Def

A proposition (statements / claims) is a statement that is either true or false.

And for a proposition, its truth value is its truth of falsity.

	Proposition?	T / F
1. $2+2=4$	Y	T
2. 33 is a prime number	Y	F
3. All swans are white	Y	F
4. $\sqrt{2}$ is a rational number	Y	F
5. Is sky blue?	N	—
6. $1+2+3=x$	N	—
7. close the door!	N	—
8. The fastest comparison based sorting algorithm has worst case running time of $O(n \log n)$	Y	T
9. There are infinite # of perfect numbers.	Y	?

Def

Atomic and compound propositions.

An atomic proposition is propositions that is conceptually indivisible.

A compound proposition is a proposition that is built out of conceptually simpler propositions.

Ex: MSU's mascot is a bobcat.

MSU's mascot is a bobcat and UM's mascot is a grizzly.

Logical connectors

A logical connectors are glue that creates more complicated proposition from simpler propositions.

There are 3-major logical connectors

- negation [not, ¬]
- conjunction [and, ∧]
- disjunction [or, ∨]

Negation

The proposition ¬p is true, when the proposition p is false.

p - MSU's mascot is a wolverine (F)

¬p - MSU's mascot is not a wolverine (T)

conjunction

The proposition p∧q ("p and q", "conjunction of p and q") is true when both p and q are true, and false otherwise.

p - MSU's mascot is a Bobcat (T)

q - UM's mascot is a Grizzly (T)

p∧q - MSU's mascot is a Bobcat and UM's mascot is a Grizzly (T)

r - Sky is Green (F)

$p \wedge r$ - MSU's mascot is a Bobcat and Sky is Green (F)

Disjunction

The proposition $p \vee q$ ["p or q", "disjunction of p or q"] is true if either p and q is true, it is false if both p and q are false.

$p \vee q$ (T)

$p \vee r$ (T)

S - sky is red (F)

$r \vee s$ (F)

In this class we will try to learn and practice methods of proving propositions are true or false.

Def.

A proof a proposition is a convincing arguement that the proposition is true.

A disproof is an arguement that the proposition is false.

Direct proofs

Direct proof of a proposition φ starts with known facts and implications and repeatedly applies logical deduction to derive new facts leading to the conclusion.

claim: If x, y is rational, then $x \cdot y$ is rational.

P Q

step 01:

A rational number is a number that can be written as $\frac{n}{d}$ where n, d are integers and $d \neq 0$

$10 = \frac{10}{1}$ ✓ $\quad \pi$ X

$-10 = \frac{-10}{1}$ ✓ $\quad \sqrt{2}$ X

step 02: try some examples

X	Y	X · Y	is x, y rational?	is x·y rational
2	$\frac{1}{2}$	1	Y	Y
2	$\frac{1}{\sqrt{2}}$	$\sqrt{2}$	N	N

Step 03 : proof

claim: If x, y is rational, then $x \cdot y$ is rational

we will use 2-column format.

Statements	Reasoning
1. Assume x, y is rational	assumption.
2. $x = \frac{n_x}{d_x}$, $y = \frac{n_y}{d_y}$, n_x, d_x, n_y, d_y are integers and $d_x, d_y \neq 0$	by the def of rational #
3. $x \cdot y = \frac{n_x \cdot n_y}{d_x \cdot d_y}$	by substitution
4. $xy = \frac{n}{d}$, where $n = n_x \cdot n_y$ $d = d_x \cdot d_y$ and $d \neq 0$, $d_x, d_x \neq 0$	product of integers are integers.
5. xy is a rational number	by def of rational numbers □