3RD LECTURE: LOOPS & RUNNING COUPLINGS

1) LOOP CORRECTIONS OF $\lambda\phi^4$

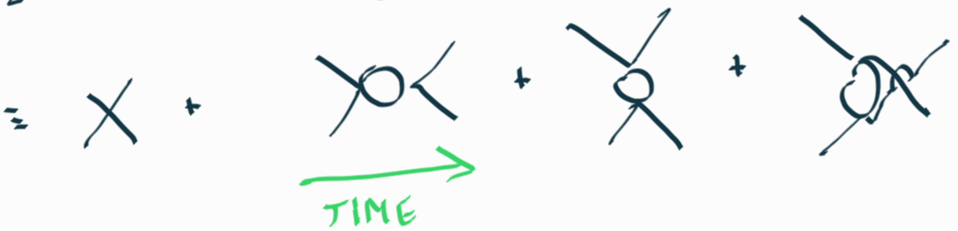
+ FORMALLY INFINITE $\Rightarrow$ REGULARIZE!
  - PAULI-VILLAR (PV)
  - OR - DIMENSIONAL REG.

+ REDEFINE IN TERMS OF MEASURED QUANTITIES $\Rightarrow$ $\lambda \to \lambda_0$ AT SOME SCALE SAY μ

+ RE-EXPRESS IN TERMS OF $\lambda_0(q^2)$

FOR $\lambda\phi^4$, THIS GIVES AT ONE-LOOP

$$i\mathcal{M} = -i\lambda + (-i\lambda)^2\left[iV(s) + iV(t) + iV(u)\right] \qquad *1$$



$$+\, O(\lambda^3)$$

WITH $V(p^2) = -\frac{1}{2}\int_0^1 dx \, \frac{\Gamma(2-D/2)}{(4\pi)^{D/2}} \, \frac{1}{(m^2 - x(1-x)p^2)^2}$

see eg ST, (10.23), PAGE 324

$$\xrightarrow[D\to 4]{} -\frac{1}{32\pi^2}\int_0^1 dx \left(\frac{2}{\varepsilon} - \gamma + \log(4\pi) - \log(m^2 - x(1-x)p^2)\right)$$

OR (MORE CLEAR)

$$\xrightarrow[PV]{} -\frac{1}{32\pi^2}\int_0^1 dx \left[\log(M^2) - \log(\cancel{m^2} - x(1-x)p^2)\right]$$

WITH $M^2 \gg m^2, p^2$

NEGLECT ~ HIGH ENERGY SCATTERING

LET $i\mathcal{M} = -i\tilde{\lambda}_0$ @ SOME (s_0, t_0, u_0)

THEN $-i\tilde{\lambda}_0 = -i\lambda + (-i\lambda)^2\left[\underbrace{iV(s_0) + iV(t_0) + iV(u_0)}\right]$

OR $\tilde{\lambda}_0 = \lambda + \lambda^2 C(\mu_0) + O(\lambda^3)$ $\quad \mu_0$ COLLECTIVE FOR (s_0, t_0, u_0)

$\hookrightarrow$ MEASURED @ μ_0

NOTA BENE: WE MEASURE CROSS-SECTIONS BUT THIS CAN BE EXPRESSED AS MEASURING AMPLITUDE $\sim$ COUPLINGS OR PARAMETERS

THEN $\tilde{\lambda}_0 = \lambda + \lambda^2 C(\mu_0) + O(\lambda^3)$ BY WHICH WE MEAN PERTURBATION THEORY

$$\Rightarrow \quad \lambda = \tilde{\lambda}_0 - \tilde{\lambda}_0^2 C(\mu_0) + O(\tilde{\lambda}_0^3)$$

SUBSTITUTING IN $*_1$ GIVES

$$i\mathcal{M} \equiv -i\tilde{\lambda}(\mu) = -i\left(\tilde{\lambda}_0 - \tilde{\lambda}_0^2 C(\mu_0)\right) + i\lambda_0^2 C(\mu) + O(\lambda_0^3)$$

OR

$$\Rightarrow \quad \underset{\substack{\text{COUPLING MEASURED} \\ @\ \mu \sim (s,t,\mu)}}{\tilde{\lambda}(\mu)} = \tilde{\lambda}_0 + \tilde{\lambda}_0^2 \underbrace{\left[C(\mu) - C(\mu_0)\right]}_{\Delta C \text{ is independent of } \Lambda \text{ (or } 1/\varepsilon) \text{ !!!}} + O(\tilde{\lambda}_0^3)$$

INDEED, $\Delta C = V(s) + V(t) + V(u) - V(s_0) - V(t_0) - V(u_0)$

WITH, EG $\quad = \frac{1}{32\pi^2}\left(\log\left(\frac{s}{s_0}\right) + \log\left(\frac{t}{t_0}\right) + \log\left(\frac{u}{u_0}\right)\right)$

AT HIGH ENERGIES, NEGLECTING $O(m^2/s)$ ETC. TERMS

REMARKS:

+ THE RESULT IS FINITE ONCE EXPRESSED IN TERMS OF OBSERVABLES. THIS IS CALLED RENORMALISATION BUT IT IS A BIT OF A MISNOMER.
+ $\tilde{\lambda}(\mu_0) = \lambda_0$ BY CONSTRUCTION
+ FOR THE PRESENT THEORY

$\tilde{\lambda}(\mu)$ INCREASES AS μ INCREASES

$\lambda(M)$ increases ...

$\Longrightarrow$ • This means that the theory is more strongly interacting at high energies...

• What happens to such a theory at high energies or, from a 4 perspective, where it comes from, is unclear.

• In brief, we don't like that!

Go back to the result in terms of λ. *1

$$i\mathcal{M} = -i\lambda - i\lambda^2\left[V(s)+V(u)+V(s)\right]$$

$$= -i\lambda + \frac{i\lambda^2}{32\pi^2}\left[3\log(M^2) + \text{finite parts}\right]$$

This depends explicitly of the cut-off scale M.

But the cut-off is ours. The amplitude cannot depend on it, ie $i\mathcal{M}(M) = i\mathcal{M}$

Let's express this. Change $M \to M + \delta M$

Then $i\mathcal{M} \to i\mathcal{M}$ is unchanged. In other words

$$\frac{d}{dM}\mathcal{M} = 0 = -i\frac{d}{dM}\lambda + i\frac{\lambda^2}{32\pi^2}\cdot\frac{6}{M} + O(\lambda^3)$$

↳ WE LOOK AT THE 1ST ORDER, SO $\lambda \frac{d\lambda}{dM}$ e.g. M^2 is higher order

$$\Rightarrow \boxed{M \frac{d}{dM} \lambda = \frac{3\lambda^2}{16\pi^2} \equiv \beta(\lambda)}$$

+ This result comes entirely from the log-divergent contribution, so independent of (s, t, u)
+ The RHS is positive
+ We can solve this equation!

$$\Rightarrow \quad -\frac{1}{\lambda(M)} + \frac{1}{\lambda(M_0)} = \frac{3}{16\pi^2} \ln\left(\frac{M}{M_0}\right)$$

or

$$\lambda(M) = \frac{\lambda(M_0)}{1 - \frac{3}{16\pi^2}\lambda(M_0) \ln \frac{M}{M_0}}$$

Rem: when this blows up is called a "Landau pole"

$\Rightarrow$ We recover the conclusion that the theory is more strongly interacting as $M \nearrow$

+ effectively

$$\lambda(M) = \lambda(M_0) \sum^{\infty} c_n \lambda^n(M_0) \left(\ln \frac{M}{M_0}\right)^n$$

$$\lambda(M) = \lambda(M_0) \sum_{n=0} c_n \cdots \left(\cdots M_0 \right)$$

We have resummed an <u>infinite number</u> of log corrections.

$\Rightarrow$ Good thing if $\lambda(M_0) \ln \frac{M}{M_0} \sim O(1)$

or if we have large logs.

+ This is part of a broader program called the Renormalization Group equations

The "bare mass"

+ We phrased it in terms of λ and M.

It can be done in terms of λ and μ.

The "renormalized mass"

The result is the same (at one-loop)

<u>Broader perspective</u>

* For a theory with couplings g_i $i = 1, \ldots m$

$$\mu \frac{d}{d\mu} g_i = \beta_i (g_j)$$

with β called β functions (a matrix $m \times m$ can be argued to be symmetric and so can be diagonalized)

* if $\beta > 0$ (positive eigenvalues)

$\Rightarrow$ $g_i \nearrow$ with energy scale

* if $\beta < 0$ (NEGATIVE EIGENVALUES)

$\Rightarrow g_0 \searrow$ WITH ENERGY SCALE

THE LATER IS CALLED ASYMPTOTIC FREEDOM

LET'S NOW CONSIDER QED. THIS IS BASED ON A U(1) GAUGE SYMMETRY

$$\mathcal{L} = i\bar{\psi}\not{D}\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

WITH $D_\mu = \partial_\mu + ieA_\mu$

INHOMOGENEOUS

THIS IS INVARIANT UNDER $\psi \to e^{i\alpha(x)}\psi$ AND $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\alpha$

THIS GAUGE SYMMETRY FORBIDS A MASS TERM FOR THE GAUGE (ie PHOTON) FIELD A_μ.

SO A LIGHT GAUGE FIELD IS POSSIBLE!

NB ψ IS THE ELECTRON FIELD BUT WE COULD CONSIDER OTHER CHARGED PARTICLES.

A QUESTION IS NOW THE PERFECT (UP TO CURRENT MEASUREMENTS) EQUALITY OF THE PROTON, SAY, AND ELECTRON IS MAINTAINED.

TO SEE THIS (AND OTHER THINGS) LET $A_\mu \to \frac{1}{e}A_\mu$

THEN

$$\mathcal{L} = i\bar{\psi}\not{\partial}\psi - A_\mu\bar{\psi}\gamma^\mu\psi - \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu}$$

COUPLING IS ONLY HERE

IN THIS SCHEME, THE CHARGED

e DICTATES HOW THE GAUGE FIELD

PROPAGATES WHILE ALL THE CHARGE ± 1 PARTICLES

PROPAGATE, WHILE ...

COUPLE THROUGH $A_\mu \bar{\Psi}\gamma^\mu \Psi$, INDEPENDENT OF e.

CONVENIENT TO SEE THAT ELECTRIC CHARGE RATIOS ARE MAINTAINED BY QUANTUM CORRECTIONS.

LET'S LOOK AT THAT IN MORE DETAILS.

FIRST THING FIRST, THE PHOTON PROPAGATOR

START WITH

$$S = \int d^4x \left(-\frac{1}{4e^2} F_{\mu\nu}F^{\mu\nu}\right)$$

$$= \frac{1}{2e^2}\int d^4x\, A_\mu(x)\left(\partial^2 \eta^{\mu\nu} - \partial^\mu\partial^\nu\right) A_\nu(x)$$

*₂ $$= \frac{1}{2e^2}\int \frac{d^4k}{(2\pi)^4} A_\mu(k)\left(-k^2\eta^{\mu\nu} + k^\mu k^\nu\right) A_\nu(-k)$$

$$= \frac{-i}{2}\int \frac{d^4k}{(2\pi)^4} A_\mu(k)\, \underbrace{D^{\mu\nu}(k)^{-1}}_{\text{INVERSE PROPAGATOR}} A_\nu(-k)$$

INVERSE PROPAGATOR IN PRINCIPLE,

IN THE SENSE

$$(-k^2\eta_{\mu\alpha} + k_\mu k_\alpha)D^{\alpha\nu} = i\,\delta_\mu^\nu \quad \text{(FORMAL)}$$

THERE IS A PROBLEM HOWEVER WITH *₂. ANY A_ν OF THE FORM $A_\nu = k_\nu \alpha(k)$ GIVE ZERO WHEN MULTIPLIED BY $(-k^2\eta^{\mu\nu} + k^\mu k^\nu)$

SO THIS LORENTZ MATRIX HAS A ZERO EIGENVALUE.

$\longrightarrow$ CANNOT BE INVERTED AND $D_{\mu\nu}$ DOES NOT EXIST.

+ This is a reflexion of the fact that A_ν can be changed by a total derivative without changing the physics.

+ This is related to the fact that a photon has only two propagating modes (ie two polarization states) while A_μ describes a priori 4 degrees of freedom.

+ This is finally related to current conservation albeit in a subtle way. Indeed we do not need a local symmetry to ensure electric charge conservation: a global symmetry is sufficient.

$\Longrightarrow$ This is for an Abelian theory. For non Abelian theories we need local gauge invariance.

<u>Ways out</u>:

1) Give a small mass to the photon $m^2 \to 0$

2) Fix the gauge.

1) <u>Massive Photon</u>

$$\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2e^2} m^2 A_\mu A^\mu - A_\mu J^\mu \quad *_3$$

$$= \frac{1}{2e^2} A_\mu(k) \left((-k^2+m^2) \eta^{\mu\nu} + k^\mu k^\nu \right) A_\nu(-k) - A_\mu(k) J^\mu(-k)$$

EX: CHECK THAT

$$\left[(-k^2+m^2)\eta^{\mu\alpha} + k^\mu k^\alpha\right] D_{\alpha\nu} = i\,\delta^\mu_\nu$$

IS SOLVED BY

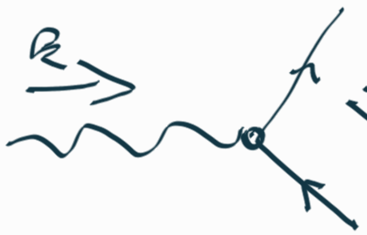
$$D_{\mu\nu} = -\frac{ie^2(\eta_{\mu\nu} - k_\mu k_\nu/m^2)}{k^2-m^2}$$

REM
SMALL e^2
MEANS WEAKER
PROPAGATING
(IE $e^2 \to 0$, NO
PROPAGATION)

IN $D_{\mu\nu}$ THE LIMIT $m^2 \to 0$ IS SINGULAR BECAUSE OF THE $k_\mu k_\nu/m^2$ TERM.

HOWEVER COUPLING TO A CONSERVED CURRENT IMPLIES THAT $k_\mu k_\nu/m^2$ DROPS IN CALCULATING AMPLITUDES.

EG



$$\Rightarrow i\mathcal{M} \sim D_{\mu\nu} J^\nu$$

$$\Rightarrow D_{\mu\nu} J^\nu = -ie^2\left(\frac{\eta_{\mu\nu}}{k^2-m^2} - \dots k_\mu k_\nu\right) J^\nu$$

AND $\partial_\mu J^\mu = 0 \xLeftrightarrow[\text{FOURIER}]{} k_\mu J^\mu(k) = 0$

SO $$D_{\mu\nu} J^\nu = -i\frac{e^2 J_\mu(k)}{k^2-m^2} \xrightarrow[m^2\to 0]{} \frac{-ie^2 J_\mu(k)}{k^2}$$

SMOOTH LIMIT AS $m^2 \to 0$.

THIS IS EASY AND CONVENIENT BUT ONLY WORKS FOR $U(1)$ GAUGE SYMMETRIES...

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ex WRITE THE EL EQUATIONS FOR $*_3$

SHOW THAT THEY IMPLY THAT $\partial_\mu A^\mu = 0$

Rem: THIS IS NOT A GAUGE CONDITION!

INSTEAD IT EXPRESS THE FACT THAT FOR A MASSIVE SPIN 1 PARTICLE, THERE ARE 3 DEGREES OF FREEDOM (DOF) CORRESPONDING TO SPIN ± 1 AND 0 IN THE REST FRAME OF THE PHOTON.

THUS $\partial_\mu A^\mu = 0$ TELLS THAT OUT OF 4 A_μ DOF, 3 ARE PHYSICAL!

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2) GAUGE FIXING

GAUGE FIXING AMOUNTS TO ELIMINATING BY HAND THE REDUNDANT DOF.

$$\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi e^2} (\partial_\mu A^\mu)^2$$

↳ GAUGE FIXING

$\xi \to \infty$ BACK TO ORIGINAL

LAGRANGIAN GOT SINGULAR
SO KEEP ξ FINITE

Ex: SHOW THAT

$$D_{\mu\nu}(k) = -\frac{ie^2}{k^2}\left(g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right)$$

NB + THE $\frac{k_\mu k_\nu}{k^2}$ TERMS DROP WHEN ACTING ON CONSERVED CURRENTS!

$\Rightarrow$ PHYSICAL AMPLITUDES ARE GAUGE INVARIANT i.e. INDEPENDENT OF ξ.

+ $\xi = 0$ IS LANDAU GAUGE
  $\xi = 1$ IS FEYNMAN GAUGE

$$\Rightarrow \quad D_{\mu\nu} = \frac{-ie^2 g_{\mu\nu}}{k^2}$$

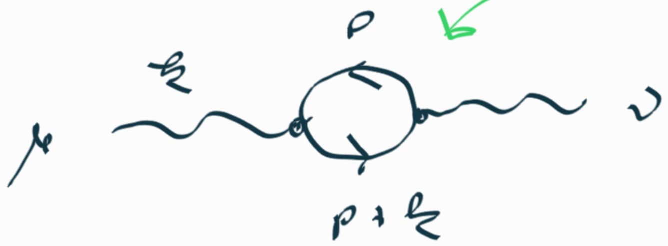
WHICH IS EASY EVEN THOUGH IT LOOKS LIKE THAT 4 DOF PROPAGATE.

QUANTUM CORRECTIONS TO PHOTON PROPAGATOR

1) TREE LEVEL (FEYNMAN GAUGE)

2) 1-LOOP

FERMION LOOP



THIS PIECE IS IMPORTANT. IT IS CALLED THE POLARIZATION TENSOR:

$$i\Pi_{\mu\nu}(k) = \langle J_\mu J_\nu \rangle(k) =$$ CORRELATOR OF 2 CURRENTS

NOW $\partial_\mu J^\mu = 0 \iff k_\mu J^\mu(k) = 0$

$$\Longrightarrow \quad k_\mu \Pi^{\mu\nu} = 0 = k_\nu \Pi^{\mu\nu}$$

NB CURRENT CONSERVATION EXPRESSED ON GREEN FUNCTIONS IS CALLED A WARD-TAKAHASHI IDENTITY

THIS IS AN IMPORTANT CONSTRAINT.

IT IMPLIES THAT $\Pi^{\mu\nu}$ MUST TAKE THE FORM

$$i\Pi^{\mu\nu}(k) = i\,(k^2 g^{\mu\nu} - k^\mu k^\nu)\,\Pi(k^2)$$

↳ THIS IS QUADRATIC SO $\Pi(k^2)$ HAS TWO POWERS LESS THAN $\Pi^{\mu\nu}$

QUANTUM CORRECTIONS SHOULD RESPECT THIS. LET US CHECK

SUM OVER ALL ELECTRON HELICITIES POSITRON

$$i\Pi^{\mu\nu}(k) = -(-i)^2 \int \frac{d^4p}{(2\pi)^4} \, \mathrm{Tr}\left[\gamma^\mu \frac{i}{\not p - m} \gamma^\nu \frac{i}{(\not p + \not k) - m}\right]$$

↳ ELECTRON MASS

COUPLING THROUGH $-i\gamma^\mu$

MINUS SIGN BECAUSE FERMION LOOP (CF QFT COURSE)

Rem: THIS INTEGRAL IS SUPERFICIALLY QUADRATICALLY DIVERGENT. CURRENT CONSERVATION MUST IMPROVE THIS.

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ex: CHECK THAT, INDEED,

$$i\Pi_{\mu\nu}(k) = i(k^2\eta_{\mu\nu} - k_\mu k_\nu)\Pi(k^2)$$

WITH

$$\Pi(k^2) = -\frac{8}{(4\pi)^{D/2}}\Gamma(2 - D/2)\int_0^1 dx \frac{x(1-x)}{(m^2 - k^2 x(1-x))^{2-D/2}}$$

THIS IS TRICKY. IT WORKS FINE WITH DIMENSIONAL REGULARISATION WHICH INDEED REMOVES A POTENTIAL QUADRATIC DIVERGENCE!

THIS IS IMPORTANT, SINCE OTHERWISE

$$m^2 A_\mu A^\mu \sim M^2 A_\mu A^\mu$$

WITH M THE CUT-OFF SCALE

EX USE THE APPENDIX OF PS TO WRITE

$$\Pi(k^2) = -\frac{1}{2\pi^2}\int_0^1 dx\, x(1-x)\left[\frac{2}{\varepsilon} - \gamma + \log 4\pi - \log\left(m^2 - q^2 x(1-x)\right)\right]$$

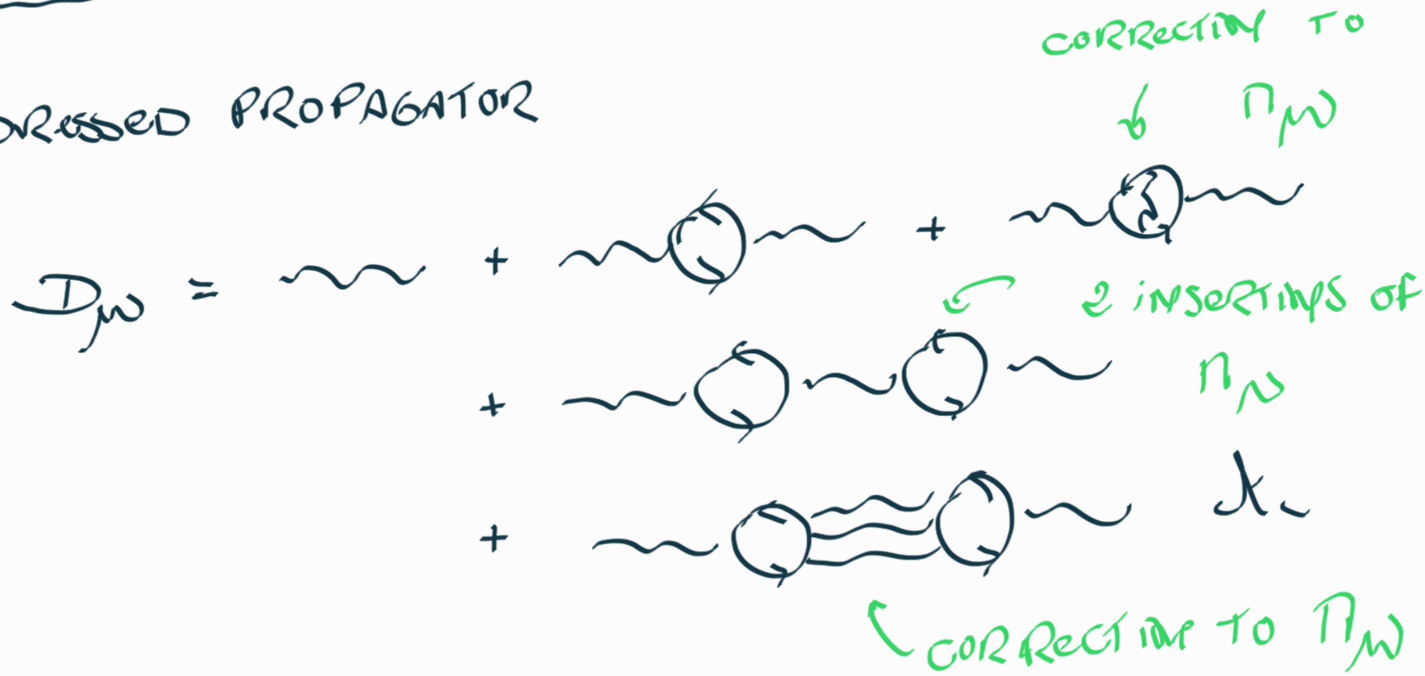
USING OUR PREVIOUS DICTIONARY, WITH PV REGULARIZATION

$$= -\frac{1}{2\pi^2}\int_0^1 dx\, x(1-x)\log\left(\frac{M^2}{m^2 - q^2 x(1-x)}\right)$$

WHICH MAKES EXPLICIT THE LOG DIVERGENCE.

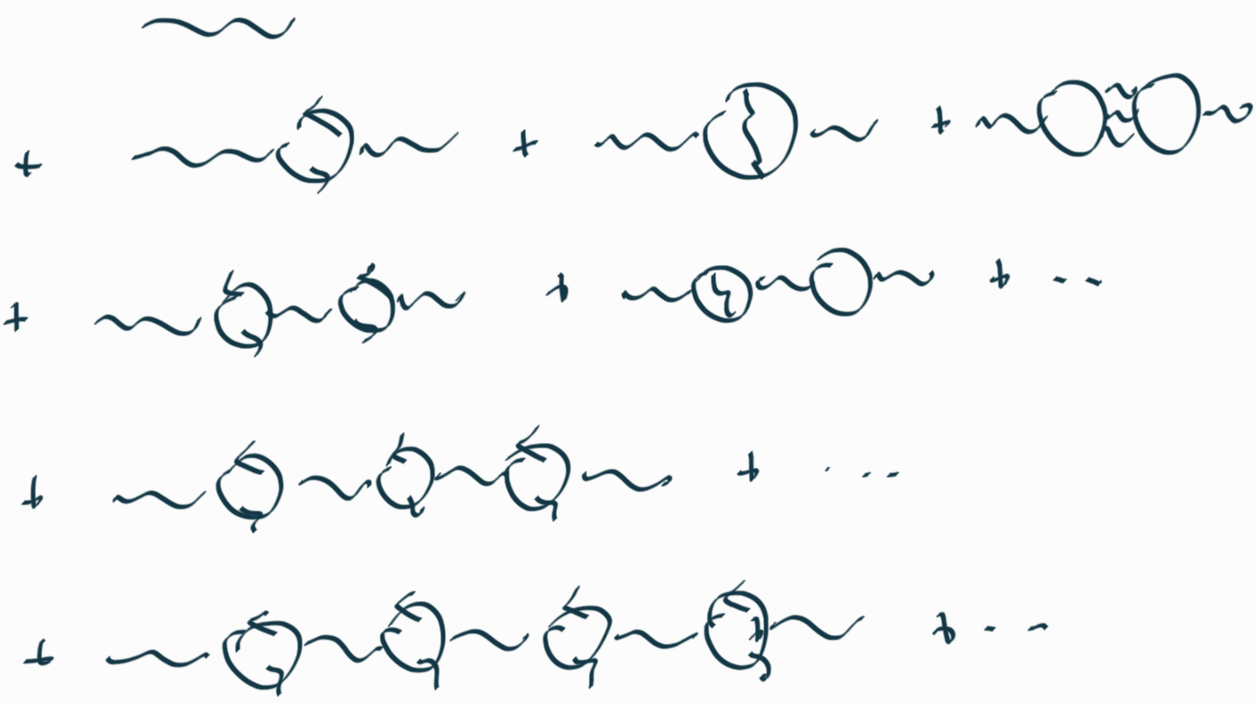
RELATION TO RENORMALIZED ELECTRIC CHARGE ?

DRESSED PROPAGATOR



RE-ORGANIZE:

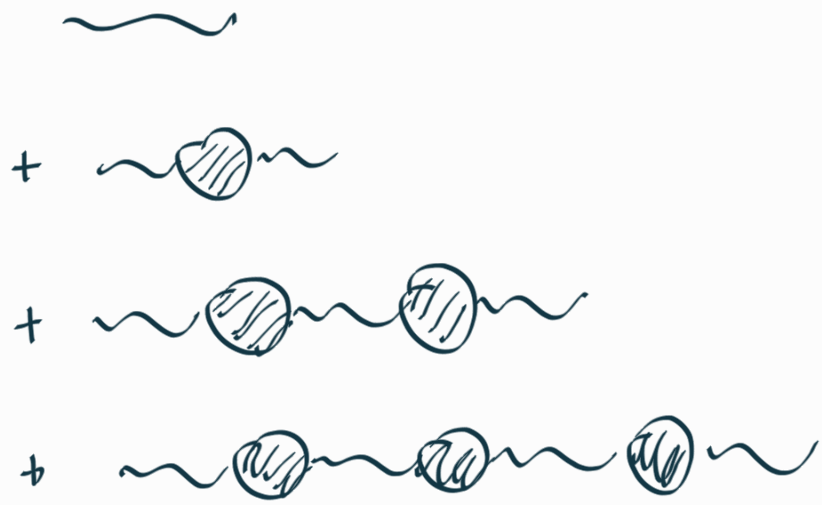
$D_{\mu\nu} =$



etc.

$\equiv$

$D_{\mu\nu} =$



+ etc.

WITH FULL POLARIZATION TENSOR WRITTEN AS

$i\Pi_{\mu\nu} =$ 

etc.

THAT SOUNDS REASONABLE. IT IS A THEOREM THAT

... (THEOREM BY DYSON)

IT CAN BE DONE (...

NOW, IN FEYNMAN GAUGE, WE HAVE

$$D_{\mu\nu} = \frac{-ie^2\eta_{\mu\nu}}{k^2} + \frac{(-ie^2\eta_{\mu\alpha})}{k^2} i\Pi^{\alpha\beta} \frac{(-ie^2\eta_{\beta\nu})}{q^2}$$

$$+ \frac{(-ie^2\eta_{\mu\alpha})}{k^2} i\Pi^{\alpha\beta} \frac{(-ie^2\eta_{\beta\rho})}{k^2} i\Pi^{\rho\sigma} (-ie^2\eta_{\sigma\nu})$$

$$+ \dots$$

WITH, BY GAUGE INVARIANCE / CURRENT CONSERVATION

$$i\Pi^{\mu\nu} = i\,(k^2\eta^{\mu\nu} - k^\mu k^\nu)\,\Pi(k^2)$$

$\hookrightarrow$ FULL POL. TENSOR

$$\Rightarrow \boxed{D_{\mu\nu} = \frac{-ie^2}{k^2(1-e^2\Pi(k^2))}\eta_{\mu\nu}} + k_\mu k_\nu \text{ TERMS}$$

*4

DROP ON CURRENTS!

VERY TEMPTING TO WRITE THIS AS

$$e^2(k^2) = \frac{e^2}{1-e^2\Pi(k^2)}$$

$$\simeq e^2 + e^4\Pi(k^2) + O(e^6)$$

+ THIS IS VERY ANALOGOUS TO WHAT WE GOT FOR

$\lambda \phi^4$!

+ THE DYSON RESUMMATION GIVES DIRECTLY THE SUM OF LOG CONTRIBUTIONS.

+ RENORMALIZATION, IN THIS SIMPLE THEORY, GOES AS FOR $\lambda\phi^4$

FOR QED TYPICALLY $\mu^2 = 0$

$$\Longrightarrow \quad \tilde{e}^2(\mu^2) = e^2 + e^4\,\Pi(\mu^2) + O(e^6)$$

TILDE FOR MEASURED / PHYSICAL

$$\text{so} \quad e^2 = \tilde{e}^2(\mu^2) - \tilde{e}^2(\mu^2)\,\Pi(\mu^2) + O(e^6)$$

$$\Longrightarrow \quad \tilde{e}^2(k^2) = \tilde{e}^2(\mu^2) + \tilde{e}^4(\mu^2)\left(\Pi(k^2) - \Pi(\mu^2)\right) + O(\tilde{e}^6)$$

NO LOG (M^2) TERMS !

WRITE $\Pi_R(k^2) = \Pi(k^2) - \Pi(\mu^2)$

$$\boxed{\Pi_R(k^2) = \frac{1}{2\pi^2}\int_0^1 dx\, x(x-1)\,\log\left(\frac{m^2 - k^2 x(1-x)}{m^2 - \mu^2 x(1-x)}\right)}$$

+ FOR $k^2 \gg \mu^2$, $\Pi_R(k^2) > 0$ AND INCREASING.

$\Longrightarrow$ ELECTRIC INCREASES WITH MOMENTUM / SCALE CONVERSELY, DECREASES AT LARGE DISTANCES.

+ THIS IS LIKE $\lambda\phi^4$, IT IS BELIEVED TO BE BAD...

+ AS FOR $\lambda\phi^4$ WE CAN PHRASE THAT AS THE BEHAVIOUR OF THE BARE ELECTRIC COUPLING WITH CUT-OFF SCALE.

$\Rightarrow$ MEASURED AMPLITUDE $i\mathcal{M}(M) = i\mathcal{M}$

↳ INDEPENDENT OF M

$$i\mathcal{M} \propto e^2 + e^4\,\Pi(k^2) + \mathcal{O}(6)$$

$$\frac{d\mathcal{M}}{dM} = 0 \Rightarrow \frac{d}{dM} e^2 = -e^4 \frac{d}{dM}\Pi(k^2)$$

WITH $\Pi(k^2) = -\frac{1}{2\pi^2}\int_0^1 dx\, x(1-x)\left[\log M^2 + \text{finite}\right]$

$$\frac{d\Pi}{dM} = -\frac{1}{\pi^2 M}\int_0^1 dx\, x(1-x)$$

$$= -\frac{1}{6\pi^2 M}$$

$$\Rightarrow M\frac{d}{dM}e^2 = \frac{e^4}{6\pi^2} \Leftrightarrow M\frac{d}{dM}\alpha = \frac{2\alpha^2}{3\pi}$$

WITH $\alpha = \frac{e^2}{4\pi}$ THE FINE STRUCTURE CONSTANT.
BARE

EQUIVALENTLY, WITH $e^2(M) \longleftrightarrow \tilde{e}(\mu)$

$M \longleftrightarrow \mu$

$$\Rightarrow \boxed{\mu \frac{d}{d\mu}\tilde{\alpha} = \frac{2}{3\pi}\tilde{\alpha}^2} \equiv \beta(\tilde{\alpha})$$

THIS INTEGRATES TO

$$-\frac{1}{\tilde{\alpha}(\mu)} + \frac{1}{\tilde{\alpha}(\mu_0)} = \frac{2}{3\pi}\ln\frac{\mu}{\mu_0}$$

OR

$$\boxed{\tilde{\alpha}(\mu) = \frac{\tilde{\alpha}(\mu_0)}{1 - \frac{2}{3\pi}\tilde{\alpha}(\mu_0)\ln\frac{\mu}{\mu_0}}}$$

WHICH IS (ESSENTIALLY) EQUIVALENT TO $*_4$

AGAIN, THERE IS A LANDAU POLE SO POTENTIALLY THE THEORY IS OUT-OF-CONTROL IN THE UV.

EX: TAKE $\mu_0 = m$ (MEASURED ELECTRIC CHARGE AT ELECTRON MASS).

AT WHICH SCALE μ_* IS $\tilde{\alpha}(\mu)$ DIVERGING?

TAKING THAT $\tilde{\alpha}(\mu) \simeq \frac{1}{137}$? COMPARE THIS TO PLANCK MASS

SUMMARY $\lambda\phi^4$, QED

ARE STRONGLY INTERACTING IN THE UV.

WE ARE NOT SURE HOW TO LEAVE WITH SUCH THEORIES.

THE SITUATION IS (POTENTIALLY) VERY DIFFERENT FOR NON-ABELIAN GAUGE THEORIES !!!