CSCI-246 Discrete Structures
HW 12 – Final Homework

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Objective

- Understanding counting.
- Understanding asymptotic analysis.
- Understanding properties of Big O.
- Understanding algorithm analysis.
- Understanding problem solving process.

Submission requirements

- ***Type or clearly hand-write*** your solutions into a ***PDF FORMAT.***
- ***DO NOT UPLOAD images.***
- ***non-pdf or emailed solutions will not be graded.***
- **If you take pictures of your handwritten homework, put it into pdf format.**
- ***Start each problem on a new page.***
- Follow the model that you have learned during the lectures for proofs.
- Do not wait until the last minute to submit the assignment.

- You can submit any number of times before the deadline.
- If you are using latex, and you do not know how to type a symbol, use the following website. You can draw the symbol here and it will give you the latex code and the packages that you have to import. `https://detexify.kirelabs.org/classify.html`
- If you are using latex to write the answer, you can use overleaf to make your life easier. **Overleaf is a free, online platform that helps users create and publish scientific and technical documents using LaTeX, a markup-based document preparation system**
- If you do not understand a problem, ask questions during/after the lectures, or during office hours or via discord.
- Go to TA office hours and talk with them and ask for help.
- ***Do not use generative AI to write answers.***

1 Q1

If we roll a fair 3-sided die 11 times, what is the number of ways that we can get 4 1's, 5 2's, and 2 3's? Show your work.

2 Q2

1. Prove that $3n^4+4n^2-2n = O(n^4)$ by constructing $c > 0$, $n_0 \geq 0 : \forall n \geq n_0 : 3n^4+4n^2-2n \leq c \cdot n^4$.

3 Q3

In class we proved that $n^3 \neq O(n^2)$. Following that proof, fill the blanks, to show that $3n^4 + 4n^2 - 2n \neq O(n^3)$ by disproving $\exists c > 0, n_0 \geq 0 : \forall n \geq n_0 : 3n^4 + 4n^2 - 2n \leq c \cdot n^3$.

To disprove $\exists c > 0, n_0 \geq 0 : \forall n \geq n_0 : 3n^4 + 4n^2 - 2n \leq c \cdot n^3$, we need to show that: ____.

So we show how to construct ______, given any ______, and __________.

Let $c > 0$ and $n_0 \geq 0$. Let $n = max(n_0, c+1)$, then consider $n = c+1$.

$$3n^4 + 4n^2 - 2n = \text{fill in here and take as much as lines you need}$$

which is ______ $> c \cdot (c+1)^3$. Therefore, we have shown how to produce a $n \geq n_0$ such that $3n^4 + 4n^2 - 2n > c \cdot n^3$, for any c, n_0 value, meaning that $3n^4 + 4n^2 - 2n$ is not $O(n^3)$.

4 Q4

1. Give two functions, f and h, such that $f = O(n^3)$ and $h = O(2^n)$ but $f(n) \neq O(h(n))$. Note that you only need to pick f, h that follows the given restriction.
2. Now that you have picked f, h, prove that $f = O(n^3)$ and $h = O(2^n)$.
3. Use that same f, h functions and prove that $f(n) \neq O(h(n))$.