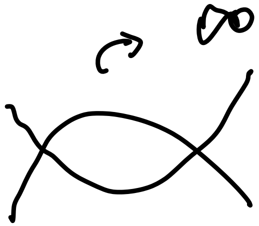
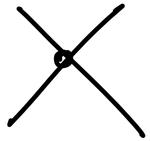
ONE-LOOP PROCESSES

$\underline{\lambda\phi^4}$



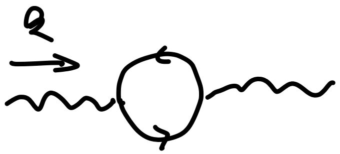


$$\tilde{\lambda}(\mu_0) = \lambda_0$$

$$\downarrow$$

$$\tilde{\lambda}(\mu)$$

$\underline{QED}$



$$= D_{\mu\nu} = -i \underbrace{\frac{e^2}{k^2}\eta_{\mu\nu}} + \text{GAUGE DEPENDENT PARTS}$$

$$\Longrightarrow \quad -i\frac{e_0^2}{k^2\left(1 - e_0^2\Pi_R(k^2)\right)}\eta_{\mu\nu} = -i\frac{e^2(k^2)}{k^2}\eta^{\mu\nu}$$

$\underline{SM}$: $SU(3) \times SU(2) \times U(1)$

$\underline{\text{2 WAYS}}$

$$e A_\mu J^\mu \Longleftrightarrow \breve{A}_\mu J^\mu$$

$$\mathcal{L} = i\bar{\psi}\not{D}\psi - m\bar{\psi}\psi - \frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} \qquad \sim SU(N)$$

$$\not{D} = \gamma^\mu D_\mu \quad \text{with} \quad D_\mu = \partial_\mu - ig\, G^A_\mu T^A$$

T^A : GENERATORS $\qquad \psi \to U\psi \qquad U \in SU(N)$

$\psi \sim$ FUNDAMENTAL REP. / DEFINING REP.

$$\psi = \begin{pmatrix} \psi_N \\ \vdots \\ \psi_1 \end{pmatrix} \qquad U = e^{i\alpha^A T^A}$$

$$U^\dagger U = U U^\dagger = \mathbb{1} \iff (T^A)^\dagger = T^A$$

$$\det U = 1 \iff \mathrm{Tr}\, T^A = 0$$

$\underline{\underline{SU(2)}}$ $\qquad T^A = \frac{\sigma^A}{2} \qquad A = 1, 2, 3 \qquad$ PAULI MATRICES

$\underline{\underline{SU(N)}}$ $\qquad A = 1, \dots, N^2 - 1$

$\underline{SU(N)} \iff T_R^A \; : \; [T_R^A, T_R^B] = i f^{ABC} T_R^C$ Lie $\underline{\text{ALGEBRA}}$

f: STRUCTURE CONSTANTS.

$f^{ABC} \in \mathbb{R}$, COMPLETLY ANTISYMMETRIC

$A = 1 \dots N^2-1$

$\underline{SU(2)}$ $[T^A, T^B] = i\varepsilon^{ABC} T^C$

$\sim \underline{SO(3)}$

$(i)^2 = -1$

* $\underline{\text{ADJOINT REPRESENTATION}}$ $f^{ABC} \implies (T_G^A)_{ij} = i f^{iAj}$

$i, j = 1 \dots N^2-1$

$\underline{\text{CHECK}}$! $[T_G^A, T_G^B] = i f^{ABC} T_G^C$

① KEYWORD: JACOBI IDENTITY

② $\underline{SU(2)}$ $(T_G^A)_{ij} = i\,\varepsilon^{iAj} = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$

WHY IS THAT IMPORTANT?

$$\bar{\psi}\,U^{+} \quad U\,\not{D}\,\psi$$

* $\psi' = U\psi \rightarrow U(x)\psi(x)$

$(D_\mu \psi)' = U(x)(D_\mu \psi)$

$\Rightarrow \mathcal{L} = i\bar{\psi}\not{D}\psi$

$= i\bar{\psi}'(\not{D}\psi)'$

$$D_\mu = \partial_\mu - i g\, G_\mu^A T^A$$

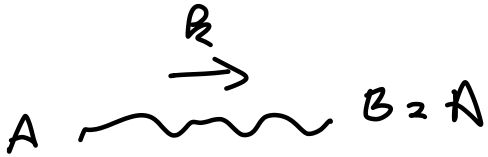
* $$\begin{cases} G_\mu'^A = G_\mu^A + f^{ABC} G_\mu^B \alpha^C + \frac{1}{g}\partial_\mu \alpha^A \\ \psi' = \psi + i\alpha^A T^A \psi \end{cases}$$

$G_\mu^A \sim$ ADJOINT REP.

EX $SU(2)$ $A = 1,2,3$ $G_\mu'^A = G_\mu^A + \varepsilon^{ABC} G_\mu^B \alpha^C$

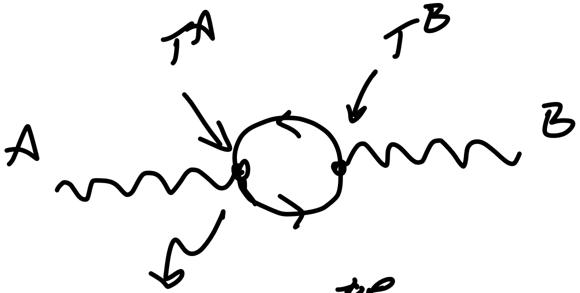
PROPAGATOR OF SU(N) THEORY

$$D^{AB}_{\mu\nu} = -i \frac{g_{\mu\nu}\,\delta_{AB}}{k^2}$$



LOOP CORRECTIONS

① $\bar{\psi}\,\not{D}\,\psi$



$\sim \delta^{AB} \sim \mathrm{Tr}[T^A T^B]$

$G^A_\mu\, \psi \gamma^\mu T^A \psi \xRightarrow{FR} i\gamma^\mu T^A$

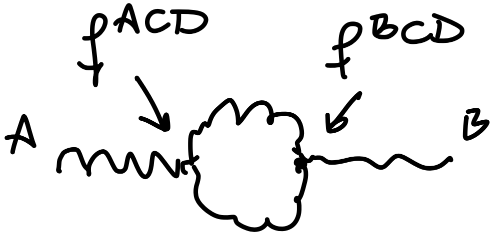
$\mathrm{Tr}[T^A_r T^B_r] = C(r)\,\delta^{AB}$

FUDAMENTAL REPS SU(N) $= \frac{1}{2}\delta^{AB}$

ex SU(2) $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$

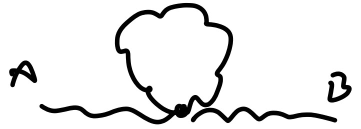
② $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \Longleftrightarrow G^A_{\mu\nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + g f^{ABC} G^B_\mu G^C_\nu$

$\mathcal{L} = -\frac{1}{4} G^A_{\mu\nu} G^{\mu\nu A}$ = FREE PART. + CUBIC + QUARTIC



CUBIC

$+$



QUARTIC

$$\sim \delta^{AB} \sim \sum_{C,D} f^{ACD} f^{BCD} = \mathrm{Tr}\left[T_G^A T_G^B\right]$$

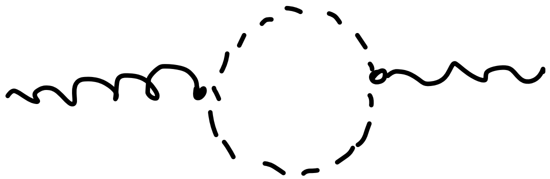
$$\mathrm{Tr}\left[T_G^A T_G^B\right] = \sum_{C,D} f^{ACD} f^{BCD} = N \, \delta_{AB}$$

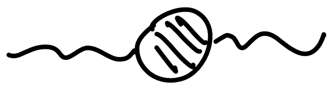
2 COMPLICATIONS

① $G'^{A}_{\mu} = G^{A}_{\mu} + \dots + \frac{1}{g}\partial_\mu \alpha^A$

$\mu = 0,1,2,3 \Leftrightarrow 4$ DOF $\Longleftrightarrow$ 2 DOF (ON-SHELL)

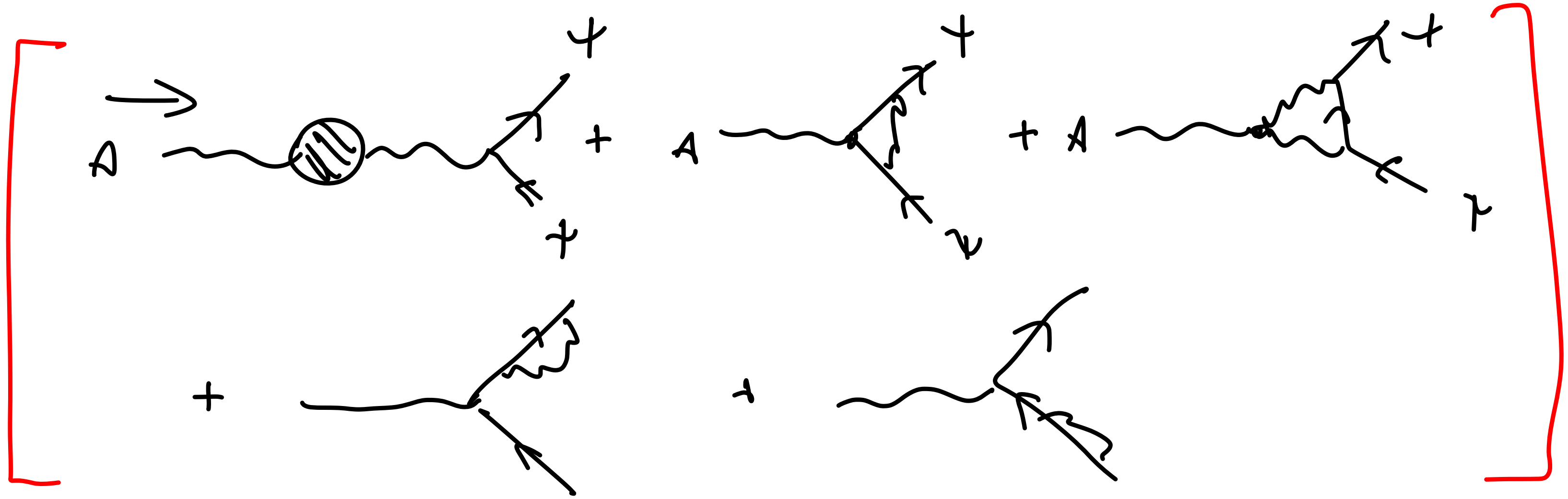
"GHOSTS"

$\Longrightarrow$ 

(2) QED  $\Longleftrightarrow$ $\boxed{e^2(k^2)} = \dfrac{e_0^2}{1 - e_0^2\,\Pi(k^2)}$

YM ~ NON-ABELIAN

PHYSICAL



PHYSICAL $\Longleftrightarrow$ DOES NOT DEPEND ON THE GAUGE CHOICE

$e^2(k^2) \Longleftrightarrow g^2(k^2)$

$\boxed{15:20}$

1) A_μ ~ CONNECTION ~ $\Gamma^{\alpha}_{\mu\nu}$ — DIFF. GEOMETRY

=> PARALLEL TRANSPORT

$$\boxed{\psi_p(x) = e^{i\int_y^x A_\mu dx^\mu}\,\psi(y)}$$

$$\psi(y) \to e^{i\alpha(y)}\,\psi(y)$$
$$\psi_p(x) \to e^{i\alpha(x)}\,\psi_p(x)$$

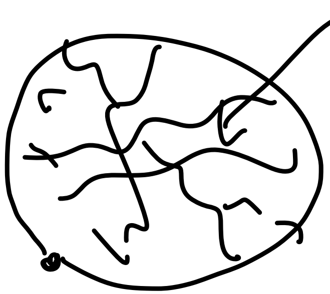
compare $\psi(x)$ with $\psi(y)$

$\psi_p(x)$ ~

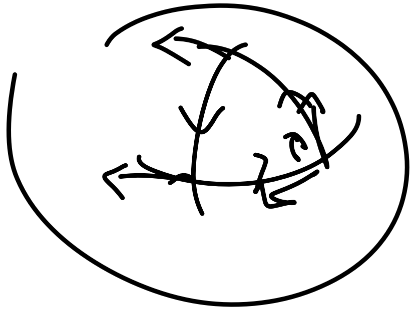
$$A_\mu \to A_\mu \pm \partial_\mu \alpha$$

$\psi(x) - \psi_p(x)$ = DERIVATIVE

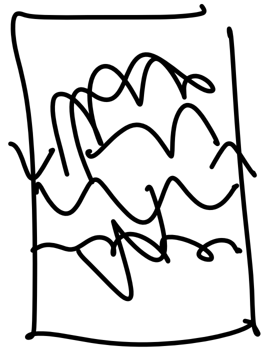
2)



$F_{\mu\nu} \neq 0$



$R \neq 0$



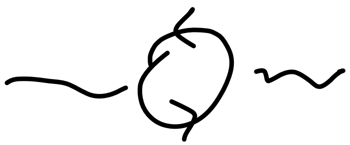
3) LATTICE

A_μ

$\boxed{g(e^2)}$ $\quad \boxed{i\mathcal{M}_{PHYS}} = \text{exp. cond.} \; (g, M)$

$= \quad " \quad " \quad (\tilde{g}, M)$

$* \; M \dfrac{d\mathcal{M}_{PHYS}}{dM} = 0 \qquad \sim$ CALLAN-SIMANZIK

$\Longrightarrow$  $\Longrightarrow \mu \dfrac{d}{d\mu} g(\mu) = \beta(g(\mu))$

BETA FUNCTION

QED $\boxed{\mu \dfrac{d}{d\mu} e^2(\mu) = \dfrac{e^4}{6\pi^2}} > 0$

$\Longrightarrow e^2(\mu) \nearrow$ AS $\mu \uparrow$

DARWIN TERM

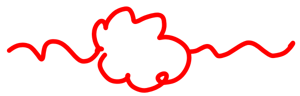
$\delta^3(r) \sim$ S-WAVE

SU(N) YM with fermions in Rep. R

$\mathrm{Tr}[T^A_R T^B_R] = C(R)\,\delta^{AB} = \frac{1}{2}\delta^{AB}$

N for SU(N)

$$\mu \frac{d}{d\mu} g = -\frac{g^3}{(4\pi)^2}\left[\frac{11}{3} C_2(G) - \frac{4}{3} N_f\, C(R)\right]$$



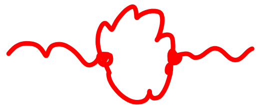
$$\Downarrow \qquad \frac{4\pi}{g^2(\mu)} - \frac{4\pi}{g^2(\mu_0)} = \frac{2}{4\pi}\left[\frac{11}{3} N - \frac{2}{3} N_f\right] \log \frac{\mu}{\mu_0}$$

$$\Rightarrow \quad \frac{g^2}{4\pi} = \alpha_g(\mu) \; : \qquad \alpha_g(\mu) = \frac{\alpha_g(\mu_0)}{1 + \frac{1}{2\pi}\alpha_g(\mu_0)\left(b_g\right) \ln \mu/\mu_0}$$

$\Rightarrow$ REN. GROUP

$$= \sum \#_k\, \alpha_g^k \left(\ln \mu/\mu_0\right)^k$$

$SU(3) \times SU(2) \times U(1)$



DIRAC FERMIONS

$$\mu \frac{d}{d\mu} g = -\frac{g^3}{(4\pi)^2}\left[\frac{11}{3}\, C_2(G) - \frac{4}{3} N_f \, C(R)\right]$$

$g = g_3 \sim SU(3)_c$ · $C_2(G) = N_c = 3$

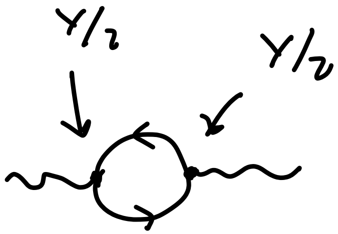
· N_f ? $\qquad N_f = 2 \times N_{FAMILIES}$

$g = g_2 \sim SU(2)_L$ · $C_2(G) = 2$

· N_f : WEYL FERMIONS

$$N_f = \frac{1}{2} \times N_{FAMILIES} \times (1 + 3) = 2 N_{FAMILIES}$$

$(Y_{e_L} = -1 \qquad Y_{e_R} = -2)$



$g = g_1 \sim U(1)_Y$

$\overset{\shortparallel}{g'}$

$$\begin{cases} \cdot\ C_2(G) = 0 \\ \cdot\ N_f = \dfrac{N_{FAMILIES}}{2} \times \left[\sum_i \left(\dfrac{Y_i}{2}\right)^2 \right] \\ \cdot\ C(1) = 1 \end{cases}$$

$\boxed{\text{SM}}$ PARTICLES! (BUT FOR TWO HIGGS)

$$\frac{1}{\alpha_3(\mu)} \hat{=} \frac{4\pi}{g_s^2} = \dots \quad \text{evol} \left(\ln \mu/\mu_0\right)$$

$$\frac{1}{\alpha_2(\mu)} = \frac{4\pi}{g^2} = \dots \quad \dots$$

$$\frac{1}{\alpha_1(\mu)} = \underbrace{\frac{5}{3}}_{\uparrow\ SU(5)} \frac{4\pi}{g'^2} = \dots$$

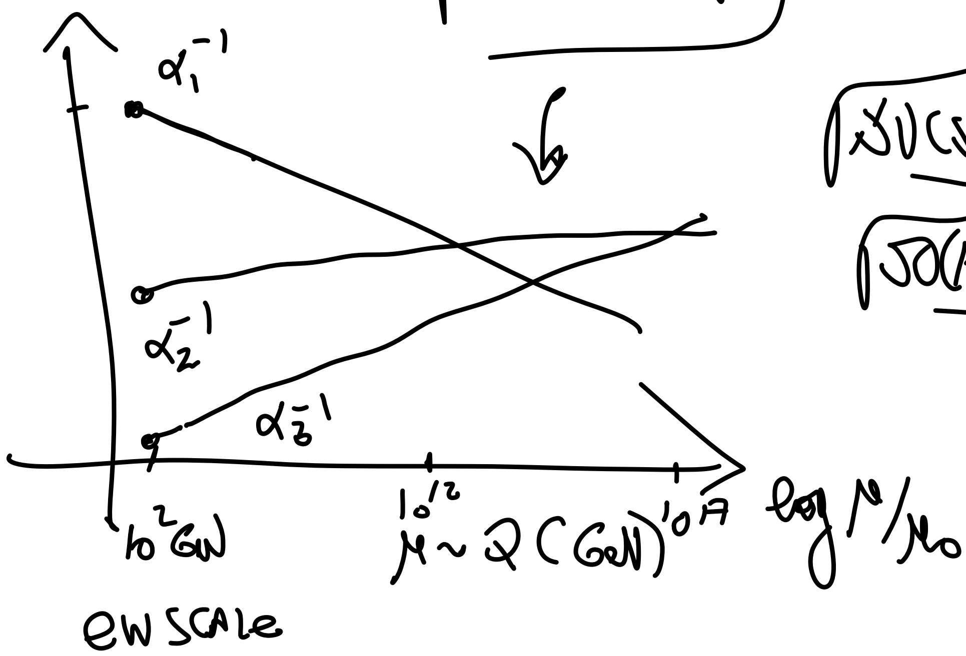
STRAIGHT LINES

PROTON DECAY

$\boxed{\sim 10^{16}\ \text{GeV}}$

$\boxed{SU(5)}$

$\boxed{SO(10)}$



LOOK AT FIGURE 22.1 PAGE 787 OF PESKIN-SCHROEDER

MYSTERY OF THE SM

$$\Rightarrow \quad Q = T_3 + Y/2 \quad \begin{cases} \text{u - QUARKS} & Q = +2/3 \\ \text{d - QUARKS} & Q = -1/3 \\ \text{e - leptons} & |Q = -1| = 1 \\ \nu - \text{"} & Q = 0 \end{cases} \iff \begin{matrix} Y \\ Y \; \ldots \\ Y \; \begin{smallmatrix} LH \\ RH \end{smallmatrix} \\ Y = -1 \end{matrix}$$

$\Rightarrow$ • CHARGE QUANTIZATION — WHY THERE ARE IN SIMPLE RATIO.

• $SU(N) \iff \boxed{\mathrm{Tr}[T^A] = 0}$

$\Rightarrow$ • THEORETICAL CONSTRAINT // CHIRAL ANOMALIES