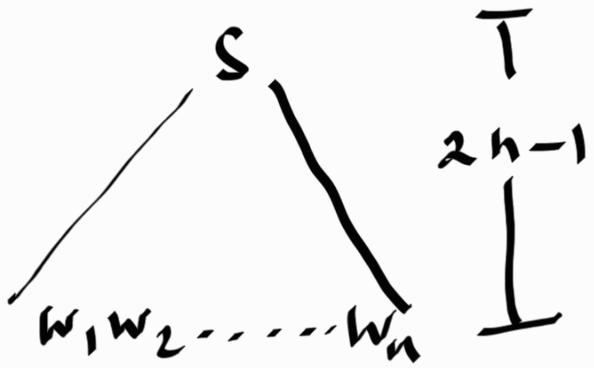
$A_{CFG} = \{ \langle G, w \rangle \mid G$ is a CFG and G generates $w \}$

- Solution that we used:

$G \longrightarrow$ Convert G to CNF

Theorem 4.7



$s = w_1 w_2 \ldots\ldots w_n$

$O(|G|^{2n-1})$

G is a CFG in CNF

This is too costly.

- Solution 02
  - Dynamic programming solution

If $A \Rightarrow BC$ is a rule and $B \overset{*}{\Rightarrow} X$ and $C \overset{*}{\Rightarrow} Y$ then $A \overset{*}{\Rightarrow} XY$

DP solution is based on this observation.

- Define a table[i, j] — which stores the collection of variables that generates the substring $w_i w_{i+1} \cdots w_j$

- Idea is simillar to matrix multiplication.

$$\underbrace{w_i\, w_{i+1} \cdots w_k}\ \Big|\ \underbrace{w_{k+1} \cdots w_j}$$

in the table[i,k] you have the variable B

in the table you have variable C

Then If I have a rule $A \to BC$

then I can add A to table[i, j]

Refer the handout for the algorithm

Example

$w = \overset{1}{a}\ \overset{2}{a}\ \overset{3}{b}\ \overset{4}{c}\ \overset{5}{c}$

G : $S \rightarrow AB$

$A \rightarrow AB \mid AA \mid a$

$B \rightarrow BC \mid b$

$C \rightarrow c$

m[1,2]

$\overset{1}{a} \mid \overset{2}{a}$

A A

$* \rightarrow AA$

i \ j	1	2	3	4	5
1	A	A			
2	–	A	A,S		
3	–	–	B	B	
4	–	–	–	c	∅
5	–	–	–	–	c

m[2,3]

$\overset{2}{a} \mid \overset{3}{b}$

A B

$* \rightarrow AB$

m[3,4]

$b \mid c$

$* \rightarrow B\ C$

m[4,5]

$\overset{4}{c} \mid \overset{5}{c}$

C C

$* \rightarrow C\ C$

G : $S \to AB$

$A \to AB \mid AA \mid a$

$B \to BC \mid b$

$C \to c$

$w = \overset{1}{a}\overset{2}{a}\overset{3}{b}\overset{4}{c}\overset{5}{c}$

i \ j	1	2	3	4	5
1	A	A	A,S		
2	—	A	A,S	S,A	
3	—	—	B	B	B
4	—	—	—	C	$\emptyset$
5	—	—	—	—	C

m[1, 3]

$\overset{1}{a}\ \overset{2}{a}\ \overset{3}{b}$

a | ab   aa | b

A   A,S   A   B

$* \to AA$

$* \to AS$ $\to AB$

m[2, 4] $\overset{2}{a}\overset{3}{b}\overset{4}{c}$

a | bc   ab | c

A   B   A,S   C

$* \to AB$ $\to AC$

$\to SC$

m[3, 5]

$\overset{3}{b}\,c\,\overset{5}{c}$

b | cc   bc | c

B $\emptyset$ B   C

× $* \to BC$

$G:\ S \rightarrow AB$

$A \rightarrow AB \mid AA \mid a$

$B \rightarrow BC \mid b$

$C \rightarrow c$

$w = aabcc$

i \ j	1	2	3	4	5
1	A	A	A,S	A,S	
2	—	A	A,S	S,A	S,A
3	—	—	B	B	B
4	—	—	—	C	ϕ
5	—	—	—	—	c

m[1,4]

$\overset{1}{a}ab\overset{4}{c}$

a | abc   aa | bc   aab | c

A   S,A   A   B   A,S   C

→ AS   → AB   → AC

→ AA   → SC

m[2,5]

abcc

a | bcc   ab | cc   abc | c

A   B   A,S ϕ S,A   C

→ AB   → SC

→ AC

$G: S \to AB$

$A \to AB \mid AA \mid a$

$B \to BC \mid b$

$C \to c$

$w = aabcc$

i \ j	1	2	3	4	5
1	A	A	A,S	A,S	A,S
2	—	A	A,S	S,A	S,A
3	—	—	B	B	B
4	—	—	—	C	ϕ
5	—	—	—	—	C

$m[1,5]$

$\overset{1}{a}\ a\ b\ c\ \overset{5}{c}$

a | abcc   aa | bcc   aab | cc   aabc | c

A   S,A   A   B   A,S ϕ A,S   C

$\to AA$ $\to AB$ $\to SC$

$\to AS$ $\to AC$

In the table[1,n], If we can find the starting variable, then $w = w_1 w_2 \ldots w_n$ can be generated by G.

In m[1,5] we have S

$\therefore$ aabcc can be generated by G.