BEYOND THE STANDARD MODEL PHYSICS

LECTURE NOTES

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CHAP. I : BASIC ELEMENTARY FIELDS

Small dist. → Largep

$\Delta x \Delta p \underset{\downarrow}{\gtrsim} \hbar$

Fundamental constituents are described by relativistic quantum mechanics laws: Quantum Field theory

↳ Lorentz invariant: fields can be classified in terms of their transform. properties ⇒ irreducible representat° of Lorentz group

- A real scalar field : $\phi(x)$

↳ spin 0 : $\phi(x) \to \phi'(x') = \phi(\Lambda^{-1}x')$ $\qquad x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu}$

↳ 1 degree of freedom (d.o.f)

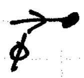
↳ possible lorentz invar. bilinears : $\partial^{\mu}\phi\partial_{\mu}\phi, \phi^2$

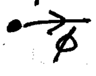
↳ free field Lagrangian : $\mathcal{L} = \frac{1}{2}(\partial_{\mu}\phi\partial^{\mu}\phi) - \frac{1}{2}\underset{\uparrow}{m^2}\phi^2$

a # with dim. of energy : a "mass"

↳ equ. of motion (eom) : $(\Box + m^2)\phi = 0$ ← Klein-Gordon equ.

↳ solut° of eom : $\phi(x) = \int d^3\tilde{k}\,[\underset{\uparrow}{a(k)}e^{ikx} + \underset{\uparrow}{a^{\dagger}(k)}e^{-ikx}]$





↳ self-conjugate : 1 d.o.f.

- A complex scalar field : $\phi(x) = \mathrm{Re}\,\phi + i\,\mathrm{Im}\,\phi$
  $\phi^\dagger(x) = \text{"} - \text{"}$ $\Big\}$ 2 d.o.f = 2 degenerate real scalars

  ↳ spin 0 : $\phi(x) = \phi'(x') = \phi(\Lambda^{-1}x')$

  ↳ 2 d.o.f.

  ↳ Lor. invar. bilinears : $\partial^\mu\phi^\dagger\partial_\mu\phi$, $\phi^\dagger\phi$ $\in$ hermitian (to conserve probability)

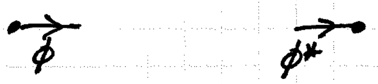
  ↳ free field Lagrangian : $\mathcal{L} = \partial^\mu\phi^\dagger\partial_\mu\phi - \underbrace{m^2\phi^\dagger\phi}_{m^2((\mathrm{Re}\phi)^2+(\mathrm{Im}\phi)^2)}$

  ↳ e.o.m : $(\Box+m^2)\phi=0$ or $(\Box+m^2)\phi^\dagger=0$ ↳ same mass: "degenerate"

  ↳ solut° of eom:

$$\phi(x) = \int d^3\tilde{k}\,[\,a(k)e^{-ikx} + b^\dagger(k)e^{ikx}\,]$$

$$\phi^\dagger(x) = \int d^3\tilde{k}\,[\,a^\dagger(k)e^{ikx} + b(k)e^{-ikx}\,]$$



  ↳ not self-conjugate : 2 d.o.f.

- A vector field : $A^\mu(x)$

  ↳ spin 1: transforms as a 4-vector : $A'^\mu(x') = \Lambda^\mu_{\ \nu} A^\nu(\underbrace{\Lambda^{-1}x'}_{x})$

  ↳ Lor. invar. bilinears : $\cdot\ \partial_\mu A^\nu\partial^\mu A_\nu$
  $\cdot\ \partial_\mu A^\nu\partial_\nu A^\mu$ $\Big\}$ only the $F_{\mu\nu}F^{\mu\nu}$ combinatio of these 2 is allowed from Hamiltonian defin. positive condit° : see QFT course

  $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$

  $\cdot\ A_\mu A^\mu$

  ↳ $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu$

  ↳ e.o.m. : $\partial^\mu F_{\mu\nu} + m^2 A_\nu = 0 \iff \underset{\text{K.G. : boson}}{(\Box+m^2)A_\nu=0}$

  $\partial_\mu A^\mu = 0$: → automatic for $m\neq 0$
  → choice of gauge for $m=0$ ↳ lorentz gauge

  cf QFT course

$\hookrightarrow$ solut^{o} of e.o.m.: $A_\mu(x)=\int d^3\tilde{k}\sum_\lambda\left[a^\lambda(k)\varepsilon^\lambda_\mu(k)\,e^{-ikx}+a^{\lambda+}(k)\,\varepsilon^{\lambda*}_\mu(k)\,e^{ikx}\right]$

$\partial^\mu \overset{\uparrow}{A}_\mu=0 \iff \varepsilon^\lambda_\mu k^\mu=0 \Rightarrow \lambda=1,2,3 \Rightarrow 3$ d.o.f.

$\hookrightarrow$ 2 transverses polariz. modes: $\varepsilon^{1,2}_\mu=(0,\vec{e}_{1,2}(k))$

$\hookrightarrow$ $\vec{e}_{1,2}(k).\vec{k}=0$

$\hookrightarrow$ 1 longitudinal polariz. mode: $\varepsilon^3_\mu=\left(\frac{|\vec{k}|}{m},\frac{E}{m}\frac{\vec{k}}{|\vec{k}|}\right)$

$\hookrightarrow$ unphysical for $m=0$

$\hookrightarrow$ $A^\mu J_\mu$ coupling in $\mathcal{L}^{INT}$ gives no contrib. from ε^3_μ: $\partial_\mu J^\mu=0 \iff EJ^0-\vec{k}\vec{J}=0$

$$\varepsilon^3_\mu J^\mu=\frac{|\vec{k}|J^0}{m}-\frac{E}{m}\frac{\vec{k}.\vec{J}}{|\vec{k}|}=J^0\frac{(|\vec{k}|^2-E^2)}{m|\vec{k}|}=\underset{\text{if } m=0\,\to\,\overset{\|}{0}}{-\frac{mJ^0}{|\vec{k}|}}$$

- **Dirac spinor**: $\psi(x)$: 4×1 matrix $\qquad S_{\alpha\beta}=\frac{i}{4}[\gamma_\alpha,\gamma_\beta]$

$\hookrightarrow$ spin 1/2: $\psi(x)\to\psi'(x)=\exp\left(-\frac{i}{2}\omega_{\alpha\beta}\underbrace{S^{\alpha\beta}}_{\text{Lor. algebra generators}}\right)\psi(\Lambda^{-1}x')$

$\hookrightarrow$ or in $\psi_{L,R}$ components: $\psi=\underbrace{P_L\psi}_{\equiv\psi_L=\binom{\chi_L}{0}}+\underbrace{P_R\psi}_{\equiv\psi_R=\binom{0}{\chi_R}}=\begin{pmatrix}\chi_L\\ \chi_R\end{pmatrix}$ ← Weyl representation

$\hookrightarrow$ $\chi_{L,R}(x)\underset{\substack{\uparrow\\ \text{2 types of dim. 2 irreduc. repres.}}}{\longrightarrow}\chi'_{L,R}(x')=\exp\Big(\overset{\text{3 boosts}}{\mp\frac{1}{2}\omega_{0i}\sigma_i}-\underset{\text{3 rotations}}{\frac{i}{4}\omega_{ij}\varepsilon^{ijk}\sigma_k}\Big)\cdot\chi_{L,R}(\Lambda^{-1}x')$

$\Rightarrow$ Dirac spinor = sum of 2 $\overset{\nearrow\text{ dim. 2}}{\text{irreducible}}$ representat^{o} of Lorentz group: one left repres. and one right repres. $\Rightarrow$ 4 d.o.f.

$\hookrightarrow$ Lorentz invar. bilinears: $i\bar{\psi}\not{\partial}\psi=i\bar{\psi}_L\not{\partial}\psi_L+i\bar{\psi}_R\not{\partial}\psi_R$

$\bar{\psi}\psi=\bar{\psi}_L\psi_R+\bar{\psi}_R\psi_L$

$\hookrightarrow$ free field Lagrangian: $\mathcal{L}=\bar{\psi}(i\not{\partial}-m)\psi$ DIRAC $\mathcal{L}$.

↳ e.o.m. : $(i\not{\partial} - m)\psi = 0$ ⇐ DIRAC EQU.

$$\Updownarrow$$

$$\begin{cases} i(\partial_t - \sigma.\nabla)\chi_L = m\chi_R \\ i(\partial_t + \sigma.\nabla)\chi_R = m\chi_L \end{cases}$$ ⇐ WEYL EQUATIONS $(*)_5$

↳ a Dirac spinor is a single 4 d.o.f. object becouse of mass term : χ_L and χ_R not independent in e.o.m becouse the mass term mixes both : $\underset{\underset{\bar\psi_R \psi_L}{\uparrow}}{\xrightarrow[\chi_L]{}\overset{m}{\times}\xrightarrow[m_R]{}}$ $\underset{\underset{\bar\psi_L \psi_R}{\uparrow}}{\xrightarrow[\chi_R]{}\overset{m}{\times}\xrightarrow[\chi_L]{}}$

⇒ a Dirac spinor : 2 degenerate Weyl spinors of opposite chiralities : χ_L, χ_R

↳ sol° of e.o.m : $\psi(x) = \int d^3\tilde{p} \sum_s (\underset{\underset{\xrightarrow[e^-]{}\bullet}{\downarrow}}{a_p^s} u^s(p) e^{-ip.x} + \underset{\underset{\bullet\xleftarrow{e^+_s}}{\updownarrow}}{b_p^{s+}} v^s(p) e^{ip.x})$

↳ 4 sol° as expected

$$\bar\psi(x) = \int d^3\tilde{p} \sum_s (\underset{\underset{\bullet\xrightarrow[e^-]{}}{\downarrow}}{a_p^{s+}} \bar{u}^s(p) e^{ip.x} + \underset{\underset{\xleftarrow[e^+_s]{}\bullet}{\downarrow}}{b_p^s \bar{v}^s(p)} e^{-ip.x})$$

with : $u^s(p) = \begin{pmatrix} \sqrt{p.\sigma}\,\xi_s \\ \sqrt{p.\bar\sigma}\,\xi_s \end{pmatrix}$ $\xi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ⇐ spin up, $\xi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ⇐ spin down

↑ as can be seen applying spin operator : rotat° generator

$\sigma = (1, \sigma_1, \sigma_2, \sigma_3)$

$\bar\sigma = (1, -\sigma_1, -\sigma_2, -\sigma_3)$

$v^s(p) = \begin{pmatrix} \sqrt{p.\sigma}\,\zeta_s \\ -\sqrt{p.\bar\sigma}\,\zeta_s \end{pmatrix}$ $\zeta_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\zeta_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

spin up and down sol° of e.o.m. are physical states (i.e. do not mix becouse → conservat° of spin) ⇒ are superposition of ψ_L and ψ_R which are not physical states if $m \neq 0$ (ψ_L and ψ_R mix through mass term) ⇒ helicity $\propto \vec{S}.\vec{p}$ is ≠ from chirality if $m \neq 0$.

- Massless weyl spinor: if no mass term χ_L does not mix with χ_R and since both are independent under lorentz transform. ⇒ both objects are totally independ. → one could have just 2.d.o.f : χ_L or χ_R but not both

⇒ χ_L : transforms as L field : particle

$\chi_L^c \equiv -i\sigma^2\chi_L^*$ $\quad$ χ_L^c : " " R field : anti-part.

⇒ 2 d.o.f : χ_L and χ_L^c

⇒ lorentz invar., Lagrangian : same as for Dirac keeping only $\underset{\underset{\chi_L}{\Updownarrow}}{\psi_L}$ (i.e no $\underset{\underset{\chi_R}{\Updownarrow}}{\psi_R}$, no mass) for instance

↳ solut° of e.o.m : same as Dirac with no spin sum

↳ keeping only for the particle the spin anti-∥ to $\vec{p}$: $u_\downarrow(\vec{p}) = \sqrt{2\omega_p}\begin{pmatrix}\xi_\downarrow\\0\end{pmatrix} \Leftrightarrow \begin{pmatrix}\chi_L\\0\end{pmatrix}$

and for anti-part. the spin ∥ to $\vec{p}$:

$$v_\uparrow(\vec{p}) = -\sqrt{2\omega_p}\begin{pmatrix}0\\ \xi_\uparrow\end{pmatrix} \Leftrightarrow \begin{pmatrix}0\\ \chi_L^c\end{pmatrix}$$

↳ N.B. : • χ_R (χ_R^c) is just the same as $\propto$ χ_L^c (χ_L) :

• See chapter 3.2 and 3.3 of Peskin-Schroeder for more details.

↳ for instance to check that spin = chirality for m=0 :

$$u_\downarrow(p) = \begin{pmatrix}\sqrt{p\cdot\sigma}\begin{pmatrix}0\\1\end{pmatrix}\\ \sqrt{p\cdot\bar{\sigma}}\begin{pmatrix}0\\1\end{pmatrix}\end{pmatrix} = \begin{pmatrix}\sqrt{\omega_p+p_3}\begin{pmatrix}0\\1\end{pmatrix}\\ \sqrt{\omega_p-p_3}\begin{pmatrix}0\\1\end{pmatrix}\end{pmatrix} \underset{\underset{\text{for } m=0}{\uparrow}}{=} \sqrt{2\omega_p}\begin{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\\ 0\end{pmatrix} \Leftarrow \text{pure left.}$$

$\sigma = (1, \sigma_1, \sigma_2, \sigma_3)$

$\bar{\sigma} = (1, -\sigma_1, -\sigma_2, -\sigma_3)$

• Massive Majorana Spinor

↳ question: can a single ψ_L or a single ψ_R have a mass?

↳ yes! ⇐ ∃ a mass term ; Lorentz invariant made of only ψ_L.

$$\Rightarrow \chi_L \to \chi_L' = \exp\left(-\frac{1}{2} w_{0i}\sigma^i - \frac{i}{4} w_{ij}\varepsilon^{ijk}\sigma^k\right)\chi_L$$

$$\chi_L^c \to \chi_L^{c\prime} = \exp\left(+ \quad '' \quad - \quad '' \quad\right)\chi_L^c$$

⇒ $\chi_L^{cT}\chi_L$: not Lorentz invar. !

$\chi_L^{c\dagger}\chi_L$: Lorentz invar !

$$\Rightarrow \mathcal{L}^{MAJ\text{-}MASS} = -\frac{1}{2}\left(\chi_L^{c\dagger}\chi_L + \chi_L^{\dagger}\chi_L^c\right)\underset{\substack{\uparrow\\ \text{a "Majorana" mass}}}{m_M}$$ is Lorentz invar. and hermit.

⇒ ∃ 2 types of mass term: Dirac : $\overset{\chi_L}{\longrightarrow}\underset{m_D}{\times}\overset{\chi_R}{\longrightarrow}$ or $\overset{\chi_R}{\longrightarrow}\underset{m_D}{\times}\overset{\chi_L}{\longrightarrow}$

$\psi_D = \chi_L$ "+" χ_R

Majorana: $\overset{\chi_L}{\longrightarrow}\underset{m_M}{\times}\overset{\chi_L^c}{\longrightarrow}$ or $\overset{\chi_L^c}{\longrightarrow}\underset{m_M}{\times}\overset{\chi_L}{\longrightarrow}$

$\psi_M = \chi_L$ "+" χ_L^c

↑ ↑

transform particle to anti-part.

⇒ $\overset{\leftrightarrow \chi_L}{\psi_L}$ is not a physical state : it mixes with $\overset{\leftrightarrow \chi_L^c}{\psi_L^c}$

↳ the physical state is: $\psi_M \equiv \chi_L \underset{\substack{\uparrow\\ \text{quantum superposit}^{\circ}}}{\text{"+"}} \chi_L^c = \begin{pmatrix}\chi_L\\ \chi_L^c\end{pmatrix} \begin{matrix}\leftarrow \text{left}\\ \leftarrow \text{right}\end{matrix}$

⇒ the Majorana mass term can be written as:

$$\mathcal{L}^{MAJ\text{-}MASS} = -\frac{1}{2}\left(\chi_L^{\dagger}\ \chi_L^{c\dagger}\right)\begin{pmatrix}0 & m_M\\ m_M & 0\end{pmatrix}\begin{pmatrix}\chi_L\\ \chi_L^c\end{pmatrix} = -\frac{m_M}{2}\bar{\psi}_M\psi_M$$

$$\downarrow$$

$$\overset{\psi_M}{\longrightarrow}\underset{m_M}{\times}\overset{\psi_M}{\longrightarrow}$$

$$\Rightarrow \underset{\substack{\uparrow\\ \text{4 compon. notat}^{\circ}}}{\psi_M} = P_L\psi_M + P_R\psi_M = \begin{pmatrix}\chi_L\\ 0\end{pmatrix} + \begin{pmatrix}0\\ \chi_L^c\end{pmatrix} = \underset{\uparrow}{\chi_L} \text{ "+" } \underset{\uparrow}{\chi_L^c}$$

2 compon. notat°

↑ remains identic. ⇒ physical state

In 2 compon. notat° clearly: $\psi_M^c = (\chi_L)^{c} \text{"}+\text{"} (\chi_L^c)^c$

$$= \chi_L^c + \chi_L = \psi_M \quad \checkmark$$

$\Rightarrow$ Major. spinor is self-conjug.

↳ particle = anti-particle

In 4 compon. notat° $\psi_M^c \neq \psi_M$ if $\psi_M^c = \begin{pmatrix} \chi_L \\ \chi_L^c \end{pmatrix}^c \equiv \begin{pmatrix} \chi_L^c \\ \chi_L^{cc} \end{pmatrix} = \begin{pmatrix} \chi_L^c \\ \chi_L \end{pmatrix}$

but if we define ψ_M^c as: $\psi_M^c \equiv \underset{\sim}{C}\, \underbrace{\bar{\psi}_M^T}_{\propto \psi^*:\text{ anti-part.}}$ then $\psi_M^c = \psi_M$ ⇐ see exercises

↳ $C \equiv \begin{pmatrix} i\sigma_2 & \\ & -i\sigma_2 \end{pmatrix} = i\gamma^2\gamma^0$

$$\Rightarrow \mathcal{L} = \underbrace{\bar{\psi}_L i\not{\partial}\psi_L}_{} - \frac{1}{2}\left(\bar{\psi}_L^c\psi_L + \bar{\psi}_L\psi_L^c\right) m_M$$ ⇐ 4 comp chiral notat°

↳ as for weyl
spinor: kin. term
is indep. of mass

$$= \frac{1}{2}\bar{\psi}_M i\not{\partial}\psi_M - \frac{m_M}{2}\bar{\psi}_M\psi_M$$ ⇐ 4 comp. Maj. notat°

$$= i\chi_L^\dagger\bar{\sigma}^\mu\partial_\mu\chi_L - \frac{m_M}{2}\left(\chi_L^{c\dagger}\chi_L + \chi_L^\dagger\chi_L^c\right)$$ ⇐ 2 comp weyl notat°

$\Rightarrow$ e.o.m: $(i\not{\partial} - m_M)\psi_M = 0 \iff i(\partial_t - \sigma\nabla)\chi_L = m_M\chi_L^c$

↑ same as $(*)_5$ replacing χ_R by χ_L^c

$\uparrow \; \xrightarrow[\psi_M]{}\bullet \; : \; \xrightarrow[\chi_L]{}\bullet$ and $\xrightarrow[\chi_L^c]{}\bullet$

$\Rightarrow$ solut. of e.o.m.: $\psi_M = \int d^3\tilde{p} \sum_s \left(a_p^s u^s(p) e^{-ikx} + a_p^{s\dagger} v^s(p) e^{ikx}\right)$

$\downarrow \; \bullet\xrightarrow[\psi_M]{} \; : \; \bullet\xrightarrow[\chi_L]{}$ and $\bullet\xrightarrow[\chi_L^c]{}$

$\Rightarrow$ ANY BSM THEORY: TO BE CONSTRUCTED OUT OF THESE VERY FEW TYPES OF BASIC FIELDS (APART FROM GRAVITY + ...)

CHAP II : A BRIEF REMINDER OF THE STANDARD MODEL

A] GAUGE SYMMETRIES

1) Abelian case : QED

The free field Lagrangian of for instance an electron displays a U(1) global symmetry:
under $\psi_e \to e^{i\alpha}\psi_e$: $\mathcal{L} = \bar{\psi}_e(i\not{\partial} - m_e)\psi_e \to \mathcal{L}$
To promote it into a local symmetry requires the introduct° of a vector field with $A_\mu \to A'_\mu = A_\mu - \frac{1}{e}\partial_\mu \alpha(x)$ transformat°, so that:

$$\mathcal{L} = \bar{\psi}_e(i\not{D} - m_e)\psi_e \to \bar{\psi}'_e(i\not{\partial})\psi'_e - e\,\bar{\psi}_e'\gamma^\mu A'_\mu \psi'_e$$

$$D_\mu \equiv \partial_\mu + ieA_\mu \qquad = \bar{\psi}_e i\not{\partial}\psi_e - \bar{\psi}_e \underline{\gamma^\mu \psi_e}\,\partial_\mu(\alpha(x))$$

$$- e\bar{\psi}_e\gamma^\mu A_\mu \psi_e + e\,\bar{\psi}_e \underline{\gamma^\mu \psi_e}\,\partial_\mu(\alpha(x))$$

$$= \mathcal{L} \quad ✓$$

The gauge local U(1) sym. allows the vector field to propagate : $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ is gauge invar. (with $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$)
This localisation principle of the free field global sym. + adjunct° of vector field kinetic term gives nothing but the QED $\mathcal{L}$:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_e(i\not{D} - m_e)\psi_e$$

where we have rewritten e as $-e\underbrace{Q_e}_{-1}$ and $\alpha(x)$ as $-\alpha(x)Q_e$ to make apparent the e^- electric charge (keeping $A_\mu \to A'_\mu = A_\mu - \frac{1}{e}\partial_\mu\alpha(x)$ unchanged)

In the interact° term the coeffic. of eA_μ is the e-m current: $\bar{\Psi}_e e Q_e \gamma_\mu A^\mu \Psi_e = e A^\mu J^{em}_\mu$ which is conserved being the Noether current of U(1) global sym.:

$$\partial^\mu J^{em}_\mu = 0 \Rightarrow Q_{em} = \int J_0^{em} d^3x = Q_e(\# \text{ of } e^- - \# \text{ of } e^+)$$

is conserved.

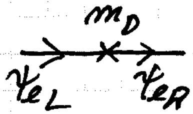
N.B.: • The local sym. allows ≠ particles to have ≠ electric charges

- a mass term $A_\mu A^\mu$ for the photon is not allowed by gauge sym. ⇒ photon massless as observed ⇒ long range force
- γ self-interact° $A_\mu A_\nu A^\mu A^\nu, \ldots$ not allowed
- For a scalar field same principle:

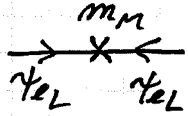
$$\mathcal{L} = (\partial_\mu \phi)^+ \partial^\mu \phi - m^2 \phi^+ \phi \rightarrow (D_\mu \phi)^+ (D^\mu \phi) - m^2 \phi^+ \phi$$

↳ with same $D_\mu \equiv \partial_\mu - ieQ_\phi A_\mu$

- Charge conservat° requires the massive e^- to be of the Dirac type

↳ 

allowed



not allowed

2) Non-abelian gauge sym.

The free field $\mathcal{L}$ of 2 fermions without masses or equal masses displays a SU(2) global sym. under which both fermions form a doublet:

$$\mathcal{L} = \sum_{i=1,2} \bar{\Psi}_i (i\not{\partial} - m) \Psi_i = \bar{\Psi}(i\not{\partial} - m)\Psi \quad \text{with } \Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$

$$\Psi \to \Psi' = \begin{pmatrix} \Psi'_1 \\ \Psi'_2 \end{pmatrix} = U \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = U\Psi \qquad U = e^{-i\frac{\tau_i}{2}\theta_i} \qquad \tau_i = \sigma_i$$

$i=1,2,3$ ↑ Pauli matr.

to promote this sym. into a local one requires the introd. of 3 vector fields to compensate for the 3 $\theta_i(x)$ terms:

$\mathcal{L} = \bar{\Psi}(i\not{D}-m)\Psi \qquad D_\mu = \partial_\mu - ig\frac{\tau_i A_\mu^i}{2}$ # of A_μ = # of gener. of group

↳ with $\Psi \to \Psi' = U\Psi$

$$\underset{\sum_i \frac{\tau_i}{2} A'^i_\mu}{\frac{\tau}{2}\cdot A'_\mu} = U\frac{\tau}{2}A_\mu U^{-1} - \frac{i}{g}[\partial_\mu U]U^{-1}$$

$$\hookrightarrow \Leftrightarrow A'^i_\mu = A^i_\mu + \underbrace{\theta_j \varepsilon_{ijk} A^k_\mu}_{\text{global SU(2) rotat° term}} \underbrace{-\frac{1}{g}\partial_\mu\theta_i}_{\text{local term}} \leftarrow \text{infinitesimal transformation}$$

$\Rightarrow A^i_\mu$ = triplet of global SU(2): adjoint representat° of SU(2)

↳ ⇒ changed under SU(2)! (unlike A_μ of U(1))

A kinetic term $-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$ with $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$ is not gauge invar. because A^i_μ are changed under SU(2) but is invar. if $F^a_{\mu\nu}$ is defined as:

$$-ig\frac{\tau_i}{2}\cdot F^i_{\mu\nu} = [D_\mu, D_\nu] \Leftrightarrow F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon_{abc}A^b_\mu A^c_\nu$$

$$\Rightarrow \mathcal{L} \ni -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu}) \leftarrow \text{vector field kinet. term}$$
$$\left.\begin{aligned} &+ (\quad '' \quad) g\varepsilon_{abc}A^b_\mu A^c_\nu \\ &+ g\varepsilon_{abc}A^b_\mu A^c_\nu(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu})\end{aligned}\right\} A^i_\mu \text{ trilinear interact° term}$$
$$+ g^2\varepsilon_{abc}A^b_\mu A^c_\nu \varepsilon_{ade}A^{d\mu}A^{e\nu} \; \Big\} \; A^i_\mu \text{ 4-linear interact. term}$$

$$\Rightarrow \mathcal{L} = \bar{\Psi}(i\not{D}-m)\Psi - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$$

N.B.: - non abel. local sym.: charges are not free as for U(1)

- same procedure can be applied for any other SU(2) represent.
- " " " " " " " other Lie group

↳ replacing: $\frac{\tau_i}{2} \to T_a$ group generat. and $i\varepsilon_{abc}$ by $i\overset{\text{group struct. const.}}{C_{abc}}$

↳ $iC_{abc}T_c = [T_a, T_b]$

c] Electroweak interact° for leptons and quarks

All electroweak interact° can be perfectly accounted for assuming the localizat° of 2 global sym. : $SU(2)_L$ and $U(1)_Y$ under which left-handed fermions are in doublets and right-handed fermions are in singlets and assuming 3 times the same particle content

$L_e = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$ $L_\mu = \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$ $L_\tau = \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$ with $Y_{L_e} = Y_{L_\mu} = Y_{L_\tau} = -1$

e_R, μ_R, τ_R with $Y_{e_R} = Y_{\mu_R} = Y_{\tau_R} = -2$

inputs of the SM →

$\nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$ are assumed not to exist

$Q_L^u = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$ $Q_L^c = \begin{pmatrix} c_L \\ s_L \end{pmatrix}$ $Q_L^t = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$ with $Y_{Q_L^u} = Y_{Q_L^c} = Y_{Q_L^t} = \frac{1}{3}$

u_R, c_R, t_R with $Y_{u_R} = Y_{c_R} = Y_{t_R} = 4/3$

d_R, s_R, b_R with $Y_{d_R} = Y_{s_R} = Y_{b_R} = -2/3$

$$\Rightarrow \mathcal{L}^{EW}_{SM} = \sum_{i=e,\mu,\tau} \left(i \bar{L}_i \not{D} L_i + i \bar{\ell}_{Ri} \not{D} \ell_{Ri} \right)$$

$$+ \sum_{i=u,c,t} \left(i \bar{Q}_L^i \not{D} Q_L^i + i \bar{q}_R^i \not{D} q_{Ri} \right)$$

$$+ \sum_{i=d,s,b} i \bar{q}_R^i \not{D} q_{Ri}$$

$$- \frac{1}{4} \underset{\overset{\uparrow}{SU(2)_L}}{W^a_{\mu\nu}} W^{a\mu\nu} - \frac{1}{4} \underset{\overset{\uparrow}{U(1)_Y}}{B_{\mu\nu}} B^{\mu\nu}$$

with $D_\mu = \partial_\mu - ig \frac{\tau_a}{2} W^a_\mu - ig' \frac{Y}{2} B_\mu$

e-m interact° turns out to be included in this $\mathcal{L}$: so far W^i_μ and B_μ are massless ⇒ any rotation of them is equally physical : gauge boson mass eigenstates

⇒ defining $\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$

and taking $\sin\theta_W \equiv \frac{g'}{\sqrt{g^2+g'^2}}$ one gets θ_W = Weinberg angle

$$\mathcal{L} \ni e J_\mu^{em} A^\mu + \frac{g}{\cos\theta_W} J_\mu^Z Z^\mu$$

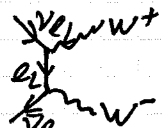
with $J_\mu^{em} = \underset{\substack{\uparrow \\ \text{all ferm.}}}{\sum_i} Q_i \bar{\Psi}_i \gamma_\mu \Psi_i$ ← ordinary e-m current

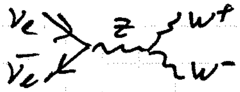
↳ with $Q_i = T_{3_i} + \frac{Y_i}{2}$

$e = g \sin\theta_W$

$J_\mu^Z = \underset{\substack{\uparrow \\ \text{all L and R ferm.}}}{\sum_i} [T_{3_i} - Q_i \sin^2\theta_W] \bar{\Psi}_i \gamma_\mu \Psi_i$: ← neutral current: prediction!

N.B.:
- e-m and weak interact°: derive from ≠ mixtures of same 2 gauge interact°s but do not unify: still coming from 2 ≠ interact°s: $SU(2)_L$ and $U(1)_Y$ each with its independent coupling strength g and g'
- gauge sym. implies gauge bosons massless
- gauge sym + the fact that L and R fermions are in ≠ $SU(2)_L$ repres ⇒ all fermions massless!
- e_L^- and e_R^- for instance have same e-m charges: purely accidental in SM! ← but will allow them to form a Dirac fermion once they will get a mass.
- to add explicitely $SU(2)_L \times U(1)_Y$ breaking gauge boson mass terms $W^a_{\mu\nu} W^{a\mu\nu}$ leads to non-renormalizability of theory: loop diagrams ⇒ ∞ : crucial constraint

- unitarity constraint: requires neutral current on top of charged currents: for instance 

gives $\sigma(\nu_e \bar\nu_e \to W^+W^-) \propto E_\nu^2$ for large energies $E_\nu >> m_W$ $\Rightarrow$ incompatible with $S^+S=1$ at large E_ν ($\Leftrightarrow$ conservation of probability) but extra diagram 

gives total cross section $\propto$ const for $E_\nu >> m_W$ $\Rightarrow S^+S=1$: unitarity is a generic problem if vector interact° fields do not stem from a gauge sym. principle.

c] QCD interactions for quarks

Strong interactions of quarks are accounted for also simply by the gauge principle, assuming that each quark comes in 3 copies forming a triplet of an SU(3) gauge symmetry:

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar q_f (i\gamma^\mu D_\mu - m_f) q_f$$

$$q_f = \begin{pmatrix} q_b \\ q_g \\ q_r \end{pmatrix}_f = q_{f_L} + q_{f_R}$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f_{abc} A^b_\mu A^c_\nu$$

↳ $a = 1, \dots, 8$: 8 massless gluons because 8 SU(3) gener.

$$[T_a, T_b] = i f_{abc} T_c$$

$T_a = \frac{\lambda_a}{2}$: Gellman matrices

$$D_\mu = \partial_\mu - i g T_a A^a_\mu$$

N.B.:
- QCD: vectorial theory (as QED): L and R in same repres.
  ↳ $\Rightarrow$ QCD allows quark masses
- most ⚠ property of QCD: asymptotic freedom and confinement

D] Spontaneous Symmetry breaking of $SU(2)_L \times U(1)_Y$

↳ necessary to have massive W, Z and fermions keeping the theory renormalizable and unitary

1) Analogy with ferromagnetism

↳ atoms with interacting spins

$$\mathcal{H} = -\lambda \sum_{i,j} \vec{S_i}.\vec{S_j} \quad (\lambda > 0)$$

↳ displays a SO(3) sym.: global rotat° of all $\vec{S_i}$

⇒ $\mathcal{H}_{MIN}$: configurat° with all $\vec{S_i}$ aligned = "vacuum"
↳ break the SO(3) sym. of $\mathcal{H}$ (⇔ of $\mathcal{L}$): prefered direction of the vacuum
↳ ≠ from configur. with random $\vec{S_i}$ which has no prefered global direct° ⇔ which does not break SO(3) sym.

- high temperature : $T \gg \mathcal{H}_{MIN}$: kinetic energy of atoms is >> energy associated to spin interact° ⇒ configurat° with random $\vec{S_i}$: SO(3) sym. ⇐ $Z = \sum \underbrace{e^{-\underset{\ll 1}{\mathcal{H}/T}}}_{1}$: entropy maximum for random $\vec{S_i}$.

phase transit° at $T \sim \mathcal{H}_{MIN}$

- low temperature : $T \ll \mathcal{H}_{MIN}$: $e^{-\mathcal{H}/T}$ peaked on $\mathcal{H}_{min}$ configur. ⇒ all spin aligned : SO(3) is spontaneously broken into SO(2) sym (rotat° around aligned spin direct°)

N.B.: direct° of alignement : undetermined : remnant of SO(3) sym. of $\mathcal{H}$: at phase transition one direct° is taken randomly (depending on thermal fluctuations)
↳ physics same whatever is the direction taken!

2) SSB of a local SU(2)

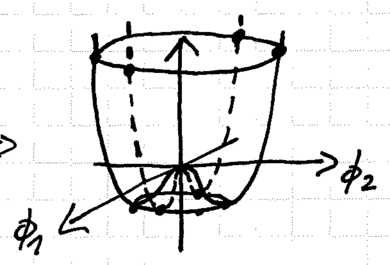
$$\mathcal{L} = (D_\mu \phi)^+ (D^\mu \phi) - V(\phi) - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

↳ one assumes a complex scalar doublet

$$\phi = \begin{pmatrix} (\phi_1 + i\phi_2)/\sqrt{2} \\ (\phi_3 + i\phi_4)/\sqrt{2} \end{pmatrix}$$

$$V(\phi) = \mu^2 \phi^+\phi + \lambda(\phi^+\phi)^2$$

↳ one assumes $\mu^2 < 0$ ⟶



↳ vacuum is a 3-dim. sphere : $\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = -\frac{\mu^2}{\lambda} \equiv \underset{\uparrow}{v}^2$

↳ here represented by a circle   vacuum expectat° value

⇒ in early universe: at $T \gg v$: $SU(2)_L \times U(1)_Y$: no SSB

at $T \sim v$: phase transition : any point of the vacuum manifold is randomly chosen (physics does not depend on which point)

⇒ let's take : $\langle\phi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \underset{\nwarrow \text{real}}{v} \end{pmatrix}$

let us also parametrize ϕ in polar coordinates

$$\phi(x) = \exp\left(i \tau_a \underset{\uparrow}{\frac{\xi^a(x)}{v}}\right) \begin{pmatrix} 0 \\ \frac{v + \eta(x)}{\sqrt{2}} \end{pmatrix} \leftarrow \phi_3 = v + \eta(x)$$

3 tangential fields   ↖ radial field

We make a gauge choice which simplify interpretation of the result : $\phi \rightarrow \phi' = \underset{\uparrow}{U} \phi = \begin{pmatrix} 0 \\ \frac{v + \eta(x)}{\sqrt{2}} \end{pmatrix}$

$$U = \exp\left(-i \tau_a \frac{\xi^a(x)}{v}\right)$$

$$\frac{\tau_a}{2} W^{a\prime}_\mu = U \frac{\tau_a}{2} W^a_\mu U^+ - \frac{i}{g} [\partial_\mu U] U^{-1}$$

$$\Rightarrow \mathcal{L} = (D_\mu \phi')^+ (D^\mu \phi') - \frac{\mu^2}{2} (\eta + v)^2 - \frac{\lambda}{4} (\eta + v)^4 - \frac{1}{4} F^{a\prime}_{\mu\nu} F^{a\mu\nu\prime}$$

$$= \frac{1}{2}\partial_\mu \eta \partial^\mu \eta + \frac{g^2}{8}\left(0 \quad \frac{\eta+v}{\sqrt{2}}\right)(\tau_a W_\mu^{a'})(\tau_b W^{\mu b'})\begin{pmatrix} 0 \\ \frac{\eta+v}{\sqrt{2}} \end{pmatrix}$$

$$-\frac{\eta^2}{2}(\mu^2+3\lambda v^2) - \eta\underbrace{(\mu^2 v+\lambda v^3)}_{\substack{=0 \\ \hookrightarrow \text{because } \langle\eta\rangle=0}} - \lambda v \eta^3 - \frac{1}{4}\eta^4 - \frac{1}{4}F_{\mu\nu}^{\prime a}F^{\prime a\mu\nu}$$

$$= \frac{1}{2}\partial_\mu \eta \partial^\mu \eta + \frac{1}{2}\left(\frac{gv}{2}\right)^2 (W_\mu^{\prime a} W^{\prime a\mu}) - \frac{\eta^2}{2}(-2\mu^2) - \lambda v \eta^3$$

$$-\frac{1}{4}\lambda\eta^4 + \frac{g^2 v^2}{8}(2\eta v+\eta^2) W_\mu^{\prime a} W^{\prime a\mu} - \frac{1}{4}F_{\mu\nu}^{\prime a} F^{\prime a\mu\nu}$$

⇒ • η field: propagates : $\partial_\mu \eta \partial^\mu \eta$ term: = 1 real scalar particle

$\uparrow$ Brout-Englert-Higgs boson

$$m_\eta^2 = -2\mu^2 = \frac{v^2}{2\lambda}$$

- W_μ^a : have a mass! : $m^2_{W_{1,2,3}} = \left(\frac{gv}{2}\right)^2$
- ξ^a : have disappeared from $\mathcal{L}$! : no kinetic term, ... : eaten by the $W_\mu^{\prime a}$ which allow the $W_\mu^{\prime a}$ to have a mass

$$\frac{\tau_a}{2}W_\mu^{\prime a} = U\frac{\tau_a}{2}W_\mu^a - \frac{i}{g}[\partial_\mu U]U^{-1}$$

$\uparrow\uparrow\uparrow$ $\ni \xi^a(x)$

⇒

before SSB		After SSB
W_μ^a : 3×2 dof		W_μ^a : 3×3 dof
ϕ : 4dof		η : 1dof
10 dof	$\Longleftrightarrow$	10 dof

3) SSB of $SU(2)_L \times U(1)_Y$

↳ same as SSB of SU(2) above but assuming in addition the scalar doublet has $Y=1$ so that the ϕ_3 field acquiring the vev (by convention) is neutral
⇒ no SSB of $U(1)_{EM}$.

⇒ changes gauge boson mass term:

$$\mathcal{L} \ni (D_\mu \phi')(D^\mu \phi') \ni \frac{1}{2}(0\ v)\left(\frac{g}{2}\tau_a W'^a_\mu + \frac{g'}{2}B_\mu\right)\left(\frac{g}{2}\tau^a W'^{a\mu} + \frac{g'}{2}B^\mu\right)\binom{0}{v}$$

$$= \frac{v^2}{8}\left\{g^2\left[(W'^1_\mu)^2 + (W^2_\mu)^2\right] + (-gW'^3_\mu + g'B_\mu)^2\right.$$

$$= \underbrace{\frac{g^2v^2}{4}}_{m^2_W} W'^+_\mu W'^{\mu -} + \underbrace{\frac{(g^2+g'^2)v^2}{8}}_{m^2_Z} \underbrace{\left(\frac{g}{\sqrt{g^2+g'^2}}W'^3_\mu - \frac{g'}{\sqrt{g^2+g'^2}}B_\mu\right)^2}_{Z_\mu Z^\mu}$$

⇒ 3 massive gauge bosons: $W^\pm$, Z ⇒ weak changed and neutral currents

+1 massless gauge boson: A_μ ⇒ QED

↳ predicts $\frac{m^2_W}{m^2_Z} = \frac{g^2}{g^2+g'^2} = \cos^2\theta_W$

$$\Rightarrow \mathcal{L}_{EW} = -\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \sum_{\text{all ferm.}} \bar{f}_i i\not{\partial} f_i$$

$$+ m^2_W W^+_\mu W^{-\mu} + \frac{m^2_Z}{2} Z_\mu Z^\mu$$

$$+ \frac{g}{\sqrt{2}}(J^+_\mu W^{+\mu} + J^-_\mu W^{-\mu}) + e J^{em}_\mu A^\mu$$

$$+ \frac{g}{\cos\theta_W} J^Z_\mu Z^\mu \qquad \Leftarrow J^Z_\mu = \sum_i (g^i_L \bar{f}_{iL}\gamma_\mu f_{iL} + g^i_R \bar{f}_{iR}\gamma_\mu f_{iR})$$

$$-\frac{m^2_\eta}{2}\eta^2 - \lambda v \eta^3 - \frac{\lambda}{4}\eta^4 \qquad g^i_L = T^i_3 - Q_i \sin^2\theta_W$$

$$+ \frac{g^2}{4}(2\eta v + \eta^2)(W^+_\mu W^{-\mu}) \qquad g^i_R = -Q_i \sin^2\theta_W$$

$$+ \frac{g^2+g'^2}{8}(2\eta v + \eta^2) Z_\mu Z^\mu$$

↗ all that with only one scalar doublet and only 4 paramet., g, g', μ^2, λ

E] FERMION MASSES

It turns out that the scalar doublet, because it has $Y=1$, is allowed to have an extra interact°:

$$\mathcal{L} \ni -Y_e \bar{L}_e \phi e_R + h.c. \quad \leftarrow \text{YUKAWA INTERACTION}$$

$$= -\underbrace{\frac{Y_e v}{\sqrt{2}}}_{m_e!!} \bar{e}_L e_R - \frac{Y_e \eta}{\sqrt{2}} \bar{e}_L e_R + h.c.$$

↳ with 3 generat°s of leptons and similarly for quarks:

$$\mathcal{L} \ni -\bar{L}_i Y_{\ell ij} \phi \ell_{Rj} + h.c.$$

$$\ni -\underbrace{\frac{v}{\sqrt{2}} Y_{\ell ij}}_{(M_\ell)_{ij} \leftarrow \text{lepton mass matrix}} \bar{\ell}_{iL} \ell_{jR} - Y_{\ell ij} \frac{\eta}{\sqrt{2}} \bar{\ell}_{iL} \ell_{jR} + h.c.$$

$$\mathcal{L} \ni -\bar{Q}_{Li} \phi Y_{Dij} d_{Rj} + h.c.$$

$$\ni -\underbrace{\frac{v}{\sqrt{2}} Y_{Dij}}_{(M_D)_{ij} \leftarrow \text{d quark mass matrix}} \bar{d}_{Li} d_{Rj} - Y_{Dij} \frac{\eta}{\sqrt{2}} \bar{d}_{Li} d_{Rj}$$

for quarks an extra interact° is also allowed; because $\tilde{\phi} \equiv i\tau_2 \phi^*$ transforms as ϕ under $SU(2)_L$

$$\mathcal{L} \ni -\bar{Q}_{Li} \tilde{\Phi} Y_{Uij} u_{Rj} + h.c.$$

$$\ni -\underbrace{\frac{v}{\sqrt{2}} Y_{Uij}}_{(M_U)_{ij} \leftarrow \text{u quark mass matrix}} \bar{u}_{Li} u_{Rj} - Y_{Uij} \frac{\eta}{\sqrt{2}} \bar{u}_{Li} u_{Rj} + h.c.$$

$(\mathcal{M}_l)_{ij}, (\mathcal{M}_D)_{ij}, (\mathcal{M}_U)_{ij}$: 3×3 complex matrices: can be diagonalized by bi-unitary transfor.

$$\mathcal{L} \ni -\bar{l}_L' \underbrace{U_{l_L}^{+} \mathcal{M}_l U_{l_R}}_{=\mathcal{M}_l^{DIAG} = \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix}} l_R' - \bar{d}_L' \underbrace{U_{d_L}^{+} \mathcal{M}_D U_{d_R}}_{\equiv \mathcal{M}_D^{DIAG} = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}} d_R' - \bar{u}_L' \underbrace{U_{u_L}^{+} \mathcal{M}_U U_{u_R}}_{\equiv \mathcal{M}_U^{DIAG} = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}} u_R'$$

=> l_i, d_i, u_i : flavour states : in doublets of SU(2)
l_i', d_i', u_i' : mass eigenstates : physical states

Rewriting then the all $\mathcal{L}$ in terms of mass eigenstates:

- for leptons: $(\bar{\nu}_L\ \bar{l}_L) i\not{\partial} \begin{pmatrix} \nu_l \\ l_L \end{pmatrix} = (\bar{\nu}_L' U_{l_L}^{+} . \bar{l}_L' U_{l_L}^{+})(i\not{\partial}) \begin{pmatrix} U_{l_L} \nu_L \\ U_{l_L} e_L \end{pmatrix}$

$$= (\bar{\nu}_L'\ \bar{l}_L')(i\not{\partial}) \begin{pmatrix} \nu_L' \\ e_L' \end{pmatrix}$$

=> no change: charged and neutral current interact° remain flavour diagon. (becouse ν_L have no mass => the 3 ν can be rotated by any unitary matrix => we choose a same rotat° for ν_L than for l_L)

- for quarks: big ≠ with leptons: both up and down quarks have a mass matrix which fix U_{d_L} and U_{u_L} => and in general $U_{d_L} \neq U_{u_L}$

=> • <u>for neutral currents</u>: remain diagonal in mass eigenst. basis becouse involve always $\bar{\psi}_i \dots \psi_i$ for same i:
for instance $\bar{u}_L \gamma_\mu u_L \to \bar{u}_L' U_{u_L}^{+} \gamma_\mu U_{u_L} u_L' = \bar{u}_L' \gamma_\mu u_L$
=> no Flavour Changing Neutral Currents in SM!

- for charged currents: not flavour diagonal in mass eigenst. basis!

↳ $\mathcal{L} \ni \frac{g}{\sqrt{2}} (\bar{u}_L' \gamma_\mu (\underbrace{U_{u_L}^{+} U_{d_L}}_{\neq 1!}) d_L') W^{+\mu} + h.c.$

$$= \frac{g}{\sqrt{2}} (\bar{u}_L', \bar{c}_L', \bar{t}_L') \gamma_\mu \underbrace{U_{u_L}^{+} U_{d_L}}_{V_{CKM}} \begin{pmatrix} d_L' \\ s_L' \\ b_L' \end{pmatrix}$$

↳ contains 3 angles and one phase: δ_{CKM}

↳ break CP

↳ unique source of CP violat^{o} in SM

$\Rightarrow$ for leptons: $\underset{\overset{\uparrow}{U(1)_{L_e}}}{L_e}, \underset{\overset{\uparrow}{U(1)_{L_\mu}}}{L_\mu}, \underset{\overset{\uparrow}{U(1)_{L_\tau}}}{L_\tau}, \underset{\overset{\uparrow}{U(1)_L}}{L = L_e + L_\mu + L_\tau}$ conserved

for quarks: only $B = (\#\text{ of quarks} - \#\text{ of antiquarks}) . \frac{1}{3}$ is conserved : $U(1)_B$

F] LIMITATIONS/RESTRICTIONS OF THE SM

The SM, based on a very limited number of concepts and principles (QFT, gauge symmetries essentially) can explain a huge diversity of phenomena.
It, nevertheless, leaves unanswered many questions including:

- why a $SU(3)_c \times SU(2)_L \times U(1)_Y$ structure?
- why this fermion content with this charge assignement?
- status of neutrinos: ν_R or not ν_R?
- origin of flavour structure: fermion mass hierarchies?
  V_{CKM} structure?
- hierarchy problem?
- θ_{STRONG} problem?
- ...

Moreover it cannot account for a series of phenomena:

- neutrino masses: origin?
- origin of baryogenesis?
- origin and status of dark matter?
- origin of cosmic inflation?
- ...

} evidence for BSM physics

Finally it doesn't include gravity: gravity $\overset{?}{\rightleftarrows}$ QFT??

CHAP III: NEUTRINO MASSES

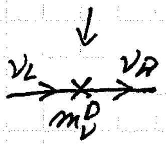
↳ ONLY clear laboratory evidence we have at the moment for BSM physics

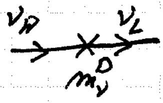
↳ it is new physics BSM because as we will see ν masses either implies a new symmetry of nature or a new physics scale (beyond the unique E-W scale of SM) and because there are several simple possibilities of explanation for them (new fields, ...)

A] TWO POSSIBLE TYPES OF ν MASSES

- **Dirac masses**: if right-hd ν exist neutrinos could have Dirac masses

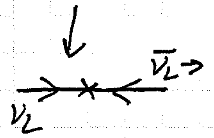
$$\mathcal{L} \ni -m_\nu^D \left(\overline{\nu_R}\,\nu_L + \overline{\nu_L}\,\nu_R\right)$$

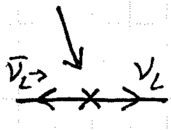




- **Majorana masses**: even if right-hd do not exist (or even if they do exist, see below) they could have instead Majorana masses (possible becouse ν: neutral)

$$\mathcal{L} \ni -\frac{1}{2}\, m_\nu^M \left(\overline{\nu_L^c}\,\nu_L + \overline{\nu_L}\,\nu_L^c\right)$$





- 3 flavour ν's : $\Rightarrow$ 3×3 ν mass matrix

  - For the Dirac mass case: $\Leftarrow$ similar to case of l_i, d_i, u_i

$$\mathcal{L} \ni -(\bar{\nu}_{eR}\ \bar{\nu}_{\mu R}\ \bar{\nu}_{\tau R}) \begin{pmatrix} m_{\nu ee} & m_{\nu e\mu} & m_{\nu e\tau} \\ m_{\nu\mu e} & m_{\nu\mu\mu} & m_{\nu\mu\tau} \\ m_{\nu\tau e} & m_{\nu\tau\mu} & m_{\nu\tau\tau} \end{pmatrix} \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} + h.c.$$

$$= -(\bar{\nu}_{1R}\ \bar{\nu}_{2R}\ \bar{\nu}_{3R})\, \underbrace{U^{+}_{\nu_R} \begin{pmatrix} & & \\ & " & \\ & & \end{pmatrix} U_{\nu_L}}_{= \mathcal{M}_{\nu}^{DIAG} = \begin{pmatrix} m_{\nu 1} & & \\ & m_{\nu 2} & \\ & & m_{\nu 3} \end{pmatrix}} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

Diagonalisat° by bi-unitary tranformat°

$\Rightarrow$ in charged current interact° of the SM, in the same way as for up quarks, one gets a CKM type mixing matrix

$$\mathcal{L} \ni -(\bar{e}'_L\ \bar{\mu}'_L\ \bar{\tau}'_L)\, \gamma^{\mu} W^{+}_{\mu}\, \overbrace{U^{+}_{\ell_L} U_{\nu_L}}^{\equiv U_{PMNS}} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix} + h.c.$$

$$= -(\bar{e}''_L\ \bar{\mu}''_L\ \bar{\tau}''_L)\, \gamma^{\mu} W^{+}_{\mu}\, \underbrace{\begin{pmatrix} e^{-i\alpha_e} & & \\ & e^{-i\alpha_\mu} & \\ & & e^{-i\alpha_\tau} \end{pmatrix} U^{+}_{\ell_L} U_{\nu_L} \begin{pmatrix} e^{i\alpha_{\nu 1}} & & \\ & e^{i\alpha_{\nu 2}} & \\ & & e^{i\alpha_{\nu 3}} \end{pmatrix}}_{U'_{PMNS}} \begin{pmatrix} \nu'_{1L} \\ \nu'_{2L} \\ \nu'_{3L} \end{pmatrix} + h.c.$$

U_{PMNS} : 3×3 unitary $\Rightarrow$ 3 angles + 6 phases

$\Rightarrow$ U'_{PMNS} : 3 angles + 1 phase $\Leftarrow$ just as in SM for quarks

$\hookrightarrow$ called the "Dirac phase"

  - For the Majorana mass case:

$$\mathcal{L} \ni -(\bar{\nu}^c_e\ \bar{\nu}^c_\mu\ \bar{\nu}^c_\tau) \underbrace{\begin{pmatrix} m_{\nu ee} & m_{\nu e\mu} & m_{\nu e\tau} \\ m_{\nu e\mu} & m_{\nu\mu\mu} & m_{\nu\mu\tau} \\ m_{\nu e\tau} & m_{\nu\mu\tau} & m_{\nu\tau\tau} \end{pmatrix}}_{\text{symmetric matrix because } \bar{\nu}^c \nu \propto \nu^T \nu} \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} + h.c.$$

$$= -(\bar{\nu}^c_{1L}\ \bar{\nu}^c_{2L}\ \bar{\nu}^c_{3L})\, \underbrace{U^{+}_{\nu_L} \begin{pmatrix} & & \\ & " & \\ & & \end{pmatrix} U_{\nu_L}}_{= \mathcal{M}_{\nu}^{DIAG} = \begin{pmatrix} m_{\nu 1} & & \\ & m_{\nu 2} & \\ & & m_{\nu 3} \end{pmatrix}} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

Diagonalisat° by single unitary transformat°

⇒ in charged current interact° of the SM we get something ≠ from Dirac case:

$$\mathcal{L} \ni -(\bar{e}'_L\ \bar{\mu}'_L\ \bar{\tau}'_L)\gamma^\mu W^+_\mu \overbrace{U^\dagger_{\ell L} U_{\nu L}}^{U_{PMNS}} \begin{pmatrix}\nu_{1L}\\ \nu_{2L}\\ \nu_{3L}\end{pmatrix} + h.c.$$

$$= -(\bar{e}''_L\ \bar{\mu}''_L\ \bar{\tau}''_L)\gamma^\mu W^+_\mu \underbrace{\begin{pmatrix} e^{-i\phi_e} & & \\ & e^{-i\phi_\mu} & \\ & & e^{-i\phi_\tau}\end{pmatrix} U^\dagger_{\ell L} U_{\nu L}}_{U'_{PMNS}} \underset{\uparrow}{\begin{pmatrix}\nu_{1L}\\ \nu_{2L}\\ \nu_{3L}\end{pmatrix}}$$

$\nu_{1,2,3}$ cannot be rephased because that would give $m_{\nu_{1,2,3}}$ imaginary

⇒ Dirac case: $3\, m_\nu$ + 3 angle + 1 phase : 7 param. in $\mathcal{M}_\nu$

Majorana case: $3\, m_\nu$ + 3 angle + 3 phases : 9 param in $\mathcal{M}_\nu$

↳ the Dirac one + 2 Majorana ones.

B] NEUTRINO OSCILLATIONS

↳ allow to determine the 3 mixing angles, 2 Δm^2 and δ_{DIRAC} but not absolute ν mass scale and not both Majorana phases

a) OSCILLATIONS BETWEEN 2 NEUTRINOS

↳ Suppose there are only 2 neutrinos, ν_μ and ν_τ with non-diagonal neutrino 2×2 mass matrix

⇒ Flavour states: $|\nu_\mu\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$

$|\nu_\tau\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$

⇑

N.B.: in 2 by 2 Dirac cases only 3 param: $m_{\nu_1}, m_{\nu_2}, \cos\theta$

in 2 by 2 Major. case there is one additional Major. phase we neglect here consider a ν_μ state produced at $t=x=0$ (from a W^+ decay for instance) with energy E: how this ν_μ evolves in time??

$$\hookrightarrow \quad |\nu_\mu(t=0,x=0)\rangle = |\nu_\mu\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle \qquad (*)$$

$$\underset{\substack{\uparrow\\ \text{mass eigenstates}}}{|\nu_{1,2}\rangle} : \text{plane waves} : |\nu_{1,2}(t,x)\rangle = e^{-i(E_{1,2}t - p_{1,2}x)}\,|\nu_{1,2}\rangle \qquad (**)$$

$$\Rightarrow |\nu_\mu(t,x)\rangle = \cos\theta\,|\nu_1(t,x)\rangle + \sin\theta\,|\nu_2(t,x)\rangle$$

$$\overset{(*)+(**)}{\hookrightarrow} = \left(\cos^2\theta\, e^{-i(E_1t - p_1x)} + \sin^2\theta\, e^{-i(E_2t-p_2x)}\right)|\nu_\mu\rangle$$

$$+ \left(-\sin\theta\cos\theta\, e^{-i(E_1t-p_1x)} + \sin\theta\cos\theta\, e^{-i(E_2t-p_2x)}\right)|\nu_\tau\rangle$$

$\Rightarrow$ probability that a ν_μ produced at $t=x=0$ is seen as a ν_τ at t,x:

$$P(\nu_\mu(t=0,x=0)\rightarrow \nu_\tau(t,x)) = |\langle\nu_\tau|\nu_\mu(t,x)\rangle|^2$$

$$= |\sin\theta\cos\theta\,(-e^{-i(E_1t-p_1x)} + e^{-i(E_2t-p_2x)}|^2$$

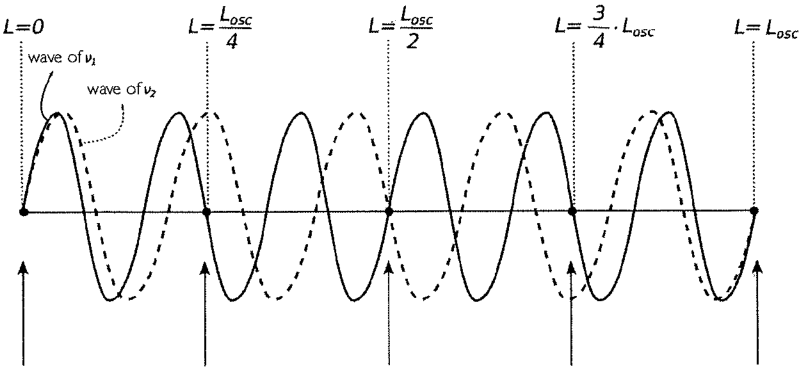
$$\hookrightarrow \; = \sin^2 2\theta\, \sin^2\frac{\Delta m^2_\nu L}{4E}$$

$$E_it - p_ix \underset{\substack{\uparrow\\ \text{relativistic } \nu : \begin{cases} t=x=L \\ E_i=p_i=E\end{cases}}}{\simeq} \frac{m^2_{\nu_i} L}{2E}$$

$$\nwarrow \; \Delta m^2_\nu = m^2_{\nu_2} - m^2_{\nu_1}$$

$$P(\nu_\mu(t=0,x=0)\rightarrow \nu_\mu(t,x)) = 1 - \sin^2 2\theta \sin^2\frac{\Delta m^2_\nu L}{4E}$$

$\Rightarrow$ example : maximum mixing : $\theta = \pi/4$ $\qquad L_{osc} \equiv \frac{4E}{\Delta m^2_\nu}\cdot\pi$



pure ν_μ : waves of ν_1 and ν_2 component are aligned

the ν_μ became a ν_μ at 50% and a ν_τ at 50%

the ν_μ became a pure ν_τ: waves of ν_1 and ν_2 component are opposed

the ν_μ became a ν_μ at 50% and a ν_τ at 50%

pure ν_μ again: waves of ν_1 and ν_2 component are aligned

Oscillations for the 3 ν case:

↳ we proceed in same way as for the 2ν case but now writing: $|\nu_\mu(t,x)\rangle = U_{\mu i}\,|\nu_i(t,x)\rangle$ $\quad U \equiv U_{\nu_L}$

$$= U_{\mu i}\, e^{-i(E_i t - p_i x)}\,|\nu_i\rangle$$

$$= U_{\mu i}\, e^{-i(\quad " \quad)}\, U^\dagger_{i\alpha}\,|\nu_\alpha\rangle \qquad \alpha = e,\mu,\tau$$

$$\Rightarrow P(\nu_\mu(t=0,x=0) \to \nu_\tau(t,x)) = |\langle \nu_\tau|\nu_\mu(t,x)\rangle|^2$$

$$= \sum_{i,j} U_{\mu i} U^*_{\tau i} U^*_{\mu j} U_{\tau j}\, e^{-i\frac{\Delta m^2_{ij} L}{2E}}$$

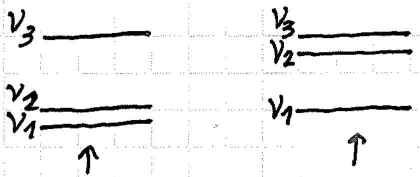
$$\Delta m^2_{ij} \equiv m^2_{\nu_i} - m^2_{\nu_j}$$

$$J^{\mu\tau}_{ij} = U_{\mu i} U_{\tau j} U^*_{\tau i} U^*_{\mu j}$$

$$= -4 \sum_{i<j} \mathrm{Re}\,|J^{\mu\tau}_{ij}|\, \sin^2 \frac{\Delta m^2_{\nu_{ij}} L}{4E}$$

$$-2 \sum_{i<j} \underbrace{\mathrm{Im}\,|J^{\mu\tau}_{ij}|}_{\hookrightarrow \cancel{CP}\text{ part}}\, \sin^2 \frac{\;"\;}{"}$$

Now in practice one splitting is much larger than the other one, say $\Delta m^2_{32} \gg \Delta m^2_{21}$ ⇒ 2 possibilities



"normal hierarchy" "inverted hierarchy"

In both case we can make the approximat°: $\Delta m^2_{31} = \Delta m^2_{32} \gg \Delta m^2_{21}$ and writing

Dirac phase ↓ $\qquad$ ∈ 3 EULER angles

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} e^{i\alpha} & & \\ & e^{i\beta} & \\ & & 1 \end{pmatrix}}_{\text{addition. phases for Major. case}}$$

we get:

$$P(\nu_e \to \nu_\mu) = s_{23}^2 \sin^2 2\theta_{13} S_{23} + c_{23}^2 \sin^2 2\theta_{12} S_{12} - P_{CP}$$

$$S_{ij} = \sin^2 \frac{\Delta m_{ij}^2 L}{4E}$$

$$P(\nu_e \to \nu_\tau) = c_{23}^2 \sin^2 2\theta_{13} S_{23} + s_{23}^2 \sin^2 2\theta_{12} S_{12} + P_{CP}$$

$$P(\nu_\mu \to \nu_\tau) = c_{13}^4 \sin^2 2\theta_{23} S_{23} - s_{23}^2 c_{23}^2 \sin^2 2\theta_{12} S_{12} - P_{CP}$$

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_{13} S_{23} - c_{13}^4 \sin^2 2\theta_{12} S_{12}$$

$$P(\nu_\mu \to \nu_\mu) = 1 - 4 c_{13}^2 s_{23}^2 (1 - c_{13}^2 s_{23}^2) S_{23} - c_{23}^4 \sin^2 2\theta_{12} S_{12}$$

$$P(\nu_\tau \to \nu_\tau) = 1 - 4 c_{13}^2 c_{23}^2 (1 - c_{13}^2 c_{23}^2) S_{23} - s_{23}^4 \sin^2 2\theta_{12} S_{12}$$

↳ replacing ν by $\bar{\nu}$ gives the same except that $P_{CP} \to -P_{CP}$
↳ see below

⇒ very brief survey on oscillation parameter determinat

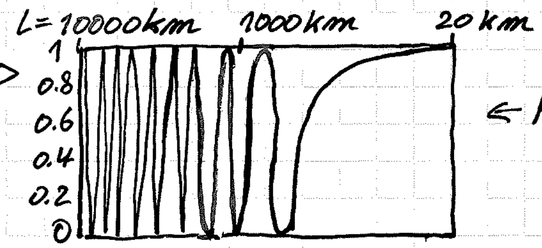
- "Atmospheric parameters": θ_{23}, Δm_{23}^2

↳ a useful formula: $\sin^2 \frac{\Delta m^2 L}{4E} = \sin^2 1{,}27 \frac{\Delta m^2}{eV^2} \frac{L}{km} \frac{GeV}{E}$

⇒ athmospheric ν_μ: $E \sim GeV$, $\Delta m^2 \sim 2{,}5 \cdot 10^{-3} eV$

↳ ⇒ $L_{osc} \approx 250 km \Leftarrow \frac{\Delta m^2 L_{osc}}{4E} \sim \pi$

⇒



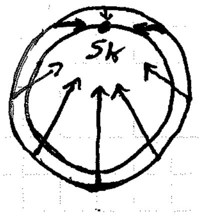
← $P(\nu_\mu \to \nu_\mu)$

$$S_{ij} \equiv \sin^2 \frac{\Delta m_{ij}^2 L}{4E}$$

$$P(\nu_\mu \to \nu_\mu) = 1 - 4 \underbrace{c_{13}^2}_{\sim 1} s_{23}^2 (1 - \underbrace{c_{13}^2}_{\sim 1} s_{23}^2) S_{23} - c_{23}^4 \underbrace{\sin^2 2\theta_{12}}_{small} S_{12}$$

$$\simeq 1 - 4 s_{23}^2 (1 - s_{23}^2) S_{23}$$

⇒ fitting the observed oscillat° pattern of $P(\nu_\mu \to \nu_\mu)$ as a function of L (⇔ zenithal angle distribut°)
SK got: $|\Delta m_{23}^2| = 2.5 \cdot 10^{-3} eV^2$, $\theta_{23} = \pi/4$



↖ The superkamiokonde 1998 breakthrough
↳ Nobel prize 2015

↳ note that to determine $\sin\theta_{23}$ it is enough to observe ν_μ with $L >> L_{osc}$ <=> $S_{23} = \sin^2 \frac{\Delta m^2_{23} L}{4E} \simeq \underset{\substack{\uparrow \\ \text{average}}}{\frac{1}{2}}$

in case $P(\nu_\mu \to \nu_\mu) \simeq 1 - 2 s^2_{23}(1-s^2_{23}) \simeq \underset{\substack{\uparrow \\ \text{observed}}}{\frac{1}{2}}$

↳ => $\theta_{23} \sim \frac{\pi}{4}$: maximal mixing

↳ note also that $P(\nu_\mu \to \nu_\mu)$ deficit has been also observed from a beam of ν_μ : K2K (~2000) : first laboratory evidence for BSM physics!

- "Solar parameters": θ_{12}, Δm^2_{12}

  ↳ solar ν_e neutrinos

  ↳ $P(\nu_e \to \nu_\mu) + P(\nu_e \to \nu_\tau) = \underbrace{\sin^2 2\theta_{13} S_{23}}_{\text{very small: see below}} + \sin^2 2\theta_{12} S_{12}$

  => $P(\nu_e \to \nu_e) \simeq 1 - \sin^2 2\theta_{12} S_{12}$

  ↳ solar ν_e experiments: ν_e deficits

  already in 1970's
  (Nobel prize 2002)
  ↑
  Homestake,
  Kamiokande,
  SNO, ...

  => $\Delta m^2_{21} = (7{,}5 \pm 02) \cdot 10^{-5} eV^2$

  $\sin^2\theta_{12} = 0.304 \pm 0.01$

  ⇑
  matter effect crucial for this determination
  ↳ (skipped here)

- θ_{13}: "reactor ν parameter": $L \sim 1$ km, $E \sim$ MeV

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2\theta_{13} \underbrace{\sin^2 \frac{\Delta m^2_{23} L}{4E}}_{\sim 1} - c^4_{13} \sin^2 2\theta_{12} \underbrace{\sin^2 \frac{\Delta m^2_{12} L}{4E}}_{<<1}$$

$\leftarrow$ for $\frac{\Delta m^2_{23} L}{4E} \sim \frac{\pi}{2}$ $\nearrow$

$\Rightarrow \sin^2 2\theta_{13} = 0.09 \pm 0.001$ DOUBLE CHOOZ, DAYA BAY } 2012

- δ: $A_{CP} \equiv \frac{P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)}{\text{" } + \text{ "}} \propto P_{CP}$

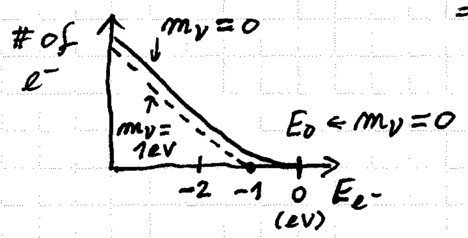
$$P_{CP} \equiv \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin\delta_{13} \sin\frac{\Delta m^2_{12} L}{4E} \cdot \sin\frac{\Delta^2_{23} L}{4E}$$

... (yet to be cleanly observed)

- absolute ν mass scale: · Tritium decay (KATRIN, ...)

$\Rightarrow m_\nu < 1.1$ eV

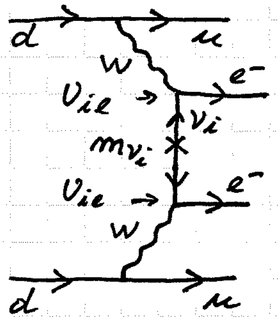
$^3H \to {}^3He + e^- + \bar{\nu}_e$ ($\uparrow$ not seen)



- cosmology: $\sum_{i=1,..,3} m_{\nu_i} < 0.4$ eV

↳ affects large scale structure formation

- Dirac or Majorana? neutrinoless double beta decay:



$\leftarrow$ does not exist if ν masses are of the Dirac type.

$\Rightarrow$ rate proportional to $\sum U^2_{ie} m_{\nu_i} \equiv m_{\nu ee}$

depends on Majorana phases $\downarrow$

$$= c^2_{13}(m_{\nu_1} c^2_{12} + m_{\nu_2} s^2_{12} e^{2i\alpha}) + m_{\nu_3} s^2_{13} e^{2i(\beta-\delta)}$$

$|m^{exp.}_{\nu ee}| < 0.23$ eV (Kamland Zen experiment) (90% CL)

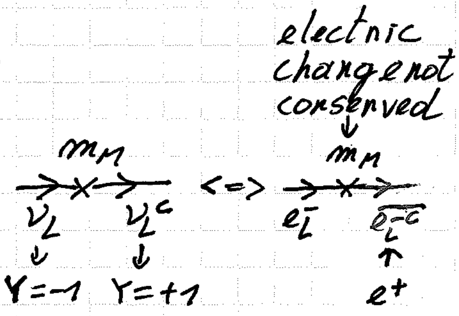
C| NEUTRINO MASS ORIGIN?

- why neutrinos are massless in the SM:

  ↳ no Dirac masses in SM because no ν_R in SM

  ↳ no Majorana masses because:

  - this breaks $SU(2)_L \times U(1)_Y$:



  - in principle could be generated through SSB of $SU(2)_L \times U(1)_Y$ from interaction involving the SM scalar doublet once replaced by its vev but no such interact in SM:

H = SM scalar doublet

$$\mathcal{L} \ni \overline{L^c} \cdot L\, H$$: not $SU(2)_L \times U(1)_Y$ invar: 3 doublets

$$\mathcal{L} \ni \frac{1}{\Lambda} \underbrace{(\overline{L^c}\tilde{H}^*)}_{Y=-2} \underbrace{(\tilde{H}^\dagger L)}_{Y=2}$$: $SU(2)_L \times U(1)_Y$ invar. but not in SM because dim-5 interact. ⇒ non-renormal. ⇒ forbidden in SM

$$\left.\begin{array}{l} \overset{SU(2)_L}{\downarrow} \\ L \to UL,\ H \to UH, \\ \tilde{H} \to U\tilde{H},\ \tilde{H}^\dagger \to \tilde{H}^\dagger U^\dagger \\ \overline{L^c} \sim L^T \to \overline{L^c} U^T \\ \tilde{H}^* \to U^* \tilde{H}^* \end{array}\right\} \Rightarrow SU(2)_L \text{ invar.} \nearrow$$

- Weinberg operator:

the interact° $\frac{1}{\Lambda}(\overline{L^c}\tilde{H}^*)(\tilde{H}^\dagger L)$ could nevertheless be induced by dim-4 interactions involving a BSM heavy field (see the 3 tree level possibilities below): if this is the case ν_L Majorana masses are induced:

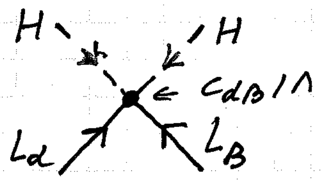
$H \equiv \phi$ doublet

$\mathcal{L} \ni \frac{c_{\alpha\beta}}{\Lambda}(\overline{L^c_\alpha}.\tilde{H}^*)(\tilde{H}^\dagger L_\beta)$

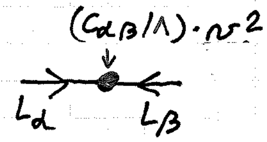
$\alpha,\beta = e,\mu,\tau$

Λ: number with dimen of mass to make $\mathcal{L}$ of dim-4 as must be

$\Downarrow$



$\Downarrow$



← Majorana mass matrix for $\nu_{\alpha,\beta}$:

↳ $\mathcal{L} \ni \frac{1}{2}\underbrace{\frac{c_{\alpha\beta}}{\Lambda} v^2}_{= m_{\nu_{\alpha\beta}}} \overline{\nu^c_\alpha}\nu_\beta$

⇒ seesaw mechanism: if $\Lambda >> v$ then $m_\nu \overset{\text{as observed}}{<<} v$

↳ ~ mass of heavy exchanged particle: see below

⇒ possibility of clear explanat° for small ν masses: because it is induced by exchange of heavy particle ⇔ because new physics scale is much larger than $v \simeq 246$ GeV

↳ for instance for $\underset{\uparrow \text{couplings}}{c_{\alpha\beta}} \sim 1$: $m_\nu \sim 0.1$ eV ⇔ $\underline{\underline{\Lambda \sim 10^{15} \text{ GeV}}}$

- Tree level ways to generate the Weinberg operator: ∃ three ways: the 3 seesaw models

any combinat° of L and H in weinberg operat. is either a singlet or a triplet of $SU(2)_L$.

⇒ LH = singlet : type-I seesaw
LH = triplet : type-III seesaw
LL and HH are triplets : type-II seesaw

(LL and HH singlets: doesn't work: $\tilde{H}^{+}\tilde{H}^{*} = H^{+}H^{0} - H^{0}H^{+} = 0$)

- Type-I seesaw: HL couples to a $SU(2)_L$ singlet of $Y=0$

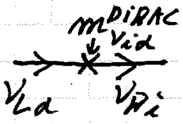
$$\mathcal{L} \ni -Y_{\nu_{i\alpha}} \underbrace{\overline{\nu_{R_i}}} \tilde{H}^{+} L_\alpha + h.c.$$

↳ a lepton with e-m charg $=0$
is called a neutrino
⇒ ν_R is a right-handed ν

↳ with a right-handed ν one can expect Dirac ν masses just as with the up quarks in SM:

$$\mathcal{L} \ni -Y_{\nu_{i\alpha}} \overline{\nu_{R_i}} \nu_{L\alpha} H^0 + h.c. \qquad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$$

$$\ni \underbrace{-Y_{\nu_{i\alpha}} \frac{v}{\sqrt{2}}}_{= m^{DIRAC}_{\nu_{i\alpha}}} \overline{\nu_{R_i}} \nu_{L\alpha} + h.c$$



↳ if there is only this interact° then $m_\nu \sim 0.1 eV$ requires $Y_\nu \sim 10^{-11}$: much smaller than Yukawa couplings of other fermions in SM: looks weird but nothing forbids that.

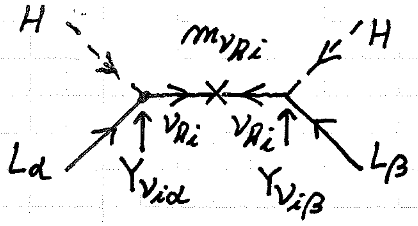
↳ however nothing forbids also (except if one assumes an extra BSM symmetry and nothing tells us this sym. exists), Majorana masses for the ν_{R_i}:

$$\mathcal{L} \ni -\frac{1}{2} m_{\nu_{R_{ij}}} \overline{\nu^c_{R_i}} \nu_{R_j} + h.c.$$

⇒ let us assume $m_{\nu_{R_{ij}}}$ diagonal here (⟺ go to the ν_{R_i} basis where $m_{\nu_{R_{ij}}}$ is diagonal)

$$\Rightarrow \mathcal{L} \ni -Y_{\nu i\alpha}\,\overline{\nu_{Ri}}\,\tilde{H}^{+}L_{\alpha} - \frac{1}{2}\,m_{\nu_{Ri}}\,\overline{\nu^{c}_{Ri}}\,\nu_{Ri} \quad + h.c.$$

↳ these 2 interactions generates nothing but the Weinberg operator interaction



$$\Rightarrow \frac{c_{\alpha\beta}}{\Lambda} = \frac{-1}{2}\sum_i \frac{Y_{\nu i\alpha}Y_{\nu i\beta}}{m_{\nu_{Ri}}} \leftarrow \text{from propag of } \nu_{Ri}$$

$$\Downarrow$$

$$m_{\nu\alpha\beta} = -\frac{1}{2}\sum_i \frac{Y_{\nu i\alpha}Y_{\nu i\beta}}{m_{\nu_{Ri}}}\,v^2$$

Another way to see that is to write the total ν mass matrix and to diagonalize it:

$$\mathcal{L} \ni -\underbrace{Y_{\nu i\alpha}\frac{v}{\sqrt{2}}}_{m_{Di\alpha}}\,\underbrace{\overline{\nu_{Ri}}\,\nu_{L\alpha}}_{\overline{\nu^{c}_{L\alpha}}\,\nu^{c}_{Ri}} - \frac{1}{2}\,m_{\nu_{Ri}}\,\overline{\nu^{c}_{Ri}}\,\nu_{Ri}$$

$$- Y^{*}_{\nu i\alpha}\frac{v}{\sqrt{2}}\,\underbrace{\overline{\nu_{L\alpha}}\,\nu_{Ri}}_{\overline{\nu^{c}_{Ri}}\,\nu^{c}_{L}} - \frac{1}{2}\,m_{\nu_{Ri}}\,\overline{\nu_{Ri}}\,\nu^{c}_{Ri}$$

$$= -\frac{1}{2}\left(\overline{\nu^{c}_{L}}\ \ \overline{\nu_{R}}\right)\begin{pmatrix} 0 & (m_D^T) \\ (m_D) & (m_{\nu_R}) \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu^{c}_{R} \end{pmatrix} + h.c.$$

$$= -\frac{1}{2}\left(\overline{\nu^{c}_{L}}\ \ \overline{\nu_{R}}\right)U_\nu U_\nu^{+}\underbrace{\begin{pmatrix} 0 & (m_D^T) \\ (m_D) & (m_{\nu_R}) \end{pmatrix}U_\nu}_{\begin{pmatrix} -m_D^T m_{\nu_R}^{-1} m_D & \\ & m_{\nu_R} + \underbrace{m_D^T m_{\nu_R}^{-1} m_D}_{\text{negligible}} \end{pmatrix}} U_\nu^{+}\begin{pmatrix} \nu_L \\ \nu^{c}_{R} \end{pmatrix} + h.c.$$

$\Rightarrow$ 3 mass eigenstates which have Major. masses $\sim m_{\nu_{Ri}}$ which are essentially the ν_{Ri} : $\nu'_{Ri} = \nu_{Ri} + 6\left(\frac{m_D}{m_{\nu_R}}\right).\nu_L$ and 3 mass eigenstates which are essentially the ν_L : $\nu'_{L\alpha} = \nu_{L\alpha} + 6\left(\frac{m_D}{m_{\nu_R}}\right).\nu_R$ with ν mass matrix which is the one we got from the diagram:

$$m_{\nu_{\alpha\beta}} = - m_D^T m_{\nu_R}^{-1} m_D = -\frac{1}{2} \frac{Y_{\nu_{i\alpha}} Y_{\nu_{i\beta}}}{m_{\nu_{Ri}}} v^2$$

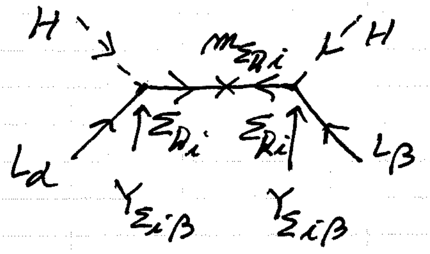
=> in summary unless there are no Major. masses (<=> there exists a symmetry forbidding them: total lepton # conservation: automatic in SM but now to be assumed "by hand" if we want to forbid the $m_{\nu_{Ri}}$) the SM left-handed $\nu_{L\alpha}$ have Majorana masses (resulting from Dirac masses and $m_{\nu_{Ri}}$ masses) and not Dirac masses!

↳ these Majorana masses are naturally small (as observed) if the scale of new physics (<=> the $m_{\nu_{Ri}}$) is much larger than the EW scale: seesaw

↳ and in general we expect $m_{\nu_{Ri}} >> v$ since the $m_{\nu_{Ri}}$ are not protected by EW scale (i.e. $m_{\nu_{Ri}} \neq 0$ even if $v = 0$ unlike all other SM fermion masses)

↳ point towards a new physics scale $\sim 10^{15}$ GeV if Yukawa of order unity: GUT scale (see below).

- Type-III seesaw: very same as type-I but taking L and H in a triplet combination:



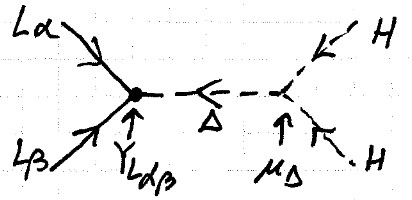
$$\Sigma_{R_i} = \begin{pmatrix} \Sigma^+_{R_i} \\ \Sigma^0_{R_i} \\ \Sigma^-_{R_i} \end{pmatrix} \Rightarrow m_\nu \propto \frac{Y_\Sigma^2 v^2}{m_\Sigma}$$

$$\mathcal{L} \ni - Y_{\Sigma_{i\alpha}} \overline{\Sigma}_{R_i} \overbrace{\tilde{H}^{+} L_\alpha}^{\text{triplet combinat.}} + h.c.$$

$$- \frac{1}{2} m_{\Sigma_i} \overline{\Sigma^c_{R_i}} \Sigma_{R_i} + h.c.$$

- Type-II seesaw: LL and HH couple to a heavy scalar triplet Δ of mass m_Δ and hypercharge = 2 :

$$\Delta = \begin{pmatrix} \delta^{++} \\ \delta^{+} \\ \delta^{0} \end{pmatrix}$$



$$\mathcal{L} \ni - Y_{L\alpha\beta} \underbrace{\overline{L^c_\alpha} L_\beta}_{\text{3-plet combin.}} \Delta - \mu_\Delta \underbrace{H^\dagger \tilde{H}}_{\text{3-plet combin.}} \Delta + h.c$$

Majorana masses here too

$$\Rightarrow \frac{c_{\alpha\beta}}{\Lambda} = - \frac{\mu_\Delta Y_{L\alpha\beta}}{m^2_\Delta} \quad \Rightarrow \quad m_{\nu_{\alpha\beta}} = - \frac{\mu_\Delta Y_{L\alpha\beta}}{m^2_\Delta} v^2$$

↳ $m_\nu \ll v$ as soon as $m_\Delta \gg v \Rightarrow$ seesaw mechanism operative too
⇒ also points towards a new physics BSM scale much larger than the E-W scale