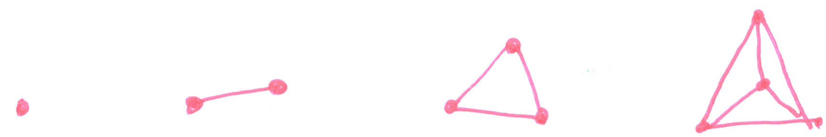
11/04/2024

Recap

- cliques or complete graphs.

Given $G = (V, E)$

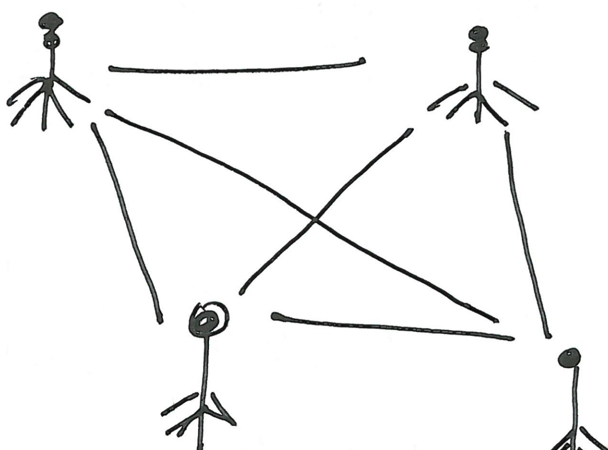
$$\forall u, v : u \neq v \Rightarrow \{u, v\} \in E$$



K_1 K_2 K_3 K_4

- Every node is connected to every other node

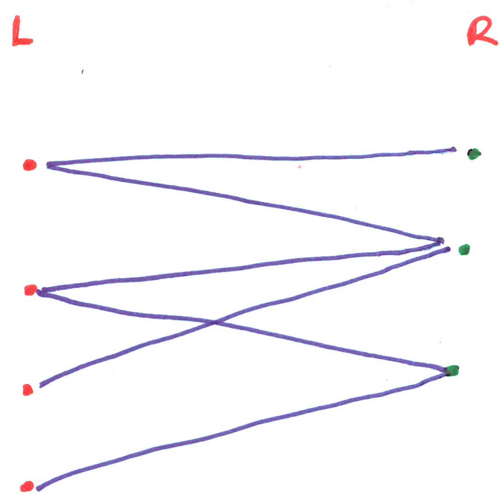
Ex:



Telecommunication towers.

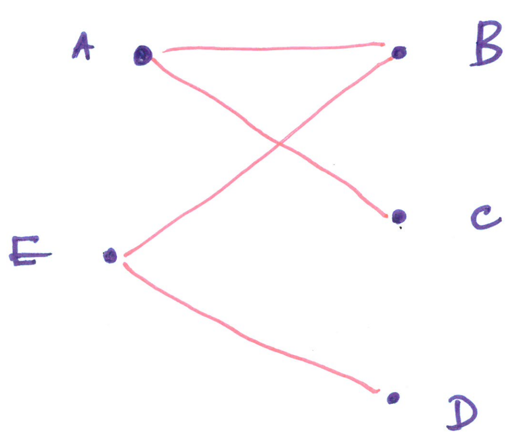
Def: Bi-partite graphs.

A graph G is called a Bi-partite graph, if $G = (L \cup R, E)$ s.t $L \cap R = \emptyset$ and $E \subseteq \{\{l, r\} : l \in L \wedge r \in R\}$



Basically, graph vertices should be able to partition into two sets such that these two sets are disjoint & all edges should go between nodes of L & nodes of R.

Ex:



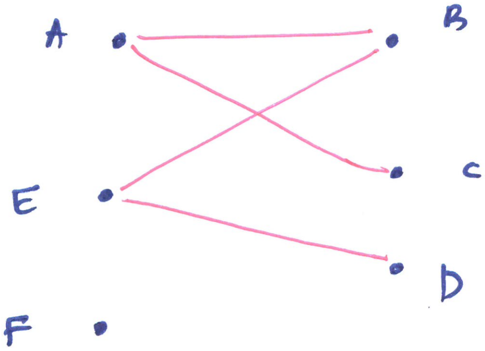
$V = \{A, B, C, D, E\}$

$L = \{A, E\}$

$R = \{B, C, D\}$

$L \cap R = \emptyset$ ✓

$E = \{\{A, B\}, \{A, C\}, \{E, B\}, \{E, D\}\}$



$L = \{A, E, F\}$

$R = \{B, C, D\}$

$L \cap R = \phi$

$L' = \{A, E\}$

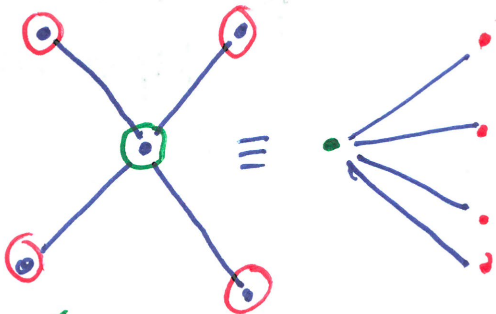
$R' = \{B, C, D, F\}$

$L' \cap R' = \phi$

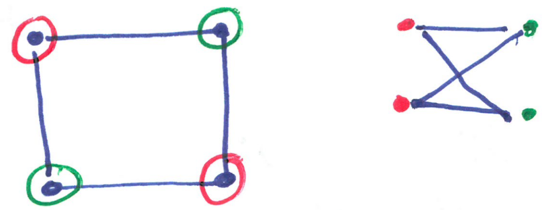
There no edges between the vertices in L' or R'

Popup test 08

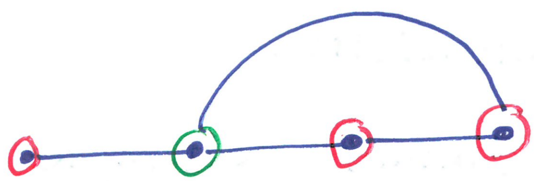
Determine the bipartite graphs.



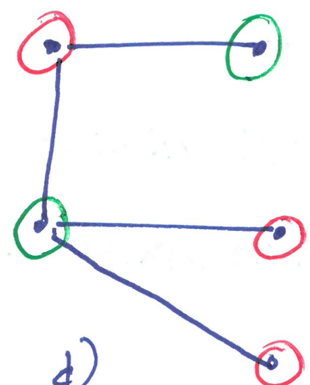
a)



b)

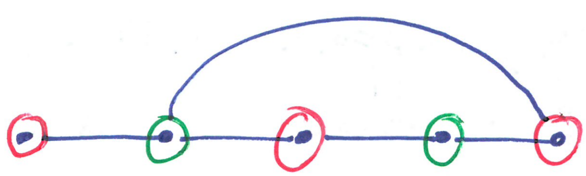


c) not-bipartite



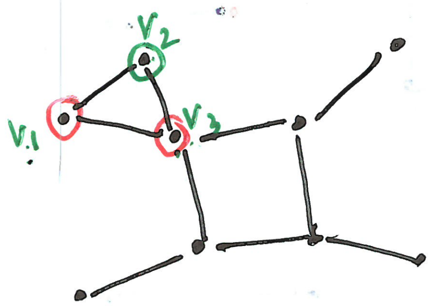
d)

e)



Claim: Let G be an undirected graph. If G contains a triangle, then it is not bipartite graph.

Ex:



G contains a traingle $\Longrightarrow$ G is <u>not</u> bipartite.

$p \Rightarrow q$

$\neg(p \Rightarrow q) \equiv p \wedge \neg q$

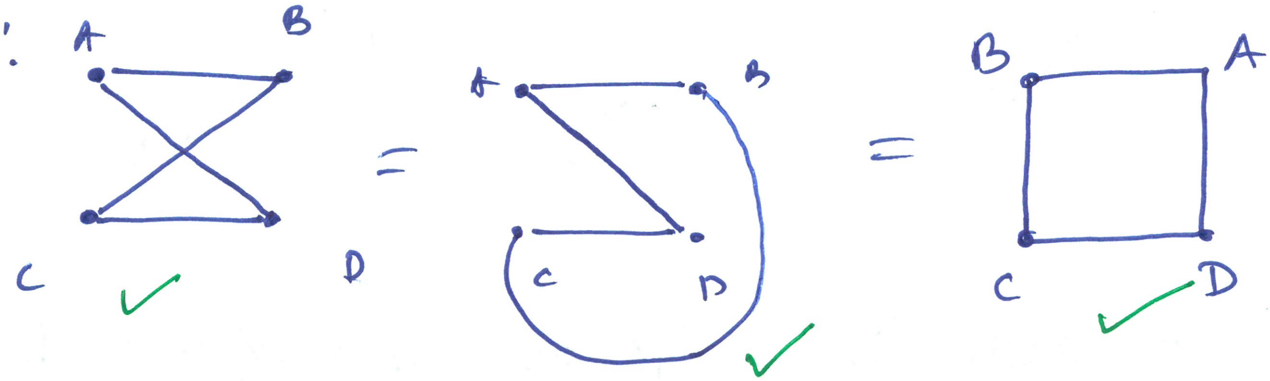
Proof: Aiming for contradiction.

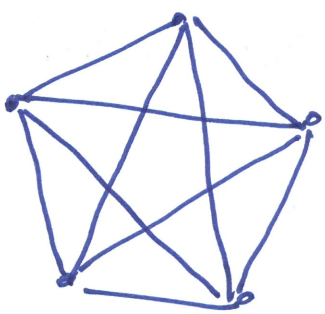
negation statement: G contains a triangle and G is bipartite.

Let v_1, v_2, v_3 be the nodes of the triangle. without loss of generality, suppose $v_1 \in L$, $v_2 \in R$. Since $v_2 \in R$, $v_3 \in L$. But there is an edge from v_1 to v_3 and ~~bothe~~ both of them are in L.

This is an contradiction. $\boxtimes$

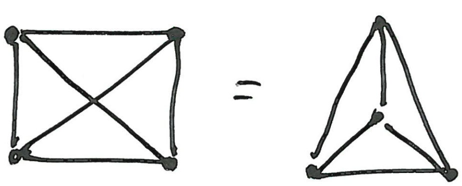
Def : A graph G is planar, if we can draw it in the 2D-plane without edge crossings.

Ex: 



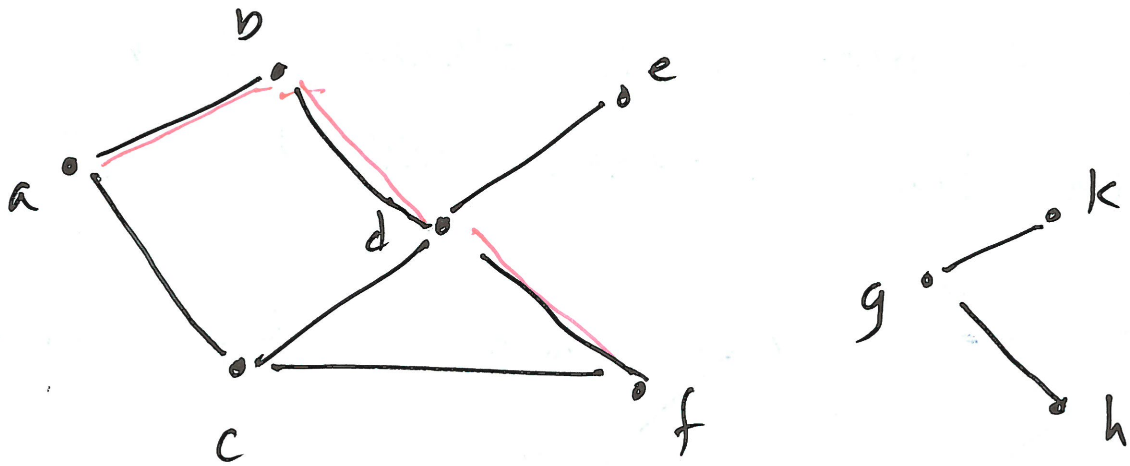
not planar.

K_4



✓ planar graph.

Ex:



Def: A path in a graph $G = (V, E)$ is a sequence of nodes $\langle u_1, u_2, u_3, \ldots, u_k \rangle$ s.t

- $\forall i \in \{1, 2, 3, \ldots, k\} : u_i \in V$
- $\forall i \in \{1, 2, 3, \ldots, k-1\} : \{u_i, u_{i+1}\} \in E$

Pink path from $a \to f$: $\langle a, b, d, f \rangle$

is $\langle a, b, e \rangle$ a path in this graph?