10/16/2024

Recap on Relations, Equivalence Relations

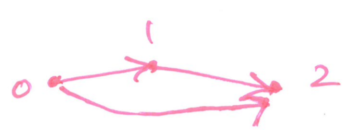
- Given ~~a~~ sets A, B, A binary Relation R is a ~~the~~ subset of $A \times B$

$$R \subseteq A \times B$$

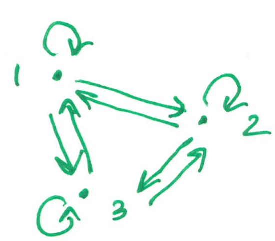
- Given a relation R on set A, we can define several properties

$$R \subseteq A \times A$$

  - Reflexive : $\forall a \in A : aRa$
  - irreflexive : $\forall a \in A : a \not R a$
  - Symmetric : $\forall a_1, a_2 \in A : a_1 R a_2 \Rightarrow a_2 R a_1$
  - Anti-symmetric : $\forall a_1, a_2 \in A : (a_1 R a_2) \wedge (a_2 R a_1) \Rightarrow (a_1 = a_2)$
  - transitive : $\forall a_1, a_2, a_3 \in A : (a_1 R a_2) \wedge (a_2 R a_3) \Rightarrow (a_1 R a_3)$



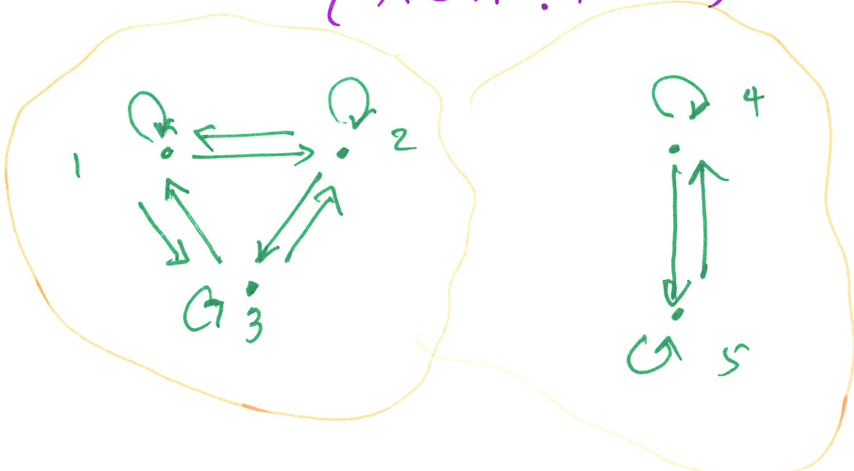
- Equivalence relations
A binary relation R on set A, is an equivalence relation, if R is reflexive, symmetric & transitive.



Equivalence class.

Given an equivalence relation $R_{\not\equiv}$ on Set A, the equivalence class of element $a \in A$

$$\{ x \in A : xRa \}$$



$[4] = \{4, 5\} = [5]$

$[1] = [2] = [3] = \{1, 2, 3\}$

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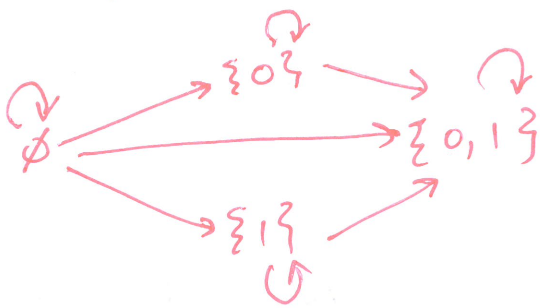
set $S = P(\{0,1\})$ $\boxed{2/2}$

$\subseteq$ relation (R_2)

$R_2 \subseteq S \times S$

$(A,B) \in R_2$, if $A \subseteq B$

$S = \{\phi, \{0\}, \{1\}, \{0,1\}\}$



$(\phi, \phi) \in R_2$ $\quad \phi \subseteq \phi$

$(\{0\}, \{0\}) \in R_2$ $\quad \{0\} \subseteq \{0\}$

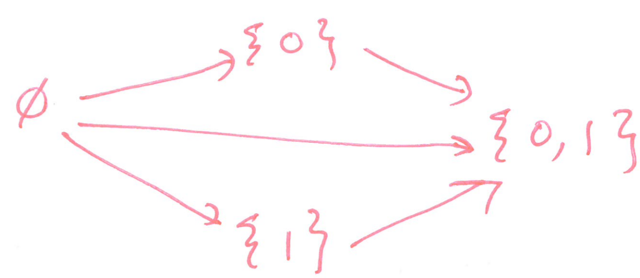
$(\phi, \{0\}) \in R_2$

$(\{0\}, \{1\}) \notin R_2$

$\subset$ relation (R_3)

$R_3 \subseteq S \times S$

$(A,B) \in R_3$, if $A \subset B$



$(\phi, \phi) \notin R_3$ $\quad \phi \not\subset \phi$

$(\phi, \{0\}) \in R_3$ $\quad \phi \subset \{0\}$

$\{0\} \subset \{0,1\}$

Def. A binary relation R on Set A is:

1. A partial order, if R is
   - Reflexive
   - transitive
   - anti-symmetric

   R_2 is a partial order

   R_3 <u>is not</u> a partial order

2. A strict partial order, if R is
   - Irreflexive
   - transitive
   - anti-symmetric

   R_3 is a strict partial order

   R_2 <u>is not</u> a strict partial order.

Def. A <u>partial order</u> is a <u>total order</u> if all pairs of different elements from A are comparable.

$$\forall a_1, a_2 \in A : (a_1 \neq a_2) \Rightarrow (a_1 R a_2) \vee (a_2 R a_1)$$

A <u>strict partial order</u> is a <u>strict total order</u> if all pairs of different elements from A are comparable.

$$\forall a_1, a_2 \in A : (a_1 \neq a_2) \Rightarrow (a_1 R a_2) \vee (a_2 R a_1)$$