CSCI-246 Discrete Structures HW 10

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November 05 2024

Objective

- Understanding graph definitions.
- Understanding the problem solving process.
- Understanding how to solve a problem using graph based approach.

Submission requirements

- ***Type or clearly hand-write*** your solutions into a ***PDF FORMAT.***
- ***DO NOT UPLOAD images.***
- ***non-pdf or emailed solutions will not be graded.***
- **If you take pictures of your handwritten homework, put it into pdf format.**
- ***Start each problem on a new page.***
- Follow the model that you have learned during the lectures for proofs.
- Do not wait until the last minute to submit the assignment.
- You can submit any number of times before the deadline.
- If you are using latex, and you do not know how to type a symbol, use the following website. You can draw the symbol here and it will give you the latex code and the packages that you have to import. `https://detexify.kirelabs.org/classify.html`

- If you are using latex to write the answer, you can use overleaf to make your life easier. **Overleaf is a free, online platform that helps users create and publish scientific and technical documents using LaTeX, a markup-based document preparation system**
- If you do not understand a problem, ask questions during/after the lectures, or during office hours or via discord.
- Go to TA office hours and talk with them and ask for help.
- ***Do not use generative AI to write answers.***

Homework 02 contains **3 questions**.

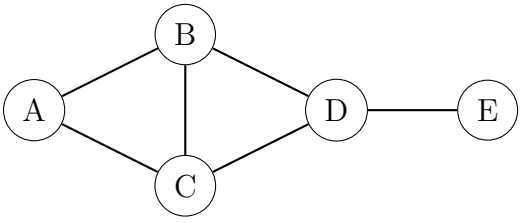
1 Q1

In the lecture on special graphs, we proved that if a graph contains a triangle, then it is not bipartite.

1. What is the converse of this statement?
2. What is the inverse of this statement?
3. Is the converse of the statement true or false?
4. Prove your answer to the previous question.

2 Q2

In the lectures, we learned that distance between two nodes is the length of the shortest path between them. The **Diameter** of a graph is the largest distance between any two nodes in the graph. For example, for the following graph, the diameter is 3, which is the distance between A and E. One of the shortest paths between A and E is (A, B, D, E).



1. What could be the largest diameter of a connected graph with n nodes can have and **explain your answer.**

2. What could be the smallest diameter that a connected graph with n nodes can have and explain the answer?

3. Draw the above two graphs for $n = 5$.

3 Q3

1. Suppose that Alice has exactly 160 friends, and each of her friends has exactly 160 friends—that is, a friend of Alice knows Alice and 159 **other** people. (Note that Alice's friends' sets of friends can overlap.) Let S denote the set of people that Alice knows directly or with whom Alice has a mutual friend. What's the largest possible value of $|S|$?. **Explain how you got this answer using graph based approach.**

   Hint: Think of these friendships in terms of a graph. You can create a node for each person and connect them with an undirected edge if they are friends. How can you create a graph such that the set S is maximized—If you answer this question then you can come up with an answer to this question?

2. For the set S defined above, what's the smallest possible value of $|S|$? Explain what type of graph would that be if people are defined as nodes and friendships are defined as an undirected edge.

3. Let u be a node in an undirected graph G. Prove that $u's$ degree is at most the sum of the degrees of $u's$ neighbors. Hint: use direct proof.