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Recap

predicate : A boolean valued function that maps elements in a set to the set {True, False}

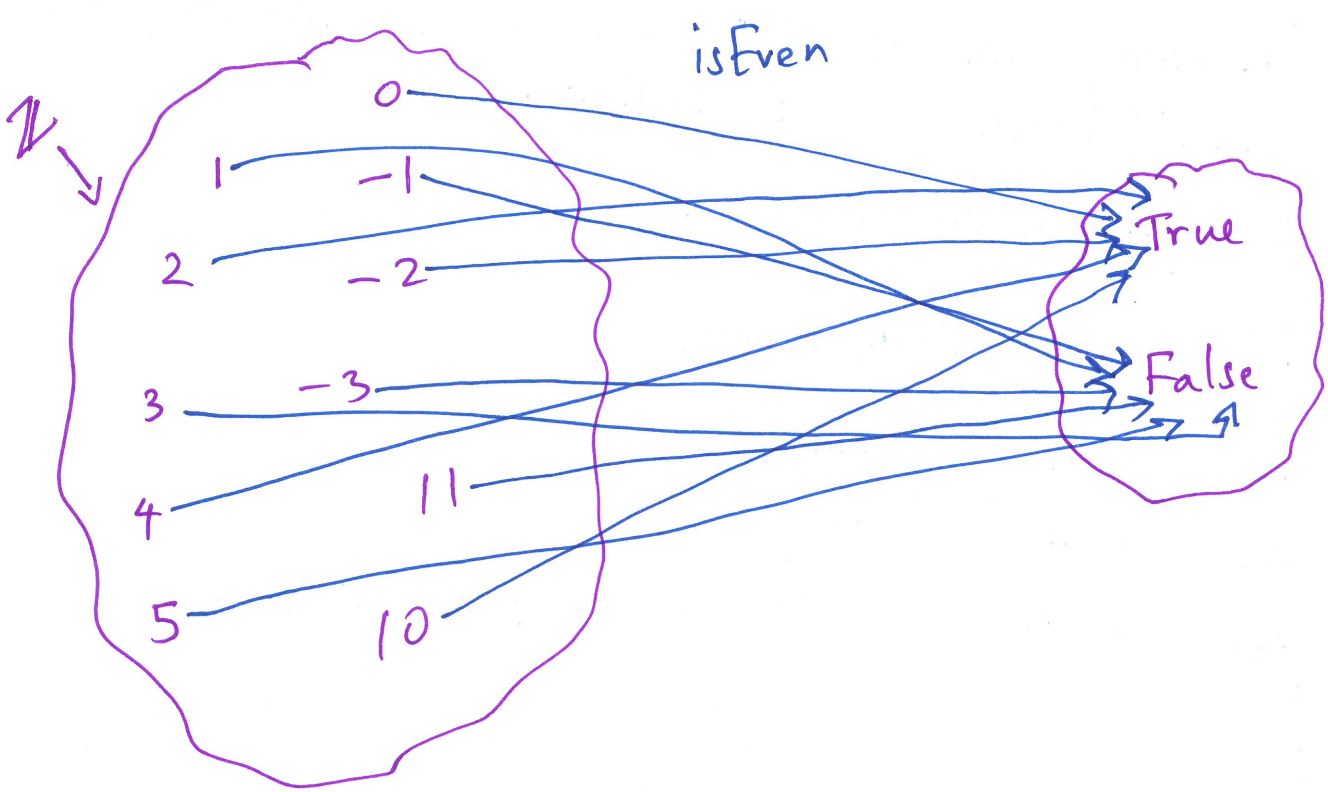
Ex: isEven : $\mathbb{Z} \longrightarrow$ {True, False}

- predicate itself is not a proposition.

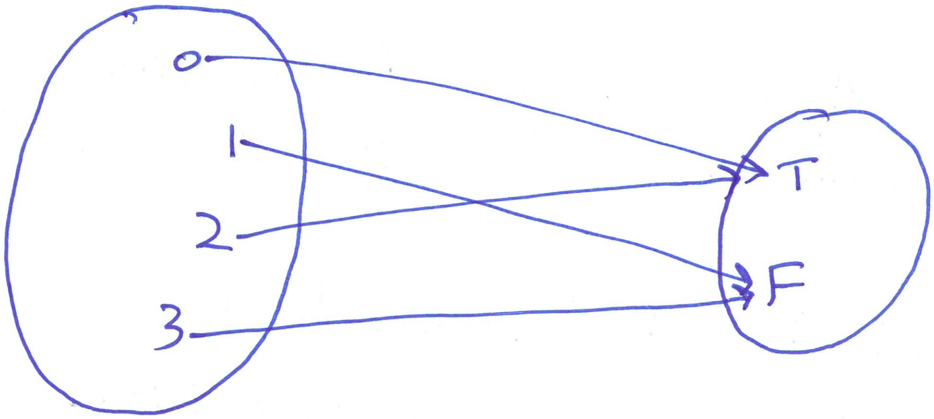
  isEven(n) is not a proposition.

  but

  isEven(1) is a proposition.



$isEven: \{0,1,2,3\} \longrightarrow \{True, False\}$



- predicate itself is not a proposition

- There are 2 ways you can create propositions from predicates.

1. Apply an element on a predicate, we can create a proposition.

$U = \{x_1, x_2, x_3, \ldots\ldots, x_n\}$

$P: U \longrightarrow \{True, False\}$

$P(x_1)$ - A proposition

$P(x_2)$ - " "

$P(x_3)$ - " " $\qquad P(x_2) \vee P(x_3)$

$\vdots$

$P(x_n)$ - " "

2. Using quantifiers.

If I want to say all of the individual propositions that I can create using the elements of the universe are true, I can write the following proposition.

$$\mathcal{U} = \{x_1, x_2, x_3, \ldots, x_n\}$$

$$P : \mathcal{U} \longrightarrow \{\text{True}, \text{false}\}$$

$$P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge \cdots \wedge P(x_n)$$

$$\equiv$$

$$[\forall x \in \mathcal{U} : P(x)]$$

This is called universal quantifier.

Now, what if I want to say at least ~~at~~ one of the propositions that I can create from the elements of $\mathcal{U}$ is true.

$$P(x_1) \vee P(x_2) \vee P(x_3) \vee \cdots \vee P(x_n)$$

$$\equiv$$

$$[\exists x \in \mathcal{U} : P(x)]$$

$$[\exists x \in \mathcal{U} : P(x)] \equiv P(x_1) \vee P(x_2) \vee P(x_3) \vee P(x_4) \vee \cdots \vee P(x_n)$$

Def. Theorems in predicate logic

A ~~#~~ fully quantified expression of predicate logic is a theorem if and only if it is true for every possible meaning of each of its predicates.

Thm 3.39

$$P : S \longrightarrow \{True, False\}$$

$$\forall x \in S : [P(x) \vee \neg P(x)]$$

$$S = \{x_1, x_2, x_3, \ldots\ldots, x_n\}$$

$$\forall x \in S : [P(x) \vee \neg P(x)] \equiv \underbrace{(P(x_1) \vee \neg P(x_1))}_{T} \wedge \underbrace{(P(x_2) \vee \neg P(x_2))}_{T} \wedge \overbrace{(P(x_3) \vee \neg P(x_3))}^{T} \wedge \ldots \wedge \underbrace{(P(x_n) \vee \neg P(x_n))}_{T}$$

$$\forall x \in S [P(x) \vee \neg P(x)] \equiv True$$

Non-Thorem 3.40

$P : S \longrightarrow \{T, F\}$

$$[\forall x \in S : P(x)] \vee [\forall x \in S : \neg P(x)]$$

$$\left[\forall x \in S : P(x)\right] \bigvee \left[\forall x \in S : \neg P(x)\right]$$

$$[P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge \cdots \; P(x_n)] \vee [\neg P(x_1) \wedge \neg P(x_2) \wedge \neg P(x_3) \wedge \cdots \wedge \neg P(x_n)]$$

$$\text{isEven} : \mathbb{Z} \longrightarrow \{T, F\}$$

$$\underbrace{[\underbrace{\text{isEven}(1)}_{F} \wedge \underbrace{\text{isEven}(2)}_{T} \wedge \cdots \; \text{isEven}(\infty)]}_{F} \vee \underbrace{[\underbrace{\neg\text{isEven}(1)}_{T} \wedge \underbrace{\neg\text{isEven}(2)}_{F} \wedge \cdots \wedge \neg \cdots}_{F}$$

Disproof by counter example. Original theorem is not true.

Theorem 3.41

$$\neg[\forall x \in S : P(x)] \Longleftrightarrow \exists x \in S : [\neg P(x)]$$

Theorem 3.42

$$\neg[\exists x \in S : P(x)] \Longleftrightarrow [\forall x \in S : \neg P(x)]$$

Theorem

$$[\forall x \in \emptyset : P(x)]$$