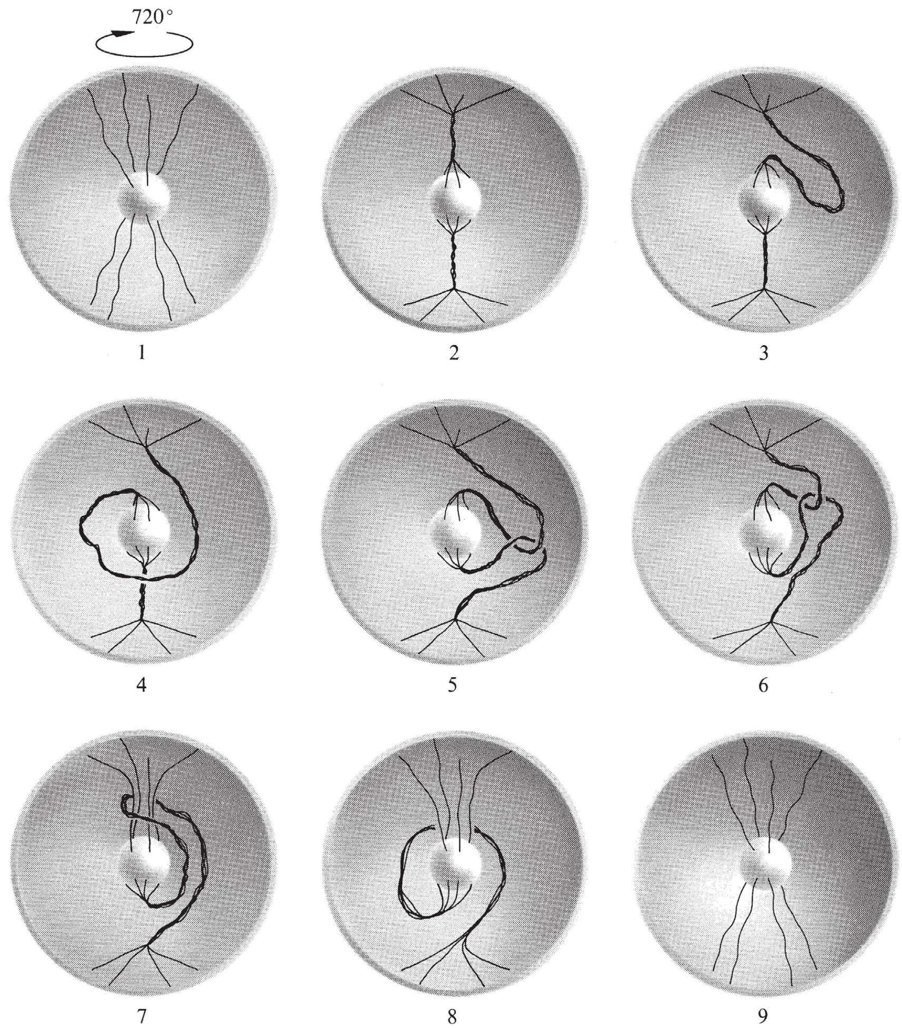


Figure 41.6.
An object is connected to its surroundings by elastic threads as in Figure 41.5. (Eight are shown here; any number could be used.) Rotating the object through 720° and then following the procedure outlined (Edward McDonald) in frames 2–8 (with the object remaining fixed), one finds that the connecting threads are left disentangled, as in frame 9 (lower right).

Such a quantity is known as a spinor. A spinor reverses sign on a 360° rotation. It therefore provides a reasonable means to keep track of the difference between the two inequivalent versions of the cube. More generally, with each orientation-entanglement relation between the cube and its surroundings one can associate a different value of the spinor ξ. Moreover, there is nothing that limits the usefulness of the spinor concept to rotations. Also, for the general combination of boost and rotation, one can write

Lorentz transformation of a spinor

$$\xi \longrightarrow \xi' = L\xi. \qquad (41.51)$$

When the boost and rotation are both infinitesimal, the explicit form of this transformation is simple:

$$\xi' = [1 - (i\,d\theta/2)(\boldsymbol{n}\cdot\boldsymbol{\sigma}) + (d\boldsymbol{\beta}/2)\cdot\boldsymbol{\sigma}]\xi,$$

or, according to (41.1),

$$\begin{pmatrix}\xi'^1\\ \\ \xi'^2\end{pmatrix} = \begin{Vmatrix} 1+\frac{1}{2}(-i\theta_{xy}+\beta_z) & \frac{1}{2}(-i\theta_{yz}-\theta_{zx}+\beta_x-i\beta_y) \\ \frac{1}{2}(-i\theta_{yz}+\theta_{zx}+\beta_x+i\beta_y) & 1+\frac{1}{2}(i\theta_{xy}-\beta_z)\end{Vmatrix} \begin{pmatrix}\xi^1\\ \\ \xi^2\end{pmatrix} \tag{41.52}$$

For any combination of a boost in the z-direction of any magnitude and a finite rotation about the z-axis, one has

$$\begin{pmatrix}\xi'^1\\ \xi'^2\end{pmatrix} = \begin{Vmatrix} e^{-1/2i\theta_{xy}+1/2\beta_z} & 0 \\ 0 & e^{1/2i\theta_{xy}-1/2\beta_z}\end{Vmatrix} \begin{pmatrix}\xi^1\\ \xi^2\end{pmatrix}. \tag{41.53}$$

To keep track of the two components of the spinor, it is convenient and customary to introduce a label (capital Roman letter near beginning of alphabet) that takes on the values 1 and 2; thus (41.51) becomes

$$\xi'^A = L^A{}_B\xi^B. \tag{41.54}$$

The spinor has acquired a significance of its own through one's having pulled out half of the transformation formula

$$X' = LXL^*. \tag{41.55}$$

To be able to recover this formula, one requires the other half as well. It contains the conjugate complex of the Lorentz transformation. Therefore introduce another spinor η that transforms according to the law

Second type of spinor

$$\eta'^{\dot U} = \bar{L}^{\dot U}{}_{\dot V}\eta^{\dot V} \tag{41.56}$$

[$\dot U = \dot 1, \dot 2$; $\dot V = \dot 1, \dot 2$; dots and capital letters near the end of the alphabet are used to distinguish components that transform according to the conjugate complex (no transpose!) of the Lorentz spin matrix].

§41.6. CORRESPONDENCE BETWEEN VECTORS AND SPINORS

Vector regarded as a Hermitian second-rank spinor

To go back from spinors to vectors, note that the spin matrix X in (41.55) has the form

$$X = t + (\boldsymbol{x}\cdot\boldsymbol{\sigma}) = \begin{Vmatrix}(t+z) & (x-iy)\\ (x+iy) & (t-z)\end{Vmatrix} = \begin{Vmatrix} X^{1\dot 1} & X^{1\dot 2}\\ X^{2\dot 1} & X^{2\dot 2}\end{Vmatrix}, \tag{41.57}$$

2

where the labels receive dots or no dots according as they are coupled in (41.55) to L^* or to L. That equation of transformation becomes

$$X'^{A\dot{U}} = L^A{}_B \bar{L}^{\dot{U}}{}_{\dot{V}} X^{B\dot{V}} \tag{41.58}$$

(transpose obtained automatically by ordering of indices; thus $\bar{L}^{\dot{U}}{}_{\dot{V}}$, not $L^{*\dot{U}}{}_{\dot{V}}$). The coefficients in this transformation are identical with the coefficients in the law for the transformation of a "second-rank spinor with one index undotted and the other dotted:"

$$\xi'^A \eta'^{\dot{U}} = L^A{}_B \bar{L}^{\dot{U}}{}_{\dot{V}} \xi^B \eta^{\dot{V}}. \tag{41.59}$$

In this sense one can say that "a 4-vector transforms like a second-rank spinor." To be completely explicit about this connection between a 4-vector and a second-rank spinor, note from (41.57) the relations

$$\begin{aligned} X^{1\dot{1}} &= x^0 + x^3, \\ X^{1\dot{2}} &= x^1 - ix^2, \\ X^{2\dot{1}} &= x^1 + ix^2, \\ X^{2\dot{2}} &= x^0 - x^3. \end{aligned} \tag{41.60}$$

In a more compact form, one has

$$X^{A\dot{U}} = [t + (\boldsymbol{x} \cdot \boldsymbol{\sigma})]^{A\dot{U}} = x^\mu \sigma_\mu{}^{A\dot{U}} \tag{41.61}$$

where σ_0 is the unit matrix. This equation tells immediately how to go from the components of a 4-vector, or "1-index tensor," to the components of the corresponding "1,1-spinor" (one undotted and one dotted index).

With each real 4-vector x^α is associated a 1,1-spinor that is *Hermitian* in the sense that

$$X^{A\dot{U}} = \overline{X^{U\dot{A}}}. \tag{41.62}$$

An example of a Hermitian 1,1-spinor is provided by (41.61). The concept of Hermiticity can be stated in other words, and more generally. Associated with any N,N-spinor Φ with components $\Phi^{A_1 \ldots A_N \dot{U}_1 \ldots \dot{U}_N}$ is the *conjugate complex spinor* $\bar{\Phi}$ with

N,N-spinors and Hermiticity

$$(\bar{\Phi})^{A_1 \ldots A_N \dot{U}_1 \ldots \dot{U}_N} = \overline{(\Phi^{U_1 \ldots U_N \dot{A}_1 \ldots \dot{A}_N})} \tag{41.63}$$

An N,N-spinor is said to be Hermitian when it is equal to its conjugate complex.

§41.7. SPINOR ALGEBRA

Equation (41.53) showed the component ξ'^1 of a spinor rising exponentially with a boost in proportion to the factor $e^{1/2\beta_z}$, and the other component, ξ'^2 falling exponentially. If from two spinors ξ and ζ, there is to be any quantity constructed which is unaffected in value by the boost, it must be formed out of such products

Spinor algebra:

as $\xi^1\zeta^2$ and $\xi^2\zeta^1$. One can restate this product prescription in other language. Introduce the alternating symbols ε^{AB} and ε_{AB} such that $\varepsilon^{12} = \varepsilon_{12} = 1$ and

(1) ε^{AB}, ε_{AB} defined

$$\varepsilon^{AB} = -\varepsilon^{BA}, \qquad \varepsilon_{AB} = -\varepsilon_{BA}, \tag{41.64}$$

the only other nonvanishing components being $\varepsilon^{21} = \varepsilon_{21} = -1$. Define the lower-label spinor ξ_A in terms of the upper-label spinor ξ^A by the equation

(2) raising and lowering spinor indices

$$\xi_A = \xi^B\varepsilon_{BA}, \tag{41.65}$$

with the inverse

$$\xi^B = \varepsilon^{BC}\xi_C. \tag{41.66}$$

Then the scalar product of one spinor by another is defined to be

(3) scalar products of spinors

$$\xi_A\zeta^A. \tag{41.67}$$

The value of this scalar product is unaffected by any boost or rotation or combination thereof:

$$\begin{aligned}\xi'_A\zeta'^A &= \xi'^B\varepsilon_{BA}\zeta'^A \\ &= (L^B{}_D\xi^D)\varepsilon_{BA}(L^A{}_C\zeta^C) \\ &= (\det L)\xi^D\varepsilon_{DC}\zeta^C \\ &= \xi_C\zeta^C.\end{aligned} \tag{41.68}$$

The proof uses the fact that the expression $L^B{}_D\varepsilon_{BA}L^A{}_C$ (1) vanishes when $D = C$, and (2) reduces to the determinant of L (unity!) or its negative when $D = 1$, $C = 2$, or $D = 2$, $C = 1$. Note that the scalar product $\xi^A\zeta_A$ is the negative of the scalar product $\xi_A\zeta^A$. The value of the scalar product of a spinor with itself is automatically zero ("built-in null character of a spinor").

(4) the mapping between vectors and 1,1-spinors

The components of a vector with upper index have been expressed in terms of the components of a 1,1-spinor with upper indices

$$X^{A\dot U} = x^\mu\sigma_\mu{}^{A\dot U}, \tag{41.69}$$

and a similar correlation holds between vector and 1,1-spinor with lower indices; thus,

$$X_{A\dot U} = x_\mu\sigma^\mu{}_{A\dot U}. \tag{41.70}$$

(5) σ^μ defined and related to σ_μ

Here the "associated basic spin matrices" have the components

$$\sigma^\mu{}_{A\dot U} = \eta^{\mu\nu}\sigma_\nu{}^{B\dot V}\varepsilon_{BA}\varepsilon_{\dot V\dot U}, \tag{41.71}$$

or, explicitly,

$$\begin{Vmatrix} \sigma^{\mu}{}_{1\dot{1}} & \sigma^{\mu}{}_{1\dot{2}} \\ \sigma^{\mu}{}_{2\dot{1}} & \sigma^{\mu}{}_{2\dot{2}} \end{Vmatrix} = \begin{cases} -\begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix} \text{ for } \mu = 0, \\ -\begin{Vmatrix} 0 & 1 \\ 1 & 0 \end{Vmatrix} \text{ for } \mu = 1, \\ +\begin{Vmatrix} 0 & -i \\ i & 0 \end{Vmatrix} \text{ for } \mu = 2, \\ -\begin{Vmatrix} 1 & 0 \\ 0 & -1 \end{Vmatrix} \text{ for } \mu = 3. \end{cases} \tag{41.72}$$

The same type of multiplication law holds for these matrices, $(\sigma^x)^2 = (\sigma^y)^2 = (\sigma^z)^2 = 1$, $\sigma^x\sigma^y = -\sigma^y\sigma^x = i\sigma^z$, etc., as for the matrices σ_x, σ_y, σ_z of (41.2). Between the "basic spin matrices," σ_μ, and the "associated basic spin matrices," σ^μ, the following orthogonality and normalization relations obtain:

$$\sigma_\mu{}^{A\dot{U}}\sigma^\mu{}_{B\dot{V}} = -2\,\delta^A_B\,\delta^{\dot{U}}_{\dot{V}} \tag{41.73}$$

and

$$\sigma_\mu{}^{A\dot{U}}\sigma^\nu{}_{A\dot{U}} = -2\,\delta^\nu_\mu. \tag{41.74}$$

One can use these relations to "go back from a quantity expressed as a 1,1-spinor ('spinor equivalent of a vector') to the same quantity expressed directly as a vector (first-rank tensor)." Thus, multiply through (41.61) on both sides by $-\frac{1}{2}\sigma^\nu{}_{A\dot{U}}$, sum over the spinor indices, and employ (41.74) to find the contravariant components of the vector,

$$x^\nu = -\frac{1}{2}\sigma^\nu{}_{A\dot{U}}X^{A\dot{U}}. \tag{41.75}$$

Similarly from (41.70) and (41.73) one finds the covariant components,

$$x_\nu = -\frac{1}{2}\sigma_\nu{}^{A\dot{U}}X_{A\dot{U}}. \tag{41.76}$$

An N-index tensor T lets itself be expressed in spinor language ("spinor equivalent of the tensor") by a generalization of (41.61) or (41.70); thus, for a mixed tensor of third order, one has

(6) the mapping between rank-N tensors and N,N-spinors

$$T_{A\dot{U}}{}^{B\dot{V}C\dot{W}} = \sigma^\alpha{}_{A\dot{U}}\sigma_\beta{}^{B\dot{V}}\sigma_\gamma{}^{C\dot{W}}T_\alpha{}^{\beta\gamma} \tag{41.77}$$

and the converse relation

$$T_\alpha{}^{\beta\gamma} = \left(-\frac{1}{2}\right)^3\sigma_\alpha{}^{A\dot{U}}\sigma^\beta{}_{B\dot{V}}\sigma^\gamma{}_{C\dot{W}}T_{A\dot{U}}{}^{B\dot{V}C\dot{W}}. \tag{41.78}$$

Box 41.1 gives the spinor representation of several simple tensors.

Box 41.1 SPINOR REPRESENTATION OF CERTAIN SIMPLE TENSORS IN THE CONTEXT OF A LOCAL LORENTZ FRAME

Quantity	*Tensor language*	*Spinor language*
General 4-vector	x^α (four complex numbers)	$X^{A\dot{U}}$ (4 complex numbers)
Real 4-vector (example: 4-momentum)	$x^\alpha = \bar{x}^\alpha$ (four real numbers)	$X^{A\dot{U}} = \overline{(X^{U\dot{A}})}$ (2 real components, 1 distinct complex component)
Null 4-vector	$\eta_{\alpha\beta}x^\alpha x^\beta = 0$	$\det X^{A\dot{U}} = 0$ [see (41.57)]; hence there exist two spinors ξ^A and η^U such that $X^{A\dot{U}} = \xi^A\eta^{\dot{U}}$.
Future-pointing real null 4-vector (such as 4-momentum of a photon)	$x^\alpha = \bar{x}^\alpha$ $\eta_{\alpha\beta}x^\alpha x^\beta = 0$ $x^0 > 0$	There exists a spinor ξ^A (two complex numbers, unique up to a common multiplicative phase factor $e^{i\delta}$) such that $X^{A\dot{U}} = \xi^A(\bar{\xi})^{\dot{U}}$
Past-pointing real null 4-vector	$x^0 < 0$	$X^{A\dot{U}} = -\xi^A(\bar{\xi})^{\dot{U}}$.
Real bivector or 2-form (such as Maxwell field)	$F_{[\alpha\beta]}$ (subscript implying $F_{\alpha\beta} = -F_{\beta\alpha}$; six distinct real components)	There exists a symmetric spinor ϕ_{AB} (three distinct complex components ϕ_{11}, ϕ_{12}, ϕ_{22}) such that $F_{A\dot{U}B\dot{V}} = \phi_{AB}\varepsilon_{\dot{U}\dot{V}} + \varepsilon_{AB}(\bar{\phi})_{\dot{U}\dot{V}}$
Real 2-form dual to foregoing real 2-form	${}^*F_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\gamma\delta}F^{\gamma\delta}$	${}^*F_{A\dot{U}B\dot{V}} = -i\phi_{AB}\varepsilon_{\dot{U}\dot{V}} + i\varepsilon_{AB}(\bar{\phi})_{\dot{U}\dot{V}}$ (duality for 2-form corresponds to multiplication of spinor ϕ_{AB} by $-i$)
Real fourth-order tensor with symmetries of Weyl conformal curvature tensor; that is, with symmetries of Riemann curvature tensor and with additional requirement of vanishing Ricci tensor ("empty space;" "vacuum Riemann tensor")	$C_{\alpha\beta\gamma\delta} = C_{([\alpha\beta][\gamma\delta])}$ (antisymmetric in first two indices; antisymmetric in last two indices; symmetric against interchange of first pair with second pair) $C^\alpha{}_{[\beta\gamma\delta]} = 0$ (20 algebraically distinct components, as for the Riemann tensor, reduced to 10 by the further vacuum condition:) $C^\alpha{}_{\beta\alpha\delta} = 0$	There exists a completely symmetric spinor ψ_{ABCD} with five distinct complex components, ψ_{1111} ψ_{1112} ψ_{1122} ψ_{1222} ψ_{2222} such that $C_{A\dot{U}B\dot{V}C\dot{W}D\dot{X}} = \psi_{ABCD}\varepsilon_{\dot{U}\dot{V}}\varepsilon_{\dot{W}\dot{X}} + \varepsilon_{AB}\varepsilon_{CD}\bar{\psi}_{\dot{U}\dot{V}\dot{W}\dot{X}}$

In some treatises on spinor analysis, the factor $(-\frac{1}{2})^N$ in equations like (41.78) is eliminated by the following double prescription: (1) insert into the matrices σ_μ and σ^μ a factor $1/\sqrt{2}$ not included above; and (2) use for the standard metric not diag $\eta_{\mu\nu} = (-1, 1, 1, 1)$ as above, but $(1, -1, -1, -1)$. This prescription was not adopted here (1) because the introduction of $1/\sqrt{2}$ in the matrices σ_x, σ_y, σ_z would put them out of line with the Pauli matrices as used for many years throughout

Quantity	*Tensor language*	*Spinor language*
Fully developed Riemann curvature tensor (space where matter is present)	$R_{\alpha\beta\gamma\delta} = R_{[\alpha\beta]\gamma\delta} = R_{\alpha\beta[\gamma\delta]} = R_{([\alpha\beta][\gamma\delta])}$ $R_{\alpha[\beta\gamma\delta]} = 0$ (20 algebraically distinct components)	There exists a completely symmetric spinor ψ_{ABCD} ("Weyl" or "conformal" part of curvature, or part of nonlocal origin) and a scalar Λ (measure of trace of part of curvature of local origin) and a spinor $\Phi_{AB\dot{U}\dot{V}} = \Phi_{(AB)(\dot{U}\dot{V})} = (\bar{\Phi})_{AB\dot{U}\dot{V}}$ (measure of trace-free part of curvature of local origin; last of the three irreducible parts of the curvature tensor) such that $R_{A\dot{U}B\dot{V}C\dot{W}D\dot{X}} = \psi_{ABCD}\varepsilon_{\dot{U}\dot{V}}\varepsilon_{\dot{W}\dot{X}}$ $+ \varepsilon_{AB}\varepsilon_{CD}(\bar{\psi})_{\dot{U}\dot{V}\dot{W}\dot{X}}$ $+ 2\Lambda(\varepsilon_{AC}\varepsilon_{BD}\varepsilon_{\dot{U}\dot{V}}\varepsilon_{\dot{W}\dot{X}}$ $+ \quad \varepsilon_{AB}\varepsilon_{CD}\varepsilon_{\dot{U}\dot{X}}\varepsilon_{\dot{V}\dot{W}})$ $+ \varepsilon_{AB}\Phi_{CD\dot{U}\dot{V}}\varepsilon_{\dot{W}\dot{X}}$ $+ \varepsilon_{CD}\Phi_{AB\dot{W}\dot{X}}\varepsilon_{\dot{U}\dot{V}}.$
Each physical quantity is described by a geometric object. Every local physical quantity is described by a mathematical quantity that transforms under a proper local Lorentz transformation as an "irreducible representation of the group $L\uparrow_{+}$ of proper Lorentz transformations."	Each local physical quantity is described by a tensor with its own rank and specific symmetry properties.	In order to provide the required finite irreducible representation of $L\uparrow_{+}$ to represent a local physical quantity, the associated spinor must be completely symmetric in all of its undotted indices, and also completely symmetric in all its dotted indices [Gel'fand (1963)].

atomic and nuclear physics, and (2) because a positive definite metric within a spacelike hypersurface has the advantage of naturalness for the analysis of the initial-value problem of geometrodynamics and for the definition of what one means by a 3-geometry. The price of the factor $(-\frac{1}{2})^N$ is paid here for these advantages. Conventions that avoid this price are preferable for extensive spinor computations; see, e.g., Pirani (1965) or Penrose (1968a).

§41.8. SPIN SPACE AND ITS BASIS SPINORS

Linear independence of spinors

The "space" of elementary spinors is two-dimensional. Therefore it is spanned by any two linearly independent spinors λ_A and μ_A. Moreover, it is easy to diagnose a pair of spinors for possible linear dependence, that is, for existence of a relation of the form $\mu_A = \text{const}\,\lambda_A$. In this event, the scalar product of μ_A with λ^A, like the scalar product of λ_A with λ^A (41.67) automatically vanishes. Therefore a nonvanishing scalar product

$$\lambda_A \mu^A \neq 0 \tag{41.79}$$

is a necessary and sufficient condition for the linear independence of two spinors.

The general 4-vector lets itself be represented as a linear combination of four basis vectors. Similarly the general spinor lets itself be represented as a linear combination of two basis spinors:

$$\omega^A = \lambda \xi^A + \mu \eta^A. \tag{41.80}$$

Basis spinors and spinor mates

Here it is understood that the term "basis spinor" implies that ξ^A and η^A satisfy the normalization condition

$$\xi_A \eta^A = 1\,(= -\eta_A \xi^A). \tag{41.81}$$

From this condition one derives simple expressions for the expansion coefficients in (41.80):

$$\begin{aligned} \lambda &= -\eta_A \omega^A\,(= \omega_B \eta^B), \\ \mu &= \xi_A \omega^A\,(= -\omega_B \xi^B). \end{aligned} \tag{41.82}$$

Inserting these expansion coefficients back into (41.80) will reproduce any arbitrarily chosen spinor ω^A. In other words, the following equation has to be an identity in the components of ω_B:

$$\omega^A = \varepsilon^{AB}\omega_B \equiv (\xi^A \eta^B - \eta^A \xi^B)\omega_B. \tag{41.83}$$

From this circumstance, it follows that the components of the two basic spinors are linked by the equations

$$\xi^A \eta^B - \eta^A \xi^B = \varepsilon^{AB}. \tag{41.84}$$

Given two basis spinors ξ^A and η^A, one can get two equally good basis spinors by writing

$$\begin{aligned} \xi^A{}_{\text{new}} &= \xi^A, \\ \eta^A{}_{\text{new}} &= \eta^A + \alpha \xi^A, \end{aligned} \tag{41.85}$$

with α any real or complex constant, as one checks at once by substitution into (41.81) or (41.84). The most general "spinor mate" to a given spinor ξ^A, satisfying the normalization condition (41.81), has this form (41.85).

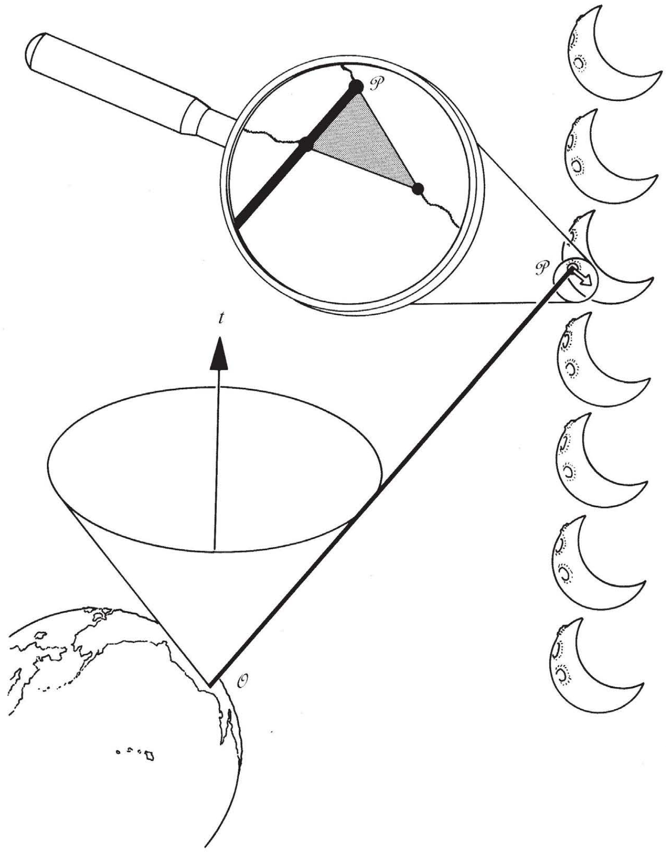


Figure 41.7.
Spinor represented by (1) "flagpole" [Penrose terminology; track of pulse of light; null vector $\mathscr{O}\mathscr{P}$] plus (2) "flag" [arrow ($\mathscr{P} \longrightarrow$) flashed onto surface of moon by laser pulse from earth or, in expanded view in the inset above, a flag itself, substituted for the arrow] plus (3) the orientation-entanglement relation between the flag and its surroundings [symbolized by strings drawn from corners of flag to surroundings]. When the spinor itself is multiplied by a factor $\rho e^{i\sigma}$, the components of the null vector (flagpole) are multiplied by the factor ρ^2 and the flag is rotated through the angle 2σ about the flagpole.

§41.9. SPINOR VIEWED AS FLAGPOLE PLUS FLAG PLUS ORIENTATION-ENTANGLEMENT RELATION

Geometric representation of a spinor:

How can one visualize a spinor? Aim the laser, pull the trigger, and send a megajoule pulse from the here and now (event $\mathscr{O}$) to the there and then (event $\mathscr{P}$: center of the crater Aristarchus, 400,000 km from $\mathscr{O}$ in space, and 400,000 km from $\mathscr{O}$ in light-travel time). The laser has been designed to produce, not a mere spot of light, but an illuminated arrow. Following Roger Penrose, speak of the null vector $\mathscr{O}\mathscr{P}$ as a "flagpole," and of the illuminated arrow as a "flag." A spinor (Figure 41.7) consists of this combination of (1) null flagpole plus (2) flag plus (3) the orientation-entanglement relation between the flag and its surroundings. "Rotate the flag" by repeatedly firing the laser, with a bit of rotation of the laser about its axis between one firing and the next. When the flag has turned through 360° and has come back to its original direction, the spinor has reversed sign. A rotation of the flag about the flagpole through any even multiple of 2π restores the spinor to its original value.

(1) null vector (flagpole), plus

One goes from a spinor ξ, a mathematical object with two complex components ξ^1 and ξ^2, to the geometric object of "flagpole plus flag plus orientation-entanglement relation" in two steps: first the pole, then the flag. Thus, go from the spinor ξ^A to the real null 4-vector of the "pole" by way of the formula

$$x^\alpha \longrightarrow X^{A\dot{U}} = \xi^A(\bar{\xi})^{\dot{U}} \tag{41.86}$$

or

$$\begin{Vmatrix} (t+z) & (x-iy) \\ (x+iy) & (t-z) \end{Vmatrix} = \begin{Vmatrix} \xi^1\bar{\xi}^{\dot{1}} & \xi^1\bar{\xi}^{\dot{2}} \\ \xi^2\bar{\xi}^{\dot{1}} & \xi^2\bar{\xi}^{\dot{2}} \end{Vmatrix}. \tag{41.87}$$

The matrix on the right has its first row identical up to a factor ξ^1/ξ^2 with its second row. Therefore the determinant of the matrix on the right vanishes. So also for the left. Therefore the 4-vector $\mathscr{O}\mathscr{P} = (t, x, y, z)$ is a null vector. One "stretches" this vector by a factor ρ^2 when one multiplies the spinor ξ^A by the nonzero complex number $\lambda = \rho e^{i\sigma}$ (ρ, σ real); however, the vector is unchanged in direction. The 4-vector is also unaffected by the choice of the angle σ. In other words, this null 4-vector is uniquely fixed by the spinor; but the spinor is not fixed with all uniqueness by the 4-vector. To a given 4-vector corresponds a whole family of spinors. They differ from one another by a multiplicative phase factor of the form $e^{i\sigma}$ ("flag factor").

(2) bivector (flag) and its orientation-entanglement relation

Looking further to see the influence of the flag factor showing up, turn from a real vector (four components) generated out of the spinor ξ^A to a real bivector (six components) generated out of the same spinor:

$$\begin{gathered} F^{\mu\nu} \longrightarrow F^{AB\dot{U}\dot{V}} = \xi^A\xi^B\epsilon^{\dot{U}\dot{V}} + \epsilon^{AB}(\bar{\xi})^{\dot{U}}(\bar{\xi})^{\dot{V}}, \\ \mu \longrightarrow A\dot{U};\ \nu \longrightarrow B\dot{V}. \end{gathered} \tag{41.88}$$

That this quantity has no more than six distinct components ($F^{\mu\nu} = -F^{\nu\mu}$) follows from interchanging A with B and $\dot{U}$ with $\dot{V}$, and noting the resultant change in sign on the righthand side of (41.88). To unfold the meaning of this bivector, look in (41.88) for every appearance of the alternating factor ε^{AB}. Wherever such a factor appears, insert the expression (41.84) for this factor in terms of the starting spinor ξ^A and insert the additional spinor η^A that is needed, along with ξ^A, to supply a basis for all spinors. In this way, find

$$\begin{aligned} F^{\mu\nu} \longrightarrow F^{AB\dot{U}\dot{V}} &= \xi^A\xi^B(\bar{\xi}^{\dot{U}}\bar{\eta}^{\dot{V}} - \bar{\eta}^{\dot{U}}\bar{\xi}^{\dot{V}}) + (\xi^A\eta^B - \eta^A\xi^B)\bar{\xi}^{\dot{U}}\bar{\xi}^{\dot{V}} \\ &= \xi^A\bar{\xi}^{\dot{U}}(\xi^B\bar{\eta}^{\dot{V}} + \eta^B\bar{\xi}^{\dot{V}}) - (\xi^A\bar{\eta}^{\dot{U}} + \eta^A\bar{\xi}^{\dot{U}})\xi^B\bar{\xi}^{\dot{V}} \\ &= X^{A\dot{U}}Y^{B\dot{V}} - Y^{A\dot{U}}X^{B\dot{V}} \longrightarrow x^\mu y^\nu - y^\mu x^\nu. \end{aligned} \tag{41.89}$$

Thus the 2,2-spinor built from ξ^A represents a bivector constructed out of the two 4-vectors $\boldsymbol{x}$ and $\boldsymbol{y}$. Of these, the first is the "real null vector of the flagpole," already seen to be determined uniquely by the spinor ξ^A. The second vector,

$$y^\alpha \longrightarrow Y^{A\dot{U}} = \xi^A \bar{\eta}^{\dot{U}} + \eta^A \bar{\xi}^{\dot{U}}, \tag{41.90}$$

is also determined by ξ^A, but not uniquely, because the "spinor mate," η^A, to ξ^A is not unique. Go from one choice of mate, η^A, to a new choice of mate (equation 41.85),

$$\eta^A{}_{\text{new}} = \eta^A + \alpha \xi^A. \tag{41.91}$$

Then the real 4-vector y^μ goes to the new real 4-vector

$$y^\mu{}_{\text{new}} = y^\mu + (\alpha + \bar{\alpha}) x^\mu. \tag{41.92}$$

Were the 4-vector $\boldsymbol{y}$ unique, there would project out from the flagpole, not a flag but an arrow. The range of values open for the real constant $\alpha + \bar{\alpha}$ makes one arrow into many arrows, all coplanar; hence the flag of Penrose. Otherwise stated, the choice of a spinor ξ^A fixes no individual arrow, but does fix the totality of the collection of arrows, and thus uniquely specifies the flag.

The 4-vector $\boldsymbol{y}$ (and with it $\boldsymbol{y}_{\text{new}}$) is orthogonal to the null 4-vector $\boldsymbol{x}$,

$$\begin{aligned} \boldsymbol{x} \cdot \boldsymbol{y} = x_\beta y^\beta &= -\frac{1}{2} X_{A\dot{U}} Y^{A\dot{U}} \\ &= -\frac{1}{2} \xi_A \bar{\xi}_{\dot{U}} (\xi^A \bar{\eta}^{\dot{U}} + \eta^A \bar{\xi}^{\dot{U}}) = 0, \end{aligned} \tag{41.93}$$

and spacelike,

$$\begin{aligned} \boldsymbol{y} \cdot \boldsymbol{y} &= -\frac{1}{2} (\xi_A \bar{\eta}_{\dot{U}} + \eta_A \bar{\xi}_{\dot{U}})(\xi^A \bar{\eta}^{\dot{U}} + \eta^A \bar{\xi}^{\dot{U}}) \\ &= -\frac{1}{2} (\xi_A \eta^A)(\bar{\eta}_{\dot{U}} \bar{\xi}^{\dot{U}}) - \frac{1}{2} (\eta_A \xi^A)(\bar{\xi}_{\dot{U}} \bar{\eta}^{\dot{U}}) = 1 \end{aligned} \tag{41.94}$$

("unit length of flag").

Multiplication of the spinor ξ^A by the "flag factor" $e^{i\sigma}$ rotates the flag about the flagpole by the angle 2σ, because the spinor mate, η^A, of ξ^A is multiplied by the factor $e^{-i\sigma}$ [see the normalization condition (41.81)]. These changes alter the vector $\boldsymbol{y}$ to a rotated vector $\boldsymbol{y}_{\text{rot}}$, with

Rotation of flag about flagpole

$$\begin{aligned} y^\alpha_{\text{rot}} \longrightarrow Y^{A\dot{U}}_{\text{rot}} &= e^{2i\sigma} \xi^A \bar{\eta}^{\dot{U}} + e^{-2i\sigma} \eta^A \bar{\xi}^{\dot{U}} \\ &= \cos 2\sigma (\xi^A \bar{\eta}^{\dot{U}} + \eta^A \bar{\xi}^{\dot{U}}) + \sin 2\sigma (i \xi^A \bar{\eta}^{\dot{U}} - i \eta^A \bar{\xi}^{\dot{U}}) \\ &\longrightarrow y^\alpha \cos 2\sigma + z^\alpha \sin 2\sigma. \end{aligned} \tag{41.95}$$

Here the 4-vector $\boldsymbol{z}$ shares with the vector $\boldsymbol{y}$ these properties: it is (1) real, (2) spacelike, (3) of unit magnitude, (4) orthogonal to the null 4-vector $\boldsymbol{x}$ of the flagpole, and (5) uniquely specified by the original spinor ξ^A up to the additive real multiple $(\alpha + \bar{\alpha})$ of $\boldsymbol{x}$. In addition, $\boldsymbol{z}$ and $\boldsymbol{y}$ are orthogonal. Thus $\boldsymbol{y}$ and $\boldsymbol{z}$ provide basis vectors in the two-dimensional space in which—to overpictorialize—the "tip of the flag" undergoes its rotation.

Equations relating spinor, flagpole, and flag

Recapitulate by returning to the laser pulse. Two numbers, such as the familiar polar angles θ (angle with the z-axis) and ϕ (azimuth around z-axis from x-axis) tell the direction of its flight. A third number, r, gives the distance to the moon and also the travel time for light to reach the moon. A fourth number, an angle ψ, tells the azimuth of the illuminated arrow shot onto the surface of the moon, this azimuth to be measured from the $\boldsymbol{e}_\theta$ direction (where $\psi = 0$), around the flagpole in a righthanded sense. Then the spinor associated with the flagpole plus flag (rotated arrow) is

$$\begin{pmatrix} \xi^1 \\ \xi^2 \end{pmatrix} = (2r)^{1/2} \begin{pmatrix} \cos(\theta/2) e^{-i\phi/2 + i\psi/2} \\ \sin(\theta/2) e^{i\phi/2 + i\psi/2} \end{pmatrix} \tag{41.96}$$

according to the conventions adopted here [see (41.87)]. The mate η_A to this spinor, unique up to an additive multiple of ξ^A, is

$$\begin{pmatrix} \eta^1 \\ \eta^2 \end{pmatrix} = (2r)^{-1/2} \begin{pmatrix} -\sin(\theta/2) e^{-i\phi/2 - i\psi/2} \\ \cos(\theta/2) e^{i\phi/2 - i\psi/2} \end{pmatrix}. \tag{41.97}$$

The 4-vector of the flagpole determined by ξ^A is found from (41.87):

$$\begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} r \\ r \sin\theta \cos\phi \\ r \sin\theta \sin\phi \\ r\cos\theta \end{pmatrix}. \tag{41.98}$$

To determine the flag itself, one requires, in addition to x^α, the unit spacelike 4-vector y^α, normal to x^α, and unique up to an additive real multiple of x^α. This vector is evaluated by use of (41.90) and has the form

$$\begin{pmatrix} y^0 \\ y^1 \\ y^2 \\ y^3 \end{pmatrix} = \begin{pmatrix} 0 \\ \cos\theta \cos\phi \cos\psi + \sin\phi \sin\psi \\ \cos\theta \sin\phi \cos\psi - \cos\phi \sin\psi \\ -\sin\theta \cos\psi \end{pmatrix}. \tag{41.99}$$

From these expressions for x^μ and y^μ, one calculates the components of the bivector ("flag") $F^{\mu\nu} = x^\mu y^\nu - y^\mu x^\nu$ by simple arithmetic.

§41.10. APPEARANCE OF THE NIGHT SKY: AN APPLICATION OF SPINORS

Attention has gone here to extracting all four pieces of information contained in a spinor: separation in time (equal to separation in space), direction in space, and

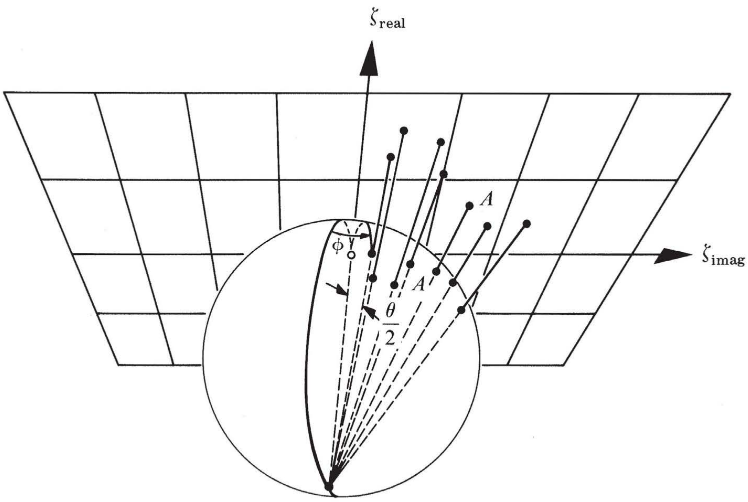


Figure 41.8.
Representation of a direction in space (one of the stars of the Big Dipper, regarded as a point on the celestial sphere) as a point in the complex ζ plane (ζ = ratio ξ^2/ξ^1 of spinor components) by stereographic projection from the South Pole.

rotation about that direction. Turn now to an application where not all that information is needed. Look at the night sky and ask (1) how to describe its appearance and (2) how to change that appearance. As one way to describe its appearance, give the direction of each star. Abandon any concern about the distance of the star, and any concern about any rotation ψ about the flagpole. In other words, the complex factor

Spinors used to analyze "Lorentz transformations" of appearance of night sky

$$(2r)^{1/2}e^{i\psi/2}$$

common to ξ^1 and ξ^2 drops from attention. All that is left as significant is the ratio ζ of these spinor components:

$$\zeta = \xi^2/\xi^1 = \tan(\theta/2)e^{i\phi}. \tag{41.100}$$

To give the one complex number ζ ("stereographic coordinate;" Figure 41.8) for each star in the sky is to catalog the pattern of the stars.

Let the observer change his stance. The celestial sphere appears to rotate. Or let him rocket past his present location in the direction of the North Star with some substantial fraction of the velocity of light. To him all that portion of the celestial sphere is opened out, as if by a magnifying glass. To compensate, the remaining stars are packed into a smaller angular compass. Any such rotation or boost or combination of rotation and boost being described in spinor language by a transformation of the form

$$\xi^A \longrightarrow \xi^A_{\rm new} = L^A{}_B\xi^B, \tag{41.101}$$

implies a transformation of the complex stereographic coordinate of any given star of the form

$$\zeta \longrightarrow \zeta_{\text{new}} = \frac{\xi^2_{\text{new}}}{\xi^1_{\text{new}}} = \frac{L^2{}_2\zeta + L^2{}_1}{L^1{}_2\zeta + L^1{}_1}. \tag{41.102}$$

In the special case of a boost in the z-direction with velocity parameter α (velocity $\beta = \tanh\alpha$), the off-diagonal components $L^1{}_2$ and $L^2{}_1$ vanish. The magnification of the overhead sky then expresses itself in the simple formula

$$\zeta_{\text{new}} = e^{\alpha}\zeta$$

or

$$\begin{aligned} \phi_{\text{new}} &= \phi, \\ \tan(\theta_{\text{new}}/2) &= e^{\alpha}\tan(\theta/2). \end{aligned} \tag{41.103}$$

Contrary to this prediction and false expectation, no magnification at all is achieved of the regions around the North Star by moving with high velocity in that direction. On the contrary, any photon coming in from a star a little off that direction, with a little transverse momentum, keeps that transverse momentum in the new frame; but its longitudinal momentum against the observer is augmented by his motion. Thus the ratio of the momenta is decreased, and the observed angle relative to the North Star is also decreased. The consequence is not magnification, but diminution ("looking through the wrong end of a telescope"). The correct formula is not (41.103) but

$$\tan(\theta_{\text{new}}/2) = e^{-\alpha}\tan(\theta/2) \tag{41.104}$$

(reversal of the sign of α). The reason for this correction is not far to seek. The spinor analysis so far had dealt with an outgoing light pulse, and a 4-vector with positive time component. That feature was built into the formula adopted to tie the spinor to the 4-vector,

$$r\mathbf{1} + (\boldsymbol{r}\cdot\boldsymbol{\sigma}) \equiv X = \|\xi^A\bar{\xi}^{\dot{U}}\|. \tag{41.105}$$

In contrast the 4-vector that reaches back to the origin of an incoming photon has a time component that is negative (or, alternatively, sign-reversed space components)! For any null 4-vector with negative time component, one employs instead of (41.105) the formula

$$X = -\|\xi^A\bar{\xi}^{\dot{U}}\|. \tag{41.106}$$

It is enough to mention here this point of principle without going through the details that give the altered sign for α in (41.104). From now on, to preserve the previous arithmetic, change the problem. Deal, not with incoming photons, but with outgoing photons. Replace the receiving telescope by the projector of a planetarium. It projects out into space a separate beam of light for each star of the Big Dipper and also one for the North Star itself. Let an observer move in the positive z-direction with velocity parameter α. In his frame of reference the beams actually will be widened out in full accord with (41.103).

"The magnification process changes the size of the Big Dipper but not its shape."

Lorentz transformations leave angles on sky unchanged ("conformal invariance")

This statement is at the same time true and false. It is true of the Dipper and of any other constellation to the extent that the angular dimensions of that constellation can be idealized to be small compared to the entire compass of the sky. It is false in the sense that any well-rounded projected constellation, however small it may appear to an observer at rest with respect to the earth, can always be so "opened out" by the observer putting on any sufficiently high velocity, the observer still being near the earth, that the constellation encompasses a major fraction of the sky.

That the "Lorentz-transformation-induced magnification" of a small object does not change its shape can be seen in three ways. (1) Stereographic projection (Figure 41.8) and "fractional linear transformation" (41.102) are both known to leave all angles unchanged ["conformal invariance;" see for example Penrose (1959)] and known even to turn every old circle into a new circle. (2) Consider a given star, M, in the constellation and immediate neighbors, L and N, just below it and just above it in the count of the members of that constellation. Consider the flagpole pointed at M and the flag pointed first from M to L, then from M to N. The flag has turned about the flagpole through an angle ψ. The two corresponding spinors therefore differ by a phase factor $e^{i\psi/2}$. They differ in no other way. After an arbitrary Lorentz transformation they still differ by the phase factor $e^{i\psi/2}$, and in no other way. The angle between the arcs ML and MN on the celestial sphere therefore remains at its original value ψ after the Lorentz transformation (again conformal invariance of patterns on the celestial sphere!). (3) An even more elementary calculation shows that infinitesimal arc lengths on the unit celestial sphere in the direction of increasing θ and arc lengths in the direction of increasing ϕ are magnified in the same proportion, thus leaving unchanged the angle between arc and arc (conformal invariance). Thus, consider a photon shot out from the planetarium projector to a point on the celestial sphere ("planetarium version of a Big-Dipper star") with inclination θ to the z-axis, as seen by an observer at rest relative to the earth. From the standard laws of transformation of angles in a Lorentz transformation ("aberration"; Box 2.4), one has for the sine of the transformed angle

$$\sin\theta_{\text{new}} = \frac{(1-\beta^2)^{1/2}}{1-\beta\cos\theta}\sin\theta \tag{41.107}$$

and (by differentiating the expression for the cosine of the transformed angle)

$$d\theta_{\text{new}} = \frac{(1-\beta^2)^{1/2}}{1-\beta\cos\theta}\,d\theta. \tag{41.108}$$

From these expressions it follows at once that the inclination, relative to a meridian line, on the transformed celestial sphere is identical to the direction, relative to the same meridian line, on the original celestial sphere:

$$\begin{aligned}\tan\begin{pmatrix}\text{new}\\\text{inclination}\end{pmatrix} &= \frac{\sin\theta_{\text{new}}\,d\phi_{\text{new}}}{d\theta_{\text{new}}} = \frac{\sin\theta\,d\phi}{d\theta}\\ &= \tan\begin{pmatrix}\text{original}\\\text{inclination}\end{pmatrix}\end{aligned} \tag{41.109}$$

(again conformal invariance!).

So much for the elementary spinor and what it has to do with a null vector, with a "flagpole" pointed to the celestial sphere, and with rotation of a "flag" about such a flagpole.

§41.11. SPINORS AS A POWERFUL TOOL IN GRAVITATION THEORY

Spinor formalism in curved spacetime

Just as vectors, tensors, and differential forms are easily generalized from flat spacetime to curved, so are spinors.

Each event $\mathscr{P}$ in curved spacetime possesses a tangent space. In that tangent space reside and operate all the vectors, tensors, and forms located at $\mathscr{P}$. The geometry of the tangent space is Lorentzian ("local Lorentz geometry at $\mathscr{P}$"), since the scalar product of any two vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ at $\mathscr{P}$, expressed in an orthonormal frame at $\mathscr{P}$, is

$$\boldsymbol{u} \cdot \boldsymbol{v} = \boldsymbol{g}(\boldsymbol{u}, \boldsymbol{v}) = \eta_{\hat{\alpha}\hat{\beta}} u^{\hat{\alpha}} v^{\hat{\beta}}.$$

Thus, there is no mathematical difference between the tangent space at $\mathscr{P}$ on the one hand, and flat spacetime on the other. Whatever mathematical can be done in the one can also be done in the other. In particular, *the entire formalism of spinors, developed originally in flat spacetime, can be carried over without change to the tangent space at the arbitrary event $\mathscr{P}$ in curved spacetime.*

Let it be done. Now spinors reside at every event in curved spacetime; and at each event one can translate back and forth between spinor language and tensor language, using the equations (valid in orthonormal frames) of §§41.6 and 41.7.

Spinors needed when analyzing fermions in gravitational fields

Spinors in curved spacetime are an indispensible mathematical tool, when one wishes to study the influence of gravity on quantized particles of half-integral spin (neutrinos, electrons, protons, . . .). Consider, for example, Hartle's (1971) proof that a black hole cannot exert any long-range, weak-interaction forces on external matter (i.e., that a black hole has no "weak-interaction hair"). His proof could not function without a spinor description of neutrino fields in curved spacetime. Similarly for Wheeler's (1971b) analysis of the quasibound states of an electron in the gravitational field of a small black hole (gravitational radius $\sim 10^{-13}$ cm): it requires solving the Dirac equation for a spin-$\frac{1}{2}$ particle in the curved spacetime geometry of Schwarzschild. For a detailed discussion of the Dirac equation in curved spacetime see, e.g., Brill and Wheeler (1957).

Equivalence of spinor and tensor formalisms

To use the mathematics of spinors, one need not be dealing with quantum theory or with particles of half-integral spin. The spinor formalism is perfectly applicable in situations where only integral-spin entities (scalars, vectors, tensors) are in view, and where in fact, the spinor formalism is fully equivalent to the tensor formalism that pervades earlier chapters of this book. Equations (41.77) and (41.78) provide the translation from one formalism to the other, once an orthonormal frame has been chosen at each event in spacetime.

Certain types of problems in gravitation theory are far more tractable in the language of spinors than in the language of tensors. Examples are as follows.

(1) Geometric Optics (the theory of "null congruences of geodesics")

Applications of spinor formalism in classical gravitation theory

Here spinors make almost trivial the lengthy tensor algebra needed in derivations of the "focusing theorem" [equation (22.37)]; and they yield an elegant, simple formalism for discussing and calculating how, with increasing affine parameter, a bundle of rays alters its size ("expansion"), its shape ("shear"), and its orientation ("rotation"). See, e.g., Sachs (1964), Pirani (1965), or Penrose (1968a) for a review and the original references.

(2) Radiation Theory in Curved Spacetime (both gravitational and electromagnetic)

Spinors provide the most powerful of all formalisms for decomposing radiation fields into spherical harmonics and for manipulating their decomposed components. See, for example, Price's (1972a,b) analysis of how a perturbed Schwarzschild black hole radiates away all its radiatable perturbations, be they electromagnetic perturbations, gravitational perturbations, or perturbations in a fictitious field of spin 17; see, similarly, the analysis by Fackerell and Ipser (1972) and by Ipser (1971) of electromagnetic perturbations of a Kerr black hole, and the analysis by Teukolsky (1972) of gravitational perturbations of a Kerr hole. Spinors also yield an elegant and powerful analysis of the "$1/r$" expansion of a radiation field flowing out from a source into asymptotically flat space. Among its results is a "*peeling theorem,*" which describes the algebraic properties of the coefficients in a $1/r$ expansion of the Riemann tensor. See, e.g., Sachs (1964) or Pirani (1965) for reviews and original references.

(3) Algebraic Properties of Curvature Tensors

The spinor formalism is a more powerful method than any other for deriving the "Petrov-Pirani algebraic classification of the conformal curvature tensor," and for proving theorems about algebraic properties of curvature tensors. See, e.g., Sachs (1964) or Pirani (1965) or Penrose (1968a) for reviews and references.

Of course, the spinor formalism, like any formalism, has its limitations. For example, many of the elementary problems of gravitation theory, and a large fraction of the most difficult ones, would be more difficult in the language of spinors than in the language of tensors! But for certain classes of problems, especially those where null vectors play a central role, spinors are a most valuable tool.

Cartan gave spinors to the world's physics and mathematics. His text (American edition, 1966) is an important reference to the subject.

CHAPTER 42

REGGE CALCULUS

§42.1. WHY THE REGGE CALCULUS?

This chapter is entirely Track 2. As preparation for it, Chapter 21 (variational principle and initial-value formalism) is needed. It is not needed as preparation for any later chapter, though it will be helpful in Chapter 43 (dynamics of geometry).

The need for Regge calculus as a computational tool

Gravitation theory is entering an era when situations of greater and greater complexity must be analyzed. Before about 1965 the problems of central interest could mostly be handled by idealizations of special symmetry or special simplicity or both. The Schwarzschild geometry and its generalizations, the Friedmann cosmology and its generalizations, the joining together of the Schwarzschild geometry and the Friedmann geometry to describe the collapse of a bounded collection of matter, the vibrations of relativistic stars, weak gravitational waves propagating in an otherwise flat space: all these problems and others were solved by elementary means.

But today one is pressed to understand situations devoid of symmetry and not amenable to perturbation theory: How do two black holes alter in shape, and how much gravitational radiation do they emit when they collide and coalesce? What are the structures and properties of the singularities at the endpoint of gravitational collapse, predicted by the theorems of Penrose, Hawking, and Geroch? Can a Universe that begins completely chaotic smooth itself out quickly by processes such as inhomogeneous mixmaster oscillations?

To solve such problems, one needs new kinds of mathematical tools—and in response to this need, new tools are being developed. The "global methods" of Chapter 34 provide one set of tools. The Regge Calculus provides another [Regge (1961); see also pp. 467–500 of Wheeler (1964a)].

§42.2. REGGE CALCULUS IN BRIEF

Approximation of smooth geometries by skeleton structures

Consider the geodesic dome that covers a great auditorium, made of a multitude of flat triangles joined edge to edge and vertex to vertex. Similarly envisage space-time, in the Regge calculus, as made of flat-space "simplexes" (four-dimensional item in this progression: two dimensions, triangle; three dimensions, tetrahedron; four dimensions, simplex) joined face to face, edge to edge, and vertex to vertex. To specify the lengths of the edges is to give the engineer all he needs in order to know the shape of the roof, and the scientist all he needs in order to know the geometry of the spacetime under consideration. A smooth auditorium roof can be approximated arbitrarily closely by a geodesic dome constructed of sufficiently small triangles. A smooth spacetime manifold can be approximated arbitrarily closely by a locked-together assembly of sufficiently small simplexes. Thus the Regge calculus, reaching beyond ordinary algebraic expressions for the metric, provides a way to analyze physical situations deprived, as so many situations are, of spherical symmetry, and systems even altogether lacking in symmetry.

Role of Einstein field equation in fixing the skeleton structure

If the designer can give the roof any shape he pleases, he has more freedom than the analyst who is charting out the geometry of spacetime. Given the geometry of spacetime up to some spacelike slice that, for want of a better name, one may call "now," one has no freedom at all in the geometry from that instant on. Einstein's geometrodynamic law is fully deterministic. Translated into the language of the Regge calculus, it provides a means to calculate the edge lengths of new simplexes from the dimensions of the simplexes that have gone before. Though the geometry is deterministically specified, how it will be approximated is not. The original spacelike hypersurface ("now") is approximated as a collection of tetrahedrons joined together face to face; but how many tetrahedrons there will be and where their vertices will be placed is the option of the analyst. He can endow the skeleton more densely with bones in a region of high curvature than in a region of low curvature to get the most "accuracy profit" from a specified number of points. Some of this freedom of choice for the lengths of the bones remains as one applies the geometrodynamic law in the form given by Regge (1961) to calculate the future from the past. This freedom would be disastrous to any computer program that one tried to write, unless the programmer removed all indefiniteness by adding supplementary conditions of his own choice, either tailored to give good "accuracy profit," or otherwise fixed.

Having determined the lengths of all the bones in the portion of skeletonized spacetime of interest, one can examine any chosen local cluster of bones in and by themselves. In this way one can find out all there is to be learned about the geometry in that region. Of course, the accuracy of one's findings will depend on the fineness with which the skeletonization has been carried out. But in principle that is no limit to the fineness, or therefore to the accuracy, so long as one is working in the context of classical physics. Thus one ends up with a catalog of all the bones, showing the lengths of each. Then one can examine the geometry of whatever spacelike surface one pleases, and look into many other questions besides. For this purpose one has only to pick out the relevant bones and see how they fit together.

§42.3. SIMPLEXES AND DEFICIT ANGLES

Figure 42.1 recalls how a smoothly curved surface can be approximated by flat triangles. All the curvature is concentrated at the vertices. No curvature resides at

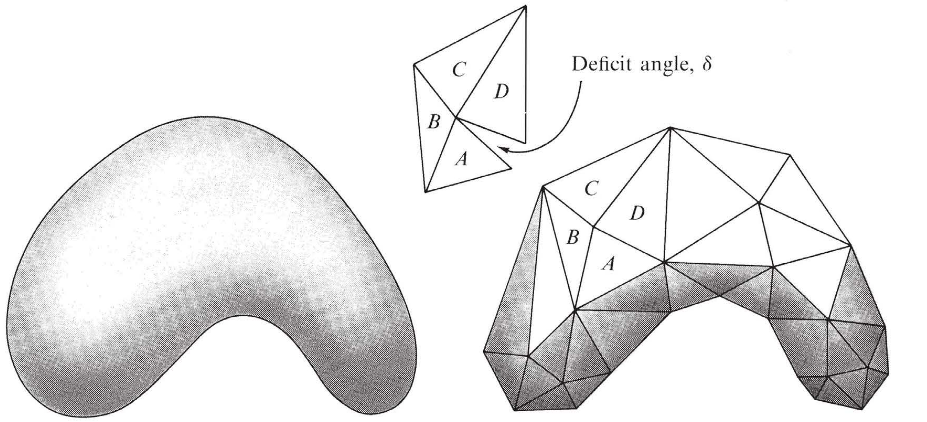


Figure 42.1.
A 2-geometry with continuously varying curvature can be approximated arbitrarily closely by a polyhedron built of triangles, provided only that the number of triangles is made sufficiently great and the size of each sufficiently small. The geometry in each triangle is Euclidean. The curvature of the surface shows up in the amount of deficit angle at each vertex (portion *ABCD* of polyhedron laid out above on a flat surface).

Deficit angle as a skeletonized measure of curvature:

(1) in two dimensions

the edge between one triangle and the next, despite one's first impression. A vector carried by parallel transport from A through B and C to D, and then carried back by another route through C and B to A returns to its starting point unchanged in direction, as one sees most easily by laying out this complex of triangles on a flat surface. Only if the route is allowed to encircle the vertex common to A, B, C, and D does the vector experience a net rotation. The magnitude of the rotation is equal to the indicated deficit angle, δ, at the vertex. The sum of the deficit angles over all the vertices has the same value, 4π, as does the half-integral of the continuously distributed scalar curvature (${}^{(2)}R = 2/a^2$ for a sphere of radius a) taken over the entirety of the original smooth figure,

$$\sum_{\substack{\text{skeleton}\\ \text{geometry}}} \delta_i = \frac{1}{2} \int_{\substack{\text{actual smooth}\\ \text{geometry}}} {}^{(2)}R \, d(\text{surface}) = 4\pi. \tag{42.1}$$

(2) in n (or four) dimensions

Generalizing from the example of a 2-geometry, Regge calculus approximates a smoothly curved n-dimensional Riemannian manifold as a collection of n-dimensional blocks, each free of any curvature at all, joined by $(n - 2)$-dimensional regions in which all the curvature is concentrated (Box 42.1). For the four-dimensional spacetime of general relativity, the "hinge" at which the curvature is concentrated has the shape of a triangle, as indicated schematically in the bottom row of Figure 42.2. In the example illustrated there, ten tetrahedrons have that triangle in common. Between one of these tetrahedrons and the next fits a four-dimensional simplex. Every feature of this simplex is determined by the lengths of its ten edges. One of the features is the angle α between one of the indicated tetrahedrons or "faces" of the simplex and the next. Thus α represents the angle subtended by this simplex

Box 42.1 THE HINGES WHERE THE CURVATURE IS CONCENTRATED IN THE "ANGLE OF RATTLE" BETWEEN BUILDING BLOCKS IN A SKELETON MANIFOLD

Dimensionality of manifold	*2*	*3*	*4*
Elementary flat-space building block:	triangle	tetrahedron	simplex
Edge lengths to define it:	3	4	5
Hinge where cycle of such blocks meet with a deficit angle or "angle of rattle" δ:	vertex	edge	triangle
Dimensionality of hinge:	0	1	2
"Content" of such a hinge:	1	length l	area A
Contribution from all hinges within a given small region to curvature of manifold:	$\sum_{\substack{\text{that}\\ \text{region}}} \delta_i$	$\sum_{\substack{\text{that}\\ \text{region}}} l_i\, \delta_i$	$\sum_{\substack{\text{that}\\ \text{region}}} A_i\, \delta_i$
Continuum limit of this quantity expressed as an integral over the same small region:	$\frac{1}{2}\int {}^{(2)}R({}^{(2)}g)^{1/2}\, d^2x$	$\frac{1}{2}\int {}^{(3)}R({}^{(3)}g)^{1/2}\, d^3x$	$\frac{1}{2}\int {}^{(4)}R(-{}^{(4)}g)^{1/2}\, d^4x$

at the hinge. Summing the angles α for all the simplexes that meet on the given hinge $\mathscr{P}\mathscr{Q}\mathscr{R}$, and subtracting from 2π, one gets the deficit angle associated with that hinge. And by then summing the deficit angles in a given small n-volume with appropriate weighting (Box 42.1), one obtains a number equal to the volume integral of the scalar curvature of the original smooth n-geometry. See Box 42.2.

§42.4. SKELETON FORM OF FIELD EQUATIONS

Einstein-Hilbert variational principle reduced to skeleton form

Rather than translate Einstein's field equations directly into the language of the skeleton calculus, Regge turns to a standard variational principle from which Einstein's law lets itself be derived. It says (see §§21.2 and 43.3) adjust the 4-geometry throughout an extended region of spacetime, subject to certain specified conditions on the boundary, so that the dimensionless integral (action in units of $\hbar$!),

$$I = (c^3/16\pi\hbar G)\int R(-g)^{1/2}\, d^4x, \tag{42.2}$$

is an extremum. This statement applies when space is free of matter and electromag-

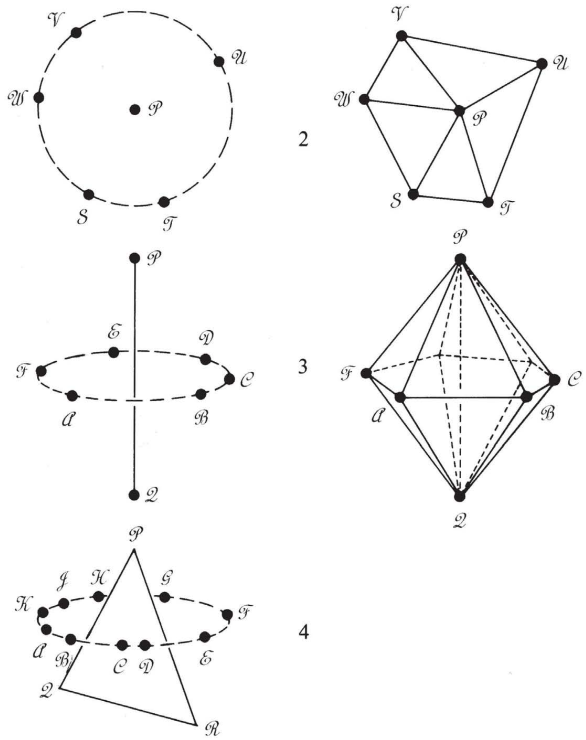


Figure 42.2.
Cycle of building blocks associated with a single hinge. Top row, two dimensions: left, schematic association of vertices $\mathcal{S}$, $\mathcal{T}$, $\mathcal{U}$, $\mathcal{V}$, $\mathcal{W}$ with "hinge" at the vertex $\mathcal{P}$; right, same, but with elementary triangles indicated in full. Middle row, three dimensions: left, schematic; right, perspective representation of the six tetrahedrons that meet on the "hinge" $\mathcal{P}\mathcal{Q}$. Bottom row, four dimensions; shown only schematically. The five vertices $\mathcal{P}\mathcal{Q}\mathcal{R}\mathcal{C}\mathcal{D}$ belong to one simplex, a four-dimensional region throughout the interior of which space is flat. The five vertices $\mathcal{P}\mathcal{Q}\mathcal{R}\mathcal{D}\mathcal{E}$ belong to the next simplex; and so on around the cycle of simplexes. The two simplexes just named interface at the tetrahedron $\mathcal{P}\mathcal{Q}\mathcal{R}\mathcal{D}$, inside which the geometry is also flat. Between that tetrahedron and the next, $\mathcal{P}\mathcal{Q}\mathcal{R}\mathcal{E}$, there is a certain hyperdihedral angle α subtended at the "hinge" $\mathcal{P}\mathcal{Q}\mathcal{R}$. The value of this angle is completely fixed by the ten edge lengths of the intervening simplex $\mathcal{P}\mathcal{Q}\mathcal{R}\mathcal{D}\mathcal{E}$. This dihedral angle, plus the corresponding dihedral angles subtended at the hinge $\mathcal{P}\mathcal{Q}\mathcal{R}$ by the other simplexes of the cycle, do not in general add up to 2π. The deficit, the "angle of rattle" or deficit angle δ, gives the amount of curvature concentrated at the hinge $\mathcal{P}\mathcal{Q}\mathcal{R}$. There is no actual rattle or looseness of fit, unless one tries to imbed the cycle into an over-all flat four-dimensional space (analog of "stamping on" the collection of triangles, and seeing them open out by the amount of the deficit angle, as indicated in inset in Figure 42.1).

netic fields, a simplification that will be made in the subsequent discussion to keep it from becoming too extended. When in addition all lengths are expressed in units of the Planck length

$$L^* = (\hbar G/c^3)^{1/2} = 1.6 \times 10^{-33}\,\text{cm}, \tag{42.3}$$

and the curvature integral is approximated by its expression in terms of deficit angles, Regge shows that the statement $\delta I = 0$ (condition for an extremum!) becomes

$$(1/8\pi)\,\delta \sum_{\substack{\text{hinges} \\ h=1}}^{H} A_h\,\delta_h = 0. \tag{42.4}$$

Box 42.2 FLOW DIAGRAMS FOR REGGE CALCULUS

A skeleton 4-geometry is completely determined by all its edge lengths. From the edge lengths one gets the integrated curvature by pursuing, for each hinge in the 4-geometry, the following flow diagram:

hinge

↓

cycle of blocks swinging on this hinge

↓

one of these blocks

↓

the two tetrahedral "faces" that set this block off from the blocks before and after it in the cycle of blocks

↓

angle α between these two faces fixed by the block's $n(n-1)/2$ edge lengths

↓

deficit angle at the given hinge is

$$\delta = 2\pi - \sum_{\substack{\text{blocks swinging} \\ \text{on that hinge}}} \alpha$$

↓

contribution to integrated curvature (Box 42.1) is δ times area of hinge

One finds it natural to apply this analysis in either of two ways. First, one can probe a given 4-geometry (given set of edge lengths!) in the sense

edge lengths

↓

curvature

Box 42.2 (*continued*)

Second—and this is the rationale of Regge calculus—one can use the skeleton calculus to deduce a previously unknown 4-geometry from Einstein's geometrodynamic law, proceeding in the direction

initial conditions translated into information about some of the edge lengths

Einstein's equations expressed as conditions on the curvature (deficit angle of each hinge)

fix remainder of the edge lengths (apart from natural options in fineness of zoning)

In the changes contemplated in this variational principle, certain edge lengths are thought of as being fixed. They have to do with the conditions specified at the boundaries of the region of spacetime under study. It is not necessary here to enter into the precise formulation of these boundary conditions, fortunately, since some questions of principle still remain to be clarified about the precise formulation of boundary conditions in general relativity (see §21.12). Rather, what is important is the effect of changes in the lengths of the edges of the blocks in the interior of the region being analyzed, as they augment or decrease the deficit angles at the various hinges. In his basic paper on the subject, Regge (1961) notes that the typical deficit angle δ_h depends in a complicated trigonometric way on the values of numerous edge lengths ℓ_p. However, he proves (Appendix of his paper) that "quite remarkably, we can carry out the variation as if the δ_h were constants," thus reducing the variational principle to the form

$$(1/8\pi) \sum_{\substack{\text{hinges} \\ h=1}}^{H} \delta_h \, \delta A_h = 0. \tag{42.5}$$

Here the change in area of the h-th triangle-shaped hinge, according to elementary trigonometry, is

$$\delta A_h = \frac{1}{2} \sum_p \ell_p \, \delta\ell_p \cot\!\mathrm{an}\, \theta_{ph}. \tag{42.6}$$

In this equation θ_{ph} is the angle opposite to the p-th edge in the triangle. Consequently, Einstein's equations in empty space reduce in skeleton geometry to the form

$$\sum_{\substack{\text{hinges that}\\ \text{have the}\\ \text{given edge}\\ p \text{ in common}}} \delta_h \cot\theta_{ph} = 0, \qquad (p = 1, 2, \ldots), \tag{42.7}$$

Einstein field equation reduced to skeleton form

one equation for each edge length in the interior of the region of spacetime being analyzed.

§42.5. THE CHOICE OF LATTICE STRUCTURE

Two questions arise in the actual application of Regge calculus, and it is not clear that either has yet received the resolution which is most convenient for practical applications of this skeleton analysis: What kind of lattice to use? How best to capitalize on the freedom that exists in the choice of edge lengths? The first question is discussed in this section, the second in the next section.

The choice of lattice structure:

(1) avoiding floppiness

It might seem most natural to use a lattice made of small, nearly rectangular blocks, the departure of each from rectangularity being conditioned by the amount and directionality of the local curvature. However, such building blocks are "floppy." One could give them rigidity by specifying certain angles as well as the edge lengths. But then one would lose the cleanness of Regge's prescription: give edge lengths, and give only edge lengths, and give each edge length freely and independently, in order to define a geometry. In addition one would have to rederive the Regge equations, including new equations for the determination of the new angles. Therefore one discards the quasirectangle in favor of the simplex with its $5 \cdot 4/2 = 10$ edge lengths. This decided, one also concludes that even in flat spacetime the simplexes cannot all have identical edge lengths. Two-dimensional flat space can be filled with identical equilateral triangles, but already at three dimensions it ceases to be possible to fill out the manifold with identical equilateral tetrahedrons. One knows that a given carbon atom in diamond is joined to its nearest neighbors with tetrahedral bonds, but a little reflection shows that the cell assignable to the given atom is far from having the shape of an equilateral tetrahedron.

(2) necessity for unequal edge lengths

Synthesis would appear to be a natural way to put together the building blocks: first make one-dimensional structures; assemble these into two-dimensional structures; these, into three-dimensional ones; and these, into the final four-dimensional structure. The one-dimensional structure is made of points, 1, 2, 3, . . . , alternating with line segments, 12, 23, 34, ... To start building a two-dimensional structure, pick up a second one-dimensional structure. It might seem natural to label its points $1'$, $2'$, $3'$, . . . , etc. However, that labeling would imply a cross-connection between 1 and $1'$, between 2 and $2'$, etc., after the fashion of a ladder. Then the elementary cells would be quasirectangles. They would have the "floppiness" that is to be excluded. Therefore relabel the points of the second one-dimensional structure as $1\frac{1}{2}'$, $2\frac{1}{2}'$, $3\frac{1}{2}'$, etc. The implication is that one cross-connects $2\frac{1}{2}'$ with points 2 and 3 of the original one-dimensional structure, etc. One ends up with something like the

(3) construction of two-dimensional structures

girder structure of a bridge, fully rigid in the context of two dimensions, as desired. The same construction, extended, fills out the plane with triangles. One now has a simple, standard two-dimensional structure. One might mistakenly conclude that one is ready to go ahead to build up a three-dimensional structure: the mistake lies in the tacit assumption that the flat-space topology is necessarily correct.

Let it be the problem, for example, to determine the development in time of a 3-geometry that has the topology of a 3-sphere. This 3-sphere is perhaps strongly deformed from ideality by long-wavelength gravitational waves. A right arrangement of the points is the immediate desideratum. Therefore put aside for the present any consideration of the deformation of the geometry by the waves (alteration of edge lengths from ideality). Ask how to divide a perfect 3-sphere into two-dimensional sheets. Here each sheet is understood to be separated from the next by a certain distance. At this point two alternative approaches suggest themselves that one can call for brevity "blocks" and "spheres."

(4) 3-D structures built from 2-D structures by "method of blocks"

(1) *Blocks.* Note that a 3-sphere lets itself be decomposed into 5 identical, tetrahedron-like solid blocks (5 vertices; 5 ways to leave out any one of these vertices!) Fix on one of these "tetrahedrons." Select one vertex as summit and the face through the other three vertices as base. Give that base the two-dimensional lattice structure already described. Introduce a multitude of additional sheets piled above it as evenly spaced layers reaching to the summit. Each layer has fewer points than the layer before. The decomposition of the 3-geometry inside one "tetrahedron" is thereby accomplished. However, an unresolved question remains; not merely how to join on this layered structure in a regular way to the corresponding structure in the adjacent "tetrahedrons"; but even whether such a regular joinup is at all possible. The same question can be asked about the other two ways to break up the 3-sphere into identical "tetrahedrons" [Coxeter (1948), esp. pp. 292–293: 16 tetrahedrons defined by a total of 8 vertices or 600 tetrahedrons defined by a total of 120 vertices]. One can eliminate the question of joinup of structure in a simple way, but at the price of putting a ceiling on the accuracy attainable: take the stated number of vertices (5 or 8 or 120) as the total number of points that will be employed in the skeletonization of the 3-geometry (no further subdivision required or admitted). Considering the boundedness of the memory capacity of any computer, it is hardly ridiculous to contemplate a limitation to 120 tracer points in exploratory calculations!

(5) 3-D structures from 2-D by "method of spheres"

(2) *Spheres.* An alternative approach to the "atomization" of the 3-sphere begins by introducing on the 3-sphere a North Pole and a South Pole and the hyperspherical angle χ ($\chi = 0$ at the first pole, $\chi = \pi$ at the second, $\chi = \pi/2$ at the equator; see Box 27.2). Let each two-dimensional layer lie on a surface of constant χ (χ equal to some integer times some interval $\Delta\chi$). The structure of this 2-sphere is already to be regarded as skeletonized into elementary triangles ("fully complete Buckminster Fuller geodesic dome"). Therefore the number of "faces" or triangles F, the number of edge lengths E, and the number of vertices V must be connected by the relation of Euler:

$$F - E + V = \begin{pmatrix}\text{a topology-dependent} \\ \text{number or "Euler character"}\end{pmatrix} = \begin{cases} 2 \text{ for 2-sphere,} \\ 0 \text{ for 2-torus.}\end{cases} \tag{42.8}$$

It follows from this relation that it is impossible for each vertex to sit at the center of a hexagon (each vertex the point of convergence of 6 triangles). This being the case, one is not astonished that a close inspection of the pattern of a geodesic dome shows several vertices where only 5 triangles meet. It is enough to have 12 such 5-triangle vertices among what are otherwise all 6-triangle vertices in order to meet the requirements of the Euler relation:

$$
\begin{aligned}
n &\quad \text{5-triangle vertices}\\
V - n &\quad \text{6-triangle vertices}\\
F &= (V-n)(6/3) + n(5/3) \text{ triangles}\\
E &= (V-n)(6/2) + n(5/2) \text{ edges}\\
V &= (V-n)(6/6) + n \text{ vertices}\\
2 = F - E + V &= n/6 \qquad \text{Euler characteristic}\\
n &= 12
\end{aligned}
\tag{42.9}
$$

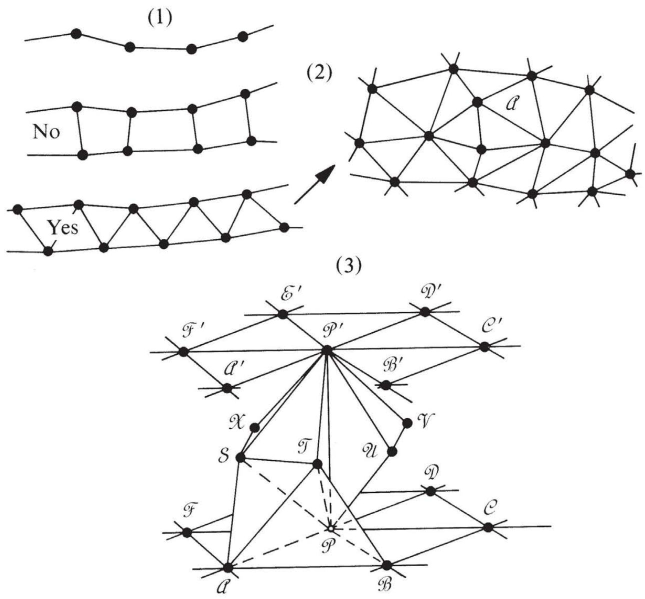
Among all figures with triangular faces, the icosahedron is the one with the smallest number of faces that meets this condition (5-triangle vertices exclusively!)

If each 2-surface has the pattern of vertices of a geodesic dome, how is one dome to be joined to the next to make a rigid skeleton 3-geometry? Were the domes imbedded in a flat 3-geometry, rigidity would be no issue. Each dome would already be rigid in and by itself. However, the 3-geometry is not given to be flat. Only by a completely deterministic skeletonization of the space between the two 2-spheres will they be given rigidity in the context of curved space geometry. (1) Not by running a single connector from each vertex in one surface to the corresponding vertex in the next ("floppy structure"!) (2) Not by displacing one surface so each of its vertices comes above, or nearly above, the center of a triangle in the surface "below." First, the numbers of vertices and triangles ordinarily will not agree. Second, even when they do, it will not give the structure the necessary rigidity to connect the vertex of the surface above to the three vertices of the triangle below. The space between will contain some tetrahedrons, but it will not be throughout decomposed into tetrahedrons. (3) A natural and workable approach to the skeletonization of the 3-geometry is to run a connector from each vertex in the one surface to the corresponding vertex in the next, but to flesh out this connection with additional structure that will give rigidity to the 3-geometry: intervening vertices and connectors as illustrated in Box 42.3.

(6) 4-D structures built from 3-D structures

In working up from the skeletonization of a 3-geometry to the skeletonization of a 4-geometry, it is natural to proceed similarly. (1) Use identical patterns of points in the two 3-geometries. (2) Tie corresponding points together by single connectors. (3) Halfway, or approximately half way between the two 3-geometries insert a whole additional pattern of vertices. Each of these supplementary vertices is "dual" to and lies nearly "below" the center of a tetrahedron in the 3-geometry immediately above. (4) Connect each supplementary vertex to the vertices of the tetrahedron immediately above, to the vertices of the tetrahedron immediately below, and to those other supplementary vertices that are its immediate neighbors. (5) In this way get the edge lengths needed to divide the 4-geometry into simplexes, each of rigidly defined dimensions.

Box 42.3 SYNTHESIS OF HIGHER-DIMENSIONAL SKELETON GEOMETRIES OUT OF LOWER-DIMENSIONAL SKELETON GEOMETRIES



(1) One-dimensional structure as alternation of points and line segments. (2) Two-dimensional structure (a) "floppy" (unacceptable) and (b) rigidified (angles of triangles fully determined by edge lengths). When this structure is extended, as at right, the "normal" vertex has six triangles hinging on it. However, at least twelve 5-triangle vertices of the type indicated at $\mathscr{A}$ are to be interpolated if the 2-geometry is to be able to close up into a 2-sphere. (3) Skeleton 3-geometry obtained by filling in between the skeleton 2-geometry ... $\mathscr{A}\mathscr{B}$ ... $\mathscr{F}\mathscr{P}\mathscr{C}$ ... $\mathscr{E}\mathscr{D}$ ... and the similar structure ... $\mathscr{A}'\mathscr{B}'$ ... $\mathscr{F}'\mathscr{P}'\mathscr{C}'$ ... $\mathscr{E}'\mathscr{D}'$ ... as follows. (a) Insert direct connectors such as $\mathscr{P}\mathscr{P}'$ between corresponding points in the two 2-geometries. (b) Insert an intermediate layer of "supplementary vertices" such as $\mathscr{S}\mathscr{T}\mathscr{U}\mathscr{V}\mathscr{W}\mathscr{X}$.... Each of these supplementary vertices lies roughly halfway between the center of the triangle "above" it and the center of the corresponding triangle "below" it. (c) Connect each such "supplementary vertex" with its immediate neighbors above, below, and in the same plane. (d) Give all edge lengths. (e) Then the skeleton 3-geometry between the two 2-geometries is rigidly specified. It is made up of five types of tetrahedrons, as follows. (1) "Right-through blocks," such as $\mathscr{P}\mathscr{P}'\mathscr{S}\mathscr{T}$ (six of these hinge on $\mathscr{P}\mathscr{P}'$ when $\mathscr{P}$ is a normal vertex; five, when it is a 5-fold vertex, such as indicated by $\mathscr{A}$ at the upper right). (2) "Lower-facing blocks," such as $\mathscr{A}\mathscr{B}\mathscr{P}\mathscr{T}$. (3) "Lower-packing blocks," such as $\mathscr{A}\mathscr{P}\mathscr{S}\mathscr{T}$. (4, 5) Corresponding "upper-facing blocks" and "upper-packing blocks" (not shown). The number of blocks of each kind is appropriately listed here for the two extreme cases of a 2-geometry that consists (a) of a normal hexagonal lattice extending indefinitely in a plane and (b) of a lattice consisting of the minimum number of 5-fold vertices ("type $\mathscr{A}$ vertices") that will permit close-up into a 2-sphere.

2-geometry of upper (or lower) face	*Hexagonal pattern of triangles*	*Icosahedron made of triangles*
Its topology	Infinite 2-plane	2-sphere
Vertices on upper face	V	12
Nature of these vertices	6-fold	5-fold
Edge lengths on upper face	$3V$	$\frac{5}{2}V = 30$
Triangles on upper face	$2V$	20
Number of "supplementary vertices"	$2V$	20
Outer facing blocks	$2V$	20
Outer packing blocks	$3V$	30
Right through blocks	$6V$	60
Inner packing blocks	$3V$	30
Inner facing blocks	$2V$	20

§42.6. THE CHOICE OF EDGE LENGTHS

The choice of edge lengths:

So much for the lattice structure of the 4-geometry; now for the other issue, the freedom that exists in the choice of edge lengths. Why not make the simplest choice and let all edges be light rays? Because the 4-geometry would not then be fully determined. The geometry $g_{\alpha\beta}(x^\mu)$ differs from the geometry $\lambda(x^\mu)\, g_{\alpha\beta}(x^\mu)$, even though the same points that are connected by light rays in the one geometry are also connected by light rays in the other geometry.

(1) choose some timelike, others spacelike

If none of the edges is null, it is nevertheless natural to take some of the edge lengths to be spacelike and some to be timelike. In consequence the area A of the triangle in some cases will be real, in other cases imaginary. In 3-space the parallelogram (double triangle) spanned by two vectors $\boldsymbol{B}$ and $\boldsymbol{C}$ is described by a vector

$$2\boldsymbol{A} = \boldsymbol{B} \times \boldsymbol{C}$$

perpendicular to the two vectors. One obtains the magnitude of $\boldsymbol{A}$ from the formula

$$4A^2 = \boldsymbol{B}^2\boldsymbol{C}^2 - (\boldsymbol{B}\cdot\boldsymbol{C})^2.$$

In 4-space, let $\boldsymbol{B}$ and $\boldsymbol{C}$ be two edges of the triangle. Then, as in three dimensions, $2\boldsymbol{A}$ is *dual* to the bivector built from $\boldsymbol{B}$ and $\boldsymbol{C}$. In other words, if $\boldsymbol{B}$ goes in the t direction and $\boldsymbol{C}$ in the z direction, then $\boldsymbol{A}$ is a bivector lying in the (x, y) plane. Consequently its magnitude A is to be thought of as a *real* quantity. Therefore the appropriate formula for the area A is (Tullio Regge)

$$4A^2 = (\boldsymbol{B}\cdot\boldsymbol{C})^2 - \boldsymbol{B}^2\boldsymbol{C}^2. \tag{42.10}$$

The quantity A is real when the deficit angle δ is real. Thus the geometrically important product $A\delta$ is also real.

2

When the hinge lies in the (x, y) plane, on the other hand, the quantity A is purely imaginary. In that instance a test vector taken around the cycle of simplexes that swing on this hinge has undergone change only in its z and t components; that is, it has experienced a Lorentz boost; that is, the deficit angle δ is also purely imaginary. So again the product $A\delta$ is a purely real quantity.

(2) choose timelike lengths comparable to spacelike lengths

Turn now from character of edge lengths to magnitude of edge lengths. It is desirable that the elementary building blocks sample the curvatures of space in different directions on a roughly equal basis. In other words, it is desirable not to have long needle-shaped building blocks nor pancake-shaped tetrahedrons and simplexes. This natural requirement means that the step forward in time should be comparable to the steps "sidewise" in space. The very fact that one should have to state such a requirement brings out one circumstance that should have been obvious before: the "hinge equations"

$$\sum_{\substack{\text{hinges } h \text{ that} \\ \text{have edge } p \\ \text{in common}}} \delta_h \operatorname{cotan} \theta_{ph} = 0 \qquad (p = 1, 2, \ldots), \tag{42.7}$$

(3) why some lengths must be chosen arbitrarily

though they are as numerous as the edges, cannot be regarded as adequate to determine all edge lengths. There are necessarily relations between these equations that keep them from being independent. The equations cannot determine all the details of the necessarily largely arbitrary skeletonization process. They cannot do so any more than the field equations of general relativity can determine the coordinate system. With a given pattern of vertices (four-dimensional generalization of drawings in Box 42.3), one still has (a) the option how close together one will take successive layers of the structure and (b) how one will distribute a given number of points in space on a given layer to achieve the maximum payoff in accuracy (greater density of points in regions of greater curvature). To prepare a practical computer program founded on Regge calculus, one has to supply the machine not only with the hinge equations and initial conditions, but also with definite algorithms to remove all the arbitrariness that resides in options (a) and (b).

Deficit angles in terms of edge lengths

Formulas from solid geometry and four-dimensional geometry, out of which to determine the necessary hyperdihedral angles α and the deficit angles δ in terms of edge lengths and nothing but edge lengths, are summarized by Wheeler (1964a, pp. 469, 470, and 490) and by C. Y. Wong (1971). Regge (1961) also gives a formula for the Riemann curvature tensor itself in terms of deficit angles and number of edges running in a given direction [see also Wheeler (1964a, p. 471)].

§42.7. PAST APPLICATIONS OF REGGE CALCULUS

Past applications of Regge calculus

Wong (1971) has applied Regge calculus to a problem where no time development shows itself, where the geometry can therefore be treated as static, and where in addition it is spherically symmetric. He determined the Schwarzschild and Reissner-Nordstrøm geometries by the method of skeletonization. Consider successive spheres surrounding the center of attraction. Wong approximates each as an icosahedron. The condition

$$^{(3)}R = 16\pi \begin{pmatrix} \text{energy density} \\ \text{on the 3-space} \end{pmatrix}$$

(§21.5) gives a recursion relation that determines the dimension of each icosahedron in terms of the two preceding icosahedra. Errors in the skeleton representation of the exact geometry range from roughly 10 percent to less than 1 percent, depending on the method of analysis, the quantity under analysis, and the fineness of the subdivision.

Partial skeletonization

Skeletonization of geometry is to be distinguished from mere rewriting of partial differential equations as difference equations. One has by now three illustrations that one can capitalize on skeletonization without fragmenting spacetime all the way to the level of individual simplexes. The first illustration is the first part of Wong's work, where the time dimension never explicitly makes an appearance, so that the building blocks are three-dimensional only. The second is an alternative treatment, also given by Wong, that goes beyond the symmetry in t to take account of the symmetry in θ and ϕ. It divides space into spherical shells, in each of which the geometry is "pseudo-flat" in much the same sense that the geometry of a paper cone is flat. The third is the numerical solution for the gravitational collapse of a spherical star by May and White (1966), in which there is symmetry in θ and ϕ, but not in r or t. This zoning takes place exclusively in the r, t-plane. Each zone is a spherical shell. The difference as compared to Regge calculus (flat geometry within each building block) is the adjustable "conicity" given to each shell. The examples show that the decision about skeletonizing the geometry in a calculation is ordinarily not "whether" but "how much."

§42.8. THE FUTURE OF REGGE CALCULUS

Hopes for the future

In summary, Regge's skeleton calculus puts within the reach of computation problems that in practical terms are beyond the power of normal analytical methods. It affords any desired level of accuracy by sufficiently fine subdivision of the spacetime region under consideration. By way of its numbered building blocks, it also offers a practical way to display the results of such calculations. Finally, one can hope that Regge's truly geometric way of formulating general relativity will someday make the content of the Einstein field equations (Cartan's "moment of rotation"; see Chapter 15) stand out sharp and clear, and unveil the geometric significance of the so-called "geometrodynamic field momentum" (analysis of the boundary-value problem associated with the variational problem of general relativity in Regge calculus; see §21.12).

CHAPTER 43

SUPERSPACE: ARENA FOR THE DYNAMICS OF GEOMETRY

Traveler, there are no paths.
Paths are made by walking.
ANTONIO MACHADO (1940)

This chapter is entirely Track 2. Chapter 21 (initial-value formalism) is needed as preparation for it. In reading it, one will be helped by Chapter 42 (Regge calculus). It is not needed as preparation for any later chapter, but it will be helpful in Chapter 44 (vision of the future).

§43.1. SPACE, SUPERSPACE, AND SPACETIME DISTINGUISHED

Superspace is the arena for geometrodynamics

Superspace [Wheeler (1964a), pp. 459 ff] is the arena of geometrodynamics. The dynamics of Einstein's curved space geometry runs its course in superspace as the dynamics of a particle unfolds in spacetime. This chapter gives one simple version of superspace, and a little impression of alternative versions of superspace that also have their uses. It describes the classical dynamics of geometry in superspace in terms of the Hamilton-Jacobi principle of Boxes 25.3 and 25.4. No version of mechanics makes any shorter the leap from classical dynamics to quantum. Thus it provides a principle ("Einstein-Hamilton-Jacobi or EHJ equation") for the propagation of wave crests in superspace, and for finding where those wave crests give one the classical equivalent of constructive interference ("envelope formation"). In this way one finds the track of development of 3-geometry with time expressed as a sharp, thin "leaf of history" that slices through superspace. The quantum principle replaces this deterministic account with a fuzzed-out leaf of history of finite thickness. In consequence, quantum fluctuations take place in the geometry of space that dominate the scene at distances of the order of the Planck length, $L^* = (\hbar G/c^3)^{1/2} = 1.6 \times 10^{-33}$ cm, and less. The present survey simplifies by considering only the dynamics of curved empty space. When sources are present and are to be taken into account, supplementary terms are to be added, some of the literature on which is listed.

In all the difficult investigations that led in the course of half a century to some understanding of the dynamics of geometry, both classical and quantum, the most

Box 43.1 GEOMETRODYNAMICS COMPARED WITH PARTICLE DYNAMICS

Concept	*Particle dynamics*	*Geometrodynamics*
Dynamic entity	Particle	Space
Descriptors of momentary configuration	x, t ("event")	$^{(3)}\mathcal{G}$ ("3-geometry")
Classical history	$x = x(t)$	$^{(4)}\mathcal{G}$ ("4-geometry")
History is a stockpile of configurations?	Yes. Every point on world line gives a momentary configuration of particle	Yes. Every spacelike slice through $^{(4)}\mathcal{G}$ gives a momentary configuration of space
Dynamic arena	Spacetime (totality of all points x, t)	Superspace (totality of all $^{(3)}\mathcal{G}$'s)

3-geometry is the dynamic object

difficult point was also the simplest: The dynamic object is not spacetime. It is space. The geometric configuration of space changes with time. But it is space, three-dimensional space, that does the changing (see Box 43.1).

Space will be treated here as "closed" or, in mathematical language, "compact," either because physics adds to Einstein's second-order differential equations the requirement of closure as a necessary and appropriate boundary condition [Einstein (1934, p. 52; 1950); Wheeler (1959; 1964c). Hönl (1962); see also §21.12] or because that requirement simplifies the mathematical analysis, or for both reasons together.

Finite-dimensional "truncated superspace"

One can approximate a smooth, closed 3-geometry by a skeleton 3-geometry built out of tetrahedrons, as indicated schematically in Figure 43.1 (see Chapter 42 on the Regge calculus). Specify the 98 edge-lengths in this example to fix all the features of the geometry; and fix these 98 edge-lengths by giving the location of a single point in a space of 98 dimensions. This 98-dimensional manifold, this "truncated superspace," goes over into superspace [Wheeler (1964a), pp. 453, 459, 463, 495] in the idealization in which the tracer points increase in density of coverage without limit. Accounts of superspace with more mathematical detail are given by DeWitt (1967a,b), Wheeler (1970), and Fischer (1970).

Let the representative point move from one location to a nearby location, either in truncated superspace or in full superspace. Then all edge-lengths alter, and the 3-geometry of Figure 43.1 moves as if alive. No better illustration can one easily supply of what it means to speak of the "dynamics of space."

The term "3-geometry" makes sense as well in quantum geometrodynamics as in classical theory. So does superspace. But spacetime does not. Give a 3-geometry, and give its time rate of change. That is enough, under typical circumstances (see Chapter 21) to fix the whole time-evolution of the geometry; enough in other words, to determine the entire four-dimensional spacetime geometry, provided one is

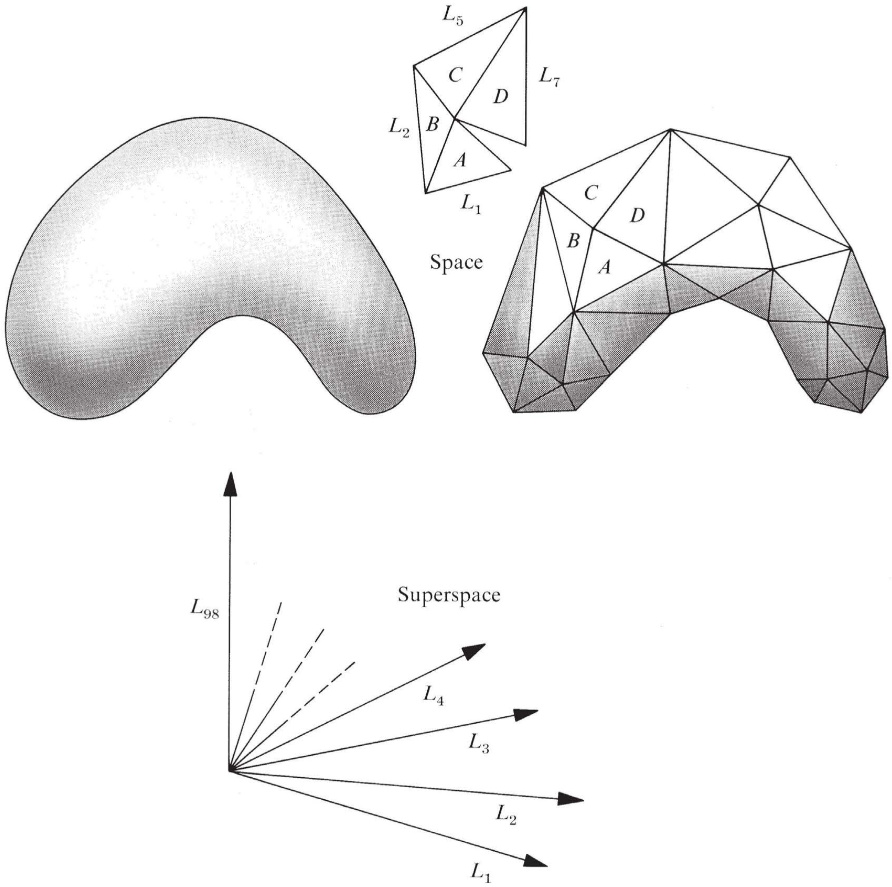


Figure 43.1.
Superspace in the simplicial approximation. Upper left, space (depicted as two-dimensional but actually three-dimensional). Upper right, simplical approximation to space. The approximation can be made arbitrarily good by going to the limit of an arbitrarily fine decomposition. The curvature at a typical location is measured by a deficit angle. This angle is completely determined by the edge lengths (L_1, L_2, . . . L_8 in the figure) of the simplexes that meet at that location. When there are 98 edge lengths altogether in the simplicial representation of the geometry, then this geometry is completely specified by a single point in a 98-dimensional space (lower diagram; "superspace").

The concept of spacetime is incompatible with the quantum principle

considering the problem in the context of classical physics. In the real world of quantum physics, however, one cannot give both a dynamic variable and its time-rate of change. The principle of complementarity forbids. Given the precise 3-geometry at one instant, one cannot also know at that instant the time-rate of change of the 3-geometry. In other words, given the geometrodynamic field coordinate, one cannot know the geometrodynamic field momentum. If one assigns the intrinsic 3-geometry, one cannot also specify the extrinsic curvature.

The uncertainty principle thus deprives one of any way whatsoever to predict, or even to give meaning to, "the deterministic classical history of space evolving in time." *No prediction of spacetime, therefore no meaning for spacetime,* is the verdict of the quantum principle. That object which is central to all of classical general relativity, the four-dimensional spacetime geometry, simply does not exist, except in a classical approximation.

These considerations reveal that the concepts of spacetime and time are not primary but secondary ideas in the structure of physical theory. These concepts are valid in the classical approximation. However, they have neither meaning nor application under circumstances where quantum geometrodynamic effects become important. Then one has to forego that view of nature in which every event, past, present, or future, occupies its preordained position in a grand catalog called "spacetime," with the Einstein interval from each event to its neighbor eternally established. There is no spacetime, there is no time, there is no before, there is no after. The question of what happens "next" is without meaning.

That spacetime is not the right way does not mean there is *no* right way to describe the dynamics of geometry consistent with the quantum principle. Superspace is the key to *one* right way to describe the dynamics (see Figure 43.2).

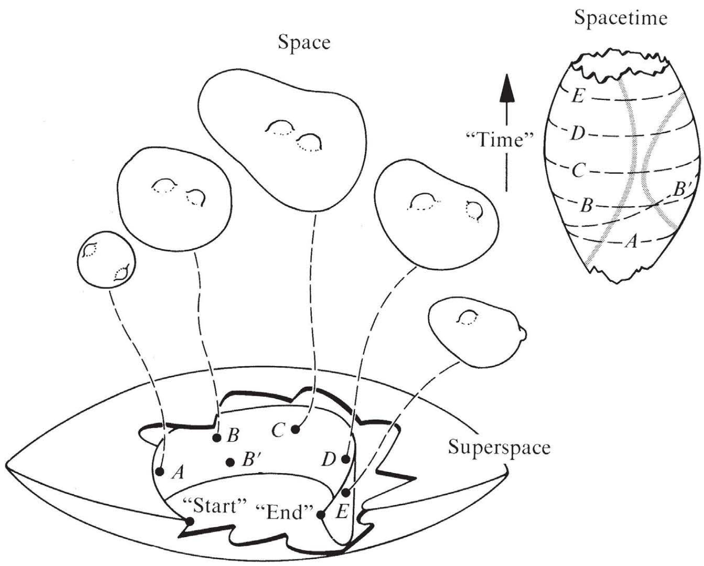


Figure 43.2.
Space, spacetime, and superspace. Upper left: Five sample configurations, A, B, C, D, E, attained by space in the course of its expansion and recontraction. Below: Superspace and these five sample configurations, each represented by a point in superspace. Upper right: Spacetime. A spacelike cut, like A, through spacetime gives a momentary configuration of space. There is no compulsion to limit attention to a one-parameter family of slices, A, B, C, D, E through spacetime. The phrase "many-fingered time" is a slogan telling one not to so limit one's slices, and B' is an example of this freedom in action. The 3-geometries B' and A, B, C, D, E, like all 3-geometries obtained by all spacelike slices whatsoever through the given classical spacetime, lie on a single bent leaf of history, indicated in the diagram, and cutting its thin slice through superspace. A different spacetime, in other words, a different solution of Einstein's field equation, means a different leaf of history (not indicated) slicing through superspace.

§43.2. THE DYNAMICS OF GEOMETRY DESCRIBED IN THE LANGUAGE OF THE SUPERSPACE OF THE $^{(3)}\mathscr{G}$'s

Spacetime is a classical leaf of history slicing through superspace

Given a spacetime, one can construct the corresponding leaf of history slicing through superspace. Conversely, given the leaf of history, one can reconstruct the spacetime.

Consider the child's toy commonly known as "Chinese boxes." One opens the outermost box only to reveal another box; when the second box is opened, there is another box, and so on, until eventually there are dozens of boxes scattered over the floor. Or conversely the boxes can be put back together, nested one inside the other, to reconstitute the original package. The packaging of $^{(3)}\mathscr{G}$'s into a $^{(4)}\mathscr{G}$ is much more sophisticated. Nature provides no monotonic ordering of the $^{(3)}\mathscr{G}$'s. Two of the dynamically allowed $^{(3)}\mathscr{G}$'s, taken at random, will often cross each other one or more times. When one shakes the $^{(4)}\mathscr{G}$ apart, one therefore gets enormously more $^{(3)}\mathscr{G}$'s "spread out over the floor" than might have been imagined. Conversely, when one puts back together all of the $^{(3)}\mathscr{G}$'s lying on the leaf of history, one gets a structure with a rigidity that might not otherwise have been foreseen. This rigidity arises from the infinitely rich interleaving and intercrossing of clear-cut, well-defined $^{(3)}\mathscr{G}$'s one with another.

In summary: (1) Classical geometrodynamics in principle constitutes a device, an algorithm, a rule for calculating and constructing a leaf of history that slices through superspace. (2) The $^{(3)}\mathscr{G}$'s that lie on this leaf of history are YES 3-geometries; the vastly more numerous $^{(3)}\mathscr{G}$'s that do not are NO 3-geometries. (3) The YES $^{(3)}\mathscr{G}$'s are the building blocks of the $^{(4)}\mathscr{G}$ that is classical spacetime. (4) The interweaving and interconnections of these building blocks give the $^{(4)}\mathscr{G}$ its existence, its dimensionality, and its structure. (5) In this structure every $^{(3)}\mathscr{G}$ has a rigidly fixed location of its own. (6) In this sense one can say that the "many-fingered time" of each 3-geometry is specified by the very interlocking construction itself. Baierlein, Sharp and Wheeler (1962) say a little more on this concept of "3-geometry as carrier of information about time."

How different from the textbook concept of spacetime! There the geometry of spacetime is conceived as constructed out of elementary objects, or points, known as "events." Here, by contrast, the primary concept is 3-geometry, *in abstracto,* and out of it is derived the idea of event. Thus, (1) the event lies at the intersection of such and such $^{(3)}\mathscr{G}$'s; and (2) it has a timelike relation to (earlier or later than, or synchronous with) some other $^{(3)}\mathscr{G}$, which in turn (3) derives from the intercrossings of all the other $^{(3)}\mathscr{G}$'s.

When one turns from classical theory to quantum theory, one gives up the concept of spacetime, except in the semiclassical approximation. Therefore, one gives up any immediate possibility whatsoever of defining the concept, normally regarded as so elemental, of an "event." The theory itself, however, here as always [Bohr and Rosenfeld (1933)], defines in and by itself, in its own natural way, the procedures-in-principle for measuring all those quantities that are in principle measurable.

Quantum theory upsets the sharp distinction between YES 3-geometries and NO

3-geometries. It assigns to each 3-geometry not a YES or a NO, but a probability amplitude,

Probability amplitude for a 3-geometry

$$\psi = \psi({}^{(3)}\mathscr{G}). \tag{43.1}$$

This probability amplitude is highest near the classically forecast leaf of history and falls off steeply outside a zone of finite thickness extending a little way on either side of the leaf.

Were one to take, instead of a physically relevant probability amplitude function, a *typical* solution of the relevant wave equation, one would have to expect to see not one trace of anything like classical geometrodynamics. The typical probability amplitude function is spread all over superspace. No surprise! Already in classical theory one has to reckon with a Hamilton-Jacobi function,

$$S = S({}^{(3)}\mathscr{G}), \tag{43.2}$$

spread out over superspace. Moreover, this "dynamic phase function" of classical geometrodynamics gives at once the phase of ψ, according to the formula

$$\psi({}^{(3)}\mathscr{G}) = \begin{pmatrix}\text{slowly varying} \\ \text{amplitude function}\end{pmatrix} e^{(i/\hbar)S({}^{(3)}\mathscr{G})}, \tag{43.3}$$

indication enough that ψ and S are both unlocalized.

Dynamics first clearly becomes recognizable when sufficiently many such spread-out probability amplitude functions are superposed to build up a localized wave packet, as in the elementary examples of Boxes 25.3 and 25.4; thus,

Wave packet recovers classical geometrodynamics

$$\psi = c_1\psi_1 + c_2\psi_2 + \cdots. \tag{43.4}$$

Constructive interference occurs where the phases of the several individual waves agree:

$$S_1({}^{(3)}\mathscr{G}) = S_2({}^{(3)}\mathscr{G}) = \cdots. \tag{43.5}$$

This is the condition that distinguishes YES 3-geometries from NO 3-geometries. It is the tool for constructing a leaf of history in superspace. It is the key to the dynamics of geometry. Moreover, it is an equation that says not one word about the quantum principle. It is not surprising that the equation of constructive interference in (43.5) makes the leap from classical theory to quantum theory the shortest.

§43.3. THE EINSTEIN-HAMILTON-JACOBI EQUATION

Should one write down a differential equation for the Hamilton-Jacobi function $S({}^{(3)}\mathscr{G})$, solve it, and then analyze the properties of the solution? The exact opposite is simpler: look at the properties of the solution, and from that inspection find out what equation the dynamic phase or action S must satisfy.

Hilberts' principle of least action reads

$$I_{\text{Hilbert}} = (1/16\pi)\int {}^{(4)}R(-g)^{1/2}\, d^4x = \text{extremum}. \tag{43.6}$$

After one separates off complete derivatives in the integrand, what is left [see equations (21.13) and (21.95)] becomes

$$\begin{aligned}(1/16\pi)I_{\text{ADM}} = I_{\text{true}} = (1/16\pi)\int \Big\{&\pi^{ij}\,\partial g_{ij}/\partial t + Ng^{1/2}R \\ &+ Ng^{-1/2}\left[\frac{1}{2}(\operatorname{Tr}\boldsymbol{\pi})^2 - \operatorname{Tr}(\boldsymbol{\pi}^2)\right] + 2N_i\pi^{ij}{}_{|j}\Big\}\, d^4x.\end{aligned} \tag{43.7}$$

In (43.7), but not in (43.6), g stands for the determinant of the three-dimensional metric tensor, g_{ij}, and R for the scalar curvature invariant of the 3-geometry; the suffix $^{(3)}$ is omitted for simplicity. The integral is extended from (1) a spacelike hypersurface on which a 3-geometry is given with metric $g_{ij}'(x, y, z)$ to (2) a spacelike hypersurface on which a 3-geometry is given with metric $g_{ij}''(x, y, z)$. Whatever is adjustable in the chunk of spacetime between is now to be considered as having been adjusted to extremize the integral. Therefore the value of the integral I_{ADM} becomes a functional of the metrics on the two hypersurfaces and nothing more.

Next, holding fixed the metric $g'_{ij}(x, y, z)$ on the earlier hypersurface, change slightly or even more than slightly the metric on the later hypersurface. Solve the new variation problem and get a new value of I_{ADM}. Proceeding further in this way, for each new g_{ij}'' one gets a new value of I_{ADM}. Call the functional I_{ADM} of the metric defined in this way "Hamilton's principal function," or the "action" or the "dynamic path length,*" $S(g_{ij}(x, y, z))$ of the "history of geometry" that connects the two given 3-geometries. The double prime suffix is dropped from g_{ij}'' here and hereafter to simplify the notation. One knows from other branches of mechanics that the quantity defined in this way, $S(g_{ij})$, when it exists, even though it is a special solution, nevertheless is *always* a solution of the Hamilton-Jacobi equation. Jacobi could look for more general solutions, but Hamilton already had one!

For (43.7) to be an extremal with respect to variations of the lapse N and the shift components N_i, it was necessary (see Chapter 21) that the coefficients of these four quantities should vanish; thus,

$$g^{-1/2}\left[\frac{1}{2}(\operatorname{Tr}\boldsymbol{\pi})^2 - \operatorname{Tr}\boldsymbol{\pi}^2\right] + g^{1/2}R = 0 \tag{43.8}$$

and

$$\pi^{ij}{}_{|j} = 0. \tag{43.9}$$

In the expression for the extremal value of the action, only one term, the first, is left:

$$S(g(x, y, z)) = I_{\text{ADM, extremal}} = \int_{g'_{ij}}^{g_{ij}} \{\pi^{ij}\,\partial g_{ij}/\partial t\}\, d^4x. \tag{43.10}$$

*Actually $S \equiv S_{\text{ADM}} \equiv 16\pi S_{\text{true}} = 16\pi$ (true dynamic path length).

The effect of a slight change, δg_{ij}, in the 3-metric at the upper limit is therefore easy to read off:

$$\delta S = \int \pi^{ij}(x, y, z)\, \delta g_{ij}(x, y, z)\, d^3x. \tag{43.11}$$

The language of "functional derivative" [see, for example, Bogoliubov and Shirkov (1959)] allows one to speak in terms of a derivative rather than an integral:

Geometrodynamic momentum as rate of change of dynamic path length with respect to 3-geometry of terminal hypersurface

$$\frac{\delta S}{\delta g_{ij}} = \pi^{ij}. \tag{43.12}$$

The "field momenta" acquire a simple meaning: they give the rate of change of the action with respect to the continuous infinitude of "field coordinates," $g_{ij}(x, y, z)$. (Here the x, y, z, as well as the i and j, serve as mere labels.)

Action depends on 3-geometry, not on metric coefficients individually

Although the phase function S appears to depend on all six metric coefficients g_{ij} individually, it depends in actuality only on that combination of the g_{ij} which is locked to the 3-geometry. To verify this point, express a particular 3-geometry ${}^{(3)}\mathcal{G}$ throughout one local coordinate patch in terms of one set of coordinates x^p by one set of metric coefficients g_{pq}. Reexpress the same 3-geometry in terms of coordinates $\bar{x}^p$ shifted by the small amount ξ^p,

$$\bar{x}^p = x^p - \xi^p. \tag{43.13}$$

To keep the 3-geometry the same, that is, to keep unchanged the distance ds from one coordinate-independent point to another, the metric coefficients have to change:

$$\bar{g}_{pq} = g_{pq} + \xi_{p|q} + \xi_{q|p}. \tag{43.14}$$

Let the phase function S (or in quantum mechanics, let the probability amplitude ψ) be considered to be expressed as a functional of the metric coefficients $g_{11}(x)$, $g_{12}(x), \ldots, g_{33}(x)$. Changes $\delta g_{pq}(x)$ in these coefficients alter the H-J phase function and the probability amplitude by the amounts

$$\begin{aligned} \delta S &= \int (\delta S/\delta g_{pq})\, \delta g_{pq}\, d^3x, \\ \delta\psi &= \int (\delta\psi/\delta g_{pq})\, \delta g_{pq}\, d^3x, \end{aligned} \tag{43.15}$$

according to the standard definition of functional derivative. Therefore the coordinate change produces an ostensible change in the dynamic path length or phase S given by

$$\begin{aligned} \delta S &= \int (\delta S/\delta g_{pq})(\xi_{p|q} + \xi_{q|p})\, d^3x \\ &= -2 \int (\delta S/\delta g_{pq})_{|q}\xi_p\, d^3x. \end{aligned} \tag{43.16}$$

This change must vanish if S is to depend on the 3-geometry alone, and not on the coordinates in terms of which that 3-geometry is expressed; and must vanish, moreover, for arbitrary choice of the ξ_p. From this condition, one concludes

$$\left(\frac{\delta S}{\delta g_{pq}}\right)_{|q} = 0. \tag{43.17}$$

Likewise, one finds three equations on the wave function ψ itself, as distinguished from its phase $S/\hbar$; thus,

$$\left(\frac{\delta \psi}{\delta g_{pq}}\right)_{|q} = 0. \tag{43.18}$$

But (43.17), by virtue of (43.12), is identical with (43.9). In this sense (43.9) merely verifies what one already knew had to be true: the classical Hamilton-Jacobi function S (like the probability amplitude function ψ of quantum theory) depends on 3-geometry, not on individual metric coefficients, and not on choice of coordinates.

All the dynamic content of geometrodynamics is summarized in the sole remaining equation (43.8), which takes the form

Law of propagation of wave crests in superspace

$$g^{-1/2}\left[\frac{1}{2}g_{pq}g_{rs} - g_{pr}g_{qs}\right]\frac{\delta S}{\delta g_{pq}}\frac{\delta S}{\delta g_{rs}} + g^{1/2}R = 0. \tag{43.19}$$

This is the Einstein-Hamilton-Jacobi equation, first given explicitly in the literature by Peres (1962) on the foundation of earlier work by himself and others on the Hamiltonian formulation of geometrodynamics. This equation tells how fronts of constant S ("wave crests") propagate in superspace.

That the one EHJ equation (43.19) contains as much information as all ten components of Einstein's field equation has been demonstrated by Gerlach (1969). The central point in the analysis is the principle of constructive interference, and the main requirement for a proper treatment of this point is the concept of a completely parametrized solution of the EHJ equation.

The problem of a particle moving in two-dimensional space, as treated by the Hamilton-Jacobi method in Boxes 25.3 and 25.4, required for complete analysis a solution containing two distinct and independently adjustable parameters, the energy per unit mass, $\widetilde{E}$, and angular momentum per unit mass, $\widetilde{L}$; thus

$$\begin{aligned} S(r, \theta, t;\ \widetilde{E}, \widetilde{L}) &= -\widetilde{E}t + \widetilde{L}\theta \\ &+ \int^r [\widetilde{E}^2 - (1 - 2M/r)(1 + \widetilde{L}^2/r^2)]^{1/2}\frac{dr}{(1 - 2M/r)} + \delta(\widetilde{E}, \widetilde{L}). \end{aligned} \tag{43.20}$$

Here the additive phase $\delta(\widetilde{E}, \widetilde{L})$ is required if one is to be able to arrange for the particle to arrive at a given r-value at a specified t value and at a specified value of θ. One thinks of superposing four probability amplitudes, as in (43.4), with dynamic phases, S, given by (43.20) and the parameters taking on, respectively, the following four sets of values: $(\widetilde{E}, \widetilde{L})$; $(\widetilde{E} + \Delta\widetilde{E}, \widetilde{L})$; $(\widetilde{E}, \widetilde{L} + \Delta\widetilde{L})$; and $(\widetilde{E} + \Delta\widetilde{E}, \widetilde{L} + \Delta\widetilde{L})$. The principle of constructive interference leads to the conditions

$$\begin{aligned} \partial S/\partial \widetilde{E} &= 0, \\ \partial S/\partial \widetilde{L} &= 0. \end{aligned} \tag{43.21}$$

The points in the spacetime (r, θ, t) that satisfy these conditions are the YES points; they lie on the world line. The ones that don't are the NO points.

The desired solution of the EHJ equation (43.19) contains not two parameters (plus an additive phase, δ, depending on these two parameters), but an infinity of parameters, and even a continuous infinity of parameters. Thus the parameters are not to be designated as $\alpha_1, \alpha_2, \ldots; \beta_1, \beta_2, \ldots$ (parameters labeled by a discrete index), but as

$$\alpha(u, v, w)$$

and

$$\beta(u, v, w)$$

(two parameters "labeled" by three continuous indices u, v, w). Accidentally omit one of this infinitude of parameters? How could one ever hope to know that what purported to be a complete solution of the EHJ equation was not in actuality complete? Happily Gerlach provides a procedure to test the parameters for completeness.

Condition of constructive interference gives classical "leaf of history" or spacetime

Granted completeness, Gerlach goes on to show that the "leaf of history in superspace" or collection of 3-geometries that satisfy the conditions of constructive interference,

$$\begin{aligned} \frac{\delta S({}^{(3)}\mathcal{G};\ \alpha(u, v, w),\ \beta(u, v, w))}{\delta\alpha} &= 0, \\ \frac{\delta S({}^{(3)}\mathcal{G};\ \alpha(u, v, w),\ \beta(u, v, w))}{\delta\beta} &= 0, \end{aligned} \tag{43.22}$$

is identical with the leaf of history, or equivalent 4-geometry, given by the ten components of Einstein's geometrodynamic law.

From the Hamilton-Jacobi equation for a problem in elementary mechanics, it is a short step to the corresponding Schroedinger equation; similarly in geometrodynamics. No one has done more than Bryce DeWitt to explore the meaning and consequences of this "Einstein-Schroedinger equation" [DeWitt (1967a,b)]. One of the most interesting consequences is the existence of a conserved current in superspace, analogous to the conserved current

$$j_\mu = \frac{\hbar}{2im}\left(\psi^* \frac{\partial\psi}{\partial x^\mu} - \psi \frac{\partial\psi^*}{\partial x^\mu}\right)$$

that one encounters in the Klein-Gordon wave equation for a particle of spin zero.

It is an unhappy feature of this Einstein-Schroedinger wave equation that it contains second derivatives; so one has to specify both the probability amplitude, and the normal derivative of the probability amplitude, on the appropriate "superhypersurface" in superspace, in order to be able to predict the evolution of this state function elsewhere in superspace. One suggested way out of this situation—it is at least an inconvenience, possibly a real difficulty—has been proposed by Leutwyler (1968): impose a natural boundary condition that reduces the number of independent solutions from the number characteristic of a second-order equation to the number characteristic of a first-order equation. Another way out is to formulate the dynamics quite differently, in the way proposed by Kuchař (see Chapter 21), in which the resulting equation is of first order in the variable analogous to time.

The exploration of quantum geometrodynamics is simplified when one treats most of the degrees of freedom of the geometry as frozen out, by imposition of a high degree of symmetry. Then one is left with one, two, or three degrees of freedom (see Chapter 30, on mixmaster cosmology), or even infinitely many, and is led to manageable problems of quantum mechanics [Misner (1972a, 1973)].

§43.4. FLUCTUATIONS IN GEOMETRY

Of all the remarkable developments of physics since World War II, none is more impressive than the prediction and verification of the effects of the vacuum fluctuations in the electromagnetic field on the motion of the electron in the hydrogen atom (Figure 43.3). That development made it impossible to overlook the effects of such fluctuations throughout all physics and, not least, in the geometry of spacetime itself.

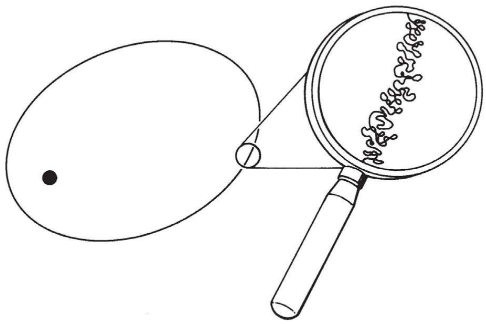


Figure 43.3.
Symbolic representation of motion of electron in hydrogen atom as affected by fluctuations in electric field in vacuum ("vacuum" or "ground state" or "zero-point" fluctuations). The electric field associated with the fluctuation, $E_x(t) = \int E_x(\omega)e^{-i\omega t}\, d\omega$, adds to the static electric field provided by the nucleus itself. The additional field brings about in the most elementary approximation the displacement $\Delta x = \int (e/m\omega^2)E_x(\omega)e^{-i\omega t}\, d\omega$. The average vanishes but the root mean square $\langle(\Delta x)^2\rangle$ does not. In consequence the electron feels an effective atomic potential altered from the expected value $V(x, y, z)$ by the amount

$$\Delta V(x, y, z) = \frac{1}{2}\langle(\Delta x)^2\rangle\, \nabla^2 V(x, y, z).$$

The average of this perturbation over the unperturbed motion accounts for the major part of the observed Lamb-Retherford shift $\Delta E = \langle \Delta V(x, y, z)\rangle$ in the energy level. Conversely, the observation of the expected shift makes the reality of the vacuum fluctuations inescapably evident.

From the zero-point fluctuations of a single oscillator to the fluctuations of the electromagnetic field to geometrodynamic fluctuations is a natural order of progression.

Fluctuations for oscillator and for electromagnetic field

A harmonic oscillator in its ground state has a probability amplitude of

$$\psi(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-(m\omega/2\hbar)x^2} \tag{43.23}$$

to be displaced by the distance x from its natural classical position of equilibrium. In this sense, it may be said to "resonate" or "fluctuate" between locations in space ranging over a region of extent

$$\Delta x \sim (\hbar/m\omega)^{1/2}. \tag{43.24}$$

The electromagnetic field can be treated as an infinite collection of independent "field oscillators," with amplitudes $\xi_1, \xi_2, \ldots$. When the Maxwell field is in its state of lowest energy, the probability amplitude—for the first oscillator to have amplitude ξ_1, and simultaneously the second oscillator to have amplitude ξ_2, the third ξ_3, and so on—is the product of functions of the form (43.23), one for each oscillator. When the scale of amplitudes for each oscillator is suitably normalized, the resulting infinite product takes the form

$$\psi(\xi_1, \xi_2, \ldots) = N \exp\left[-({\xi_1}^2 + {\xi_2}^2 + \cdots)\right]. \tag{43.25}$$

This expression gives the probability amplitude ψ for a configuration $\boldsymbol{B}(x, y, z)$ of the magnetic field that is described by the Fourier coefficients $\xi_1, \xi_2, \ldots$. One can forgo any mention of these Fourier coefficients if one desires, however, and rewrite (43.25) directly in terms of the magnetic field configuration itself [Wheeler (1962)]:

$$\psi(\boldsymbol{B}(x, y, z)) = \mathfrak{N} \exp\left(-\int\int \frac{\boldsymbol{B}(\boldsymbol{x}_1)\cdot\boldsymbol{B}(\boldsymbol{x}_2)}{16\pi^3\hbar c r_{12}^2}\, d^3x_1\, d^3x_2\right). \tag{43.26}$$

One no longer speaks of "the" magnetic field, but instead of the probability of this, that, or the other configuration of the magnetic field, even under circumstances, as here, where the electromagnetic field is in its ground state. [See Kuchař (1970) for a similar expression for the "ground state" functional of the linearized gravitational field.]

It is reasonable enough under these circumstances that the configuration of greatest probability is $\boldsymbol{B}(x, y, z) = 0$. Consider for comparison a configuration where the magnetic field is again everywhere zero except in a region of dimension L. There let the field, subject as always to the condition div $\boldsymbol{B} = 0$, be of the order of magnitude ΔB. The probability amplitude for this configuration will be reduced relative to the nil configuration by a factor exp $(-I)$. Here the quantity I in the exponent is of the order $(\Delta B)^2 L^4/\hbar c$. Configurations for which I is large compared to 1 occur with negligible probability. Configurations for which I is small compared to 1 occur with practically the same probability as the nil configuration. In this sense, one can

2

say that the fluctuations in the magnetic field in a region of extension L are of the order of magnitude

$$\Delta B \sim \frac{(\hbar c)^{1/2}}{L^2}. \tag{43.27}$$

In other words, the field "resonates" between one configuration and another with the range of configurations of significance given by (43.27). Moreover, the smaller is the region of space under consideration, the larger are the field magnitudes that occur with appreciable probability.

Still another familiar way of speaking about electromagnetic field fluctuations gives additional insight relevant to geometrodynamics. One considers a measuring device responsive in comparable measure to the magnetic field at all points in a region of dimension L. One asks for the effect on this device of electromagnetic disturbances of various wavelengths. A disturbance of wavelength short compared to L will cause forces to act one way in some parts of the detector, and will give rise to nearly compensating forces in other parts of it. In contrast, a disturbance of a long wavelength λ produces forces everywhere in the same direction, but of a magnitude too low to have much effect. Thus the field, estimated from the equation

$$\begin{pmatrix}\text{energy of electromagnetic}\\ \text{wave of wavelength } \lambda \text{ in a}\\ \text{domain of volume } \lambda^3\end{pmatrix} \sim \begin{pmatrix}\text{energy of one quantum}\\ \text{of wavelength } \lambda\end{pmatrix}$$

or

$$B^2\lambda^3 \sim \frac{\hbar c}{\lambda}$$

or

$$B \sim \frac{(\hbar c)^{1/2}}{\lambda^2} \tag{43.28}$$

is very small if λ is large compared to the domain size L. The biggest effect is caused by a disturbance of wavelength λ comparable to L itself. This line of reasoning leads directly from (43.28) to the standard fluctuation formula (43.27).

Fluctuations in geometry dominate at the Planck scale of distances

Similar considerations apply in geometrodynamics. Quantum fluctuations in the geometry are superposed on and coexist with the large-scale, slowly varying curvature predicted by classical deterministic general relativity. Thus, in a region of dimension L, where in a local Lorentz frame the normal values of the metric coefficients will be $-1, 1, 1, 1$, there will occur fluctuations in these coefficients of the order

$$\Delta g \sim \frac{L^*}{L}, \tag{43.29}$$

fluctuations in the first derivatives of the g_{ik}'s of the order

$$\Delta \Gamma \sim \frac{\Delta g}{L} \sim \frac{L^*}{L^2}, \tag{43.30}$$

and fluctuations in the curvature of space of the order

$$\Delta R \sim \frac{\Delta g}{L^2} \sim \frac{L^*}{L^3}. \tag{43.31}$$

Here

$$L^* = \left(\frac{\hbar G}{c^3}\right)^{1/2} = 1.6 \times 10^{-33}\ \text{cm} \tag{43.32}$$

is the so-called Planck length [Planck (1899)].

It is appropriate to look at orders of magnitude. The curvature of space within and near the earth, according to classical Einstein theory, is of the order

$$\begin{aligned} R &\sim \left(\frac{G}{c^2}\right)\rho \sim (0.7 \times 10^{-28}\ \text{cm/g})(5\ \text{g/cm}^3) \\ &\sim 4 \times 10^{-28}\ \text{cm}^{-2}. \end{aligned} \tag{43.33}$$

This quantity has a very direct physical significance. It measures the "tide-producing component of the gravitational field" as sensed, for example, in a freely falling elevator or in a spaceship in free orbit around the earth. By comparison, the quantum fluctuations in the curvature of space are only

$$\Delta R \sim 10^{-33}\ \text{cm}^{-2}, \tag{43.34}$$

even in a domain of observation as small as 1 cm in extent. Thus the quantum fluctuations in the geometry of space are completely negligible under everyday circumstances.

Even in atomic and nuclear physics the fluctuations in the metric,

$$\Delta g \sim \frac{10^{-33}\ \text{cm}}{10^{-8}\ \text{cm}} \sim 10^{-25}$$

and

$$\Delta g \sim \frac{10^{-33}\ \text{cm}}{10^{-13}\ \text{cm}} \sim 10^{-20}, \tag{43.35}$$

are so small that it is completely in order to idealize the physics as taking place in a flat Lorentzian spacetime manifold.

The quantum fluctuations in the geometry are nevertheless inescapable, if one is to believe the quantum principle and Einstein's theory. They coexist with the geometrodynamic development predicted by classical general relativity. The fluctuations widen the narrow swathe cut through superspace by the classical history of the geometry. In other words, the geometry is not deterministic, even though it looks so at the everyday scale of observation. Instead, at a submicroscopic scale it "resonates" between one configuration and another and another. This terminology means no more and no less than the following: (1) Each configuration $^{(3)}\mathfrak{G}$ has its own probability amplitude $\psi = \psi(^{(3)}\mathfrak{G})$. (2) These probability amplitudes have comparable magnitudes for a whole range of 3-geometries included within the limits (43.29) on either side of the classical swathe through superspace. (3) This range of 3-geometries is far too variegated on the submicroscopic sale to fit into any one 4-geometry, or any one classical geometrodynamic history. (4) Only when one overlooks these small-scale fluctuations ($\sim 10^{-33}$ cm) and examines the larger-scale features of the 3-geometries do they appear to fit into a single space-time manifold, such as comports with the classical field equations.

These small-scale fluctuations tell one that something like gravitational collapse is taking place everywhere in space and all the time; that gravitational collapse is in effect perpetually being done and undone; that in addition to the gravitational collapse of the universe, and of a star, one has also to deal with a third and, because it is constantly being undone, most significant level of gravitational collapse at the Planck scale of distances.

EXERCISES

Exercise 43.1. THE ACTION PRINCIPLE FOR A FREE PARTICLE IN NONRELATIVISTIC MECHANICS

Taking as action principle $I = \int L\, dt =$ extremum, with specified x', t' and x'', t'' at the two limits, and with $L = \frac{1}{2}m(dx/dt)^2$, find (1) the extremizing history $x = x(t)$ and (2) the dynamical path length or action $S(x'', t''; x', t') = I_{\text{extremum}}$ in its dependence on the end points. Also (3) write down the Hamilton-Jacobi equation for this problem, and (4) verify that $S(x, t; x', t')$ satisfies this equation. Finally, imagining the Hamilton-Jacobi equation not to be known, (5) derive it from the already known properties of the function S itself.

Exercise 43.2. THE ACTION FOR THE HARMONIC OSCILLATOR

The kinetic energy is $\frac{1}{2}m(dx/dt)^2$ and the potential energy is $\frac{1}{2}m\omega^2x^2$. Carry through the analysis of parts (1), (2), (3), (4) of the preceding exercise. Partial answer:

$$S = \frac{m\omega}{2}\,\frac{(x^2 + x'^2)\cos\omega(t - t') - 2xx'}{\sin\omega(t - t')}.$$

Verify that $\partial S/\partial x$ gives momentum and $-\partial S/\partial t$ gives energy.

Exercise 43.3. QUANTUM PROPAGATOR FOR HARMONIC OSCILLATOR

Show that the probability amplitude for a simple harmonic oscillator to transit from x', t' to x'', t'' is

$$\langle x'', t''; x', t'\rangle = \left(\frac{m\omega}{2\pi i\hbar\,\sin\omega(t'' - t')}\right)^{1/2} \times \exp\frac{im\omega[(x''^2 + x'^2)\cos\omega(t'' - t') - 2x''x']}{2\hbar\,\sin\omega(t'' - t')},$$

and that it reduces for the case of a free particle to

$$\langle x'', t''; x', t'\rangle = \left(\frac{m}{2\pi i\hbar(t'' - t')}\right)^{1/2} \exp\frac{im(x'' - x')^2}{2\hbar(t'' - t')}.$$

Note that one can derive all the harmonic-oscillator wave functions from the solution by use of the formula

$$\langle x'', t''; x', t'\rangle = \sum_n u_n(x'')u_n^*(x')\exp iE_n(t' - t'')/\hbar.$$

Exercise 43.4. QUANTUM PROPAGATOR FOR FREE ELECTROMAGNETIC FIELD

In flat spacetime, one is given on the spacelike hypersurface $t = t'$ the divergence-free magnetic field $B'(x, y, z)$ and on the spacelike hypersurface $t = t''$ the divergence-free magnetic field $B''(x, y, z)$. By Fourier analysis (reducing this problem to the preceding problem) or otherwise, find the probability amplitude to transit from B' at t' to B'' at t''.

Exercise 43.5. HAMILTON-JACOBI FORMULATION OF MAXWELL ELECTRODYNAMICS

Regard the four components A_μ of the electromagnetic 4-potential as the primary quantities; split them into a space part A_i and a scalar potential ϕ. (1) Derive from the action principle (in flat spacetime)

$$I = (1/8\pi) \int (E^2 - B^2)\, d^4x,$$

by splitting off an appropriate divergence, an expression qualitatively similar in character to (43.7). (2) Show that the appropriate quantity to be fixed on the initial and final spacelike hypersurface is not really A_i itself, but the magnetic field, defined by $\boldsymbol{B} = \operatorname{curl} \boldsymbol{A}$. (3) Derive the Hamilton-Jacobi equation for the dynamic phase or action $S(\boldsymbol{B}, \mathcal{S})$ in its dependence on the choice of hypersurface $\mathcal{S}$, and the choice of magnetic field $\boldsymbol{B}$ on this hypersurface,

$$-\frac{\delta S}{\delta \Omega} = \frac{1}{8\pi} \boldsymbol{B}^2 + \frac{(4\pi)^2}{8\pi} \left(\frac{\delta S}{\delta \boldsymbol{A}}\right)^2.$$

The quantity on the left is Tomonaga's "bubble time" derivative [Tomonaga (1946); see also Box 21.1].

CHAPTER 44

BEYOND THE END OF TIME

"Heaven wheels above you
Displaying to you her eternal glories
And still your eyes are on the ground"

DANTE

The world "stands before us as a great, eternal riddle"

EINSTEIN (1949a)

§44.1. GRAVITATIONAL COLLAPSE AS THE GREATEST CRISIS IN PHYSICS OF ALL TIME

This chapter is entirely Track 2. No previous Track-2 material is needed as preparation for it, but Chapter 43 will be helpful.

The universe starts with a big bang, expands to a maximum dimension, then recontracts and collapses: no more awe-inspiring prediction was ever made. It is preposterous. Einstein himself could not believe his own prediction. It took Hubble's observations to force him and the scientific community to abandon the concept of a universe that endures from everlasting to everlasting.

Later work of Tolman (1934a), Avez (1960), Geroch (1967), and Hawking and Penrose (1969) generalizes the conclusion. A model universe that is closed, that obeys Einstein's geometrodynamic law, and that contains a nowhere negative density of mass-energy, inevitably develops a singularity. No one sees any escape from the density of mass-energy rising without limit. A computing machine calculating ahead step by step the dynamical evolution of the geometry comes to the point where it can not go on. Smoke, figuratively speaking, starts to pour out of the computer. Yet physics surely continues to go on if for no other reason than this: Physics is by definition that which does go on its eternal way despite all the shadowy changes in the surface appearance of reality.

The Marchon lecture given by J. A. W. at the University of Newcastle-upon-Tyne, May 18, 1971, and the Nuffield lecture given at Cambridge University July 19, 1971, were based on the material presented in this chapter.

Some day a door will surely open and expose the glittering central mechanism of the world in its beauty and simplicity. Toward the arrival of that day, no development holds out more hope than the paradox of gravitational collapse. Why paradox? Because Einstein's equation says "this is the end" and physics says "there is no end." Why hope? Because among all paradigms for probing a puzzle, physics proffers none with more promise than a paradox.

The paradox of collapse: physics stops, but physics must go on

No previous period of physics brought a greater paradox than 1911 (Box 44.1). Rutherford had just been forced to conclude that matter is made up of localized positive and negative charges. Matter as so constituted should undergo electric collapse in $\sim 10^{-17}$ sec, according to theory. Observation equally clearly proclaimed that matter is stable. No one took the paradox more seriously than Bohr. No one worked around and around the central mystery with more energy wherever work was possible. No one brought to bear a more judicious combination of daring and conservativeness, nor a deeper feel for the harmony of physics. The direct opposite

The 1911 crisis of electric collapse

Box 44.1 COLLAPSE OF UNIVERSE PREDICTED BY CLASSICAL THEORY, COMPARED AND CONTRASTED WITH CLASSICALLY PREDICTED COLLAPSE OF ATOM

System	*Atom (1911)*	*Universe (1970's)*
Dynamic entity	System of electrons	Geometry of space
Nature of classically predicted collapse	Electron headed toward point-center of attraction is driven in a finite time to infinite energy	Not only matter but space itself arrives in a finite proper time at a condition of infinite compaction
One rejected "way out"	Give up Coulomb law of force	Give up Einstein's field equation
Another proposal for a "cheap way out" that has to be rejected	"Accelerated charge need not radiate"	"Matter cannot be compressed beyond a certain density by any pressure, however high
How this proposal violates principle of causality	Coulomb field of point-charge cannot readjust itself with infinite speed out to indefinitely great distances to sudden changes in velocity of charge	Speed of sound cannot exceed speed of light; pressure cannot exceed density of mass-energy
A major new consideration introduced by recognizing quantum principle as overarching organizing principle of physics	Uncertainty principle; binding too close to center of attraction makes zero-point kinetic energy outbalance potential energy; consequent existence of a lowest quantum state; can't radiate because no lower state available to drop to	Uncertainty principle; propagation of representative wave packet in superspace does not lead deterministically to a singular configuration for the geometry of space; expect rather a probability distribution of outcomes, each outcome describing a universe with a different size, a different set of particle masses, a different number of particles, and a different length of time required for its expansion and recontraction.

of harmony, cacophony, is the impression that comes from sampling the literature of the 'teens on the structure of the atom. (1) Change the Coulomb law of force between electric charges? (2) Give up the principle that an accelerated charge must radiate? There was little inhibition against this and still wilder abandonings of the well-established. In contrast, Bohr held fast to these two principles. At the same time he insisted on the importance of a third principle, firmly established by Planck in quite another domain of physics, the quantum principle. With that key he opened the door to the world of the atom.

Great as was the crisis of 1911, today gravitational collapse confronts physics with its greatest crisis ever. At issue is the fate, not of matter alone, but of the universe itself. The dynamics of collapse, or rather of its reverse, expansion, is evidenced not by theory alone, but also by observation; and not by one observation, but by observations many in number and carried out by astronomers of unsurpassed ability and integrity. Collapse, moreover, is not unique to the large-scale dynamics of the universe. A white dwarf star or a neutron star of more than critical mass is predicted to undergo gravitational collapse to a black hole (Chapters 32 and 33). Sufficiently many stars falling sufficiently close together at the center of the nucleus of a galaxy are expected to collapse to a black hole many powers of ten more massive than the sun. An active search is under way for evidence for a black hole in this Galaxy (Box 33.3). The process that makes a black hole is predicted to provide an experimental model for the gravitational collapse of the universe itself, with one difference. For collapse to a black hole, the observer has his choice whether (1) to observe from a safe distance, in which case his observations will reveal nothing of what goes on inside the horizon; or (2) to follow the falling matter on in, in which case he sees the final stages of the collapse, not only of the matter itself, but of the geometry surrounding the matter, to indefinitely high compaction, but only at the cost of his own early demise. For the gravitational collapse of a closed model universe, no such choice is available to the observer. His fate is sealed. So too is the fate of matter and elementary particles, driven up to indefinitely high densities. The stakes in the crisis of collapse are hard to match: the dynamics of the largest object, space, and the smallest object, an elementary particle—and how both began.

§44.2. ASSESSMENT OF THE THEORY THAT PREDICTS COLLAPSE

No one reflecting on the paradox of collapse ("collapse ends physics"; "collapse cannot end physics") can fail to ask, "What are the limits of validity of Einstein's geometric theory of gravity?" A similar question posed itself in the crisis of 1911. The Coulomb law for the force acting between two charges had been tested at distances of meters and millimeters, but what warrant was there to believe that it holds down to the 10^{-8} cm of atomic dimensions? Of course, in the end it proves to hold not only at the level of the atom, and at the 10^{-13} cm level of the nucleus, but even down to 5×10^{-15} cm [Barber, Gittelman, O'Neill, and Richter, and Bailey *et al.* (1968), as reviewed by Farley (1969) and Brodsky and Drell (1970)], a striking example of what Wigner (1960) calls the "unreasonable effectiveness of mathematics in the natural sciences."

No theory more resembles Maxwell's electrodynamics in its simplicity, beauty, and scope than Einstein's geometrodynamics. Few principles in physics are more firmly established than those on which it rests: the local validity of special relativity (Chapters 2–7), the equivalence principle (Chapter 16), the conservation of momentum and energy (Chapters 5, 15 and 16), and the prevalence of second-order field equations throughout physics (Chapter 17). Those principles and the demand for no "extraneous fields" (e.g., Dicke's scalar field) and "no prior geometry" (§17.6) lead to the conclusion that the geometry of spacetime must be Riemannian and the geometrodynamic law must be Einstein's.

Battle-tested theory of gravitation

To say that the geometry is Riemannian is to say that the interval between any two nearby events C and D, anywhere in spacetime, stated in terms of the interval AB between two nearby fiducial events, at quite another point in spacetime, has a value CD/AB independent of the route of intercomparison (Chapter 13 and Box 16.4). There are a thousand routes. By this hydraheaded prediction, Einstein's theory thus exposes itself to destruction in a thousand ways (Box 16.4).

Geometrodynamics lends itself to being disproven in other ways as well. The geometry has no option about the control it exerts on the dynamics of particles and fields (Chapter 20). The theory makes predictions about the equilibrium configurations and pulsations of compact stars (Chapters 23–26). It gives formulas (Chapters 27–29) for the deceleration of the expansion of the universe, for the density of mass-energy, and for the magnifying power of the curvature of space, the tests of which are not far off. It predicts gravitational collapse, and the existence of black holes, and a wealth of physics associated with these objects (Chapters 31–34). It predicts gravitational waves (Chapters 35–37). In the appropriate approximation, it encompasses all the well-tested predictions of the Newtonian theory of gravity for the dynamics of the solar system, and predicts testable post-Newtonian corrections besides, including several already verified effects (Chapters 38–40).

No inconsistency of principle has ever been found in Einstein's geometric theory of gravity. No purported observational evidence against the theory has ever stood the test of time. No other acceptable account of physics of comparable simplicity and scope has ever been put forward.

Continue this assessment of general relativity a little further before returning to the central issue, the limits of validity of the theory and their bearing on the issue of gravitational collapse. What has Einstein's geometrodynamics contributed to the understanding of physics?

New view of nature flowing from Einstein's geometrodynamics

First, it has dethroned spacetime from a post of preordained perfection high above the battles of matter and energy, and marked it as a new dynamic entity participating actively in this combat.

Second, by tying energy and momentum to the curvature of spacetime, Einstein's theory has recognized the law of conservation of momentum and energy as an automatic consequence of the geometric identity that the boundary of a boundary is zero (Chapters 15 and 17).

Third, it has recognized gravitation as a manifestation of the curvature of the geometry of spacetime rather than as something foreign and "physical" imbedded in spacetime.

Fourth, general relativity has reinforced the view that "physics is local"; that the analysis of physics becomes simple when it connects quantities at a given event with quantities at immediately adjacent events.

Fifth, obedient to the quantum principle, it recognizes that spacetime and time itself are ideas valid only at the classical level of approximation; that the proper arena for the Einstein dynamics of geometry is not spacetime, but superspace; and that this dynamics is described in accordance with the quantum principle by the propagation of a probability amplitude through superspace (Chapter 43). In consequence, the geometry of space is subject to quantum fluctuations in metric coefficients of the order

$$\delta g \sim \frac{(\text{Planck length, } L^* = (\hbar G/c^3)^{1/2} = 1.6 \times 10^{-33}\text{ cm})}{(\text{linear extension of region under study})}.$$

Electric charge as lines of force trapped in the topology of space

Sixth, standard Einstein geometrodynamics is partial as little to Euclidean topology as to Euclidean geometry. A multiply connected topology provides a natural description for electric charge as electric lines of force trapped in the topology of a multiply connected space (Figure 44.1). Any other description of electricity postulates a breakdown in Maxwell's field equations for the vacuum at a site where charge

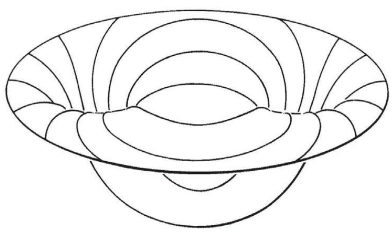


Figure 44.1.
Electric charge viewed as electric lines of force trapped in the topology of a multiply connected space [for the history of this concept see reference 36 of Wheeler (1968a)]. The wormhole or handle is envisaged as connecting two very different regions in the same space. One of the wormhole mouths, viewed by an observer with poor resolving power, appears to be the seat of an electric charge. Out of this region of 3-space he finds lines of force emerging over the whole 4π solid angle. He may construct a boundary around this charge, determine the flux through this boundary, incorrectly apply the theorem of Gauss and "prove" that there is a charge "inside the boundary." It isn't a boundary. Someone caught within it—to speak figuratively—can go into that mouth of the wormhole, through the throat, out the other mouth, and return by way of the surrounding space to look at his "prison" from the outside. Lines of force nowhere end. Maxwell's equations nowhere fail. Nowhere can one place a finger and say, "Here there is some charge." This classical type of electric charge has no direct relation whatsoever to quantized electric charge. There is a freedom of choice about the flux through the wormhole, and a specificity about the connection between one charge and another, which is quite foreign to the charges of elementary particle physics. For ease of visualization the number of space dimensions in the above diagram has been reduced from three to two. The third dimension, measured off the surface, has no physical meaning—it only provides an extra dimension in which to imbed the surface for more convenient diagrammatic representation. [For more detail see Misner and Wheeler (1957), reprinted in Wheeler (1962)].

is located, or postulates the existence of some foreign and "physical" electric jelly imbedded in space, or both. No one has ever found a way to describe electricity free of these unhappy features except to say that the quantum fluctuations in the geometry of space are so great at small distances that even the topology fluctuates, makes "wormholes," and traps lines of force. These fluctuations have to be viewed, not as tied to particles, and endowed with the scale of distances associated with particle physics ($\sim 10^{-13}$ cm) but as pervading all space ("foam-like structure of geometry") and characterized by the Planck distance ($\sim 10^{-33}$ cm). Thus a third type of gravitational collapse forces itself on one's attention, a collapse continually being done and being undone everywhere in space: surely a guide to the outcome of collapse at the level of a star and at the level of the universe (Box 44.2).

Box 44.2 THREE LEVELS OF GRAVITATIONAL COLLAPSE

1. Universe
2. Black hole
3. Fluctuations at the Planck scale of distances

Recontraction and collapse of the universe is a kind of mirror image of the "big bang," on which one already has so much evidence.

Collapse of matter to form a black hole is most natural at two distinct levels: (a) collapse of the dense white-dwarf core of an individual star (when that core exceeds the critical mass, $\sim 1 M_\odot$ or $2 M_\odot$, at which a neutron star is no longer a possible stable end-point for collapse) and (b) coalescence one by one of the stars in a galactic nucleus to make a black hole of mass up to $10^6 M_\odot$ or even $10^9 M_\odot$.

In either case, no feature of principle about matter falling into the black hole is more interesting than the option that the observer has (symbolized by the branching arrow in the inset). He can go along with the infalling matter, in which case he sees the final stages of collapse, but only at the cost of his own demise. Or he can stay safely outside, in which case even after indefinitely long time he sees only the first part of the collapse, with the infalling matter creeping up more and more slowly to the horizon.

In the final stages of the collapse of a closed model universe, all black holes present are caught up and driven together, amalgamating one by one. No one has any way to look at the event from safely outside; one is inevitably caught up in it oneself.

Collapse at the Planck scale of distances is taking place everywhere and all the time in quantum fluctuations in the geometry and, one believes, the topology of space. In this sense, collapse is continually being done and undone, modeling the undoing of the collapse of the universe itself, summarized in the term, "the reprocessing of the universe" (see text).

§44.3. VACUUM FLUCTUATIONS: THEIR PREVALENCE AND FINAL DOMINANCE

If Einstein's theory thus throws light on the rest of physics, the rest of physics also throws light on geometrodynamics. No point is more central than this, that empty space is not empty. It is the seat of the most violent physics. The electromagnetic field fluctuates (Chapter 43). Virtual pairs of positive and negative electrons, in effect, are continually being created and annihilated, and likewise pairs of mu mesons, pairs of baryons, and pairs of other particles. All these fluctuations coexist with the quantum fluctuations in the geometry and topology of space. Are they additional to those geometrodynamic zero-point disturbances, or are they, in some sense not now well-understood, mere manifestations of them?

Is matter built out of geometry?

Put the question in other words. Recall Clifford, inspired by Riemann, speaking to the Cambridge Philosophical Society on February 21, 1870, "On the Space Theory of Matter" [Clifford (1879), pp. 244 and 322; and (1882), p. 21], and saying, "I hold in fact (1) That small portions of space *are* in fact of a nature analogous to little hills on a surface which is on the average flat; namely, that the ordinary laws of geometry are not valid in them. (2) That this property of being curved or distorted is continually being passed on from one portion of space to another after the manner of a wave. (3) That this variation of the curvature of space is what really happens in that phenomenon which we call the *motion of matter,* whether ponderable or etherial. (4) That in the physical world nothing else takes place but this variation, subject (possibly) to the law of continuity." Ask if there is a sense in which one can speak of a particle as constructed out of geometry. Or rephrase the question in updated language: "Is a particle a geometrodynamic exciton?" What else is there out of which to build a particle except geometry itself? And what else is there to give discreteness to such an object except the quantum principle?

The Clifford-Einstein space theory of matter has not been forgotten in recent years. "In conclusion," one of the authors wrote a decade ago [Wheeler (1962)], "the vision of Riemann, Clifford, and Einstein, of a purely geometric basis for physics, today has come to a higher state of development, and offers richer prospects—and presents deeper problems—than ever before. The quantum of action adds to this geometrodynamics new features, of which the most striking is the presence of fluctuations of the wormhole type throughout all space. If there is any correspondence at all between this virtual foam-like structure and the physical vacuum as it has come to be known through quantum electrodynamics, then there seems to be no escape from identifying these wormholes with 'undressed electrons.' Completely different from these 'undressed electrons,' according to all available evidence, are the electrons and other particles of experimental physics. For these particles the geometrodynamic picture suggests the model of collective disturbances in a virtual foam-like vacuum, analogous to different kinds of phonons or excitons in a solid.

The richness of the physics of the vacuum

"The enormous factor from nuclear densities $\sim 10^{14}$ g/cm^3 to the density of field fluctuation energy in the vacuum, $\sim 10^{94}$ g/cm^3, argues that elementary particles represent a percentage-wise almost completely negligible change in the locally violent conditions that characterize the vacuum. ['A particle (10^{14} g/cm^3) means as little to the physics of the vacuum (10^{94} g/cm^3) as a cloud (10^{-6} g/cm^3) means to the physics of the sky (10^{-3} g/cm^3).'] In other words, elementary particles do not form a really basic starting point for the description of nature. Instead, they represent a first-order correction to vacuum physics. That vacuum, that zero-order state of affairs, with its enormous densities of virtual photons and virtual positive-negative pairs and virtual wormholes, has to be described properly before one has a fundamental starting point for a proper perturbation-theoretic analysis."

"These conclusions about the energy density of the vacuum, its complicated topological character, and the richness of the physics which goes on in the vacuum, stand in no evident contradiction with what quantum electrodynamics has to say about the vacuum. Instead the conclusions from the 'small distance' analysis (10^{-33} cm)—sketchy as it is—and from 'larger distance' analysis (10^{-11} cm) would seem to [be able] to reinforce each other in a most natural way.

"The most evident shortcoming of the geometrodynamic model as it stands is this, that *it fails to supply any completely natural place* for spin $\frac{1}{2}$ in general and *for the neutrino* in particular."

Attempts to find a natural place for spin $\frac{1}{2}$ in Einstein's standard geometrodynamics (Box 44.3) founder because there is no natural way for a *change* in connectivity to take place within the context of classical differential geometry.

A uranium nucleus undergoing fission starts with one topology and nevertheless ends up with another topology. It makes this transition in a perfectly continuous way, classical differential geometry notwithstanding.

No place in geometrodynamics for change of topology; therefore turn to "pregeometry"

There are reputed to be two kinds of lawyers. One tells the client what not to do. The other listens to what the client has to do and tells him how to do it. From the first lawyer, classical differential geometry, the client goes away disappointed, still searching for a natural way to describe quantum fluctuations in the connectivity of space. Only in this way can he hope to describe electric charge as lines of electric force trapped in the topology of space. Only in this way does he expect to be able to understand and analyze the final stages of gravitational collapse. Pondering his problems, he comes to the office of a second lawyer, with the name "Pregeometry" on the door. Full of hope, he knocks and enters. What is pregeometry to be and say? Born of a combination of hope and need, of philosophy and physics and mathematics and logic, pregeometry will tell a story unfinished at this writing, but full of incidents of evolution so far as it goes.

§44.4. NOT GEOMETRY, BUT PREGEOMETRY, AS THE MAGIC BUILDING MATERIAL

An early survey (Box 44.4) asked whether geometry can be constructed with the help of the quantum principle out of more basic elements, that do not themselves have any specific dimensionality.

The focus of attention in this 1964 discussion was "dimensionality without dimensionality." However, the prime pressures to ponder pregeometry were and remain

(continued on page 1206)

Box 44.3 THE DIFFICULTIES WITH ATTEMPTS TO FIND A NATURAL PLACE FOR SPIN $\frac{1}{2}$ IN EINSTEIN'S STANDARD GEOMETRODYNAMICS

"It is impossible" [Wheeler (1962)] "to accept any description of elementary particles that does not have a place for spin $\frac{1}{2}$. What, then, has any purely geometric description to offer in explanation of spin $\frac{1}{2}$ in general? More particularly and more importantly, what place is there in quantum geometrodynamics for the neutrino—the only entity of half-integral spin that is a pure field in its own right, in the sense that it has zero rest mass and moves with the speed of light? No clear or satisfactory answer is known to this question today. Unless and until an answer is forthcoming, *pure geometrodynamics must be judged deficient as a basis for elementary particle physics*."

A later publication [Wheeler (1968a)] takes up this issue again, noting that, "A new world opens out for analysis in quantum geometrodynamics. The central new concept is space resonating between one foamlike structure and another. For this multiple-connectedness of space at submicroscopic distances no single feature of nature speaks more powerfully than electric charge. Yet at least as impressive as charge is the prevalence of spin $\frac{1}{2}$ throughout the world of elementary particles."

Repeating the statement that "It is impossible to accept any description of elementary particles that does not have a place for spin $\frac{1}{2}$," the article adds to the discussion a new note: "Happily, the concept of spin manifold has come to light, not least through the work of John Milnor [see Lichnerowicz (1961a,b,c) and (1964); Milnor (1962), (1963), and (1965a,b); Hsiang and Anderson (1965); Anderson, Brown, and Peterson (1966a,b); and Penrose (1968a)]. This concept suggests a new and *interesting interpretation of a spinor field* within the context of the resonating microtopology of quantum geometrodynamics, *as the nonclassical two-valuedness* [Pauli's standard term for spin; see, for example, Pauli (1947)] *that attaches to the probability amplitude for otherwise identical 3-geometries endowed with alternative 'spin structures.'*" More specifically: "One does not classify the closed orientable 3-manifold of physics completely when one gives its topology, its differential structures, and its metric. One must tell which spin structure it has." [On a 3-geometry with the topology of a 3-sphere, one can lay down a continuous field of triads (a triad consisting of three orthonormal vectors). Any other continuous field of triads can be deformed into the first field by a continuous sequence of small readjustments. One says that the 3-sphere admits only one "spin structure," a potentially misleading standard word for what could just as well have been called a "triad structure." In contrast, a 3-sphere with n handles or wormholes admits 2^n "spin structures" (continuous fields of triads) inequivalent to one another under any continuous sequence of small readjustments whatsoever, and distinguished from one another in any convenient way by n "descriptors" $w_1, w_2, \ldots,$ $w_k, \ldots w_n$.] It is natural in quantum geometrodynamics to expect "separate probability amplitudes for a 3-geometry with descriptor $w_k = +1$ and for an otherwise identical 3-geometry with descriptor $w_k = -1$. Does this circumstance imply that quantum geometrodynamics supplies all the machinery one needs to describe fields of spin $\frac{1}{2}$ in general and the neutrino field in particular? . . . That is the only way that has ever turned up within the framework of Einstein's general relativity and Planck's quantum principle. Is this the right path? It is difficult to name any question more decisive than this in one's assessment of 'everything as geometry.'"

Why not spell out these concepts, reduce them to practice, and compare them with what one knows about the behavior of fields of spin $\frac{1}{2}$? There is a central difficulty in this enterprise. It assumes and demands on physical grounds that the topology of the 3-geometry shall be free to change from one connectivity to another. In contrast, classical differential geometry says, in effect, "Once one topology, always that topology." Try a question like this, "When a new handle develops and the number of descriptors rises by one, what boundary condition in superspace connects the probability amplitude ψ for 3-geometries of the original topology with the probability amplitudes ψ_+ and ψ_- for the two spin structures of the new topology?" Classical differential geometry not only gives one no help in answering this question; it even forbids one to ask it. In other words, one cannot even get the enterprise "on the road" for want of a natural mathematical way to describe the required change in topology. The idea is therefore abandoned here and now that 3-geometry is "the magic building material of the universe." In contrast, pregeometry (see text), far from being endowed with any frozen topology, is to be viewed as not even possessing any dimensionality.

Box 44.4 "A BUCKET OF DUST"—AN EARLY ATTEMPT TO FORMULATE THE CONCEPT OF PREGEOMETRY [Wheeler (1964a)]

"What line of thought could ever be imagined as *leading* to four dimensions—or any dimensionality at all—out of more primitive considerations? In the case of atoms one derives the yellow color of the sodium D-lines by analyzing the quantum dynamics of a system, no part of which is ever endowed with anything remotely resembling the attribute of color. Likewise any derivation of the four-dimensionality of spacetime can hardly *start* with the idea of dimensionality."

"Recall the notion of a Borel set. Loosely speaking, a Borel set is a collection of points ("bucket of dust") which have not yet been assembled into a manifold of any particular dimensionality. . . . Recalling the universal sway of the quantum principle, one can imagine probability amplitudes for the points in a Borel set to be assembled into points with this, that, and the other dimensionality. . . . More conditions have to be imposed on a given number of points—as to which has which for a nearest neighbor—when the points are put together in a five-dimensional array than when these same points are arranged in a two-dimensional pattern. Thus one can think of each dimensionality as having a much higher statistical weight than the next *higher* dimensionality. On the other hand, for manifolds with one, two, and three dimensions, the geometry is too rudimentary—one can suppose—to give anything interesting. Thus Einstein's field equations, applied to a manifold of dimensionality so low, demand flat space; only when the dimensionality is as high as four do really interesting possibilities arise. Can four, therefore, be considered to be the unique dimensionality which is at the same time high enough to give any real physics and yet low enough to have great statistical weight?

"It is too much to imagine that one has yet made enough mistakes in this domain of thought to explore such ideas with any degree of good judgment."

Consider a handle on the geometry. Let it thin halfway along its length to a point. In other words, let the handle dissolve into two bent prongs that touch at a point. Let these prongs separate and shorten. In this process two points part company that were once immediate neighbors. "However sudden the change is in classical theory, in quantum theory there is a probability amplitude function which falls off in the classically forbidden domain. In other words, there is some residual connection between points which are ostensibly very far away (travel from one 'tip' down one prong, then through the larger space to which these prongs are attached, and then up the other prong to the other tip). But there is nothing distinctive in principle about the two points that have happened to come into discussion. Thus it would seem that there must be a connection . . . between *every* point and every other point. Under these conditions the concept of nearest neighbor would appear no longer to make sense. Thus the tool disappears with the help of which one might otherwise try to speak [un]ambiguously about dimensionality."

two features of nature, spin $\frac{1}{2}$ and charge, that speak out powerfully from every part of elementary particle physics.

A fresh perspective on pregeometry comes from a fresh assessment of general relativity. "Geometrodynamics is neither as important or as simple as it looks. Do not make it the point of departure in searching for underlying simplicity. Look deeper, at elementary particle physics." This is the tenor of interesting new considerations put forward by Sakharov [*the* Sakharov] (1967) and summarized under the heading, "Gravitation as the metric elasticity of space," in Box 17.2. In brief, as elasticity is to atomic physics, so—in Sakharov's view—gravitation is to elementary particle physics. The energy of an elastic deformation is nothing but energy put into the bonds between atom and atom by the deformation. The energy that it takes to curve space is nothing but perturbation in the vacuum energy of fields plus particles brought about by that curvature, according to Sakharov. The energy required for the deformation is governed in the one case by two elastic constants and in the other case by one elastic constant (the Newtonian constant of gravity) but in both cases, he reasons, the constants arise by combination of a multitude of complicated individual effects, not by a brave clean stroke on an empty slate.

Sakharov: gravitation is the "metric elasticity of space"

One gives all the more favorable reception to Sakhorov's view of gravity because one knows today, as one did not in 1915, how opulent in physics the vacuum is. In Einstein's day one had come in a single decade from the ideal God-given Lorentz perfection of flat spacetime to curved spacetime. It took courage to assign even one physical constant to that world of geometry that had always stood so far above physics. The vacuum looked for long as innocent of structure as a sheet of glass emerging from a rolling mill. With the discovery of the positive electron [Anderson (1933)], one came to recognize a little of the life that heat can unfreeze in "empty" space. Each new particle and radiation that was discovered brought a new accretion to the recognized richness of the vacuum. Macadam looks smooth, but a bulldozer has only to cut a single furrow through the roadway to disclose all the complications beneath the surface.

The stratification of space

Think of a particle as built out of the geometry of space; think of a particle as a "geometrodynamic exciton"? No model—it would seem to follow from Sakharov's assessment—could be less in harmony with nature, except to think of an atom as built out of elasticity! Elasticity did not explain atoms. Atoms explained elasticity. If, likewise, particles fix the constant in Einstein's geometrodynamic law (Sakharov), must it not be unreasonable to think of the geometrodynamic law as explaining particles?

Comparison with everyday elasticity

Carry the comparison between geometry and elasticity one stage deeper (Fig. 44.2). In a mixed solid there are hundreds of distinct bonds, all of which contribute to the elastic constants; some of them arise from Van der Waal's forces, some from ionic coupling, some from homopolar linkage; they have the greatest variety of strengths; but all have their origin in something so fantastically simple as a system of positively and negatively charged masses moving in accordance with the laws of quantum mechanics. In no way was it required or right to meet each complication of the chemistry and physics of a myriad of bonds with a corresponding complication of principle. By going to a level of analysis deeper than bond strengths, one had

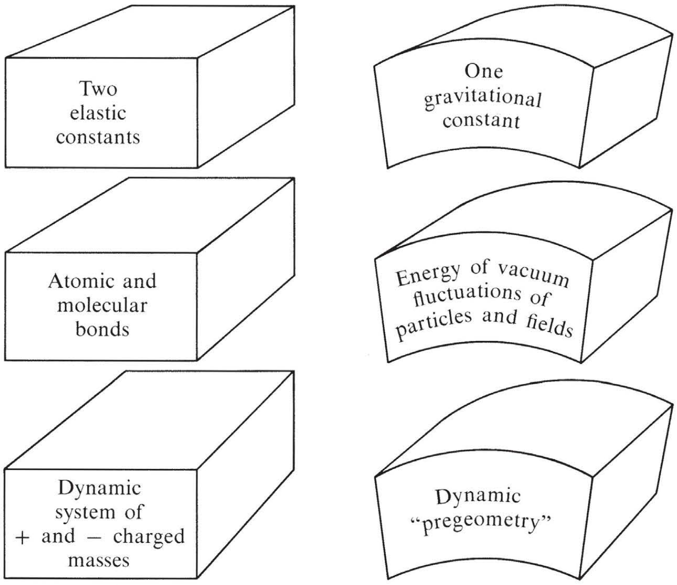


Figure 44.2.
Elasticity and geometrodynamics, as viewed at three levels of analysis. A hundred years of the study of elasticity did not reveal the existence of molecules, and a hundred years of the study of molecular chemistry did not reveal Schrödinger's equation. Revelation moved upward in the diagram, not downward.

emerged into a world of light, where nothing but simplicity and unity was to be seen.

Compare with geometry. The vacuum is animated with the zero-point activity of distinct fields and scores of distinct particles, all of which, according to Sakharov, contribute to the Newtonian G, the "elastic constant of the metric." Some interact via weak forces, some by way of electromagnetic forces, and some through strong forces. These interactions have the greatest variety of strengths. But must not all these particles and interactions have their origin in something fantastically simple? And must not this something, this "pregeometry," be as far removed from geometry as the quantum mechanics of electrons is far removed from elasticity?

If one once thought of general relativity as a guide to the discovery of pregeometry, nothing might seem more dismaying than this comparison with an older realm of physics. No one would dream of studying the laws of elasticity to uncover the principles of quantum mechanics. Neither would anyone investigate the work-hardening of a metal to learn about atomic physics. The order of understanding ran not

$$\text{Work-hardening (1 cm)} \longrightarrow \text{dislocations } (10^{-4}\text{ cm}) \longrightarrow \text{atoms } (10^{-8}\text{ cm}),$$

but the direct opposite,

$$\text{Atoms } (10^{-8}\text{ cm}) \longrightarrow \text{dislocations } (10^{-4}\text{ cm}) \longrightarrow \text{work-hardening (1 cm)}$$

One had to know about atoms to conceive of dislocations, and had to know about dislocations to understand work-hardening. Is it not likewise hopeless to go from the "elasticity of geometry" to an understanding of particle physics, and from particle physics to the uncovering of pregeometry? Must not the order of progress again be the direct opposite? And is not the source of any dismay the apparent loss of guidance that one experiences in giving up geometrodynamics—and not only geometrodynamics, but geometry itself—as a crutch to lean on as one hobbles forward? Yet there is so much chance that this view of nature is right that one must take it seriously and explore its consequences. Never more than today does one have the incentive to explore pregeometry.

§44.5. PREGEOMETRY AS THE CALCULUS OF PROPOSITIONS

Paper in white the floor of the room, and rule it off in one-foot squares. Down on one's hands and knees, write in the first square a set of equations conceived as able to govern the physics of the universe. Think more overnight. Next day put a better set of equations into square two. Invite one's most respected colleagues to contribute to other squares. At the end of these labors, one has worked oneself out into the door way. Stand up, look back on all those equations, some perhaps more hopeful than others, raise one's finger commandingly, and give the order "Fly!" Not one of those equations will put on wings, take off, or fly. Yet the universe "flies."

Search for the central principle of pregeometry

Some principle uniquely right and uniquely simple must, when one knows it, be also so compelling that it is clear the universe is built, and must be built, in such and such a way, and that it could not possibly be otherwise. But how can one discover that principle? If it was hopeless to learn atomic physics by studying work-hardening and dislocations, it may be equally hopeless to learn the basic operating principle of the universe, call it pregeometry or call it what one will, by any amount of work in general relativity and particle physics.

Thomas Mann (1937), in his essay on Freud, utters what Niels Bohr would surely have called a great truth ("A great truth is a truth whose opposite is also a great truth") when he says, "Science never makes an advance until philosophy authorizes and encourages it to do so." If the equivalence principle (Chapter 16) and Mach's principle (§21.9) were the philosophical godfathers of general relativity, it is also true that what those principles do mean, and ought to mean, only becomes clear by study and restudy of Einstein's theory itself. Therefore it would seem reasonable to expect the primary guidance in the search for pregeometry to come from a principle both philosophical and powerful, but one also perhaps not destined to be wholly clear in its contents or its implications until some later day.

Among all the principles that one can name out of the world of science, it is difficult to think of one more compelling than *simplicity;* and among all the simplicities of dynamics and life and movement, none is starker [Werner (1969)] than the *binary choice* yes–no or true–false. It in no way proves that this choice for a starting principle is correct, but it at least gives one some comfort in the choice, that Pauli's "nonclassical two-valuedness" or "spin" so dominates the world of particle physics.

It is one thing to have a start, a tentative construction of pregeometry; but how does one go on? How not to go on is illustrated by Figure 44.3. The "sewing machine" builds objects of one or another definite dimensionality, or of mixed dimensionalities, according to the instructions that it receives on the input tape in yes–no binary code. Some of the difficulties of building up structure on the binary element according to this model, or any one of a dozen other models, stand out at once. (1) Why $N = 10{,}000$ building units? Why not a different N? And if one feeds in one such arbitrary number at the start, why not fix more features "by hand?" No natural stopping point is evident, nor any principle that would fix such a stopping point. Such arbitrariness contradicts the principle of simplicity and rules out the model. (2) Quantum mechanics is added from outside, not generated from inside (from the model itself). On this point too the principle of simplicity speaks against the model. (3) The passage from pregeometry to geometry is made in a too-literal-minded way, with no appreciation of the need for particles and fields to appear along the way. The model, in the words used by Bohr on another occasion, is "crazy, but not crazy enough to be right."

A first try at a pregeometry built on the principle of binary choice

Noting these difficulties, and fruitlessly trying model after model of pregeometry to see if it might be free of them, one suddenly realizes that a machinery for the combination of yes–no or true–false elements does not have to be invented. It already exists. What else can pregeometry be, one asks oneself, than the calculus of propositions? (Box 44.5.)

A more reasonable picture: pregeometry is the calculus of propositions

§44.6. THE BLACK BOX: THE REPROCESSING OF THE UNIVERSE

No amount of searching has ever disclosed a "cheap way" out of gravitational collapse, any more than earlier it revealed a cheap way out of the collapse of the atom. Physicists in that earlier crisis found themselves in the end confronted with a revolutionary pistol, "Understand nothing—or accept the quantum principle." Today's crisis can hardly force a lesser revolution. One sees no alternative except to say that geometry fails and pregeometry has to take its place to ferry physics through the final stages of gravitational collapse and on into what happens next. No guide is evident on this uncharted way except the principle of simplicity, applied to drastic lengths.

Whether the whole universe is squeezed down to the Planck dimension, or more or less, before reexpansion can begin and dynamics can return to normal, may be irrelevant for some of the questions one wants to consider. Physics has long used the "black box" to symbolize situations where one wishes to concentrate on what goes in and what goes out, disregarding what takes place in between.

The role of the black box in physics

At the beginning of the crisis of electric collapse one conceived of the electron as headed on a deterministic path toward a point-center of attraction, and unhappily destined to arrive at a condition of infinite kinetic energy in a finite time. After the advent of quantum mechanics, one learned to summarize the interaction between

(continued on page 1213)

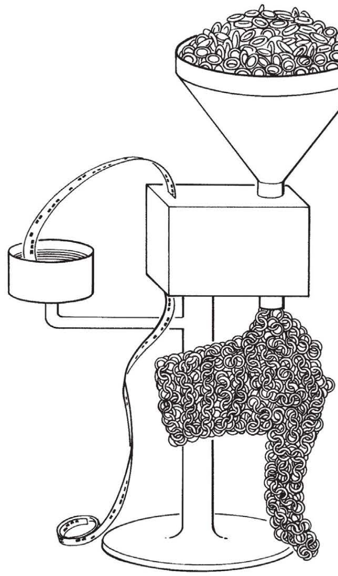


Figure 44.3.
"Ten thousand rings"; or an example of a way to think of the connection between pregeometry and geometry, wrong because it is too literal-minded, and for other reasons spelled out in the text. The vizier [story by Wheeler, as alluded to by Kilmister (1971)*] speaks: "Take $N = 10{,}000$ brass rings. Take an automatic fastening device that will cut open a ring, loop it through another ring, and resolder the joint. Pour the brass rings into the hopper that feeds this machine. Take a strip of instruction paper that is long enough to contain $N(N-1)/2$ binary digits. Look at the instruction in the (jk)-th location on this instruction tape ($j,k = 1, 2, \ldots, N$; $j < k$). When the binary digit at that location is 0, it is a signal to leave the j-th ring disconnected from the k-th ring. When it is 1, it is an instruction to connect that particular pair of rings. Thread the tape into the machine and press the start button. The clatter begins. Out comes a chain of rings 10,000 links long. It falls on the table and the machine stops. Pour in another 10,000 rings, feed in a new instruction tape, and push the button again. This time it is not a one-dimensional structure that emerges, but a two-dimensional one: a Crusader's coat of mail, complete with neck opening and sleeves. Take still another tape from the library of tapes and repeat. Onto the table thuds a smaller version of the suit of mail, this time filled out internally with a solid network of rings, a three-dimensional structure. Now forego the library and make one's own instruction tape, a random string of 0's and 1's. Guided by it, the fastener produces a "Christmas tree ornament," a collection of segments of one-dimensional chain, two-dimensional surfaces, and three-, four-, five-, and higher-dimensional entities, some joined together, some free-floating. Now turn from a structure deterministically fixed by a tape to a probability amplitude, a complex number,

$$\psi(\text{tape}) = \psi(n_{12}, n_{13}, n_{14}, \ldots\ldots, n_{N-1,N}) \qquad (n_{ij} = 0, 1), \tag{1}$$

defined over the entire range of possibilities for structures built of 10,000 rings. Let these probability amplitudes *not* be assigned randomly. Instead, couple together amplitudes, for structures that differ from each other by the breaking of a single ring, by linear formulas that treat all rings on the same footing. The separate ψ's, no longer entirely independent, will still give non-zero probability amplitudes for "Christmas tree ornaments." Of greater immediate interest than these "unruly" parts of the structures are the following questions about the smoother parts: (1) In what kinds of structures is the bulk of the probability concentrated? (2) What is the dominant dimensionality of these structures in an appropriate correspondence principle limit? (3) In this semiclassical limit, what is the form taken by the dynamic law of evolution of the geometry?" No principle more clearly rules out this model for pregeometry than the principle of simplicity (see text).

*Wheeler's story about the vizier and what the vizier had to say about superspace was told at the May 18, 1970, Gwatt Seminar on the Bearings of Topology upon General Relativity. Kilmister's (1971) published article alludes to the unpublished story, but does not actually contain it.

Box 44.5 "PREGEOMETRY AS THE CALCULUS OF PROPOSITIONS"

A sample proposition taken out of a standard text on logic selected almost at random reads [Kneebone (1963), p. 40]

$$[X \longrightarrow ((X \longrightarrow X) \longrightarrow Y)] \,\&\, (\bar{X} \longrightarrow Z) \;\mathrm{eq}\; (\bar{X} \vee Y \vee Z) \,\&\, (\bar{X} \vee Y \vee \bar{Z}) \,\&\, (X \vee Y \vee Z) \,\&\, (X \vee \bar{Y} \vee Z).$$

The symbols have the following meaning:

$\bar{A}$,	Not A;
$A \vee B$,	A or B or both ("A *vel* B");
$A \,\&\, B$,	A and B;
$A \longrightarrow B$,	A implies B ("if A, then B");
$A \longleftrightarrow B$,	B is equivalent to A ("B if and only if A").

Propositional formula $\mathfrak{A}$ is said to be equivalent ("eq") to propositional formula $\mathfrak{B}$ if and only if $\mathfrak{A} \longleftrightarrow \mathfrak{B}$ is a tautology. The letters A, B, etc., serve as connectors to "wire together" one proposition with another. Proceeding in this way, one can construct propositions of indefinitely great length.

A switching circuit [see, for example, Shannon (1938) or Hohn (1966)] is isomorphic to a proposition.

Compare a short proposition or an elementary switching circuit to a molecular collision. No idea seemed more preposterous than that of Daniel Bernoulli (1733), that heat is a manifestation of molecular collisions. Moreover, a three-body encounter is difficult to treat, a four-body collision is more difficult, and a five- or more molecule system is essentially intractable. Nevertheless, mechanics acquires new elements of simplicity in the limit in which the number of molecules is very great and in which one can use the concept of density in phase space. Out of statistical mechanics in this limit come such concepts as temperature and entropy. When the temperature is well-defined, the energy of the system is not a well-defined idea; and when the energy is well-defined, the temperature is not. This complementarity is built inescapably into the principles of the subject. Thrust the finger into the flame of a match and experience a sensation like nothing else on heaven or earth; yet what happens is all a consequence of molecular collisions, early critics notwithstanding.

Any individual proposition is difficult for the mind to apprehend when it is long; and still more difficult to grasp is the content of a cluster of propositions. Nevertheless, make a statistical analysis of the calculus of propositions in the limit where the number of propositions is great and most of them are long. Ask if parameters force themselves on one's attention in this analysis (1) analogous in some small measure to the temperature and entropy of statistical mechanics but (2) so much

Box 44.5 (*continued*)

more numerous, and everyday dynamic in character, that they reproduce the continuum of everyday physics.

Nothing could seem so preposterous at first glance as the thought that nature is built on a foundation as ethereal as the calculus of propositions. Yet, beyond the push to look in this direction provided by the principle of simplicity, there are two pulls. First, bare-bones quantum mechanics lends itself in a marvelously natural way to formulation in the language of the calculus of propositions, as witnesses not least the book of Jauch (1968). If the quantum principle were not in this way already automatically contained in one's proposed model for pregeometry, and if in contrast it had to be introduced from outside, by that very token one would conclude that the model violated the principle of simplicity, and would have to reject it. Second, the pursuit of reality seems always to take one away from reality. Who would have imagined describing something so much a part of the here and now as gravitation in terms of curvature of the geometry of spacetime? And when later this geometry came to be recognized as dynamic, who would have dreamed that geometrodynamics unfolds in an arena so ethereal as superspace? Little astonishment there should be, therefore, if the description of nature carries one in the end to logic, the ethereal eyrie at the center of mathematics. If, as one believes, all mathematics reduces to the mathematics of logic, and all physics reduces to mathematics, what alternative is there but for all physics to reduce to the mathematics of logic? Logic is the only branch of mathematics that can "think about itself."

"An issue of logic having nothing to do with physics" was the assessment by many of a controversy of old about the axiom, "parallel lines never meet." Does it follow from the other axioms of Euclidean geometry or is it independent? "Independent," Bolyai and Lobachevsky proved. With this and the work of Gauss as a start, Riemann went on to create Riemannian geometry. Study nature, not Euclid, to find out about geometry, he advised; and Einstein went on to take that advice and to make geometry a part of physics.

"An issue of logic having nothing to do with physics" is one's natural first assessment of the startling limitations on logic discovered by Gödel (1931), Cohen (1966), and others [for a review, see, for example, Kac and Ulam (1968)]. The exact opposite must be one's assessment if the real pregeometry of the real physical world indeed turns out to be identical with the calculus of propositions.

"Physics as manifestation of logic" or "pregeometry as the calculus of propositions" is as yet [Wheeler (1971a)] not an idea, but an idea for an idea. It is put forward here only to make it a little clearer what it means to suggest that the order of progress may not be

$$\text{physics} \longrightarrow \text{pregeometry}$$

but

$$\text{pregeometry} \longrightarrow \text{physics}.$$

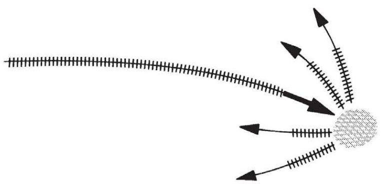


Figure 44.4.
The "black-box model" applied (1) to the scattering of an electron by a center of attraction and (2) to the collapse of the universe itself. The deterministic electron world line of classical theory is replaced in quantum theory by a probability amplitude, the wave crests of which are illustrated schematically in the diagram. The catastrophe of classical theory is replaced in quantum theory by a probability distribution of outputs. The same diagram illustrates the "black-box account" of gravitational collapse mentioned in the text. The arena of the diagram is no longer spacetime, but superspace. The incident arrow marks no longer a classical world line of an electron through spacetime, but a classical "leaf of history of geometry" slicing through superspace (Chapter 43). The wave crests symbolize no longer the electron wave function propagating through spacetime, but the geometrodynamic wave function propagating through superspace. The cross-hatched region is no longer the region where the one-body potential goes to infinity, but the region of gravitational collapse where the curvature of space goes to infinity. The outgoing waves describe no longer alternative directions for the new course of the scattered electron, but the beginnings of alternative new histories for the universe itself after collapse and "reprocessing" end the present cycle.

center of attraction and electron in a "black box:" fire in a wave-train of electrons traveling in one direction, and get electrons coming out in this, that, and the other direction with this, that, and the other well-determined probability amplitude (Figure 44.4). Moreover, to predict these probability amplitudes quantitatively and correctly, it was enough to translate the Hamiltonian of classical theory into the language of wave mechanics and solve the resulting wave equation, the key to the "black box."

Probability distribution of the outcomes of collapse

A similar "black box" view of gravitational collapse leads one to expect a "probability distribution of outcomes." Here, however, one outcome is distinguished from another, one must anticipate, not by a single parameter, such as the angle of scattering of the electron, but by many. They govern, one foresees, such quantities as the size of the system at its maximum of expansion, the time from the start of this new cycle to the moment it ends in collapse, the number of particles present, and a thousand other features. The "probabilities" of these outcomes will be governed by a dynamic law, analogous to (1) the Schrödinger wave equation for the electron, or, to cite another black box problem, (2) the Maxwell equations that couple together, at a wave-guide junction, electromagnetic waves running in otherwise separate wave guides. However, it is hardly reasonable to expect the necessary dynamic law to spring forth as soon as one translates the Hamilton-Jacobi equation of general relativity (Chapter 43) into a Schrödinger equation, simply because geometrodynamics, in both its classical and its quantum version, is built on standard differential geometry. That standard geometry leaves no room for any of those quantum fluctuations in connectivity that seem inescapable at small distances and therefore also inescapable in the final stages of gravitational collapse. Not geometry, but pregeometry, must fill the black box of gravitational collapse.

"Reprocessing" the universe

Little as one knows the internal machinery of the black box, one sees no escape from this picture of what goes on: the universe transforms, or transmutes, or transits, or is *reprocessed* probabilistically from one cycle of history to another in the era of collapse.

However straightforwardly and inescapably this picture of the reprocessing of the universe would seem to follow from the leading features of general relativity and the quantum principle, the two overarching principles of twentieth-century physics, it is nevertheless fantastic to contemplate. How can the dynamics of a system so incredibly gigantic be switched, and switched at the whim of probability, from one cycle that has lasted 10^{11} years to one that will last only 10^6 years? At first, only the circumstance that the system gets squeezed down in the course of this dynamics to incredibly small distances reconciles one to a transformation otherwise so unbelievable. Then one looks at the upended strata of a mountain slope, or a bird not seen before, and marvels that the whole universe is incredible:

> mutation of a species,
> metamorphosis of a rock,
> chemical transformation,
> spontaneous transformation of a nucleus,
> radioactive decay of a particle,
> reprocessing of the universe itself.

If it cast a new light on geology to know that rocks can be raised and lowered thousands of meters and hundreds of degrees, what does it mean for physics to think of the universe as being from time to time "squeezed through a knothole," drastically "reprocessed," and started out on a fresh dynamic cycle? Three considerations above all press themselves on one's attention, prefigured in these compressed phrases:

> destruction of all constants of motion in collapse;
> particles, and the physical "constants" themselves, as the
> "frozen-in part of the meteorology of collapse;"
> "the biological selection of physical constants."

All conservation laws transcended in the collapse of the universe

The gravitational collapse of a star, or a collection of stars, to a black hole extinguishes all details of the system (see Chapters 32 and 33) except mass and charge and angular momentum. Whether made of matter or antimatter or radiation, whether endowed with much entropy or little entropy, whether in smooth motion or chaotic turbulence, the collapsing system ends up as seen from outside, according to all indications, in the same standard state. The laws of conservation of baryon number and lepton number are transcended [Chapter 33; also Wheeler (1971b)]. No known means whatsoever will distinguish between black holes of the most different provenance if only they have the same mass, charge, and angular momentum. But for a closed universe, even these constants vanish from the scene. Total charge is automatically zero because lines of force have nowhere to end except upon charge. Total mass and total angular momentum have absolutely no definable meaning whatsoever for a closed universe. This conclusion follows not least because there is no asymptotically flat space outside where one can put a test particle into Keplerian orbit to determine period and precession.

Of all principles of physics, the laws of conservation of charge, lepton number, baryon number, mass, and angular momentum are among the most firmly established. Yet with gravitational collapse the content of these conservation laws also collapses. The established is disestablished. No determinant of motion does one see left that could continue unchanged in value from cycle to cycle of the universe. Moreover, if particles are dynamic in construction, and if the spectrum of particle masses is therefore dynamic in origin, no option would seem left except to conclude that the mass spectrum is itself reprocessed at the time when "the universe is squeezed through a knot hole." A molecule in this piece of paper is a "fossil" from photochemical synthesis in a tree a few years ago. A nucleus of the oxygen in this air is a fossil from thermonuclear combustion at a much higher temperature in a star a few 10^9 years ago. What else can a particle be but a fossil from the most violent event of all, gravitational collapse?

Three hierarchies of fossils: molecules, nuclei, particles

That one geological stratum has one many-miles long slope, with marvelous linearity of structure, and another stratum has another slope, is either an everyday triteness, taken as for granted by every passerby, or a miracle, until one understands the mechanism. That an electron here has the same mass as an electron there is also a triviality or a miracle. It is a triviality in quantum electrodynamics because it is assumed rather than derived. However, it is a miracle on any view that regards the universe as being from time to time "reprocessed." How can electrons at different times and places in the present cycle of the universe have the same mass if the spectrum of particle masses differs between one cycle of the universe and another?

Inspect the interior of a particle of one type, and magnify it up enormously, and in that interior see one view of the whole universe [compare the concept of monad of Leibniz (1714), "The monads have no window through which anything can enter or depart"]; and do likewise for another particle of the same type. Are particles of the same pattern identical in any one cycle of the universe because they give identically patterned views of the same universe? No acceptable explanation for the miraculous identity of particles of the same type has ever been put forward. That identity must be regarded, not as a triviality, but as a central mystery of physics.

Reason for identity in mass of particles of the same species?

Not the spectrum of particle masses alone, but the physical "constants" themselves, would seem most reasonably regarded as reprocessed from one cycle to another. Reprocessed relative to what? Relative, for example, to the Planck system of units,

Reprocessing of physical constants

$$L^* = (\hbar G/c^3)^{1/2} = 1.6 \times 10^{-33}\ \text{cm},$$
$$T^* = (\hbar G/c^5)^{1/2} = 5.4 \times 10^{-44}\ \text{sec},$$
$$M^* = (\hbar c/G)^{1/2} = 2.2 \times 10^{-5}\ \text{g},$$

the only system of units, Planck (1899) pointed out, free, like black-body radiation itself, of all complications of solid-state physics, molecular binding, atomic constitution, and elementary particle structure, and drawing for its background only on the simplest and most universal principles of physics, the laws of gravitation and black-body radiation. Relative to the Planck units, every constant in every other part of physics is expressed as a pure number.

No pure numbers in physics are more impressive than $\hbar c/e^2 = 137.0360$ and the so-called "big numbers" [Eddington (1931, 1936, 1946); Dirac (1937, 1938); Jordan (1955, 1959); Dicke (1959b, 1961, 1964b); Hayakawa (1965a,b); Carter (1968b)]:

$$\sim 10^{80} \text{ particles in the universe,}^*$$

$$\sim 10^{40} \sim \frac{10^{28}\text{ cm}}{10^{-12}\text{ cm}} \sim \frac{\begin{pmatrix}\text{radius of universe at}\\ \text{maximum expansion}\end{pmatrix}^*}{\begin{pmatrix}\text{"size" of an elementary}\\ \text{particle}\end{pmatrix}},$$

$$\sim 10^{40} \sim \frac{e^2}{GmM} \sim \frac{(\text{electric forces})}{(\text{gravitational forces})},$$

$$\sim 10^{20} \sim \frac{e^2/mc^2}{(\hbar G/c^3)^{1/2}} \sim \frac{\begin{pmatrix}\text{"size" of an elementary}\\ \text{particle}\end{pmatrix}}{(\text{Planck length})},$$

$$\sim 10^{10} \sim \frac{\begin{pmatrix}\text{number of photons}\\ \text{in universe}\end{pmatrix}}{\begin{pmatrix}\text{number of baryons}\\ \text{in universe}\end{pmatrix}}.$$

Some understanding of the relationships between these numbers has been won [Carter (1968b)]. Never has any explanation appeared for their enormous magnitude, nor will there ever, if the view is correct that reprocessing the universe reprocesses also the physical constants. These constants on that view are not part of the laws of physics. They are part of the initial-value data. Such numbers are freshly given for each fresh cycle of expansion of the universe. To look for a physical explanation for the "big numbers" would thus seem to be looking for the right answer to the wrong question.

In the week between one storm and the next, most features of the weather are ever-changing, but some special patterns of the wind last the week. If the term "frozen features of the meteorology" is appropriate for them, much more so would it seem appropriate for the big numbers, the physical constants and the spectrum of particle masses in the cycle between one reprocessing of the universe and another.

Values of physical constants as related to the possibilities for life

A per cent or so change one way in one of the "constants," $\hbar c/e^2$, will cause all stars to be red stars; and a comparable change the other way will make all stars be blue stars, according to Carter (1968b). In neither case will any star like the sun be possible. He raises the question whether life could have developed if the determinants of the physical constants had differed substantially from those that characterize this cycle of the universe.

Dicke (1961) has pointed out that the right order of ideas may not be, here is the universe, so what must man be; but here is man, so what must the universe

*Values based on the "typical cosmological model" of Box 27.4; subject to much uncertainty, in the present state of astrophysical distance determinations, not least because the latitude in these numbers is even enough to be compatible with an open universe.

be? In other words: (1) What good is a universe without awareness of that universe? But: (2) Awareness demands life. (3) Life demands the presence of elements heavier than hydrogen. (4) The production of heavy elements demands thermonuclear combustion. (5) Thermonuclear combustion normally requires several 10^9 years of cooking time in a star. (6) Several 10^9 years of time will not and cannot be available in a closed universe, according to general relativity, unless the radius-at-maximum-expansion of that universe is several 10^9 light years or more. So why on this view is the universe as big as it is? Because only so can man be here!

In brief, the considerations of Carter and Dicke would seem to raise the idea of the "biological selection of physical constants." However, to "select" is impossible unless there are options to select between. Exactly such options would seem for the first time to be held out by the only over-all picture of the gravitational collapse of the universe that one sees how to put forward today, the *pregeometry black-box model of the reprocessing of the universe.*

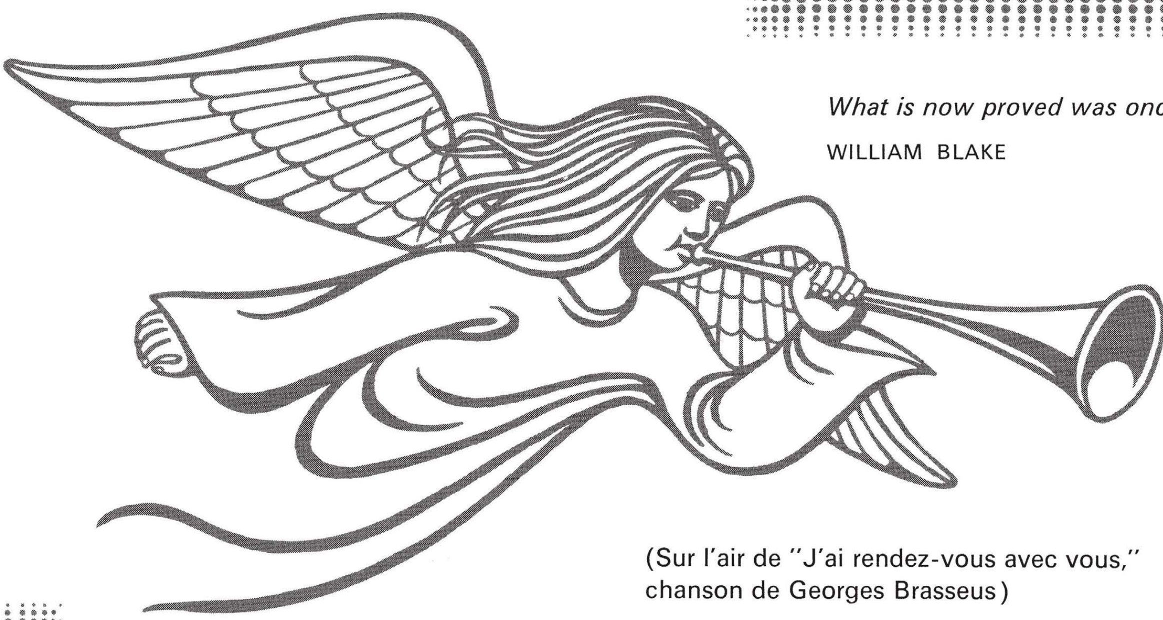
Proceeding with all caution into uncharted territory, one must nevertheless be aware that the conclusions one is reaching and the questions one is asking at a given stage of the analysis may be only stepping stones on the way to still more penetrating questions and an even more remarkable picture. To speak of "reprocessing and selection" may only be a halfway point on the road toward thinking of the universe as Leibniz did, as a world of relationships, not a world of machinery. Far from being brought into its present condition by "reprocessing" from earlier cycles, may the universe in some strange sense be "brought into being" by the participation of those who participate? On this view the concept of "cycles" would even seem to be altogether wrong. Instead the vital act is the act of participation. "Participator" is the incontrovertible new concept given by quantum mechanics; it strikes down the term "observer" of classical theory, the man who stands safely behind the thick glass wall and watches what goes on without taking part. It can't be done, quantum mechanics says. Even with the lowly electron one must participate before one can give any meaning whatsoever to its position or its momentum. Is this firmly established result the tiny tip of a giant iceberg? Does the universe also derive its meaning from "participation"? Are we destined to return to the great concept of Leibniz, of "preestablished harmony" ("Leibniz logic loop"), before we can make the next great advance?

Black hole as "laboratory" model for collapse of universe

Rich prospects stand open for investigation in gravitation physics, from neutron stars to cosmology and from post-Newtonian celestial mechanics to gravitational waves. Einstein's geometrodynamics exposes itself to destruction on a dozen fronts and by a thousand predictions. No predictions subject to early test are more entrancing than those on the formation and properties of a black hole, "laboratory model" for some of what is predicted for the universe itself. No field is more pregnant with the future than gravitational collapse. No more revolutionary views of man and the universe has one ever been driven to consider seriously than those that come out of pondering the paradox of collapse, the greatest crisis of physics of all time.

All of these endeavors are based on the belief that existence should have a completely harmonious structure. Today we have less ground than ever before for allowing ourselves to be forced away from this wonderful belief.

EINSTEIN (1934)



What is now proved was once only imagin'd.

WILLIAM BLAKE

We will first understand
How simple the universe is
When we realize
How strange it is.

ANON.

To some one who could grasp the
universe from a unified standpoint,
the entire creation would appear
as a unique truth and necessity.

J. D'ALEMBERT

Yo ho, it's hot . . . the sun is not
A place where we could live
But here on earth there'd be no life
Without the light it gives

H. ZARET

Probable-Possible, my black hen,
She lays eggs in the Relative When.
She doesn't lay eggs in the Positive Now
Because she's unable to postulate How.

F. WINSOR

(Sur l'air de "J'ai rendez-vous avec vous,"
chanson de Georges Brasseus)

Le Rayonnement dipolaire
On sait qu'il n'est pas pour nous
C'est pour Maxwell, oui mais Maxwell on s'en fout
Tout est r'latif après tout

Un argument qu'on révère
Celui de Synge pour dire le tout
Nous promet le quadrupolaire
Tout est r'latif après tout

Les sources quasi stellaires
Disparaissent comme dans un trou
Dans le Schwarzschild, oui mais Schwarzschild on s'en fout
Tout est r'latif après tout

Aux solutions singulières
On préfère et de beaucoup
Une métrique partout régulière
Tout est r'latif après tout

Les physiciens nucléaires
Comme ils nous aiment pas beaucoup
Y gardent tout l'fric, oui mais le fric on s'en fout
Tout est r'latif après tout

Les expériences de Weber
Le gyroscope, ça coûtent des sous
Celles de pensées sont moins chères
Tout est r'latif après tout

M. A. TONNELAT

"Omnibus ex nihil ducendis sufficit unum!"
(One suffices to create Everything of nothing!)

GOTTFRIED WILHELM VON LEIBNIZ

(Sur l'air de Auprès de ma blonde)

Dans les jardins d'Asnières
La science a refleuri
Tous les savants du monde
Apportent leurs écrits

Refrain:

Auprès de nos ondes
Qu'il fait bon, fait bon, fait bon
Auprès de nos ondes
Qu'il fait bon rêver

Tous les savants du monde
Apportent leurs écrits
Loi gravitationnelle
Sans tenseur d'énergie

Loi gravitationnelle
Sans tenseur d'énergie
De ravissants modèles
Pour la cosmologie

De ravissants modèles
Pour la cosmologie
Pour moi ne m'en faut guère
Car j'en ai un joli

Pour moi ne m'en faut guère
Car j'en ai un joli
Il est dans ma cervelle
Voici mon manuscrit

Le champ laisse des plumes
Aux bosses de l'espace-temps
En prendrons quelques unes
Pour décrire le mouvement

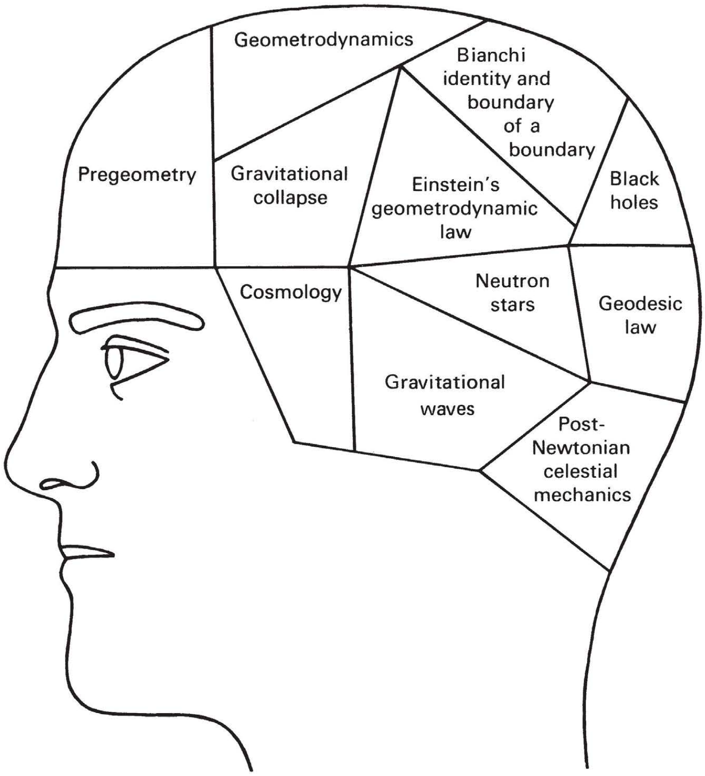
C. CATTANEO, J. GÉHÉNIAU
M. MAVRIDES, and M. A. TONNELAT

When Arthur Evans began this excavation
neither he nor anyone knew that he
would uncover an unknown world.

PAT (Mrs. Hypatia Vourloumis
at Knossos (1971))

And as imagination bodies forth
The form of things unknown, the poet's pen
Turns them to shapes, and gives to airy nothing
A local habitation and a name.

SHAKESPEARE



Appreciation and farewell to our patient reader.

Charles W. Misner   Kip S. Thorne   John A. Wheeler

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