~~Recap~~ Recap

Propositional Logic

- Proposition/statement/claim
  - Any statement that is always true or always false.
- Propositions
  - → Atomic propositions
  - → Compound propositions
- Logical connectors
  - and ($\wedge$)
  - or ($\vee$) - "Inclusive or"
  - negation ($\neg$, $\sim$)

Given two propositions p, q

connector	compound proposition	
$\wedge$	$p \wedge q$	true; If p and q is true
$\vee$	$p \vee q$	true; If p or q is true.
$\neg$	$\neg p$	true; If p is false

- Any proposition can be denoted using a boolean variable.

p: Sky is blue
$\neg$p: Sky is not blue

Syntax vs Semantic in propositional Logic

Syntax → whether given proposition is gramatically correct

Semantic → meaning of a gramatically correct statement

$1 \subseteq \mathbb{Z}$ (gramatically incorrect) → X

$1 \in \mathbb{Z}$ gramatically correct → 1 is an integer, true

$1 \notin \mathbb{Z}$ gramatically correct → 1 is not an integer, false

Def Implication ($\Rightarrow$) (This is a logical connector)

Given two propositions p and q, we can create compound proposition $p \Rightarrow q$ (denoted as "p implies q"), is true If the truth of p implies truth of q.

p - It rains in Bozeman.
q - Bozeman is wet.

$p \Rightarrow q$

If "It rains in Bozeman", then "Bozeman is wet"

In order for this compound proposition to be true, Bozeman must be wet, whenever It rains in Bozeman.

$$p \Rightarrow q$$

The p is called antecedent (or premise/hypothesis)
The q is called consequent (or the conclusion)

$p: 1+1 = 2$ $\quad$ $q: 2+3=6$

$r = (p \Rightarrow q)$

p is true but q is false

Therefore $p \Rightarrow q$ is false.

r: $1+1=3$ t: $2+2=4$

$r \Rightarrow t$ ✓

a: $1+1=2$ b: $1+3=4$

$a \Rightarrow b$ ✓

c: $1+1=6$ d: $2+3=10$

$c \Rightarrow d$ ✓

Remember: In an implication premise and conclusion do not need to be related in order for it to be true. What matters is whenever the ~~premsi~~ premise is true, the conclusion is true.

$$p \Rightarrow q \equiv \neg p \vee q$$

<u>Def</u> Exclusive or ($\oplus$)

Given two propositions p and q, the compound proposition $p \oplus q$ ("p exclusive or q", "p xor q"), is true, when one of p or q is true, but not both.

In other words, $p \oplus q$ is false, whenver p and q is true and p and q is false.

p: Alex is holding a cup of tea on his right hand

q: Alex is holding a cup of coffee on his right hand.

consider r: $p \oplus q$

- Alex is holding a cup of tea or a cup of coffee on his right hand, but not on both.
- Note that in English, it is very hard to distinguish between "inclusive or" and "exclusive or"

Def If and only If ($\Leftrightarrow$, Iff)

Given two propositions p and q, the compound proposition $p \Leftrightarrow q$ ("p if and only if") is true when the proposition p and q has the same truth ~~va~~ value.

Also: $p \Leftrightarrow q$ can be written as

$$(p \Rightarrow q) \wedge (q \Rightarrow p)$$

$n \in \mathbb{Z}$

P: The integer n is divisible by 4

Q: The integer n such that n^2 is divisible by 16.

claim $P \Leftrightarrow Q$

P if and only if Q ✓ True.

$P \Leftrightarrow Q$ means

If P, then Q and If not P, then not Q

If It rains in Bozeman, then Bozeman is wet.

If I properly set my alarm, then I will make up at correct time. (Ambiguous statement)

If the moon is made out of cheese, then $2+2=4$ (True)

I will wake up at correct time if and only if I properly set my alarm.

Truth Tables (TT)

Def A truth table (TT) for a given proposition lists, for each possible truth assignment for that proposition (with one truth assignment per row), the truth value of the entire proposition.

$$(p \vee r) \wedge (p \vee q)$$

\# variables $= 3$

\# unique truth assignments for 3 variables $= 2 \times 2 \times 2$
$= 8$

If a proposition has n variables, then the number of unique truth assignments would be 2^n.

Ex: Given two proposition p and q, let's consider several compound proposition that uses p and q.

p	q	$p \vee q$	$p \wedge q$	$p \oplus q$	$p \Rightarrow q$	$p \Leftrightarrow q$	$\neg(p \oplus q)$	$\neg p$	$\neg p \vee q$
F	F	F	F	F	T	T	T	T	T
F	T	T	F	T	T	F	F	T	T
T	F	T	F	T	F	F	F	F	F
T	T	T	T	F	T	T	T	F	T

$$p \Leftrightarrow q \equiv \neg(p \oplus q)$$

$$p \Rightarrow q \equiv \neg p \vee q$$

Def Tautology

A proposition is a tautology if it is true under every possible truth assignment

Ex: Is $p \Rightarrow p$ a tautology?

\# variables $= 1$

\# unique truth assignments $= 2^1 = 2$

p	$p \Rightarrow p$
F	T
T	T

$p \Rightarrow p$ is a tautology

Is $\underbrace{p \wedge (p \Rightarrow q)}_{\text{True}} \Rightarrow q$ a tautology?

\# variables $= 2$

\# unique truth assignments $= 2^2 = 4$

p	q	$p \Rightarrow q$	$p \wedge (p \Rightarrow q)$	$\overbrace{p \wedge (p \Rightarrow q)}^{\text{Premise}} \Rightarrow \overbrace{q}^{\text{conclusion}}$
F	F	T	F	T
F	T	T	F	T
T	F	F	F	T
T	T	T	T	T

This is a tautology

Ex: $(p \vee q) \wedge (p \vee r) \wedge (p \oplus q)$

\# variables $= 3$

\# unique truth assignments $= 2^3 = 8$

p	q	r	$p \vee q$	$p \vee r$	$p \oplus q$	$(p \vee q) \wedge (p \vee r) \wedge (p \oplus q)$
F	F	F	F	F	F	F
F	F	T	F	T	F	F
F	T	F	T	F	T	F
F	T	T	T	T	T	T
T	F	F	T	T	T	T
T	F	T	T	T	T	T
T	T	F	T	T	F	F
T	T	T	T	T	F	F

This is <u>not</u> a tautology