09/11/2025

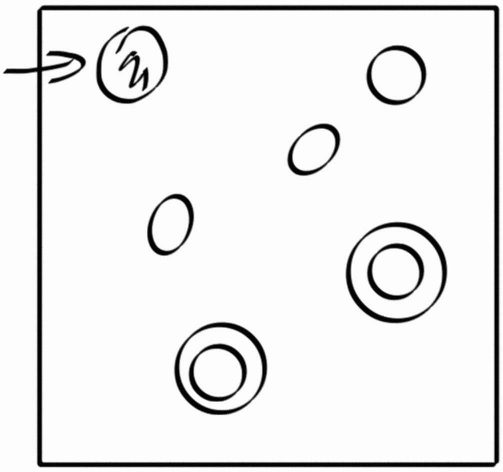
Corollary 1.40

A language is regular iff some NFA accepts it.

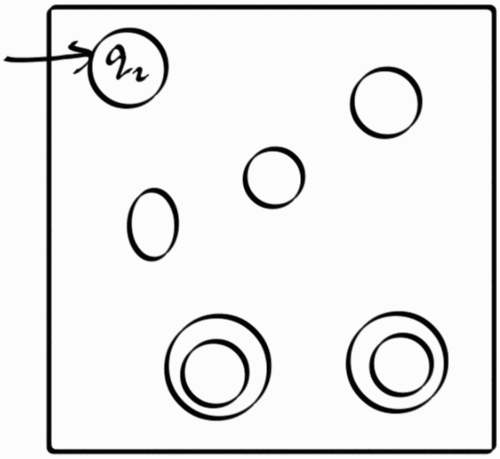
Theorem 1.45

The class of regular languages is closed under union operation.

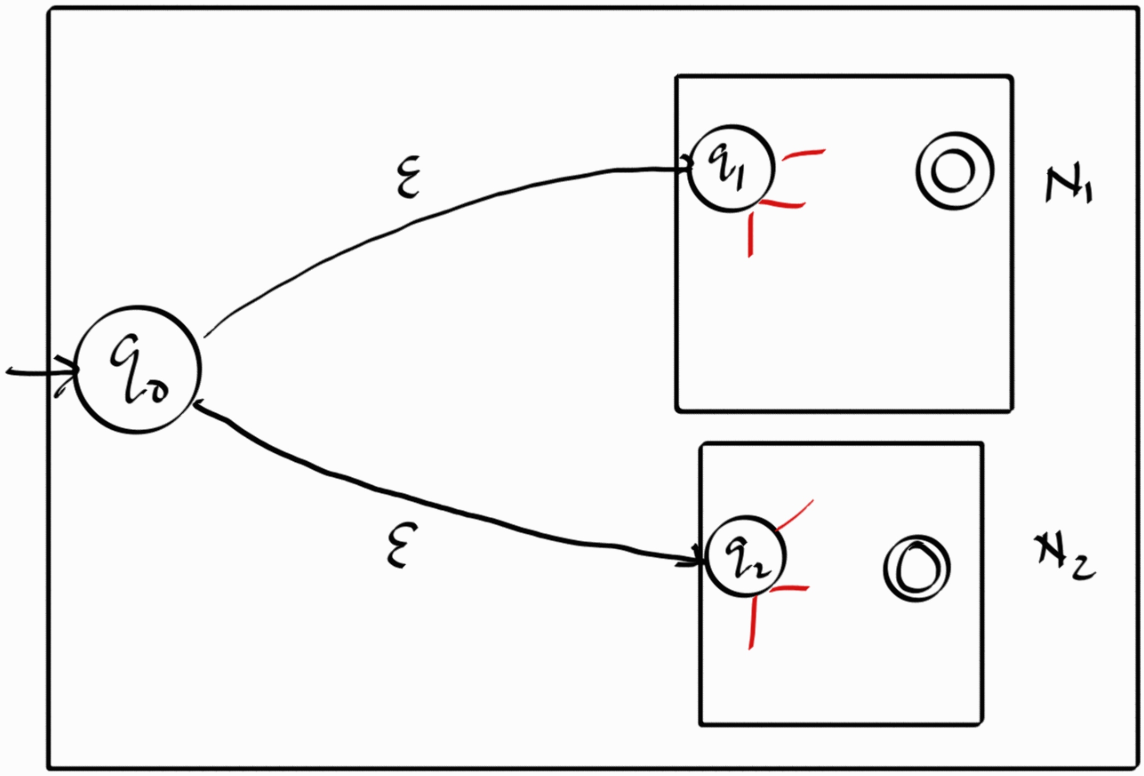
If A_1 and A_2 are regular languages then $A_1 \cup A_2$ is a regular language.



N_1 recognizes A_1



N_2 recognize A_2



N that recognize $A_1 \cup A_2$

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ be an NFA that recognizes A_1

Let $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ be an NFA that recognize A_2

We construct NFA N that recognizes $A_1 \cup A_2$, where $N = (Q, \Sigma, \delta, q_0, F)$

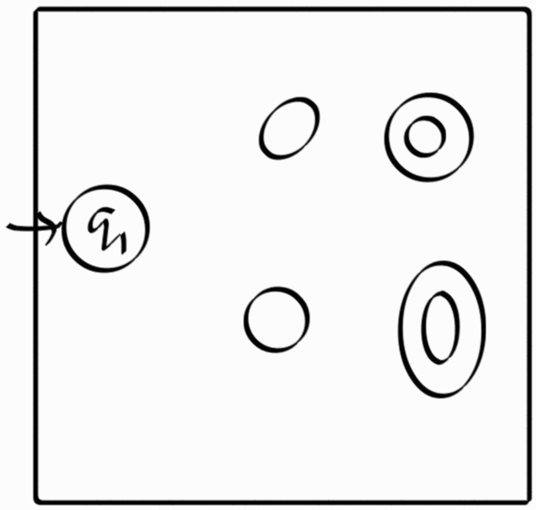
1. $Q = \{q_0\} \cup Q_1 \cup Q_2$
2. $\Sigma = \Sigma$
3. $q_0 \in Q$ is the start state.
4. $F = F_1 \cup F_2$
5. $$\delta(q, a) = \begin{cases} \delta_1(q, a) & \text{if } q \in Q_1 \\ \delta_2(q, a) & \text{if } q \in Q_2 \\ \{q_1, q_2\} & \text{if } q = q_0 \text{ and } a = \varepsilon \\ \emptyset & q = q_0 \text{ and } a \neq \varepsilon \end{cases}$$

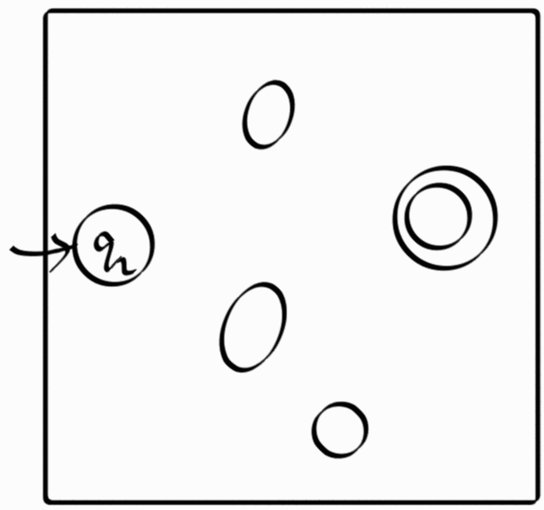
Theorem 1.47

The class of regular languages are closed under concatenation operation.

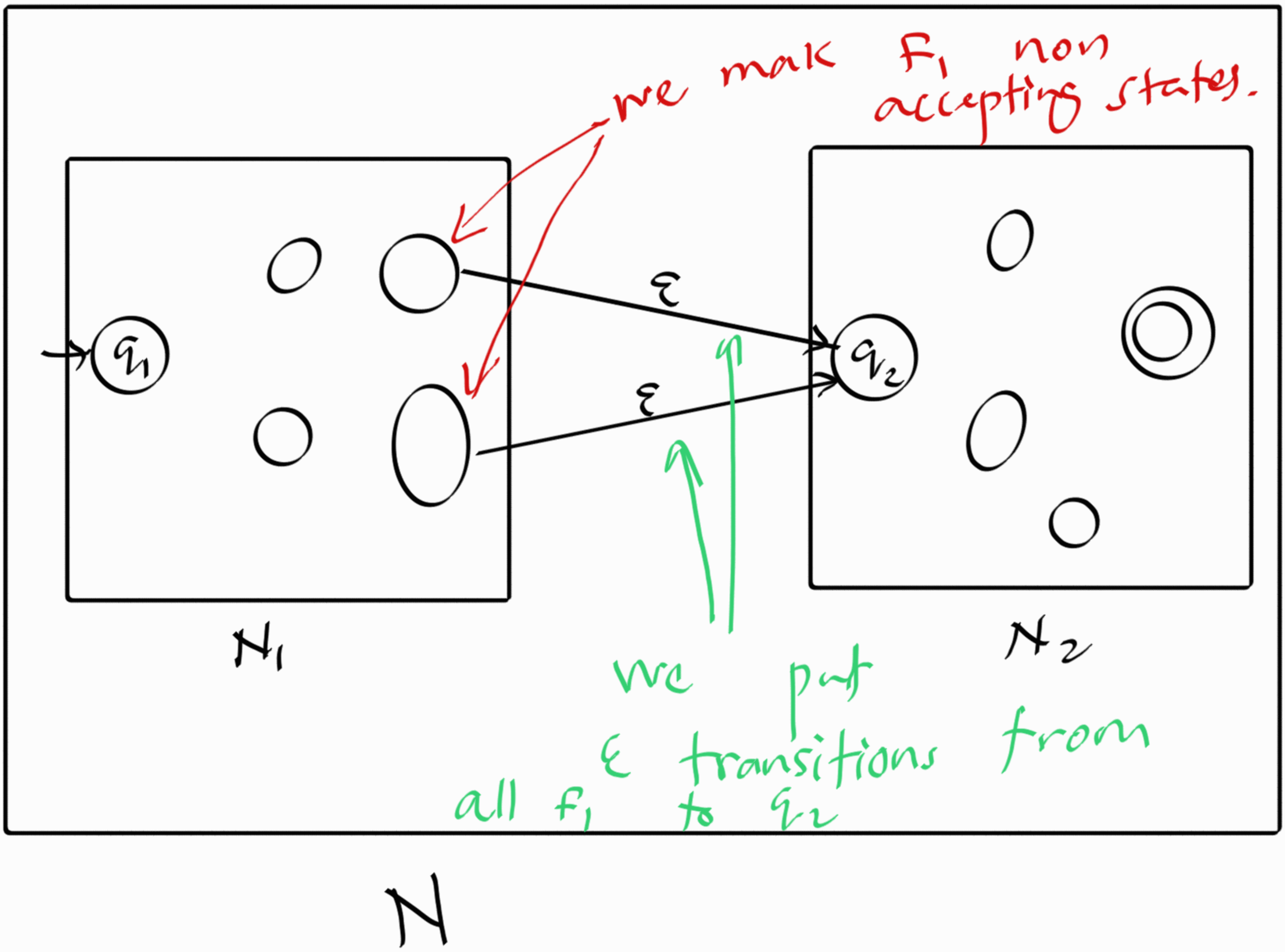
If A_1 and A_2 are regular, then $A_1 \circ A_2$ is regular.

Proof: Basic idea





N_1 A_1 N_2 A_2


we mak F_1 non accepting states.
ε
ε
q_1
q_2
N_1
N_2
we put ε transitions from all F_1 to q_2
N

Use the N_1 machine to check the prefix of an input string is accepted.

Then put ε transitions from N_1's accepting states to N_2's start state.

And make N_1's accept states non-accepting states.

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ be an NFA that recognizes A_1

Let $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ be an NFA that recognizes A_2

Then we create $N = (Q, \Sigma, \delta, q_0, F)$ that recognizes $A_1 \circ A_2$

1. $Q = Q_1 \cup Q_2$
2. $\Sigma = \Sigma$
3. $q_0 = q_1$
4. $F = F_2$ * Important.

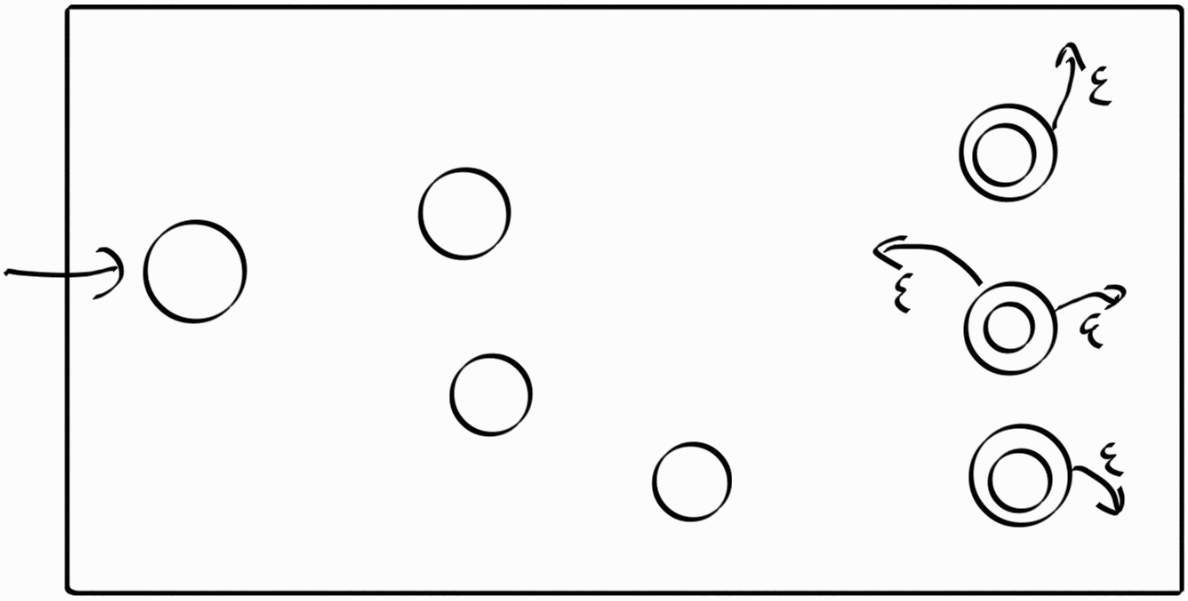
$$\delta(q,a) = \begin{cases} \delta_1(q,a) & \text{if } q \in Q_1 \text{ and } q \notin F_1 \\ \delta_1(q,a) & \text{if } q \in Q_1 \text{ and } q \in F_1 \text{ and } a \neq \varepsilon \\ \{q_2\} \cup \delta_1(q,a) & \text{if } q \in F_1 \text{ and } a = \varepsilon \\ \delta_2(q,a) & \text{if } q \in Q_2 \end{cases}$$

Theorem 1.49

The class of regular languages are closed under star operation

Here is the basic details.

If A_1 is a regular language, then A_1^* is a regular language.



N_1

N_1 recognizes A_1