08/30/2024

Sets

Def

A set is an unordered collection of distinct objects (elements).

We denote sets using capital letters.

$D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ has 10 elements

$\text{bits} = \{0, 1\}$ has 2 elements

$\mathbb{Z}$ = set of integers

$\mathbb{Z} = \{\ldots\ldots, -2, -1, 0, 1, 2, 3, \ldots\ldots\}$ has infinite number of elements.

$\mathbb{Q}$ = set of all rational numbers.

$\mathbb{R}$ = set of all real numbers.

$T = \{\text{"Apple"}, \square, 2, x\}$

$C = \{0, 1, 0\}$ ← not a set.

Def

Two sets A and B are equal ~~(dentoe~~ (denoted as $A=B$), if A and B contains exactly the same elements.

$A = \{0, 1, 2\}$ $B = \{1, 2, 3\}$

$A \neq B$

$\{0, 1\} = \{1, 0\}$

Def Set Membership

For a set S and an object x, the expression $x \in S$ is true when x is one of the objects contained in set S.

$$x \in S$$

"x is an element of S"

$$x \notin S$$

"x is not an element of S"

$0 \in \{0, 1\}$ ✓

$2 \notin \{1, 0\}$

$\pi \notin \mathbb{Z}$

Def Cardinality ~~of~~ or Size of the set S, is the number of distinct elements in S.

$$\boxed{|S|}$$

bits $= \{0, 1\}$

$|\text{bits}| = 2$

$$\left| \{2, \text{"Apple"}, \{2, 3\}\} \right| = 3$$

Question: Can we have a set such that $|S| = 0$? Yes, the empty set

Empty set is represented by $\{\}$ or ϕ

$|\phi| = 0$

$|\{\}| = 0$

$|\{\phi\}| = 1$

$\cancel{\{\phi, \{\phi\}\}}$

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Q: If $A = B$, can I say $|A| = |B|$? True ✓

How about,

If $|A| = |B|$, can we say $A = B$?

counter Example

$A = \{0\}$ $B = \{1\}$

$|A| = |B|$ but $A \neq B$

Def: Set builder notation

Set builder notation define a set as follows:

The set $S = \{x : \text{a rule about } x\}$

The set S contains element x, such that x follows the rule given.

Example:

set of integers

$$EVEN = \{x : x \in \mathbb{Z} \text{ and } 2 \mid x\}$$

$$EVEN = \{y : y = 2c \text{ for } c \in \mathbb{Z}\}$$

$$\mathbb{Q} = \{x : x = \tfrac{p}{q},\ p, q \in \mathbb{Z},\ q \neq 0\}$$

$$\mathbb{Z}^{+} = \{x : x \text{ is an integer and } x > 0\}$$

$$\mathbb{Z}^{*} = \{x : x \text{ is an integer and } x \geq 0\}$$

Def

A set A is a Subset of B (denoted as $A \subseteq B$), if every element of A is also an element of B.

$A = \{0, 1, 2, 3\}$ $\qquad$ $B = \{1, 3\}$

$A \not\subseteq B$ $\qquad$ $B \subseteq A$ ✓

"A is not a subset of B" $\qquad$ "B is a subset of A"

$\text{EVENS} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

$\mathbb{R} \subseteq \mathbb{Q}$

This is not true, because
$\pi \in \mathbb{R}$ but $\pi \notin \mathbb{Q}$

Question

$\emptyset \subseteq \mathbb{R}$? True

There are no elements in the empty set.
Therefore, all elements in empty set appears in $\mathbb{R}$.

$\emptyset \subseteq S$ for all sets S. ✓

$S \subseteq S$ for all sets S. ✓

Def. Proper subsets

A set A is a proper subset of B (denoted as $A \subset B$), if $A \subseteq B$ and $A \neq B$

$S \subset S$? wrong $S \not\subset S$

Ex: $S = \{0, 1, 2\}$ $T = \{0, 1\}$

$T \subset S$

Question

if $A \subseteq B$, can you say $|A| \leq |B|$?

True

If $|A| \leq |B|$, can we say $A \subseteq B$?

$A = \{1\}$ $\quad$ $B = \{2, 3\}$

$|A| \leq |B|$ but $A \not\subseteq B$

Claim $\underbrace{\{x : x \in \mathbb{Z} \text{ and } 18 \mid x\}}_{A} \subseteq \underbrace{\{x : x \in \mathbb{Z} \text{, and } 6 \mid x\}}_{B}$

$A \subseteq B$

$A = \{x : x \in \mathbb{Z} \text{ and } 18 \mid x\}$

$B = \{x : x \in \mathbb{Z} \text{ and } 6 \mid x\}$

x	$18 \mid x$?	$6 \mid x$?
0	T	T
18	T	T

Every number that is divisible by 18 is divisible by 6.

Proof

Let us assume "a" an arbitrary element of set "A". ~~a ∈ A~~,
~~is divisible by 18~~

$a \in A$ — by assumption.

$a = 18 \cdot c$ — by def. of divisibility.

$a = 6 \cdot 3 \cdot c$ — by factoring

$a = 6 \cdot k$, $k \in \mathbb{Z}$ — by algebra

$6 \mid a$ — by def. of divisibility.

$a \in \underline{B}$ — $a \in \mathbb{Z}$ and $6 \mid a$

note! $B = \{x : x \in \mathbb{Z} \text{ and } 6 \mid x\}$
~~$a = 6 \cdot k$~~

$A \subseteq B$ ☒

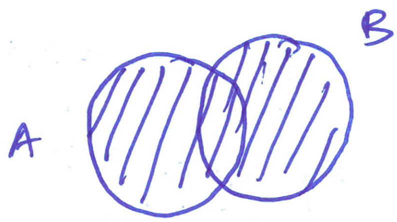
$a \in A$ and $a \in \mathbb{Z}$ and $6 \mid a$
(Basically any arbitrary element of set A follows the rule that of any element in B.)

Def

union symbol

$A \cup B$, "A union B", is the set that contains all the elements in Set A and all the elements in Set B.

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$



$A = \{2, 4, 6\}$

$B = \{2, 3, 4\}$

$A \cup B = \{2, 4, 6, 3\}$

$$\text{EVENS} \cup \text{ODDS} = \mathbb{Z}$$

$$\mathbb{R}^{\geq 0} \cup \mathbb{R}^{\leq 0} = \mathbb{R}$$

$$A \cup \emptyset = A$$

$$A \cup A = A$$

Question

Can I say $2 \cup \{1, 3\}$ ← does not make sense.