HIERARCHY PROBLEMS

THE ONLY RELEVANT OPERATOR OF THE SM (WE SHOULD "THE THEORY") IS THE SCALAR MASS TERM:

$$\mathcal{L}_{SM} \supset +\mu^2 H^\dagger H - \lambda (H^\dagger H)^2$$

FROM THIS, ASSUMING $\mu^2 > 0$ (AND SO NEGATIVE MASS TERM IN THE POTENTIAL), WE GET SETTING (UNITARY GAUGE)

$$H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} \qquad v^4 + 4v^3 h$$

$$\begin{array}{c} 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \end{array}$$

$$V = -\frac{1}{2}\mu^2 (v+h)^2 + \frac{\lambda}{4}(v+h)^4$$

$$= -\frac{1}{2}\mu^2 v^2 + \frac{\lambda}{4} v^4 + \underbrace{\left(-\mu^2 v + \lambda v^3\right)}_{=\frac{\partial V}{\partial v}} h \qquad = 0 \quad \text{AT MINIMUM (VANISHING TADPOLE)}$$

$$+ \overbrace{\left(-\frac{\mu^2}{2} + \frac{3}{2}\lambda v^2\right)}^{\frac{1}{2} m_h^2} h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4$$

- $\frac{\partial V}{\partial v} = 0 \Rightarrow v^2 = \mu^2/\lambda \iff v = 246$ GeV
- $m_h^2 = -\mu^2 + 3\lambda v^2 = 2\lambda v^2 \qquad \lambda = \frac{m_h^2}{2v^2} = \frac{125^2}{2(246)^2} = 0.13$
- $M_W = \frac{gv}{2} = 80$ GeV, etc.

REM: AT EW SCALE

$\Rightarrow$ μ SETS THE SCALE FOR THE MASS OF GAUGE BOSONS AND BARE MASS OF FERMIONS

↳ QUARKS ALSO HAVE A CONSTITUENT MASS.

FROM A BROADER PERSPECTIVE:

- GUT eg $SU(5)$ (BUT SIMILAR ISSUES WITH $SO(10)$)

BREAKING OF $SU(5) \to SU(3) \otimes SU(2) \otimes U(1)$

BY $H_1 \sim 24$ OF $SU(5)$

$\langle H_1 \rangle \sim$ Y GENERATOR $\sim 10^{16}$ GeV OR SO

THEN BREAKING OF SM $\to U(1)_Q$

$\hookrightarrow$ $H_2 \sim 5$ OF $SU(5)$

$\langle H_2 \rangle$ SIMILAR TO SM HIGGS.

SO $\langle H_2 \rangle \sim v \sim 10^{2-3}$ GeV

THE ISSUE IS $\boxed{\langle H_2 \rangle \ll \langle H_1 \rangle}$

WHY SO? $V(H_1, H_2) = V(H_1) + V(H_2) + V_{INT}(H_1, H_2)$

MINIMIZING $V(H_1) \Rightarrow$ GUT BREAKING

$V(H_2) \Rightarrow$ EW SCALE

BUT $V_{INT} = \lambda_{INT} (H_1)^2 (H_2)^2$ WITH $\lambda_{INT} \sim O(1)$

(NATURALNESS)

$\Longrightarrow \quad V_{INT} = \lambda_{INT} \, N_1^2 \, H_{SM}^2$ AFTER GUT BREAKING.

FROM $V_2 = \mu_2^2 H_2^2 + \dots$

$\Longrightarrow$ ε CONTRIBUTES TO MASS TERM AT EW SCALE:

$$\mu_{SM}^2 = \underbrace{\lambda_{INT} N_1^2}_{\text{GUT SCALE}} + \mu_2^2$$

SO EITHER λ_{INT} VERY SMALL (BUT NO REASON) SO UNNATURAL

OR $\lambda_{INT} = O(1)$ AND LARGE CANCELLATION BETWEEN $\lambda_{INT} N_1^2 + \mu_2^2 \sim N_{SM}^2$

WHICH IS ALSO UNNATURAL

ES LET $N_1 = 10^{16}$ GeV

AND $N_{SM} = 246$ GeV

WRITE μ_2^2 IN TERMS OF λ_{INT} AND CHECK THE ABOVE STATEMENT.

THIS IS AT TREE LEVEL. IF WE TUNE THINGS AT TREE LEVEL, TUNING (OF λ_{INT} OR OF SCALES) STRIKES BACK!

CONSIDER A SIMPLE TOY THEORY

$$\mathcal{L} = i\bar{\psi}\not{\partial}\psi - m\bar{\psi}\psi + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}M^2\varphi^2 - \frac{\lambda}{4}\varphi^4 + g\varphi\bar{\psi}\psi$$

ALL PARAMETERS ARE DEFINED AT SOME SCALE.

WE CONSIDER THE LOOP CORRECTIONS TO M^2.

CORRECTION TO MASS TERM

(A)

$$-i\Delta M^2(p^2) = -i3\lambda\int\frac{d^Dk}{(2\pi)^D}\frac{i}{k^2-M^2}$$

$$\frac{i}{p^2-M^2-\Delta M^2(p^2)}$$

$$+(-)(ig)^2\int\frac{d^Dk}{(2\pi)^D}\mathrm{Tr}\left[\frac{i}{\not{k}+\not{p}-m}\frac{i}{\not{k}-m}\right]$$

(B)

FERMION LOOP, SO EXTRA MINUS SIGN

NB:

$$\langle 0|\bar{\psi}(y)\psi(y)\bar{\psi}(x)\psi(x)|0\rangle =$$

NEED TO ANTICOMMUTE 3 TIMES = - SIGN

3 → ODD NUMBER

(A) SCALAR LOOP

$$-i\Delta M_A^2 = 3\lambda\int\frac{d^Dk}{(2\pi)^D}\frac{1}{k^2-M^2} = \frac{-i3\lambda}{(4\pi)^{D/2}}\Gamma(1-D/2)\left(\frac{1}{M^2}\right)^{1-D/2}$$

$$\Gamma\left(1-\frac{D}{2}\right) = \Gamma\left(-1+\frac{\varepsilon}{2}\right) = -\left(\frac{2}{\varepsilon}-\gamma+1+O(\varepsilon)\right)$$

$$-i\Delta M^2_\lambda(p) = \frac{3i\lambda M^2}{16\pi^2}\left(\frac{2}{\epsilon} - \gamma + 1 + \log(4\pi) - \log M^2\right)$$

PROPORTIONAL TO M^2

PAULI-VILLARS

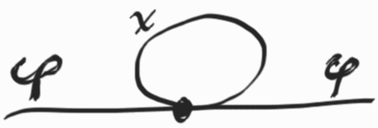
$$= \frac{3i}{16\pi^2}\lambda M^2 \log\frac{\Lambda^2}{M^2} \longrightarrow \propto \tilde{\lambda} M_X^2 \log \Lambda^2/M_X^2$$

NB: THIS IS p-INDEPENDENT. SO SETTING

$$M^2_{OBS} = M^2 + \Delta M^2$$

MAKES  WITH NO OBSERVABLE EFFECT.

NOTE HOWEVER THAT $\Delta M^2 \propto M^2$. IN A BROADER PERSPECTIVE, IF $M_\chi \gg M_\varphi$



$\Longrightarrow \Delta M^2_\varphi \propto M^2_\chi$

LARGE CORRECTION!

(2) FERMION LOOP

$$-i\Delta M^2_B = (ig)^2\int \frac{d^Dk}{(2\pi)^D}\, \mathrm{Tr}\left[\frac{1}{\not{k} - m}\,\frac{1}{\not{k}+\not{p} - m}\right]$$

$$= -g^2 D\int\frac{d^Dk}{(2\pi)^D}\frac{\left(k\cdot(k+p)+m^2\right)}{(k^2-m^2)\left((k+p)^2-m^2\right)}$$

$$= -g^2 D \int \frac{d^Dk}{(2\pi)^D}\frac{\left[\frac{k^2}{2} + \frac{1}{2}(k^2+2k\cdot p+p^2) - p^2/2 + m^2\right]}{(k^2-m^2)\left((k+p)^2-m^2\right)}$$

$$-i\,\Delta M_B^2 = -g^2 D \int \frac{d^D k}{(2\pi)^D} \left[\frac{1}{2} \frac{1}{k^2-m^2} + \frac{1}{2} \frac{1}{((k+p)^2-m^2)} - \frac{(p^2/2 - 2m^2)}{(k^2-m^2)((k+p)^2-m^2)} \right]$$

$$= -g^2 D \int \frac{d^D k}{(2\pi)^D} \left[\frac{1}{k^2-m^2} + \frac{2m^2}{(k^2-m^2)^2} + \underbrace{O(p^2)}_{\text{WF Ren.}} \right]$$

$$= -g^2 D \left[\frac{-i}{(4\pi)^{D/2}} \Gamma\left(1-D/2\right) \frac{1}{(m^2)^{1-D/2}} + \frac{2m^2\, i}{(4\pi)^{D/2}} \Gamma\left(2-D/2\right) \frac{1}{(m^2)^{2-D/2}} \right]$$

$$\Gamma(2-D/2) = (1-D/2)\,\Gamma(1-D/2)$$

$$= -g^2\, i D \frac{m^2}{(4\pi)^{D/2}} \Gamma(1-D/2) \left[\underbrace{-1+2-D}_{1-D} \right] \frac{1}{(m^2)^{2-D/2}}$$

$$\boxed{\begin{aligned} -i\Delta M_B^2 &= g^2\, i D\,(D-1) \frac{1}{(4\pi)^{D/2}} \frac{\Gamma(1-D/2)}{(m^2)^{1-D/2}} \\ &\text{vs} \\ -i\Delta M_A^2 &= \frac{-i\,3\lambda}{(4\pi)^{D/2}} \Gamma(1-D/2) \left(\frac{1}{M^2}\right)^{1-D/2} \\ &\Rightarrow \boxed{\text{opposite signs!}} \end{aligned}}$$

NET RESULT

$$\Delta M^2_{(A)} \sim \lambda M^2 \ln M^2$$

$$\Delta M^2_{(B)} \sim -g^2 m^2 \ln m^2$$