CSCI-246 Discrete Structures HW 6

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Objective

- Understanding functions
- Understanding Onto and One-To-One functions.
- Understanding how to prove a function is Onto and One-to-One.

Submission requirements

- ***Type or clearly hand-write*** your solutions into a ***PDF FORMAT.***
- ***DO NOT UPLOAD images.***
- ***non-pdf or emailed solutions will not be graded.***
- **If you take pictures of your handwritten homework, put it into pdf format.**
- ***Start each problem on a new page.***
- Follow the model that you have learned during the lectures for proofs.
- Do not wait until the last minute to submit the assignment.
- You can submit any number of times before the deadline.
- If you are using latex, and you do not know how to type a symbol, use the following website. You can draw the symbol here and it will give you the latex code and the packages that you have to import. `https://detexify.kirelabs.org/classify.html`

- If you are using latex to write the answer, you can use overleaf to make your life easier. **Overleaf is a free, online platform that helps users create and publish scientific and technical documents using LaTeX, a markup-based document preparation system**
- If you do not understand a problem, ask questions during/after the lectures, or during office hours or via discord.
- Go to TA office hours and talk with them and ask for help.
- ***Do not use generative AI to write answers.***

Homework 02 contains **3 questions**.

1 Q1

Let $f(x) = \frac{7x}{5}$. Moreover let $2\mathbb{Z}$ be the set of integers that are divisible by $c \in \mathbb{Z}$. For example:

$$2\mathbb{Z} = \{\ldots, -4, -2, 0, 2, 4, 6, \ldots\}$$
$$3\mathbb{Z} = \{\ldots, -6, -3, 0, 3, 6, 9, \ldots\}$$

1. Suppose f is a mapping defined as $f : \mathbb{Z} \to \mathbb{Z}$. Show that f is not a function.
2. Suppose f is defined as $f : 5\mathbb{Z} \to \mathbb{Z}$, then show that f is a function.
3. Suppose f is defined as $f : 5\mathbb{Z} \to \mathbb{Z}$, then show that f is **not a ONTO function.**

Hint 1: Use the 3 properties of a function that you learnt in the class. Show that one of the properties is violated by f.

Hint 2: If you want to show that f is a function, then show that f follows 3 properties of a function.

Hint 3: If you want to show that a given function $f : A \to B$ is not a ONTO function, then what you need to show is that $\exists b \in B : [\forall a \in A : [f(a) \neq b]]$. In other words, there is at least one element in the codomain, where there is no element in the domain that is mapped to it. Basically, pick an element in B that you can see there is no value that is mapped to it, and prove that there cannot be any element in the domain that can be mapped to it.

2 Q2

1. Define the $f : \{0, 1, 2, 3\} \to \{0, 1, 2, 3\}$ as $f(x) = x$. Is f onto?
2. Define the $f : \{0, 1, 2, 3\} \to \{0, 1, 2, 3\}$ as $f(x) = x^2 \ mod \ 4$. Is f onto?
3. Define the $f : \{0, 1, 2, 3\} \to \{0, 1, 2, 3\}$ defined as $f(x) = (x^2 - x) \ mod \ 4$? Is f onto?
4. Define the $f(x) = x^2 \ mod \ 8$ as a function $f : \{0, 1, 2, 3\} \to \{0, 1, 2, 3, 4, 5, 6, 7\}$, is f one-to-one?
5. Define the $f(x) = x^3 \ mod \ 8$ as a function $f : \{0, 1, 2, 3\} \to \{0, 1, 2, 3, 4, 5, 6, 7\}$, is f one-to-one?
6. Define $f(0) = 3$, $f(1) = 1$, $f(2) = 4$, and $f(3) = 1$. For this function $f : \{0, 1, 2, 3\} \to \{0, 1, 2, 3, 4, 5, 6, 7\}$, is f one-to-one?

3 Q3

Let $A = \{1, 2, 3, 4\}$, $B = \{6, 7, 8\}$, $C = \{4, 5, 6, 7\}$. Give an example of a function that satisfy:

1. An Onto function $f : A \to B$.
2. An one-to-one function $g : A \to C$.
3. A not Onto function $h : A \to C$.
4. A not one-to-one function $p : A \to C$.
5. A bijection (both onto and one-to-one) function $t : A \to C$.

Hint: A function does not need to have a neatly defined mathematical expression to qualify as a function. As long as it assigns each element in the domain to a unique value in codomain, it satisfies the definition of a function. One possibility of defining a function would be to create a table that maps every element in your domain to a unique element in the codomain.