11/13/2024

Ex: consider the random process of flipping 2 coins.

$S = \{H, T\} \times \{H, T\}$

$S = \{\langle H, H\rangle, \langle H, T\rangle, \langle T, H\rangle, \langle T, T\rangle\}$

$Pr[\langle H, H\rangle] = 1/4$

$Pr[s] = 1/4 = 0.25 : \forall s \in S$

Uniform probability.

Ex: Let $S = \{0, 1, 2, 3, 4, 5, 6, 7\}$

choose from S by flipping 7 coins & counting the number of heads.

$\langle H, H, H, H, H, H, H\rangle$ 7

$\langle T, H, H, H, H, H, H\rangle$ 6

$\langle T, T, T, T, T, T, T\rangle$ 0

$Pr[7] = 1/2^7 = 1/128$

$Pr[4] = \frac{35}{128}$

$S = \{H, T\} \times \{H, T\} \times \{H, T\} \times \{H, T\} \times \{H, T\} \times \{H, T\} \times \{H, T\}$

$|S| = 2^7$

$\langle H, H, H, H, T, T, T, T\rangle \to 4$ $\quad$ $\langle H, T, H, T, H, T, H, T\rangle \to 4$

Def. A set of outcomes is called an event.

$$E \subseteq S$$

$$Pr[E] = \sum_{s \in E} Pr[s]$$

Ex: When flipping 2-coins, what is the probability that at least one of them are heads.

$S = \{\langle H,H\rangle, \langle H,T\rangle, \langle T,H\rangle, \langle T,T\rangle\}$

$E =$ the set of outcomes that has at least one heads.

$E = \{\langle H,H\rangle, \langle H,T\rangle, \langle T,H\rangle\}$

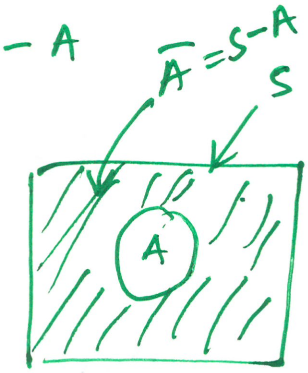
$E \subseteq S$

$$Pr[E] = Pr[\langle H,H\rangle] + Pr[\langle H,T\rangle] + Pr[\langle T,H\rangle]$$

$$= 0.25 + 0.25 + 0.25 = 0.75$$

Theorem 10.4 (properties of event probability)

Let S be your sample space and $A \subseteq S$, let $B \subseteq S$, and $\bar{A} = S - A$



$Pr[S] = 1 \qquad Pr[\emptyset] = 0$

$Pr[A] = 1 - Pr[\bar{A}]$

$Pr[\bar{A}] = 1 - Pr[A]$

Ex: when drawing 1 card from a 52-deck, What is the probability that the card we pick is <u>not</u> an Ace?

A – Picking an Ace from the Deck

$\bar{A}$ – picking a card that is not an Ace

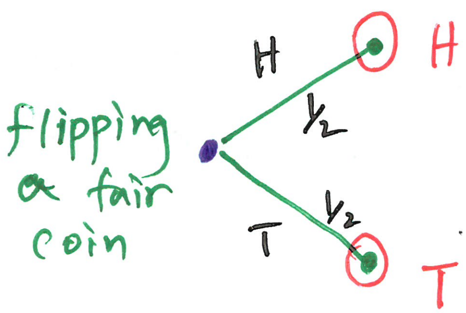
$Pr[A] = \frac{4}{52} \qquad Pr[\bar{A}] = \frac{48}{52}$

$$Pr[\bar{A}] = 1 - \frac{4}{52} = \frac{52-4}{52} = \frac{48}{52}$$

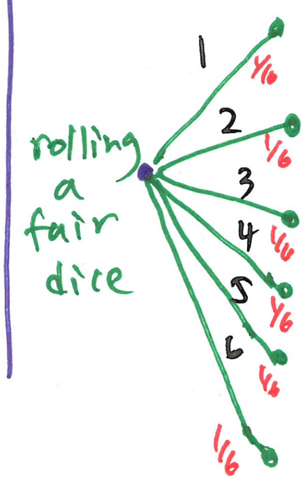
Tree Diagrams in probability.

- Internal nodes of a tree diagram represents random choice, labeled with the probability of each outcome.
- The leaves represents an outcome.

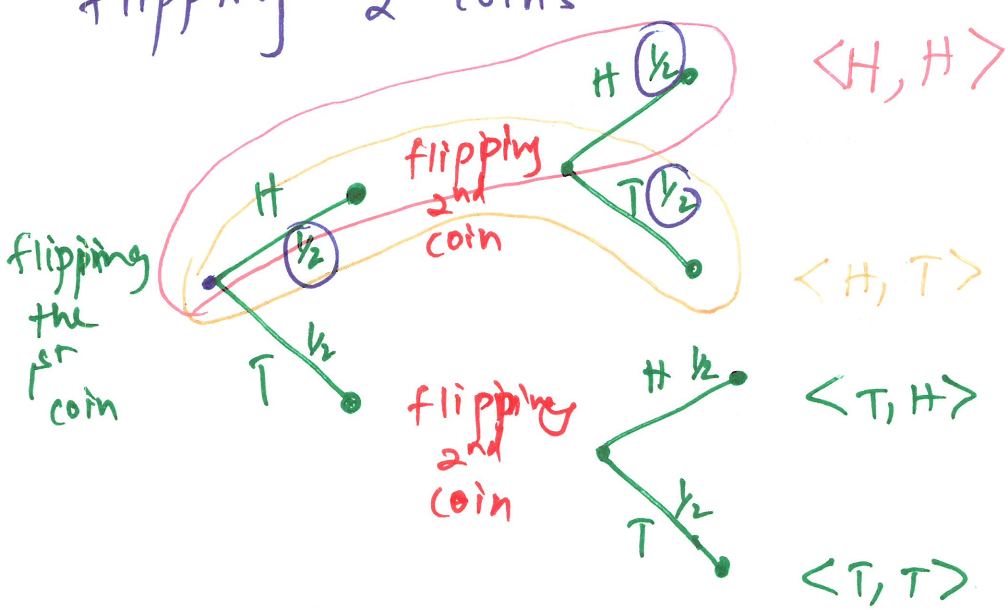
- Flipping a coin.



rolling a dice



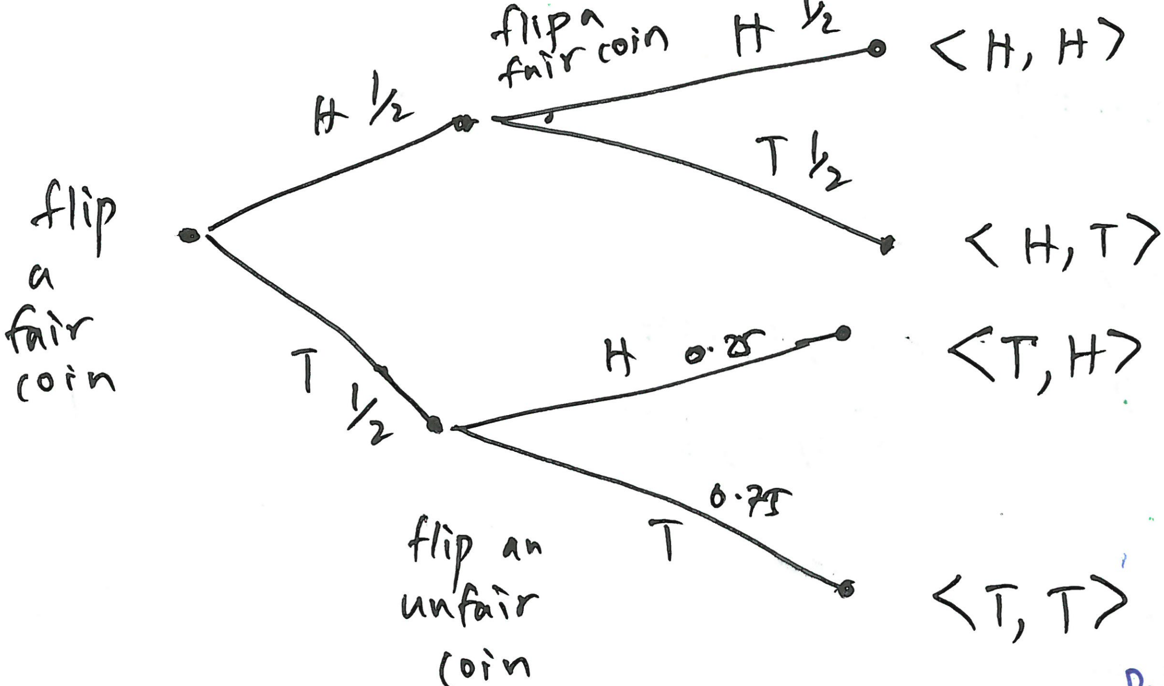
flipping 2 coins



$Pr[\langle H,H\rangle] = \frac{1}{2}\cdot\frac{1}{2}$

$Pr[\langle H,T\rangle] = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$

Ex! flip 1 fair coin, If I get "H" flip 2^{nd} fair coin. If I get "T", flip an unfair coin with 0.75 probability of having T.



$\Pr[\langle T,T\rangle] = \frac{1}{2}\cdot\frac{3}{4}$

$$\Pr[\langle T,T\rangle] = \frac{1}{2}\cdot\frac{3}{4} = \frac{3}{8}$$

$$\Pr[\text{at least one head}] = \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{4}$$

$$= \frac{4}{8} + \frac{1}{8} = \frac{5}{8}$$

Def A permutation of a set S is a length $|S|$ sequence of elements of S with no repititions.

ex: $S = \{1, 2, 3, 4\}$

✓ $\langle 1, 2, 3, 4 \rangle$

✓ $\langle 2, 3, 4, 1 \rangle$

✓ $(3, 2, 1, 4)$

$(2, 2, 1; 4)$ ✗

$(1, 2, 3)$ ✗

Theorem 9.8

Let S be a set with $|S| = n$. The number of permutations of S is $n!$ (n factorial)

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 1$$