Lecture 02

Direct proofs and disproof by counter example

Def
A proposition (statement/claim) is statement that is either true or false.

And for a proposition, its truth value is its truth of falsity!

	Proposition?	True or false
1. $2+2=4$	Y	T
2. 33 is a prime number	Y	F
3. All swans are white	Y	F
4. $\sqrt{2}$ is a rational number	Y	F
5. Is sky blue?	N	–
6. $1+2+3=x$	N	–
7. close the door!	N	–
8. The fastest comparison based sorting algorithm has worst case running time of $O(n \log n)$	Y	T
9. There are an infinite # of perfect numbers	Y	?

(A perfect # is a positive integer that is equal to its proper divisors)

Def: Atomic and compound propositions.

An atomic proposition is a proposition that is conceptually indivisible.

A compound proposition is a proposition that is built out of conceptually simpler propositions.

Ex: MSU's mascot is a bobcat.

MSU's mascot is a bobcat or UM's mascot is a grizz.

Logical connectors.

Logical connectors are glue that creates more complicated compound propositions, from simpler propositions.

3 major logical connectors.

- negation [not, $\neg$]
- conjunction [and, $\wedge$]
- disjunction [or, $\vee$]

Negation [not, P,]

The proposition $\neg p$ is true when the proposition p is false

p - MSU's mascot is a wolverine (F)

$\neg p$ - MSU's mascot is not a wolverine (T) conjunction [and, $\wedge$]

The proposition $p \wedge q$ ("p and q", "conjunction of p and q") is true when both p and q are true, false otherwise.

p - MSU's mascot is a bobcat (T)
q - UM's mascot is a Grizz. (T)

$p \wedge q$: MSU's mascot is a bobcat and UM's mascot is a Grizz. (T)

r - Sky is green (F)

$p \wedge r$: (F)

Disjunction [or, $\vee$]

The proposition $p \vee q$ ["p or q", "disjunction of p or q"] is true if either of p and q is true, it is false if ~~bothe~~ both p and q are false.

$p \vee q$ (T)

$p \vee r$

s: sky is red (F)

$r \vee s$: F

Def:

A proof of a proposition is a convincing arguement that the proposition is true.

A disproof is an arguement that a propos-ition is false.

If $\underbrace{x, y \text{ is rational}}_{p}$ then $\underbrace{x \cdot y \text{ is } \cancel{\text{also}} \text{ rational}}_{q}$.

Step 01:

a rational number is a number that can be written as $\frac{n}{d}$ where n, d are integers and $d \neq 0$

$10 \, ? \, \frac{10}{1}$

$-10 \, ? \, -\frac{10}{1}$ or $\frac{10}{-1}$

π X

$\sqrt{2}$ X

step 02: try some exaples

x	y	$x \cdot y$	x, y ?	$x \cdot y$?
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	✓	✓
$\frac{1}{2}$	$\sqrt{2}$	$\frac{1}{\sqrt{2}}$	X	X

Proof

Assume x, y ~~is~~ are rational

statements	reasoning
$x = \frac{n_x}{d_x}$ $y = \frac{n_y}{d_y}$ n_x, n_y, d_x, d_y are integers and $d_x, d_y \neq 0$	By the def. of rational #.
$xy = \frac{n_x \cdot n_y}{d_x \cdot d_y}$	Prod. of integers.
$xy = \frac{n_x \cdot n_y}{d_x \cdot d_y} = \frac{n}{d}$, $n = n_x \cdot n_y$ $d = d_x \cdot d_y$ ~~n is an~~ n, d is integers and $d \neq 0$	Prod. of integers are integers.
$xy = \frac{n}{d}$, xy is rational	By def of rational #

$\square$