CSCI-246 Discrete Structures HW 01

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Objective

- Understanding direct proofs, proof by cases, proof by counter example and propositions.

- Mathematical definitions.

- How to approach solving a problem.

Submission requirements

- *Type or clearly hand-write* your solutions into a **PDF FORMAT.**

- *DO NOT UPLOAD images.*

- *non-pdf or emailed solutions will not be graded.*

- **If you take pictures of your handwritten homework, put it into pdf format.**

- *Start each problem in a new page.*

- Follow the model that you have learned during the lectures for proofs.

- Do not wait until the last minute to submit the assignment.

- You can submit any number of times before the deadline.

- If you are using latex, and you do not know how to type a symbol, use the following website. You can draw the symbol here and it will give you the latex code and the packages that you have to import. `https://detexify.kirelabs.org/classify.html`

- If you are using latex to write the answer, you can use overleaf to make your life easier. **Overleaf is a free, online platform that helps users create and publish scientific and technical documents using LaTeX, a markup-based document preparation system**

- If you do not understand a problem, ask questions during/after the lectures, or during office hours or via discord.

- ***Do not use generative AI to write answers.***

1   Q1

What are the ***truth values*** of the following propositions? If you believe the claim is false, provide an counter example. (You don't need to write a formal proof for each of the sub-questions in Q1, If you think claim is correct, mention that it is True, if not provide an counter example) .

a . $3^2 + 4^2 = 5^2$

b . 'Answer this question.'

c . 'You shall not pass'

d . $p \implies q$ (p, q can be any proposition.)

e If $x - y$ and x are both rational, then y is too.

Grading Notes: While detailed rubric cannot be provided in advance as it would give away the solution, use the following direction to understand how the points are distributed for the problem.

- Correctness
    1. For Q1, you need to provide true or false as the solution, along with your explanation for falsity. If the truth value is incorrect, 0 points will be given.

- Communication
    1. The counter example for false answer should be clear.

2 Q2

What is the negation of these propositions?

- Linda is younger than Bert

- Andrew makes more money than Isabella

- Tadeo is taller than Monica

- Abby is richer than Richard

- $2 + 1 = 3$

Grading Notes: While detailed rubric cannot be provided in advance as it would give away the solution, use the following direction to understand how the points are distributed for the problem.

- Correctness

    1. For Q2, you need to provide the negation of the statements given

- Communication

    1. Clearly write the negated statement.

3 Q3

Prove that, if x and y are rational, then $x - y$ is also rational. (**You need to provide a formal proof for this**).

Hint: Follow the same idea we talked about in the class when we proved xy is rational when x, y is rational. Use direct proof method.

Grading Notes: While detailed rubric cannot be provided in advance as it would give away the solution, use the following direction to understand how the points are distributed for the problem.

- Correctness

    1. For Q3, If your proof is not correct, points will be docked.
    2. You need to clearly formulate your proof.

3. Regardless of your proof method, there will be some facts without which your proof would not work. These facts needs to be stated in your proof. If your proof jumps to conclusion without proper facts, then points will be docked.

   4. Order of these facts must make sense.

- Communication

   1. You should follow the format that I taught you during the class, left hand side for the statements and right hand side for the reasoning. This would make the proof easier to grasp to the reader.

   2. Do no skip too many steps at once. Since this is a fundamental class, at least in the first two homework you need to write all the steps in a proof.

4   Q4

Let n be any integer. **Prove by cases** that $n^3 - n$ is evenly divisible by 3. (**You need to provide a formal proof for this**).

Hint: Use proof by cases method to prove this claim.

Grading Notes: While detailed rubric cannot be provided in advance as it would give away the solution, use the following direction to understand how the points are distributed for the problem.

- Correctness

   1. For Q4, If your proof is not correct, points will be docked.

   2. You need to clearly formulate your proof.

   3. Regardless of your proof method, there will be some facts without which your proof would not work. These facts needs to be stated in your proof. If your proof jumps to conclusion without proper facts, then points will be docked.

   4. Order of these facts must make sense.

- Communication

   1. You should follow the format that I taught you during the class, left hand side for the statements and right hand side for the reasoning. This would make the proof easier to grasp to the reader.

   2. Do no skip too many steps at once. Since this is a fundamental class, at least in the first two homework you need to write all the steps in a proof.

5   Q5

Prove that a positive integer n is divisible by 5 if and only if its last digit is 0 or 5. **(You need to provide a formal proof for this).**

Hint: Any integer n can be expressed using the digits of n as $\langle a_k, a_{k-1}, a_{k-2}, \ldots, a_1, a_0 \rangle \in \{0, 1, 2, \ldots, 9\}^{k+1}$ for some $k \geq 1$.

$$n = 10^k a_k + 10^{k-1} a_{k-1} + \cdots 10a_1 + a_0.$$

Try different integers in this format. Then try to focus on what happens when you divide this expression by 5.

Grading Notes: While detailed rubric cannot be provided in advance as it would give away the solution, use the following direction to understand how the points are distributed for the problem.

- Correctness

    1. For Q5, If your proof is not correct, points will be docked.
    2. You need to clearly formulate your proof.
    3. Regardless of your proof method, there will be some facts without which your proof would not work. These facts needs to be stated in your proof. If your proof jumps to conclusion without proper facts, then points will be docked.
    4. Order of these facts must make sense.

- Communication

    1. You should follow the format that I taught you during the class, left hand side for the statements and right hand side for the reasoning. This would make the proof easier to grasp to the reader.
    2. Do no skip too many steps at once. Since this is a fundamental class, at least in the first two homework you need to write all the steps in a proof.

6   Q6

Consider the following proposition:

$$\{a \in \mathbb{Z} : 3|a\} \cap \{b \in \mathbb{Z} : 10|b\} \subseteq \{a \in Z : 6|a\} \cap \{b \in \mathbf{Z} : 15|b\}$$

Hint: We saw proofs involving divisibility of numbers. First try to write the left hand side of the subset equal in a way that easier to understand. Do the same thing for the set on the right hand side of the subset equal symbol. Try to figure out the behaviour of the elements in the left hand side and right hand side. Then try to show that any element of left hand side set follows the rule of the right hand side set.

Grading Notes: While detailed rubric cannot be provided in advance as it would give away the solution, use the following direction to understand how the points are distributed for the problem.

- Correctness

    1. If your proof is not correct you will points will be docked. Regardless of the proof, there are some facts that has to be stated in your proof. If those facts are not stated, a reader will feel that there are holes in your proof.

    2. Moreover, order of the facts must make sense.

- Communication

    1. You should use statement and reasoning format for your proof. For example, you state your claim using mathematical statement or in English depending on the context, then immediately you state the reasoning why your statement is true.

7 Q7

Draw Venn Diagrams for the following sets.

(a) $\overline{A \oplus B}$

(b) $(\overline{A - B}) \cup (A - C)$

(c) $\overline{B} \cap (A - C)$

(d) $(A \cap C) \cup (\overline{B})$

(e) $(A) \cap (B \cup C) \cap (D)$

8 Q8

Let $A = \{1, 3, 4, 5, 7, 8, 9\}$ and let $B = \{0, 4, 5, 9\}$. Define $C = \{0, 3, 6, 9\}$. Where relevant, assume that the universe is the set $U = \{0, 1, 2, \ldots, 9\}$. What are the following sets?

1. $A \cap B$

2. $A \cup B$

3. $A \oplus B$

4. $A - B$

5. $B - A$

6. $\overline{A} - B$

9 Q9

Determine whether each of these statements is true or false.

1. $0 \in \emptyset$

2. $\emptyset \in \{\emptyset, \{\emptyset\}\}$

3. $\emptyset \subset \{0\}$

4. $\{\emptyset\} \in \{\emptyset\}$

5. $\{0\} \subset \{0\}$