10/04/2024

Relations

Def Sequence/List/Tuple

A sequence - also known as a list or tuple - is an ordered collection of objects, typically called components or entries. When the # of components in the collection is 2, 3, 4, or n, the sequence is called an (ordered) pair, triple, quadruple or n-tuple, respectively.

- we write sequences, Lists, tuples using angle brackets $\langle \ldots\ldots \rangle$

Ex: $\langle 1, 2, 6 \rangle$ - 3-tuple or triple

$\langle$Montana, Texas$\rangle$ - 2-tuple, pair

$\langle$USA, Canada, Mexico, Cuba$\rangle$ - 4-tuple, quadruple

$\langle 1, 2 \rangle \neq \langle 2, 1 \rangle$ | $\langle 0, 1, 2 \rangle \neq \langle 2, 0, 1 \rangle$

Also some people use paranthesis to represent tuples $(1, 2)$ or (Montana, Texas)

$\langle 1, 1 \rangle$ is possible

Def Cartesian product of two sets

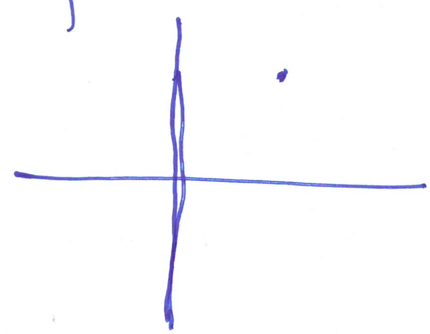
The cartesian product of two sets A and B is the set $A \times B = \{\langle a, b\rangle : a \in A \wedge b \in B\}$ containing all ordered pairs where the first element comes from set A and the second element comes from set B.

$A = \{0, 1, 2\}$ $\quad$ $B = \{2, 3\}$

$A \times B = \{\langle 0,2\rangle, \langle 0,3\rangle, \langle 1,2\rangle, \langle 1,3\rangle, \langle 2,2\rangle, \langle 2,3\rangle\}$

$B \times A = \{\langle 2,0\rangle, \langle 2,1\rangle, \langle 2,2\rangle, \langle 3,0\rangle, \langle 3,1\rangle, \langle 3,2\rangle\}$

Ex! $\mathbb{R} \times \mathbb{R}$ = 2D plane / cartesian plane



$\langle 0, 0\rangle \in \mathbb{R} \times \mathbb{R}$

$\langle 1.1112, 3.0123\rangle \in \mathbb{R} \times \mathbb{R}$

$\boxed{\mathbb{R} \times \mathbb{R} = \underline{\mathbb{R}^2}}$

$\{red, blue\} \times \{1, 2, 3\} = \{\langle red, 1\rangle, \langle red, 2\rangle, \langle red, 3\rangle, \langle blue, 1\rangle, \langle blue, 2\rangle, \langle blue, 3\rangle\}$

$\langle 1, red\rangle \notin \{red, blue\} \times \{1, 2, 3\}$

$\mathbb{R}^3 = 3D$ space

$\mathbb{R}^2 \times \mathbb{R} = \mathbb{R}^3$

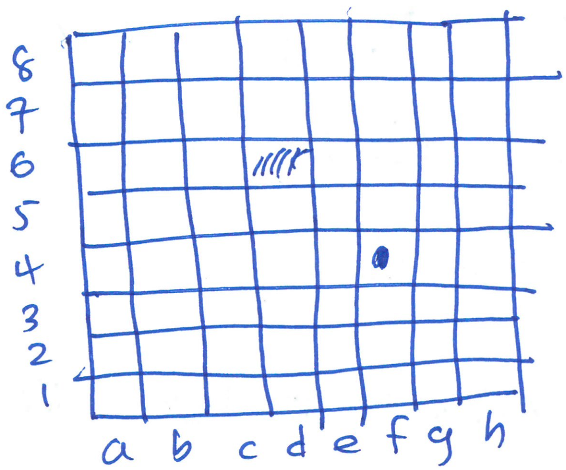
$\{0,1\} \times \{2,1\} = \{\langle 0,2\rangle, \langle 0,1\rangle, \langle 1,2\rangle, \langle 1,1\rangle\}$

$\{0,1\} \times \{2,1\} \times \{2,0\} = \{\langle 0,2\rangle, \langle 0,1\rangle, \langle 1,2\rangle, \langle 1,1\rangle\} \times \{2,0\}$

$= \{\langle 0,2,2\rangle, \langle 0,2,0\rangle, \langle 0,1,2\rangle, \langle 0,1,0\rangle, \langle 1,2,2\rangle, \langle 1,2,0\rangle, \langle 1,1,2\rangle, \langle 1,1,0\rangle\}$

They are functionally equivalent, but not exactly the same.

$(\{0,1\} \times \{2,1\}) \times \{2,0\} = \{\langle\langle 0,2\rangle,2\rangle, \langle\langle 0,2\rangle,0\rangle, \langle\langle 0,1\rangle,2\rangle, \langle\langle 0,1\rangle,0\rangle, \ldots\}$



d6

f4

$\{a,b,c,d,e,f,g,h\} \times \{1,2,3,4,5,6,7,8\}$

Q: What is the size of $|A \times B| = ?$

$$|A \times B| = |A| \cdot |B|$$

Def. Binary Relation

A Binary Relation R on sets A, B is a subset $R \subseteq A \times B$.

$\langle x, y \rangle \in R$ as xRy

$\langle x, y \rangle \notin R$ as $x \not R y$

Examples. Suppose P is the set of all people.

Then let us define the binary relation R_1 on set P, P, R_1 "___ is (blood) related to ___"

$R_1 \subseteq P \times P$

$R_1 = \{ \langle x, y \rangle : x \in P \wedge y \in P \wedge xRy \}$

$\langle$Serena Williams, Venus Williams$\rangle \in R_1$

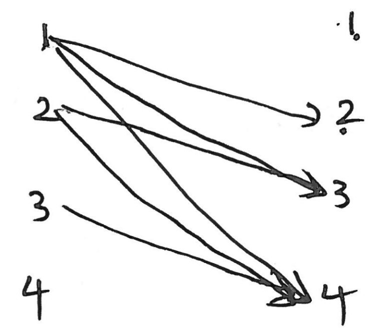
$\langle$Venus Williams, Serena Williams$\rangle \in R_1$

"Serena Williams is (blood) related to Venus Williams"

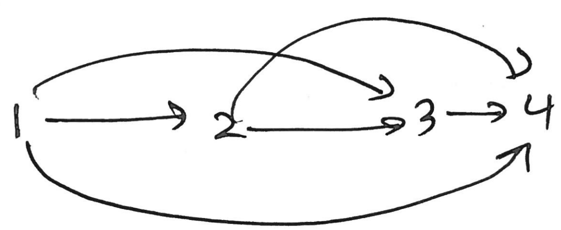
$\langle$Robin Williams, Serena Williams$\rangle \notin R_1$

(2) Let $A = \{1, 2, 3, 4\}$, then we define $<$ relation as " ___ is less than ___ "

$$< \subseteq A \times A$$



$$< = \{\langle 1,2\rangle, \langle 1,3\rangle, \langle 1,4\rangle, \langle 2,3\rangle, \langle 2,4\rangle, \langle 3,4\rangle\}$$



(3) Let $f: A \to B$, Let f be a function, Then let us define the set M as follows.

$$M = \{\langle a, f(a)\rangle : a \in A\}$$

is M a relation on sets A, B?

$$M \subseteq A \times B$$

for any <u>function</u> f, we can define a binary relation.

Let R be a binary Relation on

Sets A, B.

$$R \subseteq A \times B$$

$$R = \{ \langle x, y \rangle : x \in A \wedge y \in B \wedge x R y \}$$

question $f : A \to B$, $f(x) = y$ is this a function?

This not true

Look at the $<$ binary relation:

$\langle 1, 2 \rangle$, $\langle 1, 3 \rangle$ exists

$f(a)$ is not unique.