$SU(3) \times SU(2) \times U(1) \longrightarrow SU(5) \longrightarrow \boxed{SO(10)}$

GRAND UNIFIED THEORY

15 LH FERMIONS $\longrightarrow$ $\boxed{\bar{5} \oplus 10}$ $\longrightarrow$ $\boxed{16}$ $\boxed{32}$

$16 \longrightarrow \boxed{\bar{5} \oplus 10} \oplus \boxed{1}$

of $SO(10)$ — of $SU(5)$

↳ SINGLET OF $SU(5)$?

$SO(10)$ $\left\{ \begin{array}{l} \text{45 GENERATORS} \\ \quad \equiv \\ \text{45 GAUGE BOSONS} \end{array} \right.$

$SU(3)$ $\left\{ \begin{array}{l} \text{24 GEN.} \\ \quad \equiv \\ \text{24 GAUGE BOSONS.} \end{array} \right.$

$\boxed{\text{MINIMAL } SU(5)}$

- UNIFICATION OF GAUGE COUPLINGS
- $m_{\text{"}d\text{"}} = m_{\text{"}e\text{"}}$
- PROTON IS UNSTABLE

$\Rightarrow$
- ① UNIFICATION OF GAUGE COUPLINGS
- "m_d" = "m_e"
- PROTON IS UNSTABLE

② $m_d \neq m_e$

↳ HOW WE BREAK SU(5) ?

24 OF SU(5)

$\downarrow$

$SU(3) \otimes SU(2) \otimes U(1)$

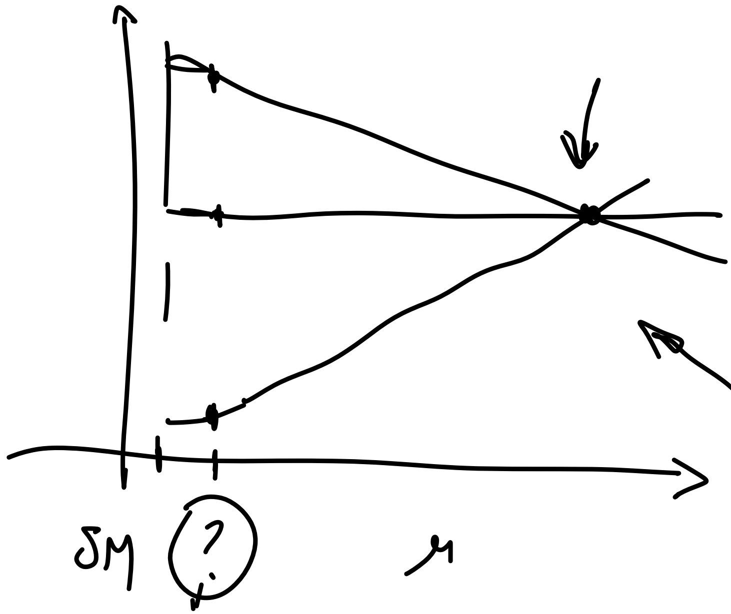
$\downarrow$ $\boxed{\overset{(-)}{5}}$ + OTHER REPS

$U(1)_Q$

①. SU(5) ~ RANK 4 ~ SM

↳ MORE DEGREES OF FREEDOM AT LOW SCALE.

SUPERSYMMETRY ~ NEW DEGREES OF PARTICLES.



③ PROTON

TOO SHORT LIVED...

$\underline{SO(10)}$

$$SU(5) \supset SU(3) \times SU(2) \times U(1)$$

$$\begin{pmatrix} \boxed{3\times 3} & \\ & \boxed{2\times 2} \end{pmatrix}$$

$$\begin{tikzcd}
SO(10) \arrow[r, "R_{H_1}"] \arrow[rr, bend right, "R_{H_3}"'] \arrow[dr, "R_{H_4}"'] & SU(5) \times U(1) \arrow[r, "R_{H_2}"] & SU(5) \arrow[r, "R_{H_2}"] & SU(3) \times SU(2) \times U(1) \\
& SU(4) \times SU(2)_L \times SU(2)_R \arrow[r, "R_{H_5}"] \arrow[d] & SU(3) \times SU(2)_L \times U(1)_Y & \\
& SU(3) \times U(1)_{B-L} & &
\end{tikzcd}$$

SU(5) ROAD / WAY

PATI-SALAM GROUP

$$Q\ ,\ L \sim (\cdot, 2, 1, \cdot)$$

$$\begin{pmatrix} u_R^c \\ d_R^c \end{pmatrix} \begin{pmatrix} \nu_R^c \\ e_R^c \end{pmatrix} \sim (\cdot, 1, 2, \cdot)$$

$\underbrace{16}$ Fermions $= \bar{5} \oplus 10 \oplus 1$

SM $\qquad Q = T_{3L} + \frac{Y}{2}$

$\boxed{\text{PS}}$ $\qquad Q = T_{3L} + \underbrace{T_{3R} + \frac{B-L}{2}}$

$Q_L = {}_1\begin{pmatrix} u_L \\ d_L \end{pmatrix}_{1/3} \qquad L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}_{-1}$

$Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}_{1/3} \qquad L_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}_{-1}$

- PURE SINGLET STATE ~ SM (SU(2))

$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$ $\quad \tilde{H} = i\sigma_2 H^*$

$\langle \tilde{H} \rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$

$\boxed{\nu_R}$

- $$\mathcal{L} = y_u \bar{Q} \tilde{H} u_R + y_d \bar{Q} H d_R + h.c.$$
$$+ y_e \bar{L} H e_R + y_\nu \underbrace{\bar{L} \tilde{H}}\, \nu_R + h.c.$$

$\updownarrow$

SU(3) x SU(2) x U(1) SINGLET

DIM < 4

$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$

$(\underbrace{H^\dagger H})$

$$\mathcal{L} \supset y_\nu \bar{L} \tilde{H} \nu_R \xrightarrow{\langle \tilde{H} \rangle} y_\nu \frac{v}{\sqrt{2}} \bar{\nu}_L \nu_R + h.c.$$

$v = 246\ \text{GeV} \simeq \frac{10^3}{4}\ \text{GeV}$

$$= m_\nu \bar{\nu}_L \nu_R + h.c.$$

* HISTORICAL → ν ARE VERY LIGHT $\simeq$ MASSLESS

* EXP → ν ARE VERY LIGHT $\neq$ MASSLESS

3 GENERATIONS

$$\Delta m^2_{12} = 7.6 \times 10^{-5}\ eV^2$$

$$\Delta m^2_{23} = 2.4 \times 10^{-3}\ eV^2$$

$$\Rightarrow \boxed{m_\nu \lesssim eV}$$

EFT LANGUAGE

$\nu_L, \nu_R \Rightarrow y \lesssim 10^{-11}$? VERY UNNATURAL!

- MASS TERM À LA DIRAC

$$\mathcal{L}_{SM} \supset y\frac{v}{\sqrt{2}}\,\bar{\nu}_L \nu_R + h.c.$$

$$\mathcal{L}_{DIRAC} = i\bar{\psi}\not{\partial}\psi - m\bar{\psi}\psi$$

$$\psi_{DIRAC} = \begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix} \qquad \hookrightarrow \bar{\psi}\psi = \chi_L^\dagger \chi_R + h.c.$$

- $$\chi_R = \sigma_2 \chi_L'^{*} = \chi_L'^{c}$$

$$\mathcal{L}_{DIRAC} = \dots - m\,\chi_L^\dagger \sigma_2 \chi_L' + h.c.$$

- WHAT if $\chi_L' = \chi_L$; $\chi_R = \sigma_2 \chi_L^*$ $\quad \psi_M = \begin{pmatrix} \chi_L \\ \sigma_2 \chi_L^* \end{pmatrix}$

$$\mathcal{L}_{\text{MAJORANA}} = \dots - m\, \chi_L^{\dagger} \sigma_2 \chi_L^{*} + h.c.$$

DIRAC = 4 DEG. OF FREEDOM 2 HELICITIES

PARTICLES & ANTIPARTICLES

MAJORANA = 2 DEG. OF FREEDOM 2 HELICITIES

$$\psi_D \to e^{i\alpha} \psi_D \quad = \quad \chi_{L,R} \to e^{i\alpha} \chi_{L,R}$$

FERMION NUMBER

$$\chi_L \to e^{i\alpha} \chi_L \qquad \chi_R = \sigma_2 \chi_L^{*} \to e^{-i\alpha} \chi_R$$

$$m\, \chi_L^{\dagger} \sigma_2 \chi_L^{*} \longrightarrow e^{-i2\alpha}\, m\, \chi_L^{\dagger} \sigma_2 \chi_L^{*}$$

ν_R ~ SINGLET

ν_L ~ ELECTRICALLY NEUTRAL

---

① FIRST PERSPECTIVE IS EFT

$$\mathcal{L}_{SM} \supset y_d \bar{Q} H d_R + y_u \bar{Q} \tilde{H} u_R + y_e \bar{L} H e_R$$

DIM 4 OPERATORS

DIM 5 $+ \frac{1}{\Lambda} \bar{L} \tilde{H} \tilde{H}^* L^c$ $\sim \chi_L^+ \quad \sim \chi_R$

$\bar{L} \tilde{H}$ $SU(2) \times U(1)$

DIM 6 $+ \frac{1}{\Lambda^2} \bar{\psi}\psi\bar{\psi}\psi$ $\longleftrightarrow$ PROTON DECAY

$$\mathcal{L}_{SM} \supset \frac{1}{\Lambda} \bar{L} \tilde{H} \tilde{H}^* L^c + h.c. = \frac{v^2}{2\Lambda} \nu_L^{+} c_2 \nu_L^{*} + h.c.$$

$$\tilde{H} = \begin{pmatrix} \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix}$$

MAJORANA MASS TERM!

$$m_\nu \simeq \frac{v^2}{\Lambda} \sim eV$$

$$v \sim 10^3 \text{ GeV}$$

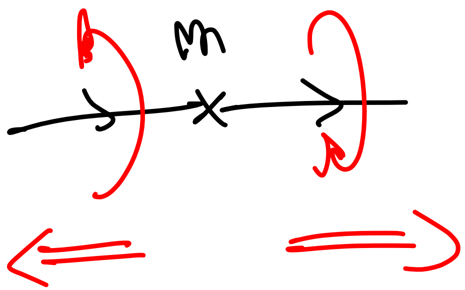
$$\boxed{\Lambda \sim 10^{15} \text{ GeV}}$$

β-DECAY

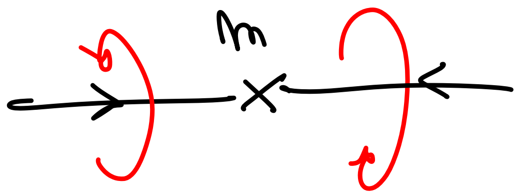
$$n \to p + e^- + \bar{\nu}$$

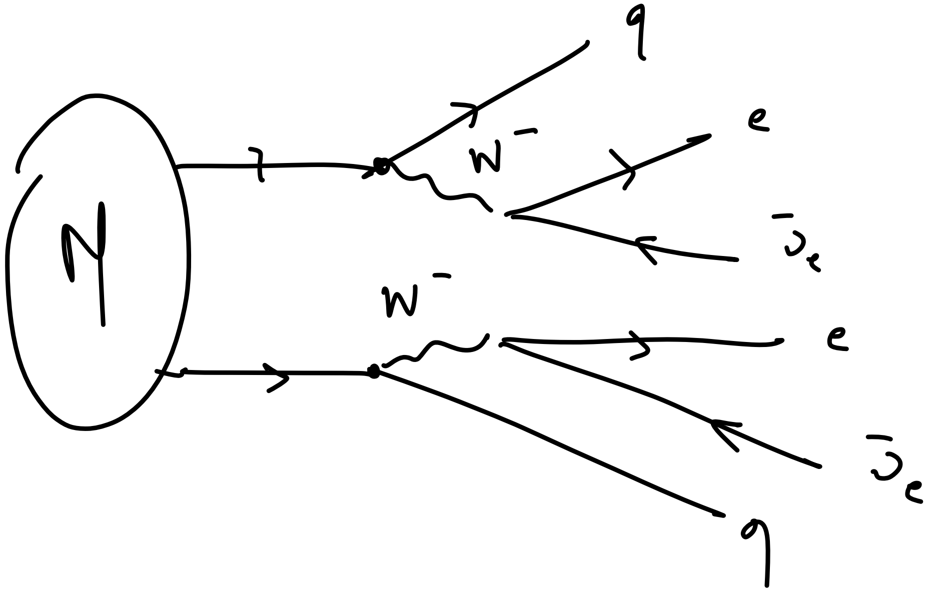
PERHAPS ν ARE MAJORANA $\Rightarrow$ MAJORANA MASS TERM

DIRAC TERM

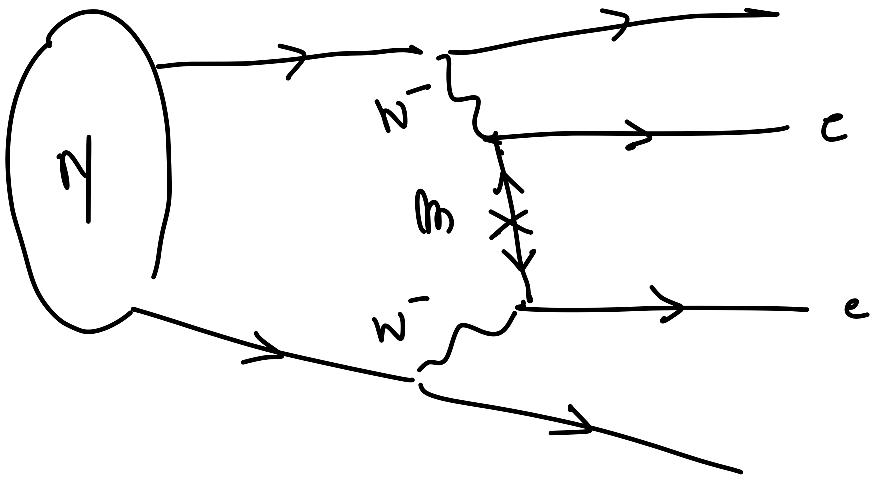


MAJORANA





DOUBLE BETA DECAY



NEUTRINOLESS DOUBLE BETA DECAY

SO(10) $\quad \nu_R$ SINGLET STATE!

$$\mathcal{L} \supset y \, \bar{L} \tilde{H} \nu_R + h.c.$$

$$+ \frac{m}{2} \nu_R^+ \sigma_2 \nu_R^* + h.c.$$

B-L BREAKING

$\frac{1}{2} m \phi^2$

$m \phi^+ \phi$

$$\mathcal{L} \supset y \frac{v}{\sqrt{2}} \nu_L^+ \nu_R + \frac{M}{2} \nu_R^+ \nu_R^c + h.c.$$

$$\supset \begin{pmatrix} \nu_L^{c\,+} & \nu_R^+ \end{pmatrix} \begin{pmatrix} 0 & yv \\ yv & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$$

$$\mathcal{L} \supset \begin{pmatrix} \nu_L^{+c} & \nu_R^{+} \end{pmatrix} \underbrace{\begin{pmatrix} 0 & yN \\ yN & M \end{pmatrix}}_{\mathcal{M}} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$$

3 GENERATIONS

$\longrightarrow$

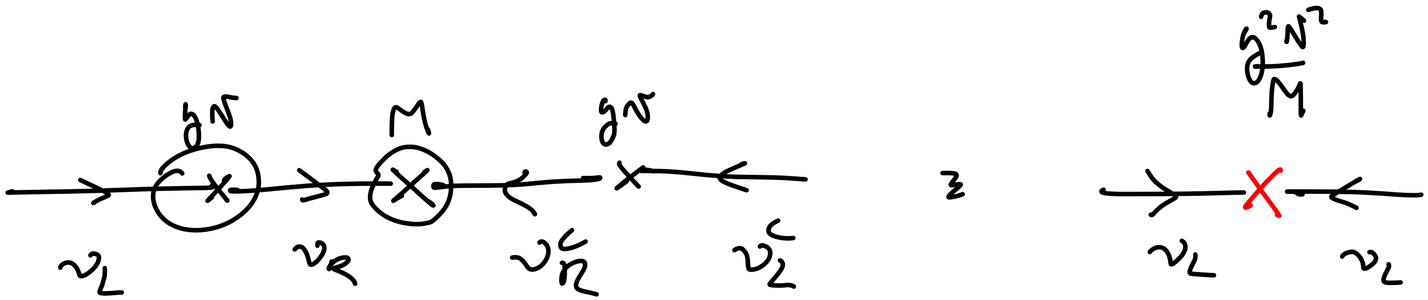
$$\det \mathcal{M} = -y^2 N^2 = \lambda_1 \lambda_2 \qquad \lambda_2 = -\frac{y^2 N^2}{M}$$

$$\mathrm{Tr}\, \mathcal{M} = M = \lambda_1 + \lambda_2 \approx \lambda_1$$

$\lambda_2 = m_\nu$ $\Longleftrightarrow$ $\boxed{m_\nu \simeq \frac{y^2 N^2}{M} = \frac{N^2}{\Lambda}}$

- LIGHT NEUTRINOS
- SM NEUTRINOS
- SU(2) INTERACTIONS!

• SM ν MASS



LIGHT SM ν $\Longleftrightarrow$ MAJORANA

• SO(10) (SU(5) or PS)

+ UNI. OF COUPL / PARTICLES

+ ν MASSES

+ FLAVOUR

+ PROTON DECAY / LEPTON NUMBER VIOLATION

- GUT PICTURE $\boxed{\Lambda_{GUT} \sim 10^{15}\ \text{GeV}}$

BIG GAP $M_{PL} = 10^{19}\ \text{GeV}$

- $\boxed{\text{EW SCALE} \qquad v \sim 10^{3}\ \text{GeV}}$?

HIERARCHY PROBLEM

* FERMIONS >< SYMMETRY CHIRAL
* GAUGE FIELDS >< GAUGE SYMMETRY
* $\boxed{\text{SCALAR (S)}}$ $\boxed{\text{BEH}}$

$\Updownarrow$

v

* $\underline{SU(5)}$ 24 , 5 TO BREAK $SU(5) \to U(1)_{EM}$

$\Rightarrow \phi_1 , \phi_2$

$$V = -\mu_1^2 \phi_1^2 - \mu_2^2 \phi_2^2 + \lambda_1 \phi_1^4 + \lambda_2 \phi_2^4$$

$$\langle \phi_1^2 \rangle = \frac{\mu_1^2}{\lambda_1} = O(\mathrm{eW})$$

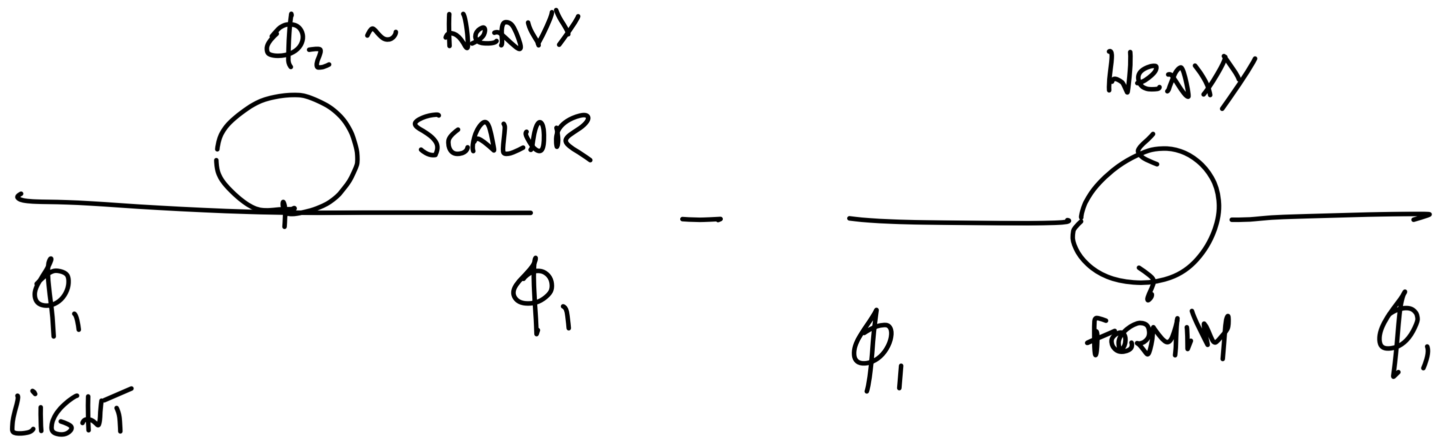
$$\langle \phi_2^2 \rangle = \frac{\mu_2^2}{\lambda_2} \quad O(\mathrm{GUT})$$

$$+ \bar{\lambda} \phi_1^2 \langle \phi_2^2 \rangle + V_0$$

V_0 = COSMOLOGICAL CONSTANT

$$\boxed{\langle \phi_1^2 \rangle = \frac{\mu_1^2 - \bar{\lambda} \langle \phi_2^2 \rangle}{\lambda_1}}$$

- $\bar{\lambda} = 0$: VERY SMALL
- $\bar{\lambda}$ NOT SMALL BUT CANCELLATION



$$\Rightarrow \quad \delta M^2_{\phi_1} \simeq O\left(M^2_{HEAVY}\right)$$

$\updownarrow$

GUT SCALE / M_{PLANCK}

BIG PROBLEM WITH SCALAR FIELDS

FERMIONS, GAUGE BOSONS GET THEIR MASS FROM SCALARS...