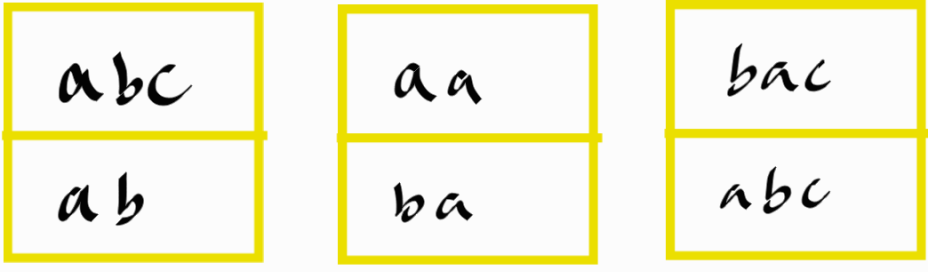
Practical undecidability problems.

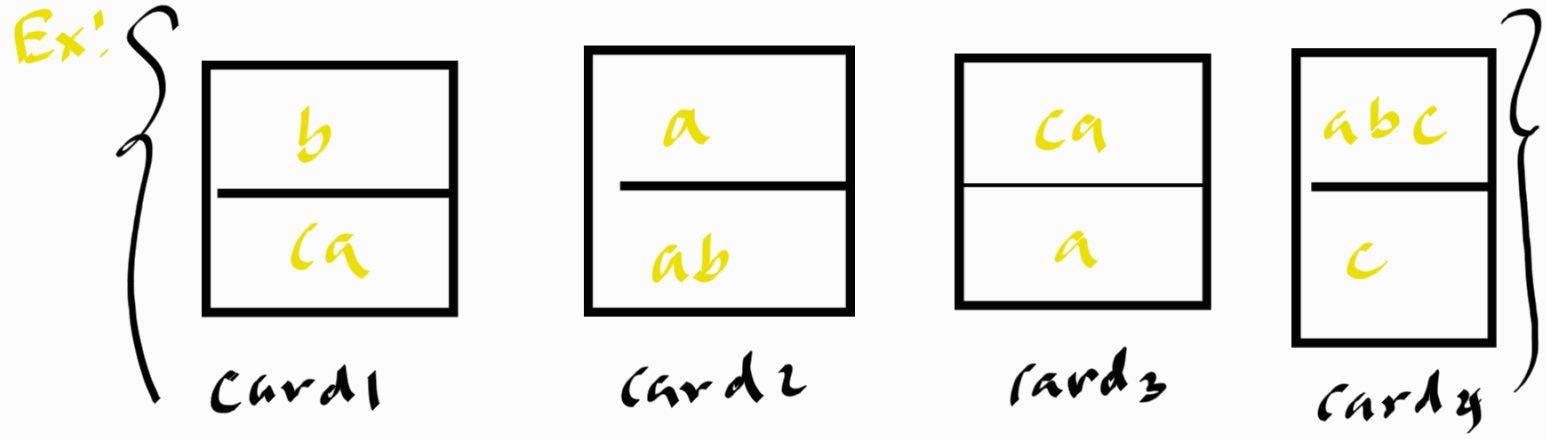
Post correspondence problem (PCP)

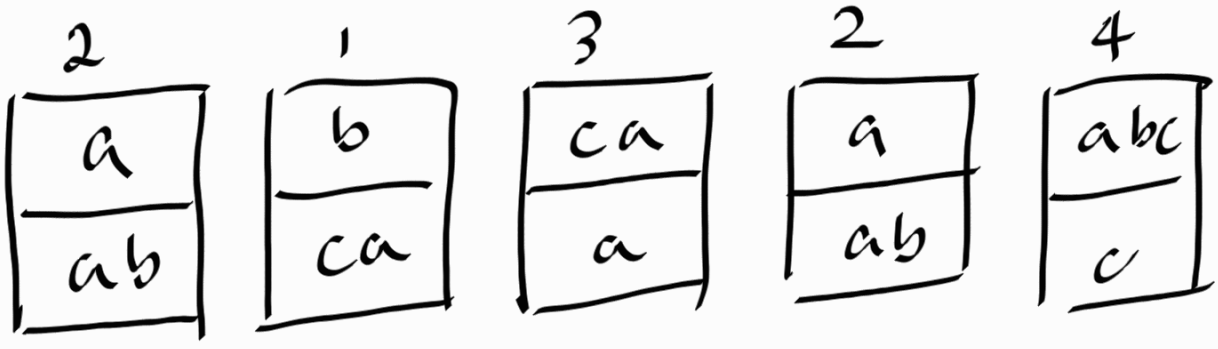
- You are given a collection of domino cards, with string written below and above the mid-line at each card.



- List these cards one by one, so that the strings above match strings below.
- You can use the same card any number of times including zero times.
- You can think of this as having infinite copies of each domino.

Ex:



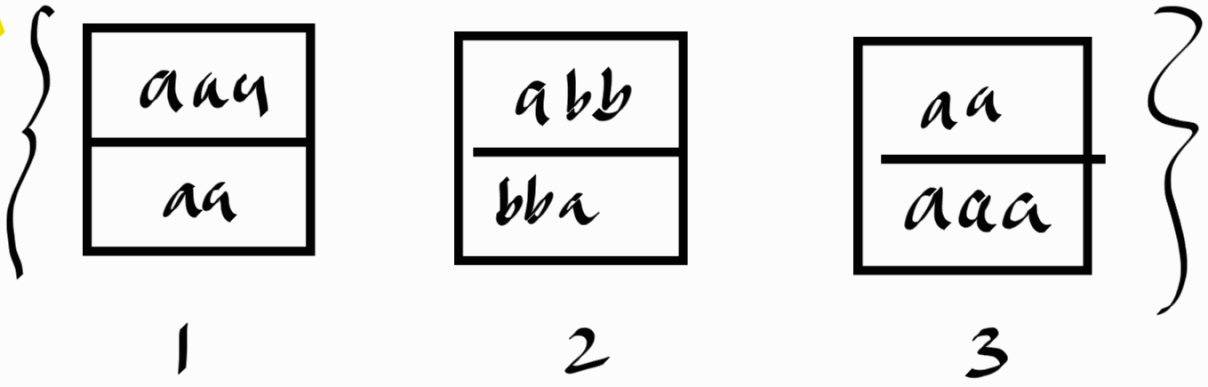


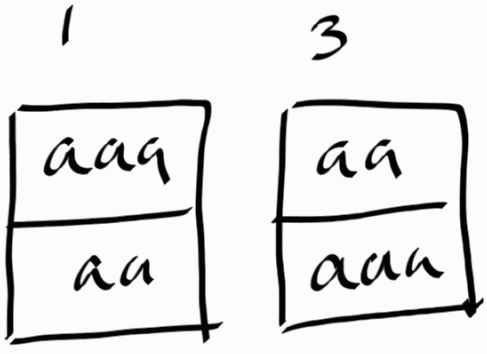
a b ca a abc

a b c a a ab c

<2, 1, 3, 2, 4> ✓

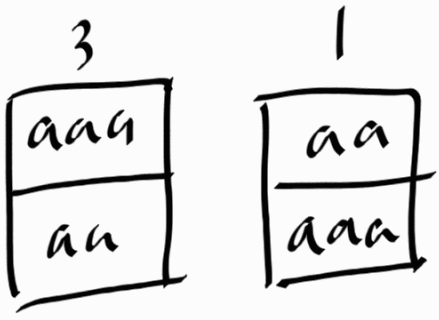
Ex 2'





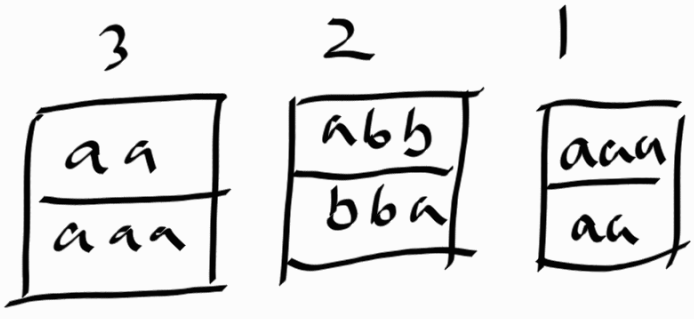
aaaaa
aaaaa

<1,3>



aaaaa
aaaaa

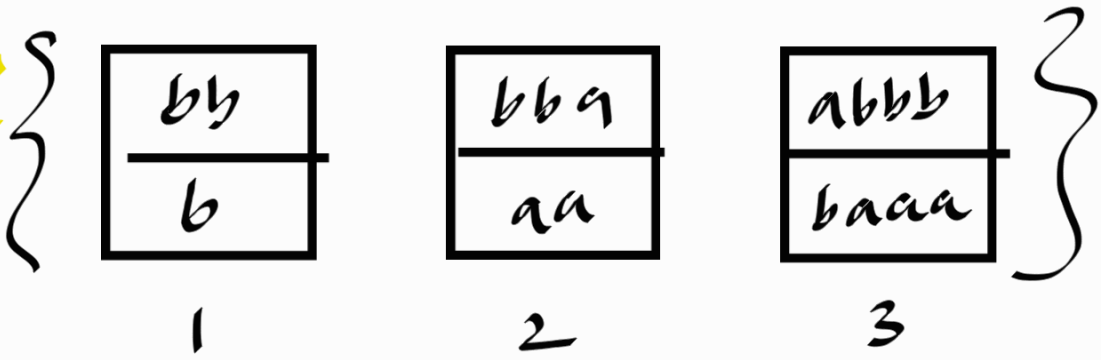
<1,3>



aaabbaaa
aaabbaaa

<3,2,1>

Ex 3:



No solution

$$P = \{\langle D_1\rangle, \langle D_2\rangle, \ldots, \langle D_k\rangle\}$$

$$PCP = \{\langle P\rangle \mid P \text{ is an instance of Post correspondence problem with a match}\}$$

The PCP is undecidable

Proof is too long

Please read the proof in the book.

Mapping Reducibility

– Map Reductions $\leq$, α, $\leq_m$

Definitions

A function $f: \Sigma^* \rightarrow \Sigma^*$ is a computable function if some TM M, on every input w, halts with f(w) on its tape.

A TM computes a function by starting with the input to the function on the tape and halting with the output of the function on the tape.

Ex: Arithmetic operations on integers are computable functions.

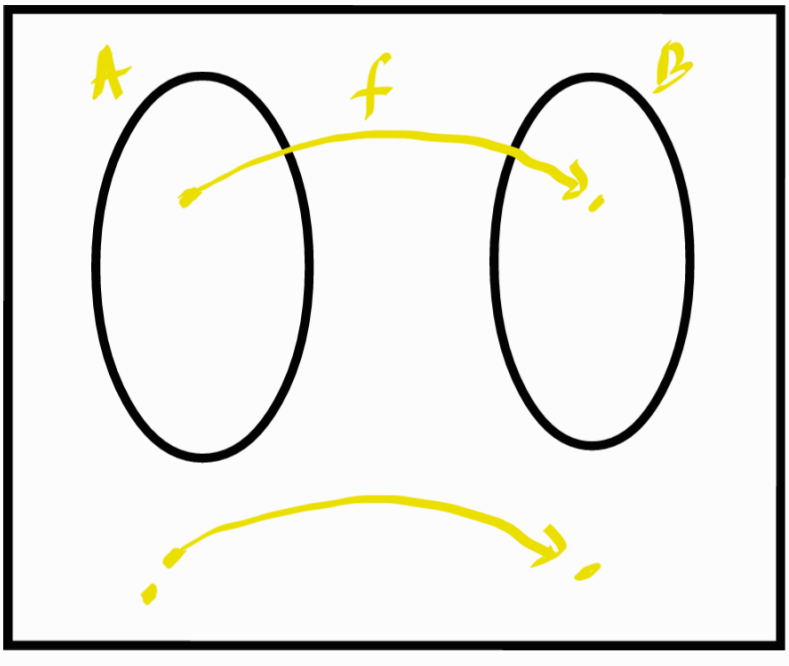
Definition

Mapping Reducibility

A language A is mapping reducible to language B, written as $A \leq_m B$, If there is a computable function $f: \Sigma^* \to \Sigma^*$, where every w

$$w \in A \iff f(w) \in B$$

The function f is called the reduction from A to B.



All previous reductions are map reductions.

* In a map reduction, you solve an instance of A by calling a solution to B only once

Theorem 5.22

If $A \leq_m B$ and B is decidable, then A is decidable.

Corollary 5.23

If $A \leq_m B$, and A is undecidable, then B is undecidable.

Theorem 5.28

If $A \leq_m B$ and B is Turing-recognizable, then A is Turing-Recognizable.