CSCI-246 Discrete Structures HW 8

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Objective

- Understanding partial orders.
- Understanding steps of a induction.
- Understanding the problem solving process.

Submission requirements

- ***Type or clearly hand-write*** your solutions into a ***PDF FORMAT.***
- ***DO NOT UPLOAD images.***
- ***non-pdf or emailed solutions will not be graded.***
- **If you take pictures of your handwritten homework, put it into pdf format.**
- ***Start each problem on a new page.***
- Follow the model that you have learned during the lectures for proofs.
- Do not wait until the last minute to submit the assignment.
- You can submit any number of times before the deadline.
- If you are using latex, and you do not know how to type a symbol, use the following website. You can draw the symbol here and it will give you the latex code and the packages that you have to import. `https://detexify.kirelabs.org/classify.html`

- If you are using latex to write the answer, you can use overleaf to make your life easier. **Overleaf is a free, online platform that helps users create and publish scientific and technical documents using LaTeX, a markup-based document preparation system**
- If you do not understand a problem, ask questions during/after the lectures, or during office hours or via discord.
- Go to TA office hours and talk with them and ask for help.
- ***Do not use generative AI to write answers.***

Homework 02 contains **3 questions**.

1 Q1

List a partial order, strict partial order and a equivalence relation that you can create from the following set $A = \{0, 1, 2, 3\}$.

2 Q2

For a given integer $n \geq 0$, consider the sum of first n cubes: $0^3 + 1^3 + 2^3 + \cdots + n^3$. You can write the sum of first n cubes as $\sum_{i=0}^{n} i^3$. We can hypothesize that this sum of cubes of first n natural numbers are $\sum_{i=0}^{n} i^3 = (\frac{n \cdot (n+1)}{2})^2$.

1. Show that this formula works for $0 \leq n \leq 3$. (This is very simple try to manually check the sum of cubes for $0 \leq n \leq 3$ and check whether you get the same value from the formula.)
2. Use mathematical induction to prove that this formula works for any integer $n \geq 0$.

   Hint: Try to model this problem into the induction framework that we learnt during the class.

   - define the predicate $P(n)$.
   - State the variable that you are performing the induction over.
   - State the base case.
   - prove the base case.
   - state the inductive case.

- prove the inductive case.
  - assume the inductive hypothesis $P(n-1)$
  - start with the **LHS** of the $P(n)$ and manipulate it to get the **RHS** (or vice versa.) **Do not start with LHS = RHS**.
  - fina a way to get the $P(n-1)$ in your algebra somewhere, so you can apply the inductive hypothesis.
  - correctly apply the inductive hypothesis.
  - clearly say that you have applied the inductive hypothesis.
  - Then derive the **RHS** of the $P(n)$ (if you starrt with the **LHS**) or **LHS** (if you start with the **RHS**).
- Finish the proof by tying everything together.

3 Q3

Suppose you want to calculate the sum of first n odd numbers. For example, the sum of first 3 odd numbers would be $1+3+5=9$. We can label this sum as $\sum_{i=1}^{n}(2i-1)$. We can hypothesize that this sum is equal to n^2.

1. Show that this formula works for $1 \leq n \leq 3$. (This is simple, manually check whether the **LHS** and **RHS** of the claim is equal).
2. Use mathematical induction to prove that this formula is correct for $n \geq 1$.

   Hint: Try to model this problem into the induction framework that we learnt during the class.

   - define the predicate $P(n)$.
   - State the variable that you are performing the induction over.
   - State the base case.
   - prove the base case.
   - state the inductive case.
   - prove the inductive case.
     - assume the inductive hypothesis $P(n-1)$
     - start with the **LHS** of the $P(n)$ and manipulate it to get the **RHS** (or vice versa.) **Do not start with LHS = RHS**.
     - fina a way to get the $P(n-1)$ in your algebra somewhere, so you can apply the inductive hypothesis.
     - correctly apply the inductive hypothesis.

  - clearly say that you have applied the inductive hypothesis.
  - Then derive the **RHS** of the $P(n)$ (if you starrt with the **LHS**) or **LHS** (if you start with the **RHS**).
- Finish the proof by tying everything together.