Popup test 4

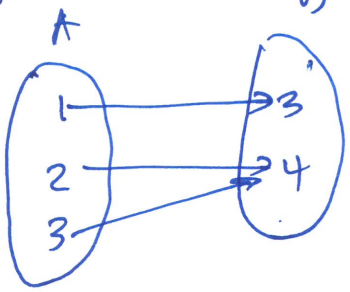
- Form groups of 3.
- write your name.

Let $A = \{1, 2, 3\}$ $B = \{3, 4\}$

Give example of function that satisfy:

1. an Onto function $f: A \longrightarrow B$

A	f(a)
1	3
2	4
3	4

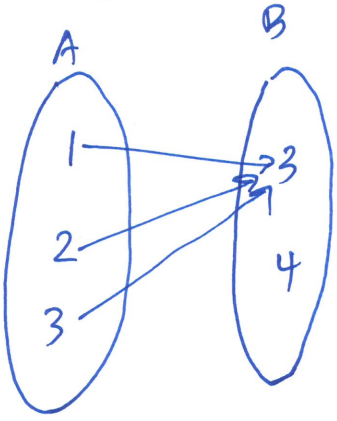


1. $\forall a \in A$, $f(a)$ is defined
2. $\forall a \in A$, $f(a)$ is unique
3. $\forall a \in A$, $f(a) \in B$

Def of onto:
$[\forall b \in B : \exists a \in A : f(a) = b]$

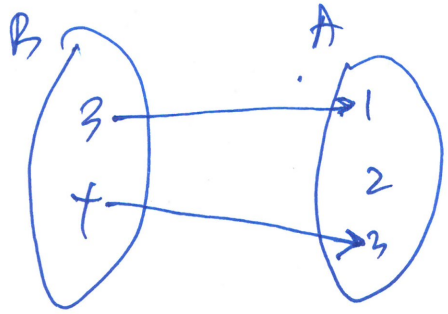
2. A function $g: A \longrightarrow B$ that is <u>not</u> onto.

A	g(a)
1	3
2	3
3	3



3. a function $h: \underline{B} \longrightarrow \underline{A}$ that is <u>onto</u>

B	
3	
4	



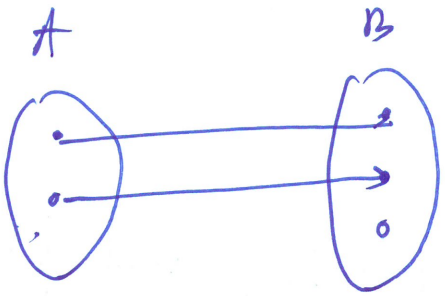
function h is not possible.

Announcement

Quiz/Test 1

- You will be given proofs, and you have to determine whether they are valid or not.

- Truth tables
- translate predicate logic statements to English statements
- All the quantifiers $\forall$, $\exists$
- Fill in the blanks in proofs.
- functions
  - Determine whether they are onto 1:1 or a bijection
  - create onto, 1:1 or bijection functions
- Disproove by counterexample.


A
B
0

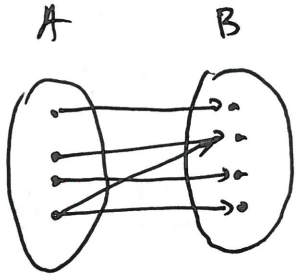
10/02/2024

Recap!

Given a function $f: A \longrightarrow B$

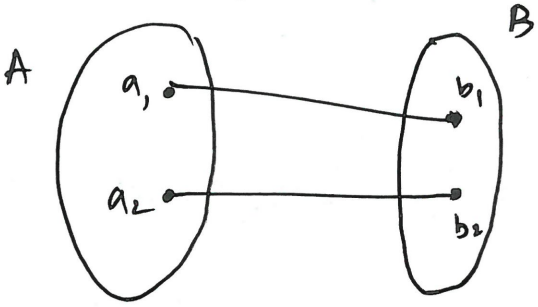
1. f is onto If

$$\left[\forall b \in B : \left[\exists a \in A : f(a) = b\right]\right]$$



2. f is 1:1 if

$$\left[\forall a_1, a_2 \in A : a_1 \neq a_2 \Longrightarrow f(a_1) \neq f(a_2)\right]$$



if $a_1 \neq a_2 \Rightarrow b_1 \neq b_2$

Let $f: A \longrightarrow B$ (f is a function)

$f: A \to B$ is onto $\Longrightarrow$ $|A| \geq |B|$

$f: A \to B$ is 1:1 $\Longrightarrow$ $|A| \leq |B|$

$f: A \to B$ is a bijection $\Longrightarrow$ $|A| = |B|$

Theorem 9.13

Pigeon Hole Principle (PCP)

Let A and B be sets with a function $f: A \to B$;

If $|A| > |B|$, then there exist 2 distinct $a_1, a_2 \in A$ such that $f(a_1) = f(a_2)$.

1. Give me a fully quantified expression for this.

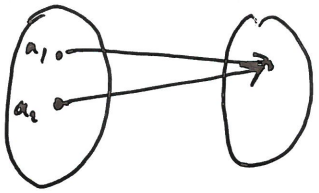
$$|A| > |B| \Rightarrow \left(\exists a_1, a_2 \in A : f(a_1) = f(a_2)\right)$$

Is there a ~~simati~~ Similarity with this theorem and def of 1:1 functions?

~~If~~ $f: A \to B$ is 1:1 $\Rightarrow |A| \leq |B|$ — ①

Note: $P \Rightarrow q \equiv (\neg q \Rightarrow \neg P)$

$|A| > |B| \Rightarrow f: A \to B$ is not 1:1.



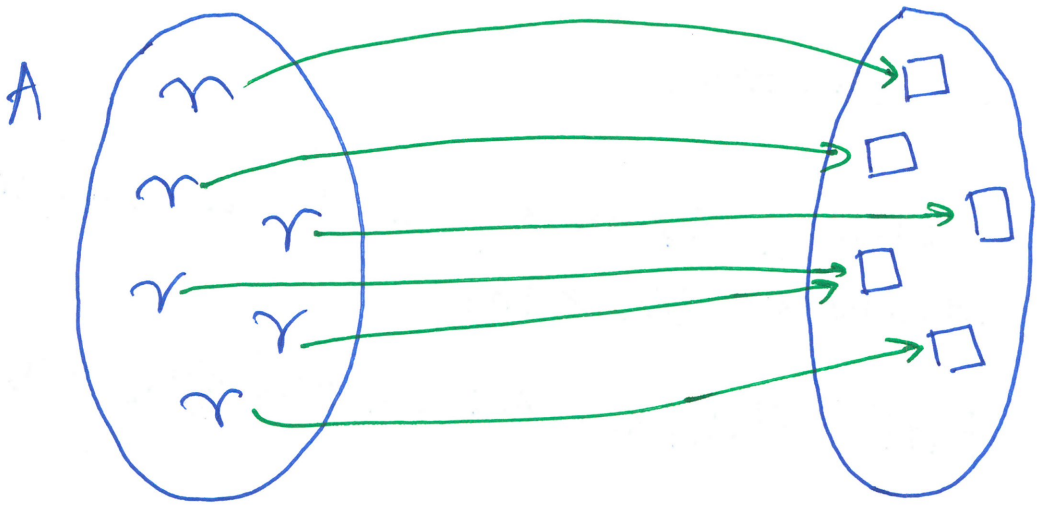
PEP with pigeons:

Suppose you have a set A of pigeons

$|A| = n+1$

Also, you have a set of cubbies (B)

$|B| = n$



≥ 2 pigeons share a cubby

claim! Prove; Among 13 people, at least 2 of them share the same birth month.

Proof:

Let A (pigeons) be the set of 13 people.

Let B (pigeonholes/cubbies) be the set of 12 months.

Let $f: A \longrightarrow B$, $f(a)$ = a's birth month.

we need to prove that f is a function.

1. $\forall a \in A$, $f(a)$ is defined because everyone has a birth month.

2. $\forall a \in A$, $f(a)$ is unique because a person can only have one birth month.

3. $\forall a \in A$, $f(a) \in B$ because the birth month has to be one of the 12 months.

$\therefore f$ is a function.

We have 2 sets, A & B, and a function $f : A \to B$. Furthermore $|A| > |B|$

Hence, using PHP, $\exists a_1, a_2 \in A : (a_1 \neq a_2) \wedge (f(a_1) = f(a_2))$