CSCI-246 Discrete Structures HW 7

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Objective

- Understanding relations.
- Understanding properties of a relation.
- Understanding the problem solving process.

Submission requirements

- ***Type or clearly hand-write*** your solutions into a ***PDF FORMAT.***
- ***DO NOT UPLOAD images.***
- ***non-pdf or emailed solutions will not be graded.***
- **If you take pictures of your handwritten homework, put it into pdf format.**
- ***Start each problem on a new page.***
- Follow the model that you have learned during the lectures for proofs.
- Do not wait until the last minute to submit the assignment.
- You can submit any number of times before the deadline.
- If you are using latex, and you do not know how to type a symbol, use the following website. You can draw the symbol here and it will give you the latex code and the packages that you have to import. `https://detexify.kirelabs.org/classify.html`

- If you are using latex to write the answer, you can use overleaf to make your life easier. **Overleaf is a free, online platform that helps users create and publish scientific and technical documents using LaTeX, a markup-based document preparation system**
- If you do not understand a problem, ask questions during/after the lectures, or during office hours or via discord.
- Go to TA office hours and talk with them and ask for help.
- ***Do not use generative AI to write answers.***

Homework 02 contains **3 questions**.

1 Q1

Suppose you are given three relations R_1, R_2, R_3 defined on the set $S = \mathbb{P}(\{0, 1, 2, 3\})$. Note that $R_1, R_2, R_3 \subseteq S \times S$. For each of these relations determine whether:

- it is reflexive, irreflexive, or neither,
- it is symmetric, anti-symmetric, both, or neither,
- it is transitive or not, and
- all pairs of elements are comparable (that is, $\forall a \neq b \in S : (aRb \vee bRa)$).

1. $R_1 \subseteq S \times S$ such that $(A, B) \in R_1$ if (i) A and B are nonempty and the largest element in A equals the largest element in B, or (ii) if $A = B = \emptyset$.
2. $R_2 \subseteq S \times S$ such that $(A, B) \in R_2$ if the sum of elements in A is equal to the sum of elements in B. (Formally, $A\, R_2\, B \iff \sum_{x \in A} x = \sum_{y \in B} y$.)
3. $R_3 \subseteq S \times S$ such that $(A, B) \in R_3$ if $A \cap B \neq \emptyset$.

Hint: Write down the set S and $S \times S$. Then try to see which elements in $S \times S$ are related to each other under R_1, R_2, R_3 relations.

2 Q2

In this problem, you will learn some new definitions. Note that we can define the size of a relation to be the number of elements in the corresponding set. For example, consider the set $A = \{1, 2, 3, 4\}$ and the relation $R = \{\langle 2, 4\rangle, \langle 4, 3\rangle, \langle 4, 4\rangle\}$. We say that $|R| = 3$.

We define the closure of a relation R with respect to some property as the smallest relation that includes everything from R but also has the property. To be precise:

- The reflexive closure of a relation R is the smallest possible $R' \supseteq R$ such that R' is reflexive.
- The symmetric closure of a relation R is the smallest possible $R' \supseteq R$ such that R' is symmetric.
- The transitive closure of a relation R is the smallest possible $R' \supseteq R$ such that R' is transitive.

If R already has the property, then the closure is just R. For example, the reflexive closure of the $=$ relation on a set is just the $=$ relation, since $=$ is already reflexive.

For the set A and relation $R \subseteq A \times A$ given above, give the following:

1. The reflexive closure of R.
2. The symmetric closure of R.
3. The transitive closure of R.