10/30/2024

Let $e = \{u,v\}$ or (u,v)

- nodes u, v are adjacent or neighbours.

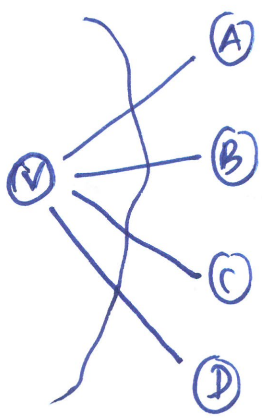
- In a directed graph, v is an out-neighbour of u & u is an in-neighborr of v.

- u, v are endpoints of e
- u, V are incident to e

Let v be a node in a simple undirected graph.

$$\text{degree}(v) = \deg(v) = d(v) = \#\text{ of neighbors of } v.$$

$$= \left| \{ u \in V : \{u,v\} \in E \} \right|$$

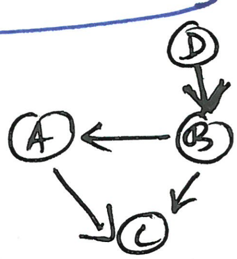


$\deg(v) = 4$

for directed graphs,

$\text{indeg}(v) = \#$ in-neighbors of v.

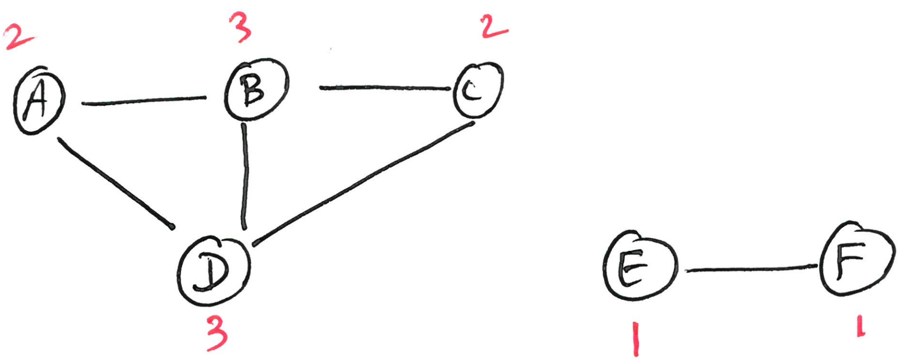
$\text{outdeg}(v) = \#$ out-neighbors of v.



$\text{indeg}(B) = 1$ $\text{outdeg}(B) = 2$

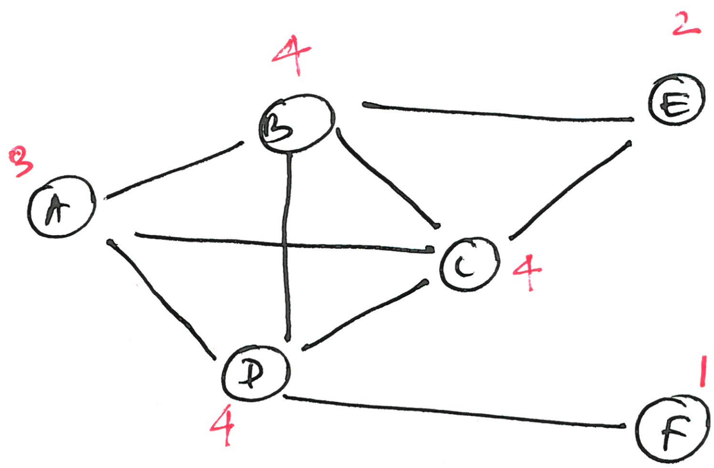
In class Activity

For each graph, label each node v with $\deg(v)$, and give $\sum_{v \in V} \deg(v)$, the total degree of the graph, and $|E|$, the number of edges in the graph.



$|E| = 6$

$\sum_{v \in V} \deg(v) = 12$



$|E| = 9$

$\sum_{v \in V} \deg(v) = 18$

$2 \cdot |E| = \sum_{v \in V} \deg(v)$

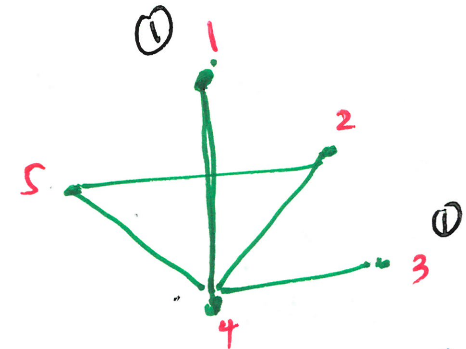
What do you think about the relationship between $\sum_{v \in V} \deg(v)$ & $|E|$

Theorem 11.8

Handshaking Lemma

Let $G=(V,E)$ be an undirected graph.

$$\sum_{v \in V} \deg(v) = 2 \cdot |E|$$



Proof: Let $G=(V,E)$ an undirected graph.
Notice that every edge is connected to exactly two nodes, meaning that it contributes 1 to the degree of two nodes.

Therefore $$\sum_{v \in V} \deg(v) = 2 \cdot |E|$$

$V = \{1,2,3,4,5\}$ $E = \{\{1,4\},\{2,4\},\{2,5\},\{3,4\},\{4,5\}\}$

$G=(V,E)$

Corollary → fact that follows simply from a previous lemma or theorem.

Let n_{odd} denote the # of nodes whose degree is odd. Then n_{odd} is even.

Proof: Aiming for a contradiction

Assume the negation of the claim is true.

n_{odd} is odd.

odd

sum of odd # odds is odd.

$$\underbrace{\sum_{v\in V} \deg(v)}_{\underbrace{2|E|}_{\text{even}}} = \sum_{x\in\{y\in V:\ \deg(y) \text{ is odd}\}} \deg(x) + \underbrace{\sum_{x\in\{z\in V:\ \cancel{\deg}\,\deg(z) \text{ is even}\}} \deg(x)}_{\text{even}}$$

$$V = \{y\in V: \deg(y) \text{ is odd}\} \cup \{z \in V: \deg(z) \text{ is even}\}$$

even = odd + even

This is a contradiction

we cannot create an even number by summing odd number and an even number.

Therefore, the initial claim is true $\blacksquare$.