09/09/2024

Def Power Set.

Given a set S, the power set of S is the set of all subsets of S.

$$P(S) = \{A : A \subseteq S\}$$

Note: $\emptyset \subseteq B$, for all sets B

$B \subseteq B$ for all sets B.

Ex: $S = \{1, 2, 3\}$

$$P(S) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

$$|P(S)| = 8 = 2^3$$

$$|S| = 3$$

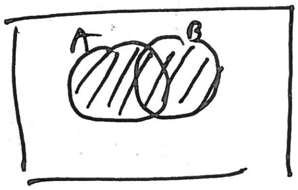
Fact, for a given set S, $\binom{n}{r}$

$$|P(S)| = 2^{|S|}$$

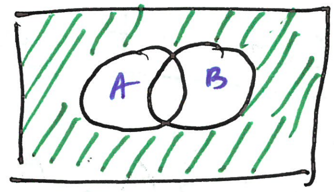
Theorem: De morgan's Law 1

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

$A \cup B$ -



$\overline{A \cup B}$



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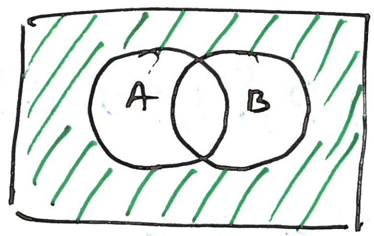
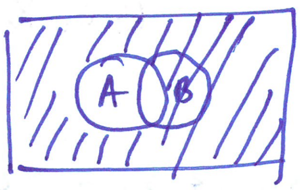
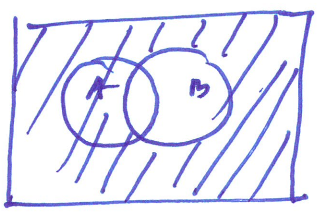


$\bar{A}$



$\bar{B}$



If I want to show $X = Y$, what can I do?

~~a, b, c~~ $a, b \in \mathbb{R}$

$a \leq b$ and $a \geq b \Rightarrow a = b$

What if we show

① — $\overline{A \cup B} \subseteq \bar{A} \cap \bar{B}$.

and

② — $\bar{A} \cap \bar{B} \subseteq \overline{A \cup B}$

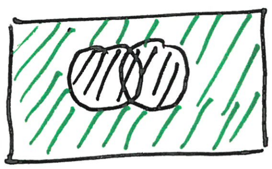
① $\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$

Proof

Assume x is an arbitrary element in $\overline{A \cup B}$

WTS (want to show) x is in $\overline{A} \cap \overline{B}$

Statements	Reasoning
$x \in \overline{A \cup B}$	by assumption
$x \notin A \cup B$	by def. of complement



$x \notin A$ and $x \notin B$	because $x \notin A \cup B$
$x \in \overline{A}$ and $x \in \overline{B}$	by def of complement
$x \in \overline{A} \cap \overline{B}$	by def of $\cap$

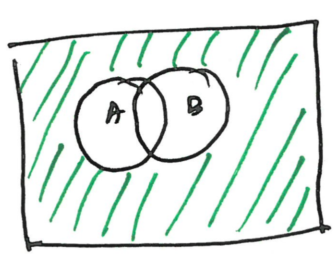
Proved case 01

② $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$

Assume y is an element of $\overline{A} \cap \overline{B}$

WTS $y \in \overline{A \cup B}$

statements	Reasoning
$y \in \overline{A} \cap \overline{B}$	by assumption
$y \in \overline{A}$ and $y \in \overline{B}$	by def of $\cap$
$y \notin A$ and $y \notin B$	by def. of complement



$y \notin A \cup B$ because $y \notin A$ and $y \notin B$

$y \in \overline{A \cup B}$ by def of complement

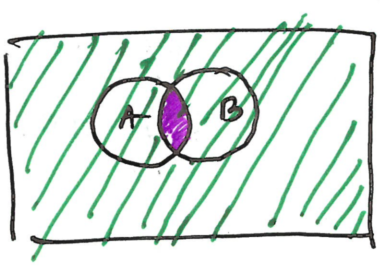
we proved case 2

Therefore, $\overline{A \cup B} \subseteq \bar{A} \cap \bar{B}$ and $\bar{A} \cap \bar{B} \subseteq \overline{A \cup B}$

Then $\overline{A \cup B} = \bar{A} \cap \bar{B}$ ∎.

De morgan's Law 2

$\overline{A \cap B} = \bar{A} \cup \bar{B}$



The green colored set is the $\overline{A \cap B}$

Claim: If $P(A) \subseteq P(B)$ then $A \subseteq B$

Examples

$A = \{1\}$ $\quad$ $B = \{2\}$

$P(A) = \{\{\}, \{1\}\}$ $\quad$ $P(B) = \{\{\}, \{2\}\}$

$P(A) \not\subseteq P(B)$

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$A = \{1\}$ $\quad$ $B = \{1, 2\}$

$P(A) = \{\emptyset, \{1\}\}$ $\quad$ $P(B) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$

$P(A) \subseteq P(B)$ ✓ $\Rightarrow$ $A \subseteq B$ ✓

Proof Idea:

$A \subseteq B$

Proof: Given $P(A) \subseteq P(B)$, Assume x is an arbitrary element of A

WTS $x \in B$

statement	Reasoning
$x \in A$	by assumption
~~$\{x\} \subseteq A$~~	
$\{x\} \subseteq A$	because $x \in A$
$\{x\} \in P(A)$	by def. of power set
$\{x\} \in P(B)$	given that $P(A) \subseteq P(B)$
$x \in B$	by def of powerset.

Any arbitrary element in A exists in B when $P(A) \subseteq P(B)$

Hence $A \subseteq B$ $\blacksquare$.