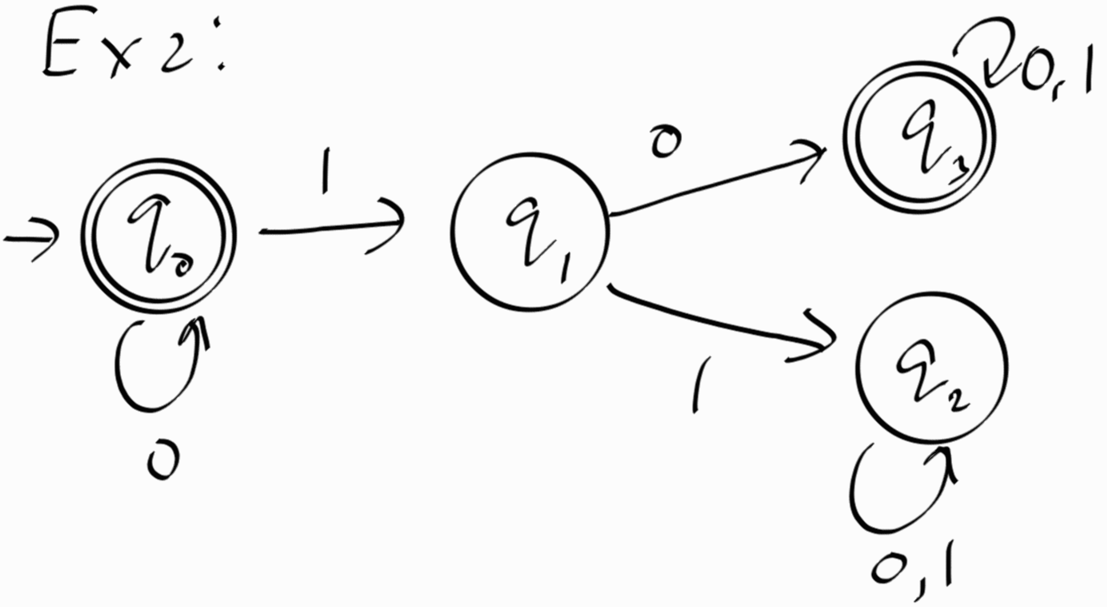
09/04/2025

Ex2:



ε ✓

0

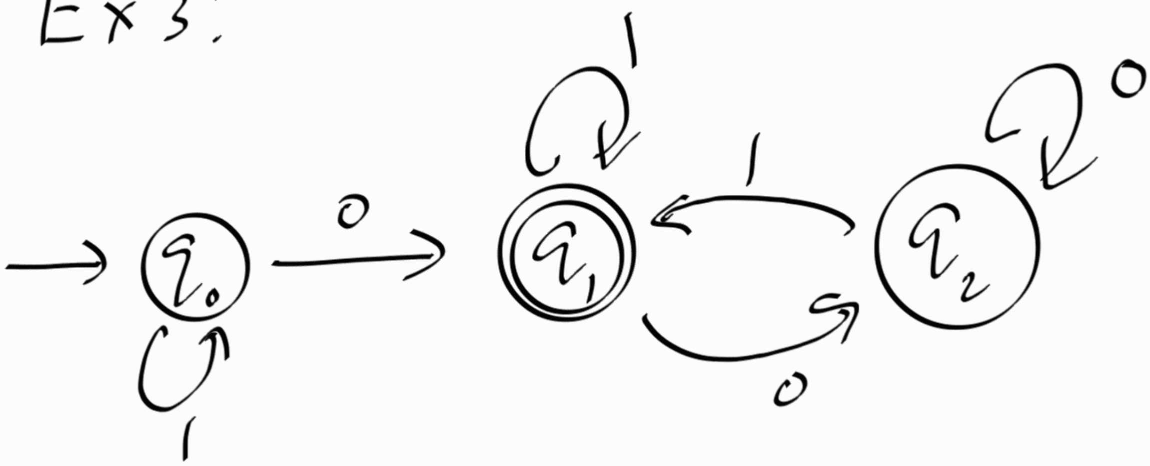
00

0000

0^*

$$\{\, 0^*,\ 0^*10(0|1)^* \,\}$$

Ex 3:



1^*01^*

$\{ 1^*01^*, \ 1^*01^*(00^*11^*)^* \}$ $\quad 1^*01^*(00^*1)^*$

$\{ 1^*01(0^+1^+)^* \}$ ✓ $\quad 100$

Definition 1.16

Regular Language:

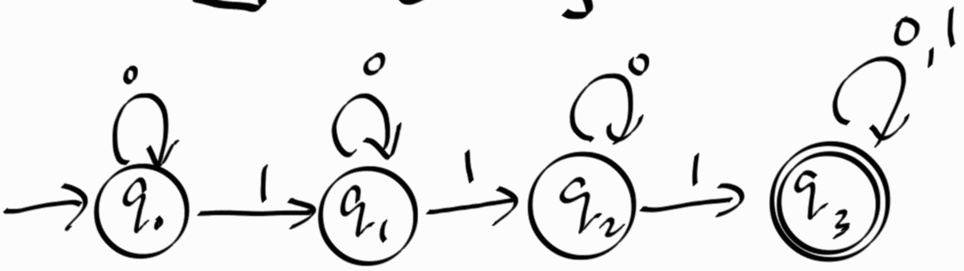
A language is called regular if some DFA accepts the language.

How to design finite automatons?

Ex: Construct a DFA that accepts language

$$A = \{ w \mid w \text{ contains at least three 1's} \}$$

$$\Sigma = \{0, 1\}$$



ε ×

01 ×

11 ×

111 ✓

1111 ✓

0100110011 ✓

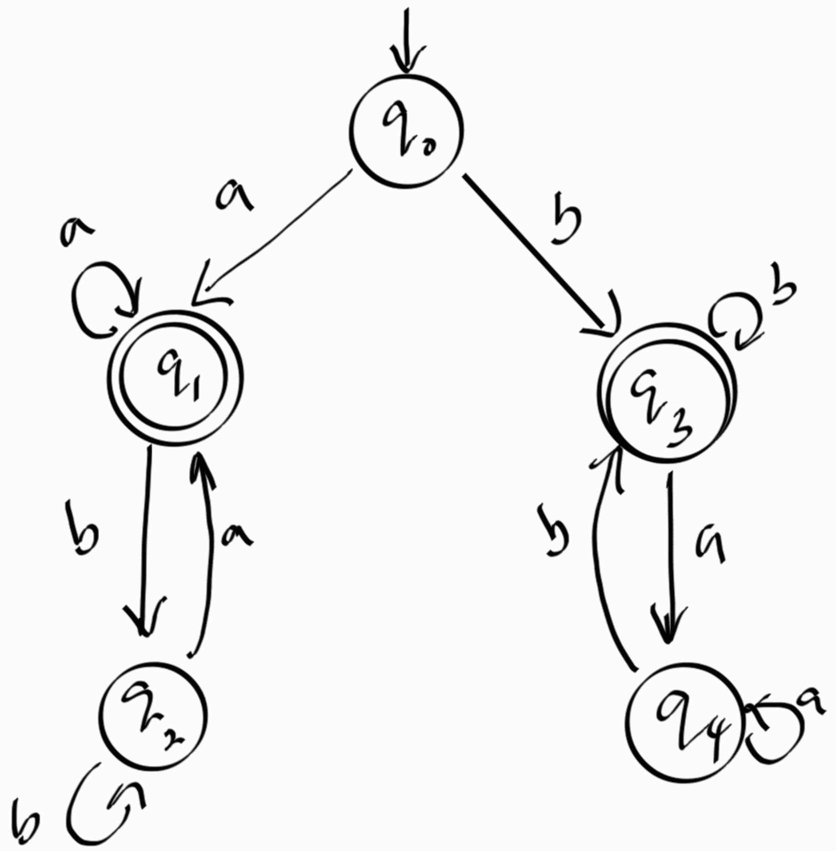
Ex 2: $A = \{ w \mid w$ starts and ends with the same symbol $\}$

$$\Sigma = \{a, b\}$$

ε ✗

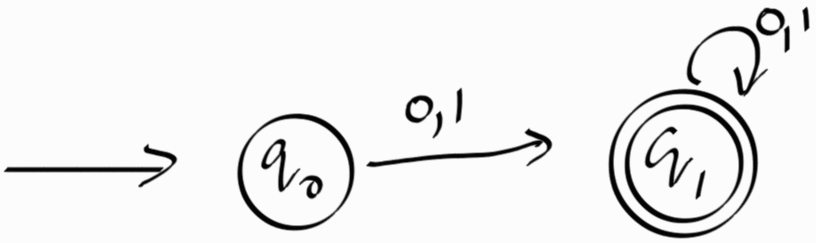
a ✓

aaaa



Ex 5: DFA that accepts every string except the empty string.

$$\Sigma = \{0, 1\}$$



Regular Operations

Let A, B be languages

1. Union : $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
2. Concatenation : $A \circ B = \{xy \mid x \in A \wedge y \in B\}$
3. Star : $A^* = \{x_1 x_2 \cdots x_k \mid k \geq 0 \text{ and each } x_i \in A\}$

Let $\Sigma = \{a, b, c, d, \ldots z\}$

$A = \{good, bad\}$ $\quad B = \{dog, cat\}$

$A \cup B = \{good, bad, dog, cat\}$

$A \circ B = \{gooddog, goodcat, baddog, badcat\}$

$A \circ B \neq B \circ A$ when $A \neq B$

$A^* = \{\varepsilon, good, goodgood, \ldots\ldots, goodbad,$

$= (good \mid bad)^*$

Side Note

If $x \in \mathbb{N}$, $y \in \mathbb{N}$, $x+y \in \mathbb{N}$

we say that natural numbers are closed under addition operator

natural numbers are not closed under division operator

$7 \in \mathbb{N}$, $8 \in \mathbb{N}$, but $8/7 \notin \mathbb{N}$

Theorem 1.25

The class of regular languages are closed under the union operation.

In other words,
If A_1, A_2 are regular languages so is $A_1 \cup A_2$

Proof $\Rightarrow$

Assume A_1, A_2 are regular and M_1, M_2 recognize A_1 and A_2 respectively.

By definition let M_1 be a DFA recognizing A_1, i.e.,
$M_1 = (Q_1, \Sigma_1, \delta_1, q_1, F_1)$

Similarly, $M_2 = (Q_2, \Sigma_2, \delta_2, q_2, F_2)$ recognizes A_2.

We construct $M = (Q, \Sigma, \delta, q_0, F)$

1. $Q = Q_1 \times Q_2$
2. $\Sigma = \Sigma_1 \cup \Sigma_2$
3. $\delta : Q \times \Sigma \longrightarrow Q$

   $\delta((r_1, r_2), a) = (\delta_1(r_1, a), \delta_2(r_2, a))$
4. $q_0 = (q_1, q_2)$
5. $F = \{(r_1, r_2) : r_1 \in F_1 \text{ or } r_2 \in F_2\}$