09/06/2024

## Recap

- subset ($\subseteq$)
- $A \subseteq B$
- Set builder notation
- Ex: The~~e~~ set ODDs, which contains all the odd integers,

  ~~ODDS = {x : x mod 2 = 1}~~

  $ODDS = \{x : x \in \mathbb{Z} \text{ and } x \bmod 2 = 1\}$
- A union B ($A \cup B$)

**Question:**

not a set     set

$2 \cup \{1, 3\} = ?$ ← Does not make sense.

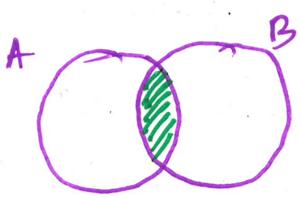
$\{2\} \cup \{1, 3\} = \{1, 2, 3\}$

**Def** Set intersection ($\cap$)

A intersection B, "$A \cap B$", the set that contains elements that appear in both A and B.

Ex: $\{2, 3, 4\} \cap \{1.5, 2, 3\} = \{2, 3\}$

$EVENS \cap ODDS = \phi = \{\}$




$A \cap A = A$

$\mathbb{R}^{\geq 0} \cap \mathbb{R}^{\leq 0} = \{0\}$

**Def** Disjoint sets

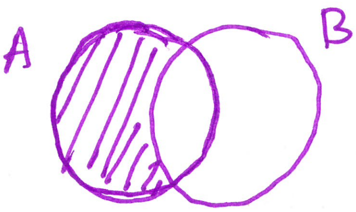
Sets A and B are called disjoint, if $A \cap B = \emptyset = \{\ \}$

is $\mathbb{R}^{\geq 0}$ and $\mathbb{R}^{\leq 0}$ disjoint? No

is EVENS and ODDs disjoint? Yes

**Def**

$A - B$ or $A \setminus B$, "A minus B", is the set of elements which are in Set A but not in B!"




$$A - B = \{y : y \in A \wedge y \notin B\}$$

$$\{2,4,6\} - \{2,3,4\} = \{6\}$$

$$\{2,3,4\} - \{2,4,6\} = \{3\}$$

EVENS − ODDS = EVENS

$A - B \subseteq A$ ✓ $\quad A - \emptyset = A$

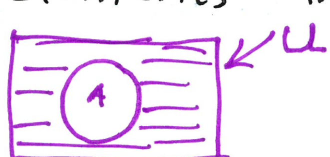
$A - \{\emptyset\} =$

$A = \{\emptyset, \{2\}, \{3\}\}$

**Def** Complement of Set A ( $\overline{\ }$ ) ~~compliment~~

The complement of set A, $\sim A$, $\overline{A}$, "A complement" is the set of all elements that are not in A.

$$\overline{A} = \{x : x \notin A\}$$



$\subseteq$

$\mathcal{U} = \{1, 2, 3, 4, 5, 6\}$

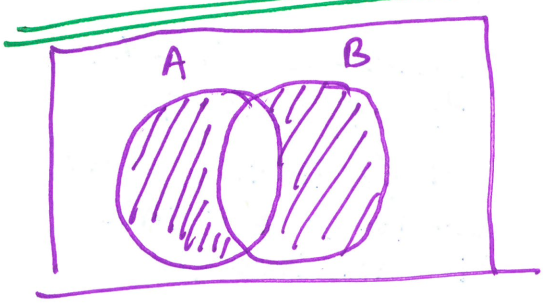
$A = \{2, 4, 6\}$

$\bar{A} = \{1, 3, 5\}$

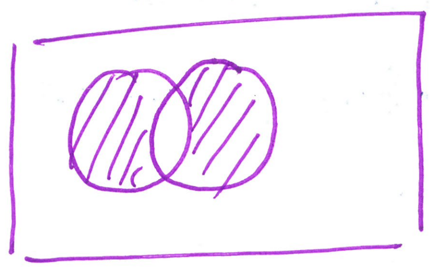
$\overline{\text{EVENS}} = \text{ODDS}$ $\qquad \mathcal{U} = \mathbb{Z}$

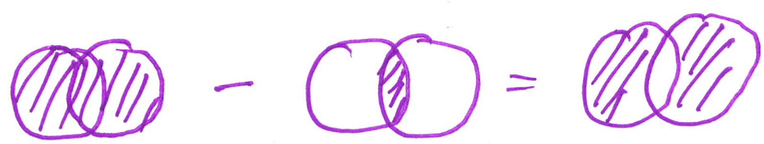
**Def** Exclusive OR ($\oplus$)

$A \oplus B$, "A exclusive or B" is the set of all elements that are either in A or B but not in both"




$A \cup B - A \cap B$





Exercise

claim: $\overbrace{\{x : x \in \mathbb{Z} \wedge 2 | x\}}^{A} \cap \overbrace{\{x : x \in \mathbb{Z} \wedge 9 | x\}}^{B} \subseteq \underbrace{\{a : a \in \mathbb{Z} \wedge 6 | a\}}_{C}$

Any element that is divisible by 2 and 9, is also divisible by 6.

claim: $A \cap B \subseteq C$

$A \cap B = \{x : x \in \mathbb{Z} \wedge 2 | x \wedge 9 | x\}$

If I say $X \subseteq Y$, means all elements of $X$ appears in $Y$.

If $X$ is a finite set, then we can go through each element and prove it exists in $Y$, But if $X$ is infinite we cannot do that. Hence, we pick an arbitrary element in $X$ and show that it can be an element of $Y$.

$$A \cap B \subseteq C$$

$$C = \{y : y \in \mathbb{Z} \wedge 6 \mid y\}$$

Strategy: pick arbitrary element $c$ in $A \cap B$, then show that $6 \mid c$ & $c \in \mathbb{Z}$.

Proof

Let us take an arbitrary element $c$ in $A \cap B$.

| statements | Reasoning |
|---|---|
| $c \in A \cap B$ | by assumption |
| $2 \mid c \wedge 9 \mid c \wedge c \in \mathbb{Z}$ | by def. of $\cap$ |
| $c = 18k \quad k \in \mathbb{Z}$ | because LCM of 9, 2 is 18 |
| $c = 6 \cdot (3 \cdot k), \quad 3 \cdot k \in \mathbb{Z}$ | by factoring and product of ints is an int. |
| $\vdots$ | |
| $6 \mid c \wedge c \in \mathbb{Z}$ | by def of divisibility |
| $c \in C$ | because $6 \mid c$ and $c \in \mathbb{Z}$ |

We proved that any arbitrary element of $A \cap B$ exists in $C$.

Therefore, claim is true $\blacksquare$.