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Recap:

1. Proposition
2. Logical Connector $[\vee, \wedge, \neg]$
3. Proof
4. Direct proof method.

Disproof by counterexample

✓ original claim: If x and y are rational, then xy is rational.

claim: If xy is rational, then x and y is rational.

$x = \sqrt{2}$ $\quad y = \sqrt{2}$ $\quad xy = \sqrt{2} \cdot \sqrt{2} = 2$

Now xy is rational but, x and y is not rational.

Therefore, this claim does not hold.

Proof by cases

Def Let n, m be integers, then n is divisible by m, if there exist an integer k, such that $n = mk$

ex: is 10 divisible by 2?

$10 = 2 \cdot 5 \leftarrow k$ (which is an itegor)

10 is divisible by 2.

is 11 divisible by 3?

$11 = 3 \cdot \left(\frac{11}{3}\right) \leftarrow$ not an integer is 0 divisible by 3?

$0 = 3 \cdot 0 \leftarrow$ integer

0 is divisible by 3.

Def we say, if n is divisible by m,

we also say m divides n.

$m \mid n$ "m divides n"

$2 \mid 10$ ✓ $3 \nmid 10$

2 divides 10

3 does not divide 10

claim: Let n be an integer, then $n \cdot (n+1)^2$ is even.

step 01: understand the claim.

what is even? if n is divisible by 2, then n is even.

If n is even, then n can be written as $2 \cdot k$

$n = 2 \cdot k$

Step 02: Let's do some examples

n	$n \cdot (n+1)^2$	is $n(n+1)^2$ even?
0	$0 \cdot 1^2 = 0$	✓
1	$1 \cdot 2^2 = 4$	✓
2	$2 \cdot 3^2 = 18$	✓
-2	$-2 \cdot (-1)^2 = -2$	✓
3	$3 \cdot 4^2 = 48$	✓

observe that n can be any integer.
Any integer has to be either an odd integer or an even integer.
Therefore, if we show that when is is odd $n \cdot (n+1)^2$ is even and when n is even is even $n \cdot (n+1)^2$ is even, then we can claim that our proposition is true.

Case 01: n is even.

statements	reasoning
$n = 2 \cdot c$	by def of even.
$n(n+1)^2 = 2 \cdot c \cdot (n+1)^2$	by substituition.
$c \cdot (n+1)^2$ is an integer	product of integers is an integer.
$n(n+1)^2 = 2 \cdot K_o$, $K_o = c(n+1)^2$	by rearranging.
$n(n+1)^2$ is even	by def. of even

Case 02: n is odd

statements	reasoning
n+1 is even	because n is odd.
$n+1 = 2 \cdot c$	by def. of even.
$n(n+1)^2 = n \cdot (2c)^2 = 4c^2 n$	by substituition
$n(n+1)^2 = 2 \cdot (2c^2 n)$	by rearranging
$2c^2 n$ is an integer	because product of integers in an integer
$n(n+1)^2$ is ~~odd~~ even	by the def. of even

Since n is either odd or even and in either case $n(n+1)^2$ is even, therefore our claim is true. $\square$.