Recap

Propositional Logic

- Proposition / statement / claim
  - Any statement that is always true or always false.
- Propositions
  - → Atomic propositions
  - → Compound propositions
- Logical connectors
  - and ($\wedge$)
  - or ($\vee$) - "Inclusive or"
  - negation ($\neg$, $\sim$)

Given two propositions p, q

connector	compound proposition	
$\wedge$	$p \wedge q$	true; If p and q is true
$\vee$	$p \vee q$	true; If p or q is true.
$\neg$	$\neg p$	true; If p is false

- Any proposition can be denoted using a boolean variable.

p: Sky is blue
$\neg p$: Sky is not blue

Syntax Vs Semantic in propositional Logic

Syntax → whether given proposition is gramatically correct

Semantic → meaning of a gramatically correct statement

$1 \subseteq \mathbb{Z}$ (gramatically incorrect) → ✗

$1 \in \mathbb{Z}$ gramatically correct → 1 is an integer, true

$1 \notin \mathbb{Z}$ gramatically correct → 1 is not an integer, false

Def Implication ($\Rightarrow$) (This is a logical connector)

Given two propositions p and q, we can create compound proposition $p \Rightarrow q$ (denoted as "p implies q"), is true If the truth of p implies truth of q.

p - It rains in Bozeman.

q - Bozeman is wet.

$p \Rightarrow q$

If "It rains in Bozeman", then "Bozeman is wet"

In order for this compound proposition to be true, Bozeman must be wet, whenever It rains in Bozeman.

$p \Rightarrow q$

The p is called antecedent (or premise/hypothesis)

The q is called consequent (or the conclusion)

p: $1+1=2$

q: $2+3=6$

$r = (p \Rightarrow q)$

p is true but q is false

Therefore $p \Rightarrow q$ is false.

r: $1+1=3$ t: $2+2=4$

$r \Rightarrow t$ ✓

a: $1+1=2$ b: $1+3=4$

$a \Rightarrow b$ ✓

c: $1+1=6$ d: $2+3=10$

$c \Rightarrow d$ ✓

Remember: In an implication premise and conclusion do not need to be related in order for it to be true. What matters is whenever the ~~premsi~~ premise is true, the conclusion is true.

$$p \Rightarrow q \equiv \neg p \vee q$$

Def Exclusive or ($\oplus$)

Given two propositions p and q, the compound proposition $p \oplus q$ ("p exclusive or q", "p xor q"), is true, when one of p or q is true, but not both.

In other words, $p \oplus q$ is false, whenver p and q is true and p and q is false.

p: Alex is holding a cup of tea on his right hand

q: Alex is holding a cup of coffee on his right hand.

consider r: $p \oplus q$

- Alex is holding a cup of tea or a cup of coffee on his right hand, but not on both.
- Note that in English, it is very hard to distinguish between "inclusive or" and "exclusive or"

Def If and only If ($\Leftrightarrow$, Iff)

Given two propositions p and q, the compound proposition $p \Leftrightarrow q$ ("p if and only if") is true when the proposition p and q has the same truth ~~valt~~ value.

Also: $p \Leftrightarrow q$ can be written as

$$(p \Rightarrow q) \wedge (q \Rightarrow p)$$