and (iii) } |vy| > 0.$

- $\{w = a^n b^n c^n\}; \quad s = a^p b^p b^p = uvxyz. vxy$ can't contain all of a, b, c thus $uv^2 xy^2 z$ must pump one of them less

- than the others.
- $\{uw : w \in \{a, b\}^*\};$

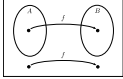
$$L \in \text{DECIDABLE} \iff (L \in \text{REC. and } L \in \text{co-REC.}) \iff \exists M_{\text{TM}} \text{ decides } L.$$

<ul style="list-style-type: none"> (TM) $M = (Q, \Sigma_{\text{input}} \subseteq \Gamma, \Gamma_{\text{tape}}, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$, where $\sqcup \in \Gamma$ (blank), $\sqcup \notin \Sigma$, $q_{\text{reject}} \neq q_{\text{accept}}$, and $\delta: Q \times \Gamma \longrightarrow Q \times \Gamma \times \{L, R\}$ (recognizable) accepts if $w \in L$, rejects/loops if $w \notin L$. <ul style="list-style-type: none"> $L \in \text{RECOGNIZABLE} \iff L \subseteq_m A_{\text{TM}}$. A is co-recognizable if \bar{A} is recognizable. 	<ul style="list-style-type: none"> Every inf. recognizable lang. has an inf. dec. subset. (decidable) accepts if $w \in L$, rejects if $w \notin L$. $L \in \text{DECIDABLE} \iff L \subseteq_m 0^*1^*$. $L \in \text{DECIDABLE} \iff L^R \in \text{DECIDABLE}$. (decider) TM that halts on all inputs. (Rice) Let P be a lang. of TM descriptions, s.t. (i) P is nontrivial (not empty and not all TM desc.) and (ii) for 	<ul style="list-style-type: none"> each two TM M_1 and M_2, we have $L(M_1) = L(M_2) \implies (\langle M_1 \rangle \in P \iff \langle M_2 \rangle \in P)$. Then P is undecidable. {all TMs} is countable; Σ^* is countable (for every finite Σ); {all languages} is uncountable; {all infinite binary sequences} is uncountable. DFA \equiv NFA \equiv GNFA \equiv REG \subset NPDA \equiv CFG \subset DTM \equiv NTM
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$$\text{FINITE} \subset \text{REGULAR} \subset \text{CFL} \subset \text{CSL} \subset \text{DECIDABLE} \subset \text{RECOGNIZABLE}$$

<ul style="list-style-type: none"> (unrecognizable) $\overline{A_{\text{TM}}}, \overline{EQ_{\text{TM}}}, EQ_{\text{CFG}}, \overline{HALT_{\text{TM}}}$, $\text{REGULAR}_{\text{TM}} = \{M \text{ is a TM and } L(M) \text{ is regular}\}$, $E_{\text{TM}}, EQ_{\text{TM}} = \{M_1, M_2 \text{ are TMs and } L(M_1) = L(M_2)\}$, $ALL_{\text{CFG}}, EQ_{\text{CFG}}$ (recognizable but undecidable) $A_{\text{TM}}, HALT_{\text{TM}} = \{\langle M, w \rangle \mid M \text{ is a TM halts on } w\}$, 	$D = \{p \mid p \text{ is an int. poly. with an int. root}\}, \overline{EQ_{\text{CFG}}}, \overline{E_{\text{TM}}}$, <ul style="list-style-type: none"> (decidable) $A_{\text{DFA}}, A_{\text{NFA}}, A_{\text{REG}}, E_{\text{DFA}}, EQ_{\text{DFA}}, A_{\text{CFG}}, E_{\text{CFG}}, A_{\text{LBA}}, ALL_{\text{DFA}} = \{\langle M \rangle \mid M \text{ is a DFA, } L(A) = \Sigma^*\}$, $A_{\varepsilon\text{CFG}} = \{\langle G \rangle \mid G \text{ is a CFG that generates } \varepsilon\}$, $\text{INFINITE}_{\text{DFA}}, \text{INFINITE}_{\text{PDA}}$ 	<ul style="list-style-type: none"> (not CFL) $\{a^i b^j c^k \mid 0 \leq i \leq j \leq k\}, \{a^n b^n c^n \mid n \in \mathbb{N}\}, \{ww \mid w \in \{a, b\}^*\}, \{a^{n^2} \mid n \geq 0\}, \{w \in \{a, b, c\}^* \mid \#_a(w) = \#_b(w) = \#_c(w)\}, \{a^p \mid p \text{ is prime}\}, L = \{ww^R w \mid w \in \{a, b\}^*\}$ (CFL but not REGULAR) $\{w \in \{a, b\}^* \mid w = w^R\}, \{ww^R \mid w \in \{a, b\}^*\}, \{a^n b^n \mid n \in \mathbb{N}\}, \{w \in \{a, b\}^* \mid \#_a(w) = \#_b(w)\}, L = \{a^n b^m \mid n \neq m\}$
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$$\text{Mapping Reduction: } A \subseteq_m B \text{ if } \exists f: \Sigma^* \rightarrow \Sigma^*: \forall w \in \Sigma^*, w \in A \iff f(w) \in B \text{ and } f \text{ is computable.}$$

<ul style="list-style-type: none"> $f: \Sigma^* \rightarrow \Sigma^*$ is computable if there exists a TM M s.t. for every $w \in \Sigma^*$, M halts on w and outputs $f(w)$ on its tape. If $A \subseteq_m B$ and B is decidable, then A is dec. If $A \subseteq_m B$ and A is undecidable, then B is undec. If $A \subseteq_m B$ and B is recognizable, then A is rec. If $A \subseteq_m B$ and A is unrecognizable, then B is unrec. (transitivity) If $A \subseteq_m B$ and $B \subseteq_m C$, then $A \subseteq_m C$. $A \subseteq_m B \iff \bar{A} \subseteq_m \bar{B}$ (esp. $A \subseteq_m \bar{A} \iff \bar{A} \subseteq_m A$) 	<p style="text-align: center;">EXAMPLES</p> <ul style="list-style-type: none"> $A_{\text{TM}} \subseteq_m S_{\text{TM}} = \{\langle M \rangle \mid w \in L(M) \iff w^R \in L(M)\}$, $f(\langle M, w \rangle) = \langle M' \rangle$, where $M' =$"On x, if $x \notin \{01, 10\}$, reject; if $x = 01$, return $M(x)$; if $x = 10$, accept;" $A_{\text{TM}} \subseteq_m L = \{\langle M, D \rangle \mid L(M) = L(D)\}$, $f(\langle M, w \rangle) = \langle M', D \rangle$, where $M' =$"On x: if $x = w$ return $M(x)$; otherwise, reject;" and D is DFA s.t. $L(D) = \{w\}$. $A \subseteq_m HALT_{\text{TM}}$, $f(w) = \langle M, \varepsilon \rangle$, where $M =$"On x: if $w \in A$, halt; if $w \notin A$, loop forever," 	<ul style="list-style-type: none"> $A_{\text{TM}} \subseteq_m CF_{\text{TM}} = \{\langle M \rangle \mid L(M) \text{ is CFL}\}$, $f(\langle M, w \rangle) = \langle N \rangle$, where $N =$"On x: if $x = a^n b^n c^n$, accept; otherwise, return $M(w)$," $A \subseteq_m B = \{0w \mid w \in A\} \cup \{1w \mid w \notin A\}$, $f(w) = 0w$. $E_{\text{TM}} \subseteq_m \text{USELESS}_{\text{TM}}$; $f(\langle M \rangle) = \langle M, q_{\text{accept}} \rangle$ $A_{\text{TM}} \subseteq_m EQ_{\text{TM}}$; $f(\langle M, w \rangle) = \langle M_1, M_2 \rangle$, where $M_1 =$"Accept everything"; $M_2 =$"On x: return $M(w)$," $A_{\text{TM}} \subseteq_m \overline{EQ_{\text{TM}}}$; $f(\langle M, w \rangle) = \langle M_1, M_2 \rangle$, where $M_1 =$"Reject everything"; $M_2 =$"On x: return $M(w)$," $ALL_{\text{CFG}} \subseteq_m EQ_{\text{CFG}}$; $f(\langle G \rangle) = \langle G, H \rangle$, where $L(H) = \Sigma^*$.
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$$\text{Polytime Reduction: } A \leq_P B \text{ if } \exists f: \Sigma^* \rightarrow \Sigma^*: \forall w \in \Sigma^*, w \in A \iff f(w) \in B \text{ and } f \text{ is polytime computable.}$$

<ul style="list-style-type: none"> (Running time) decider M is a $f(n)$-time TM. $f: \mathbb{N} \rightarrow \mathbb{N}$, where $f(n)$ is the max. num. of steps that DTM (or NTM) M takes on any n-length input (and any branch of any n-length input. resp.). $\text{TIME}(t(n)) = \{L \mid L \text{ is dec. by } O(t(n)) \text{ DTM}\}$. $\text{NTIME}(t(n)) = \{L \mid L \text{ is dec. by } O(t(n)) \text{ NTM}\}$. $\mathbf{P} = \bigcup_{k \in \mathbb{N}} \text{TIME}(n^k)$ (verifier for L) TM V s.t. $L = \{w \mid \exists c: V(\langle w, c \rangle) = \text{accept}\}$. <ul style="list-style-type: none"> (certificate for $w \in L$) str. c s.t. $V(\langle w, c \rangle) = \text{accept}$. $\mathbf{NP} = \bigcup_{k \in \mathbb{N}} \text{NTIME}(n^k)$ $\mathbf{NP} = \{L \mid L \text{ is decidable by a PT verifier}\}$. $\mathbf{P} \subseteq \mathbf{NP}$. $f: \Sigma^* \rightarrow \Sigma^*$ is PT computable if there exists a PT TM M s.t. for every $w \in \Sigma^*$, M halts with $f(w)$ on its tape. If $A \leq_P B$ and $B \in \mathbf{P}$, then $A \in \mathbf{P}$. If $A \leq_P B$ and $B \leq_P A$, then A and B are PT equivalent, denoted $A \equiv_P B$. \equiv_P is an equivalence 	<ul style="list-style-type: none"> relation on NP. $\mathbf{P} \setminus \{\emptyset, \Sigma^*\}$ is an equivalence class of \equiv_P. NP-complete $= \{B \mid B \in \text{NP}, \forall A \in \text{NP}, A \leq_P B\}$. CLIQUE, SUBSET-SUM, SAT, 3SAT, VERTEX-COVER, HAMPATH, UHAMATH, 3COLOR \in NP-complete. $\emptyset, \Sigma^* \notin \text{NP-complete}$. If $B \in \text{NP-complete}$ and $B \in \mathbf{P}$, then $\mathbf{P} = \text{NP}$. If $B \in \text{NP-complete}$ and $C \in \text{NP}$ s.t. $B \leq_P C$, then $C \in \text{NP-complete}$. If $\mathbf{P} = \text{NP}$, then $\forall A \in \mathbf{P} \setminus \{\emptyset, \Sigma^*\}, A \in \text{NP-complete}$. <p style="text-align: center;">EXAMPLES</p> <ul style="list-style-type: none"> SAT \leq_P DOUBLE-SAT; $f(\phi) = \phi \wedge (x \vee \neg x)$ SUBSET-SUM \leq_P SET-PARTITION; $f(\langle x_1, \dots, x_m, t \rangle) = \langle x_1, \dots, x_m, S - 2t \rangle$, where S sum of x_1, \dots, x_m, and t is the target subset-sum. 3COLOR $\leq_P 3COLOR_{\text{almost}}$; $f(\langle G \rangle) = \langle G' \rangle$, where $G' = G \cup K_4$ 	<ul style="list-style-type: none"> VERTEX-COVER \leq_P WVC; $f(\langle G, k \rangle) = \langle G, w, k \rangle$, $\forall v \in V(G), w(v) = 1$ HAM-PATH \leq_P 2HAM-PATH; $f(\langle G, s, t \rangle) = \langle G', s', t' \rangle$, where $V' = V \cup \{s', t', a, b, c, d\}$, $E' = E \cup \{(s', a), (a, b), (b, s)\} \cup \{(s', b), (b, a), (a, s)\} \cup \{(t, c), (c, d), (d, t')\} \cup \{(t, d), (d, c), (c, t')\}$. CLIQUE \leq_P HALF-CLIQUE; undir. G has k-clique undir. G has $V /2$-clique $f(\langle G = (V, E), k \rangle) = \langle G' = (V', E') \rangle$, if $k = \frac{ V }{2}$, $E = E'$, $V' = V$. if $k > \frac{ V }{2}$, $V' = V \cup \{j = 2k - V \text{ new nodes}\}$. if $k < \frac{ V }{2}$, $V' = V \cup \{j = V - 2k \text{ new nodes}\}$ and $E' = E \cup \{\text{edges for new nodes}\}$ UHAMPATH \leq_P PATH$_{\geq k}$; $f(\langle G, a, b \rangle) = \langle G, a, b, k = V(G) \rangle$ CLIQUE \leq_P INDEPENDENT-SET SET-COVER \leq_P VERTEX-COVER 3SAT \leq_P SET-SPLITTING INDEPENDENT-SET \leq_P VERTEX-COVER VERTEX-COVER \leq_P CLIQUE
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Counterexamples

<ul style="list-style-type: none"> $A \subseteq_m B$ and $B \in \text{REG}$, but, $A \notin \text{REG}$: $A = \{0^n 1^n \mid n \geq 0\}, B = \{1\}, f: A \rightarrow B, f(w) = \begin{cases} 1 & \text{if } w \in A \\ 0 & \text{if } w \notin A \end{cases}$ $L \in \text{CFL}$ but $\bar{L} \notin \text{CFL}$: $L = \{x \mid \forall w \in \Sigma^*, x \neq ww\}, \bar{L} = \{ww \mid w \in \Sigma^*\}$. $L_1, L_2 \in \text{CFL}$ but $L_1 \cap L_2 \notin \text{CFL}$: $L_1 = \{a^n b^n c^m\}, L_2 = \{a^m b^n c^n\}, L_1 \cap L_2 = \{a^n b^n c^n\}$. $L_1 \in \text{CFL}, L_2$ is infinite, but $L_1 \setminus L_2 \notin \text{REG}$: $L_1 = \Sigma^*, L_2 = \{a^n b^n \mid n \geq 0\}, L_1 \setminus L_2 = \{a^m b^n \mid m \neq n\}$. 	<ul style="list-style-type: none"> $L_1, L_2 \in \text{REG}, L_1 \not\subseteq L_2, L_2 \not\subseteq L_1$, but, $(L_1 \cup L_2)^* = L_1^* \cup L_2^*$: $L_1 = \{a, b, ab\}, L_2 = \{a, b, ba\}$. $L_1 \in \text{REG}, L_2 \notin \text{REG}$, but, $L_1 \cap L_2 \in \text{REG}$, and $L_1 \cup L_2 \in \text{REG}$: $L_1 = L(a^* b^*)$, $L_2 = \{a^n b^n \mid n \geq 0\}$. $L_1, L_2, L_3, \dots \in \text{REG}$, but, $\bigcup_{i=1}^{\infty} L_i \notin \text{REG}$: $L_i = \{a^i b^i\}, \bigcup_{i=1}^{\infty} L_i = \{a^n b^n \mid n \geq 0\}$. $L_1 \cdot L_2 \in \text{REG}$, but $L_1 \notin \text{REG}$: $L_1 = \{a^n b^n \mid n \geq 0\}, L_2 = \Sigma^*$. $L_2 \in \text{CFL}$, and $L_1 \subseteq L_2$, but $L_1 \notin \text{CFL}$: $\Sigma = \{a, b, c\}, L_1 = \{a^n b^n c^n \mid n \geq 0\}, L_2 = \Sigma^*$. 	<ul style="list-style-type: none"> $L_1, L_2 \in \text{DECIDABLE}$, and $L_1 \subseteq L \subseteq L_2$, but $L \in \text{UNDECIDABLE}$: $L_1 = \emptyset, L_2 = \Sigma^*, L$ is some undecidable language over Σ. $L_1 \in \text{REG}, L_2 \notin \text{CFL}$, but $L_1 \cap L_2 \in \text{CFL}$: $L_1 = \{\varepsilon\}, L_2 = \{a^n b^n c^n \mid n \geq 0\}$. $L^* \in \text{REG}$, but $L \notin \text{REG}$: $L = \{a^p \mid p \text{ is prime}\}, L^* = \Sigma^* \setminus \{a\}$. $A \not\subseteq_m \bar{A}$: $A = A_{\text{TM}} \in \text{RECOGNIZABLE}, \bar{A} = \overline{A_{\text{TM}}} \notin \text{RECOG}$. $A \notin \text{DEC.}, A \subseteq_m \bar{A}$:
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