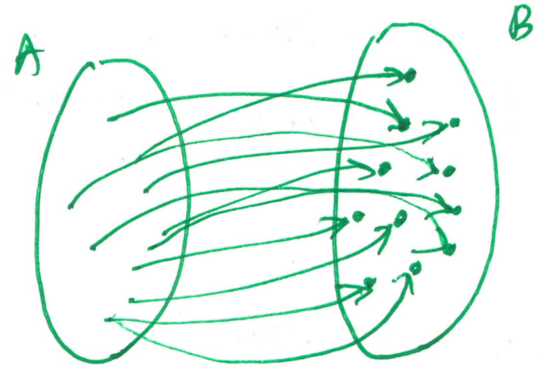
04/30/2024

Recap: onto function:

Given a <u>function</u> $f : A \longrightarrow B$

f is onto if

$$\left[\forall b \in B : \left[\exists a \in A : f(a) = b \right] \right]$$

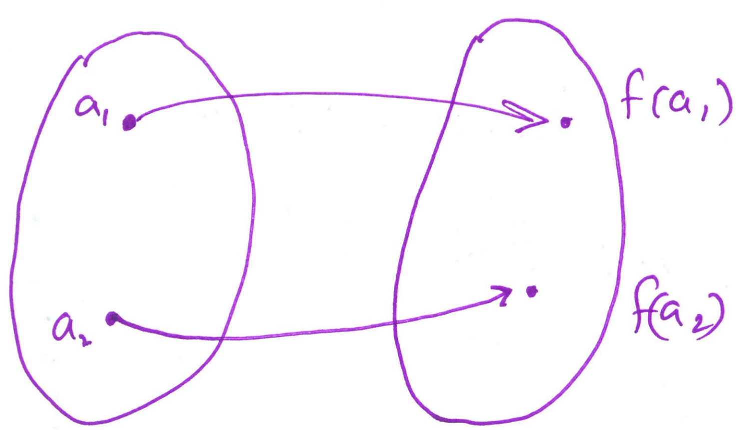


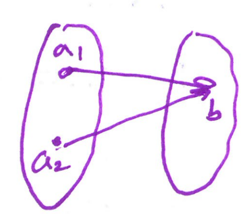
one-to-one functions:

Given a <u>function</u> $f : A \longrightarrow B$

f is 1:1 if

$$\left[\forall a_1, a_2 \in A : a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2) \right]$$





How to prove a function $f : A \to B$ is a <u>not onto</u> function.

If the function f is <u>onto</u>:

$$\left[\forall b \in B : [\exists a \in A : f(a) = b] \right].$$

Then if the function f is <u>not</u> onto the negation of earlier proposition must be true.

$$\neg \left[\forall b \in B : [\exists a \in A : f(a) = b] \right]$$

Note that

$$\neg [\forall x \in S : P(x)] \equiv [\exists x \in S : \neg P(x)]$$

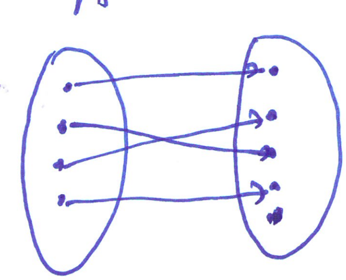
Using this theorem.

$$\equiv \left[\exists b \in B : \neg [\exists a \in A : f(a) = b] \right]$$

Note that

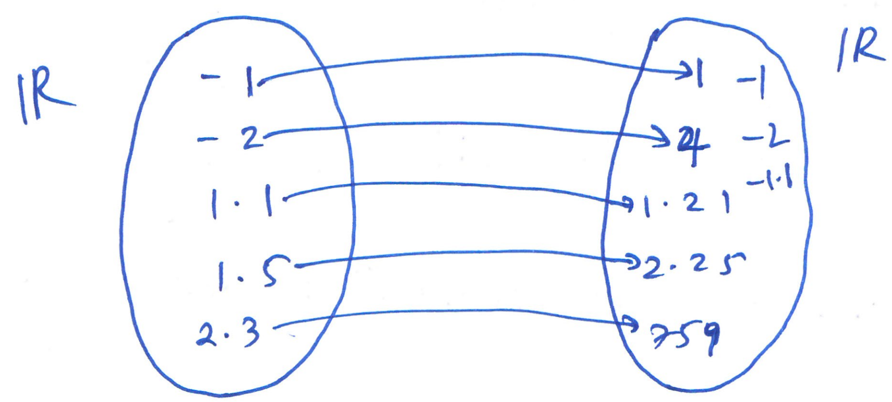
$$\neg [\exists x \in S : P(x)] \equiv [\forall x \in S : \neg P(x)]$$

$$\equiv \left[\exists b \in B : [\forall a \in A : f(a) \neq b] \right]$$



prove; $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$ is <u>not a onto function</u>.

WTS: $[\exists b \in \mathbb{R} : [\forall a \in \mathbb{R} : f(a) \neq b]]$



23
23
69
69
75 4

~~b=1 b=-1~~

Let $b = -1$,

We want to show that $[\forall a \in \mathbb{R} : f(a) \neq b]$

$A = \mathbb{R}$

$\forall a \in A : f(a) = a^2$ by def of f.

$\forall a \in A : a^2 \geq 0$ by the property of squares

$\forall a \in A : a^2 \neq -1$ because $-1 < 0$

$\forall a \in A : f(a) \neq b$ by sustituition.

We proved an existance of an element ~~b∈R~~ $b \in \mathbb{R}$, such that $[\forall a \in A : f(a) \neq b]$

$\therefore$ function f is <u>not</u> <u>onto</u>. ▫

How to prove a function f is <u>not</u> 1:1.

Let $f : A \to B$

If function f is 1:1, then

$$\left[\forall a_1, a_2 \in A : a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2) \right]$$

If function f is <u>not</u> 1:1, then

$\neg \left[\forall a_1, a_2 \in A : a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2) \right]$ must be true.

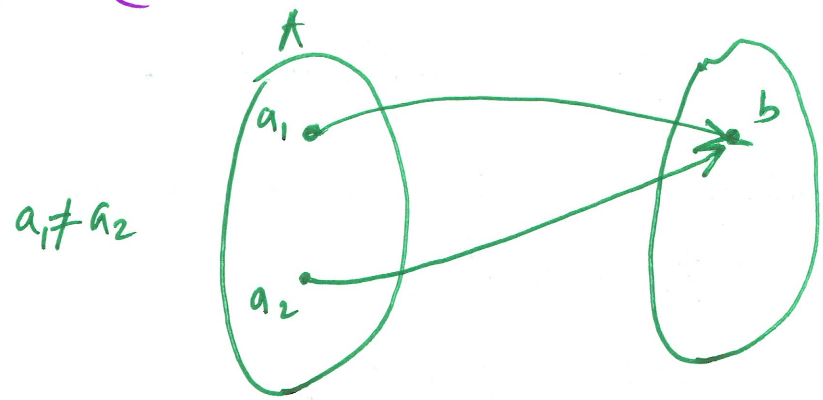
using predicate theorem

$$\neg \left[\forall x \in S : P(x) \right] \equiv \left[\exists x \in S : \neg P(x) \right]$$

$$\equiv \left[\exists a_1, a_2 \in A : \neg \left(a_1 \neq a_2 \Rightarrow \left(f(a_1) \neq f(a_2) \right) \right) \right]$$

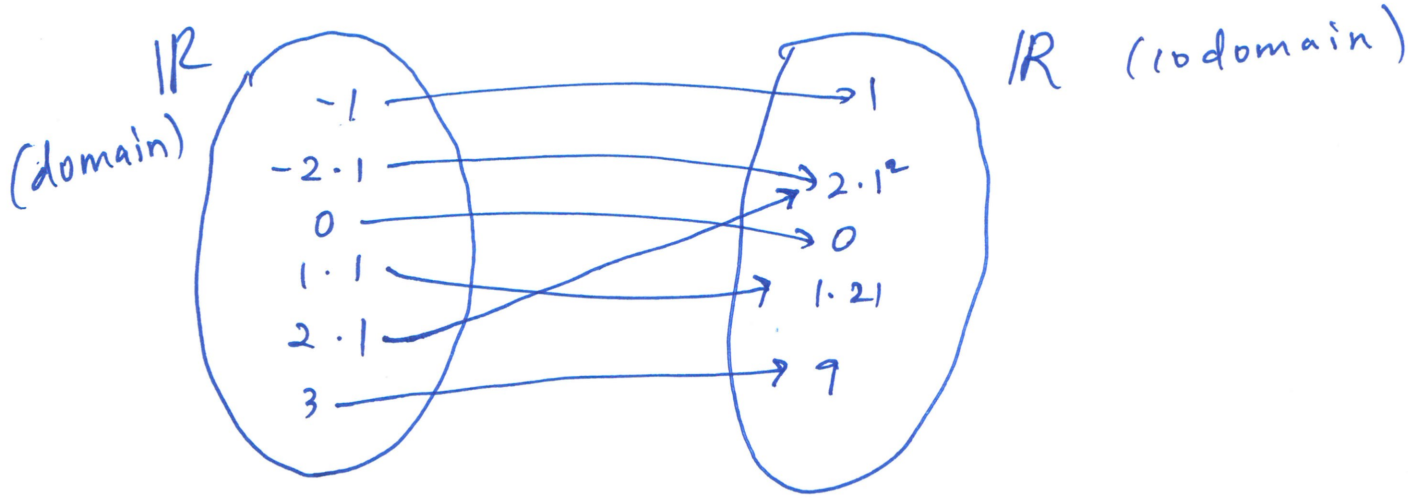
$$\neg (p \Rightarrow q) \equiv p \wedge \neg q$$

$$\equiv \left[\exists a_1, a_2 \in A : (a_1 \neq a_2) \wedge (f(a_1) = f(a_2)) \right]$$



Prove: function $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$ is <u>not</u> 1:1.

WTS: $[\exists a_1, a_2 \in \mathbb{R} : (a_1 \neq a_2) \wedge (f(a_1) = f(a_2))]$



Proof: consider $a_1 = -2.1$, $a_2 = 2.1$

$f(a_1) = 2.1^2$, $f(a_2) = 2.1^2$

$(a_1 \neq a_2) \wedge (f(a_1) = f(a_2))$

We showed the existance of two distinct elements in domain that is mapped to the same element in the codomain.

$\therefore$ function f is <u>not</u> 1:1. ☒

Popup test 4

- form groups of 3.
- write your names.

Let $A = \{1, 2, 3\}$ $\quad B = \{3, 4\}$

Give example of function that satisfy:

1. an Onto function $f: A \longrightarrow B$

2. A function $g: A \longrightarrow B$ that is <u>not</u> onto.

3. a function $h: B \longrightarrow A$ that is <u>onto</u>