# Quantum field theory II: path integrals and renormalization


1 Path integral formulation of QM
1.1 Recall of QM
1.2 Operators and representation
1.3 Amplitude $S_H$
1.4 Canonical N and gaussian integrals $S_L$
1.5 Operators under the path integral Time-ordered correlation function
1.6 Towards field theory
p5 2 Path int. for a scalar field theory
2.1 Formal def. of QFT
2.2 Free real scalar theory $Z[J]$, $D_F(x-y)$
2.3 Real scalar field with interactions $W[J]$, 1PI, $\Gamma$, $\varphi_{cl}$
p17 3 Path. int. for fermionic fields $S_{Dirac}$
3.1 Anticommuting Numbers (grassman #)
3.2 Dirac propagator and generating functional $\langle \psi_x \bar{\psi}_y \rangle$, $Z[\eta, \bar{\eta}]$
3.3 Interacting fermions Yukawa interaction
p20 4 Path. int. for vector fields $S_A$
4.1 Gauge freedom Feynman and Landau gauge
4.2 Faddeev-Popov procedure $G(A)$, $^\alpha A$
4.3 Adding sources
4.4 Example: QED $S_F$, $D_\mu$, $S_{QED}$
4.5 Scalar QED $S_{scalar\ QED}$
p26 5 Symmetries, Ward Id and the path int.
5.1 Noether thm
5.2 Quantum conservation equation Ward-Takahashi Id
5.3 Quantum EOM Schwinger-Dyson eqs.
p29 6 Radiative corrections: loops and divergences NLO, rad. corr.
6.1 A 1st computation: 2-pt function in $\lambda\phi^4$ Wick rotation, Euclidean coord.
6.2 2nd computation: vertex in $\lambda\phi^4$ Mandelstam variables, Feynman param.
6.3 3rd computation: $\mathcal{L}_I = g\,\phi\,\bar{\psi}\psi$

p35 **7 Physics of renormalization**

7.1 Field-Strenght renormalization spectral density, Kallen-Lehmann rep.

7.2 Physical and bare quantities bare

7.3 LSZ reduction formula LSZ

p38 **8 Power counting, divergencies and renormalizability**

8.1 Example: $\lambda\phi^4$ theory

8.2 Power counting (L,N,V,P), degree of divergence

8.3 Renormalizability (non, $\mathbb{1}$, super)- renormalizability

p42 **9 Counter-terms and renormalisation condition**

9.1 Renormalized perturbation theory bare Lagrangian, $\mathcal{L}_R$, $\mathcal{L}_{ct}$, RPT, $\otimes$

9.2 Renormalization conditions

9.3 Fix $\delta_X$ through NLO of 4-pt function

9.4 Dimensional regularisation B-fct, $\Gamma$-fct, $\gamma$

9.5 Field strenght and mass renormalization

p51 **10 Renormalization and gauge symmetry: QED**

10.1 Counterterms and gauge symmetry $\mathcal{L}^{QED}$

10.2 Counterterms and Ward id.

10.3 One-Loop structure of QED $i\Pi^{\mu\nu}$

p57 **11 Energy scale and evolution of couplings**

11.1 Renormalization scale $M$ $\overline{M}$

11.2 The Callan-Symanzik equation $\beta(\lambda)$, $\gamma(\phi)$

11.3 Computation of $\beta$ and $\gamma$ in $\lambda\phi^4$

11.4 generalization to $\lambda\phi^n$ $\beta_n$

11.5 An application to QED $\beta_e$

11.6 Renormalization group flow $\Lambda$, $g^*$, $\lambda^*$, $Z_\Lambda$

p66 **12 Non abelian gauge theories**

12.1 global symmetry $\delta_{abc}$, $D_\mu$, $\delta A_\mu$, $(T_i)^a{}_c$

12.2 Local symmetry

12.3 Field strength tensor $\mathcal{L}_I$, $F_{\mu\nu a}$, $\mathcal{L}_A$

12.4 Yang-Mills Lagrangian

p70 **13 Quantization and ghosts**

13.1 Gauge fixing in QED $\mathcal{L}_{gf}$

13.2 Gauge fixing in QCD

13.3 Faddeev-Popov ghost $D_\mu$, $c^a$, $\bar{c}^a$

13.4 Feynman rules

p74 **14 Renormalization and sgn{β}**

14.1 Renormalization

14.2 A long walk to the β-function $T(\rho)$, $C(\rho)$

14.3 Asymptotic freedom $\Box$, $\bar{\Box}$, $b_0$, $\Lambda_{IR}$, $\Lambda_{QCD}$