

Let's see whether we have languages we cannot recognize by an NFA/DFA.

consider the following languages.

$A = \{w : w \text{ has more } o's \text{ than } i's\}$

$B = \{a^n b^n \mid n \geq 0\}$

$C = \{w \mid w \text{ has equal number of } o's \text{ and } i's\}$

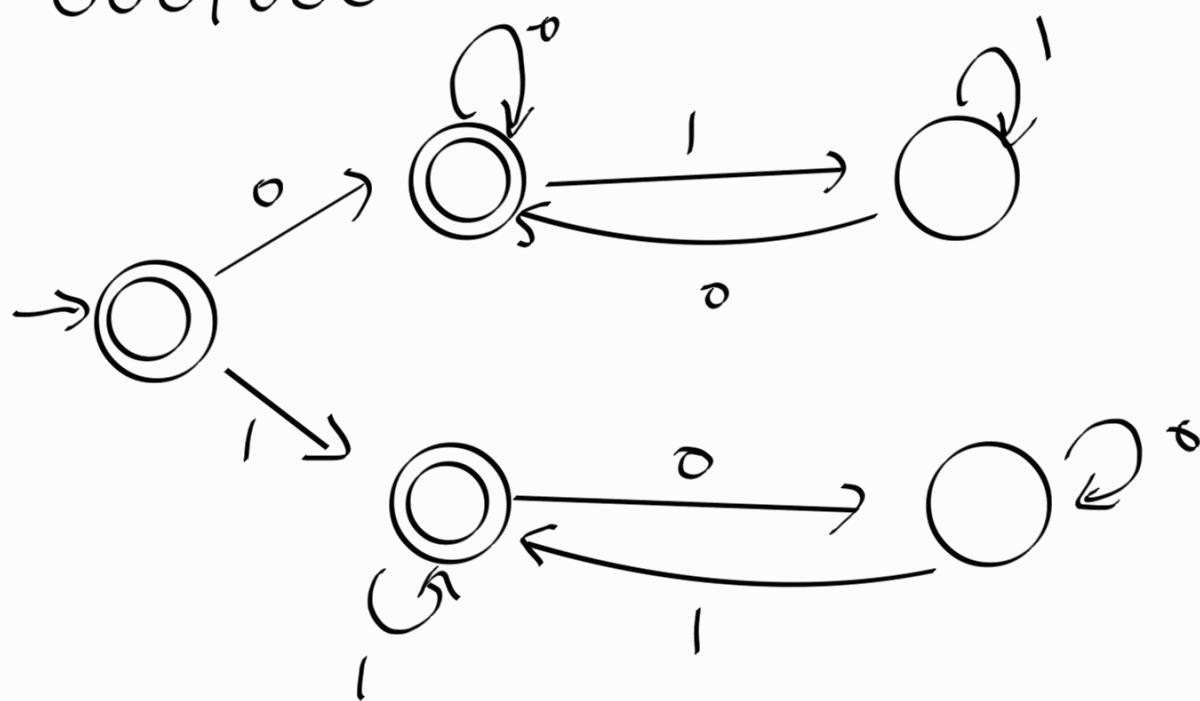
When you consider language A,
If you want to build a DFA
that recognizes A, the machine
needs to keep track of # of
0's or 1's, and depending on the
string this value could be different.
Therefore, a machine with limited
amount of memory cannot do it.

However, just because a language appears to require unbounded memory does not mean that the language is not regular.

$$C = \{ w \mid w \text{ has an equal \# of } 0's \text{ and } 1's \}$$

$$D = \{ w \mid w \text{ has equal \# of occurrences of } 01 \text{ and } 10 \}$$

$\epsilon \checkmark$ $0 \checkmark$ $00000 \checkmark$ $1 \checkmark$ $11111 \checkmark$
 0001000



Pumping Lemma

- Basically, this theorem states that regular languages have a special property.
- If we can show that a language does not have this property, then the language is not regular.

The basic idea of this property is that all strings in a regular language can be "pumped". If they are at least as long as a certain special value, called the pumping length.

Pumping Lemma

If A is a regular language, then there is a number p (the pumping length) where if s is any string in A of length at least p , then s may be divided into three pieces, $s = xyz$, satisfying the following conditions.

1. for each $i \geq 0$, $xy^i z \in A$

2. $|y| > 0$, and

3. $|xy| \leq p$

- $|s|$ represents the length of the string
- y^i means i copies of string y .

A is regular $\Leftrightarrow \exists p \in \mathbb{N} : s \in A \wedge |s| \geq p \Rightarrow s = xyz$

1.

2.

3.

Proof idea:

Assume A is regular.

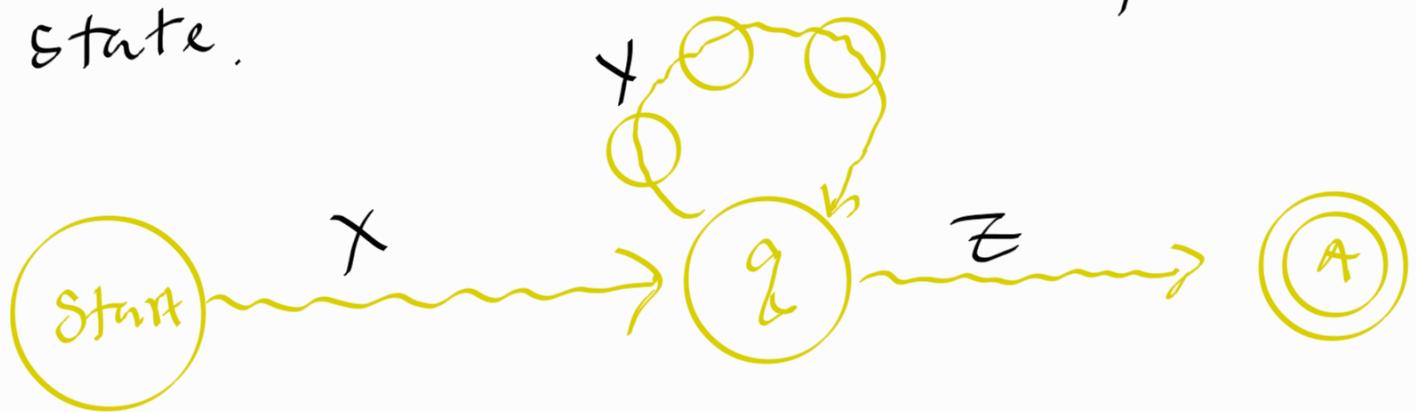
Then A can be recognized by some DFA, let's call it M .

We pick p to be the # of states in DFA M (that recognize language A).

Then pick any string s in A of length at least p .

* If no such string exist, then the claim is vacuously true.

Since the size of S is larger than p , then in its execution path to an accepting state, we must see a repeated state.



$$S = xyz$$

$$xz$$

$$xyyz$$

$$xy^iz \in A$$

Since this path leads to an accepting state, this path should accept strings, $xz, xyz, xy^2z, xy^3z, \dots, xy^iz, \dots$

Question: Can we use pumping lemma to show that a particular language is regular?

You do not use pumping lemma to show a language is regular.

But we can use this to show a language is not regular.

Applications of pumping lemma.

Pumping lemma is used to show that a particular language is not regular.

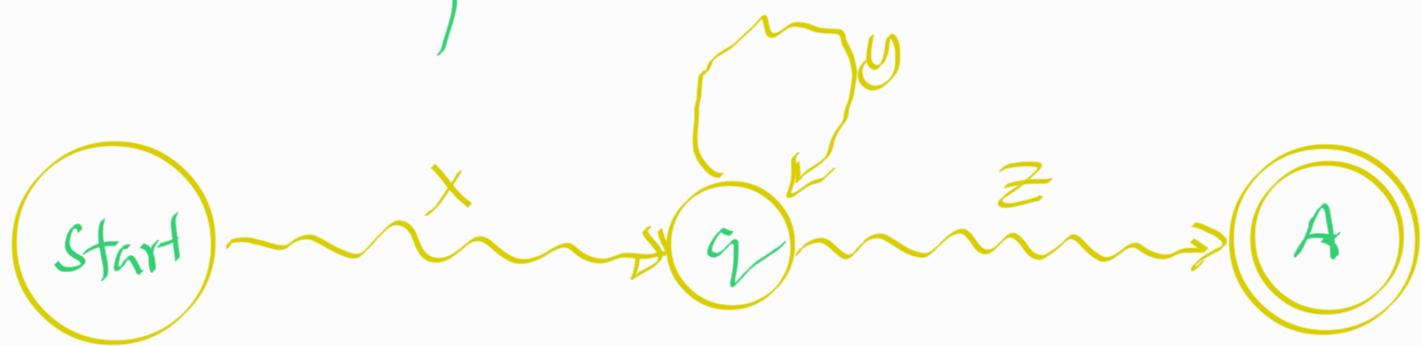
Following is the procedure that we can follow to show a particular language is not regular.

Suppose you are given the language A.

Language A

1. Language A^* should be infinite, because if it was finite, then A should be regular.
2. Let's assume A is regular.
3. Then, there should be a DFA that recognizes language A . Let's call this machine M .
4. Since M is a DFA, # states in M should be finite.
5. Since A is an infinite language, there must be at least one string in A that is of length at least p .

6. let's pick some s , s.t $|s| \geq p$.



Existence of s in A implies that there is a path to accepting state that passes at least one state in the machine.

then any string that uses this path must be accepted by this machine, e.g.,

$xz, xy\bar{z}, xy^2\bar{z}, xy^3\bar{z}, \dots$

The trick is to pick a string s from A , s.t, once it is decomposed into $s = xyz$, the strings that we generate by pumping y are no longer in language A .

* Remember there could be multiple ways to decompose s into xyz .

Therefore, we need to show that every decomposition of s leads to creating new strings that are not in A .

If you create strings that are accepted by M but not in A , then we have a contradiction.

Example: prove $A = \{0^n 1^n \mid n \geq 0\}$
is not regular.

Assume A is regular.

Suppose pumping length of language
 A is p .

(Remember this is a fancy
way of saying the DFA that
recognize A has p states)

Now, the goal is to pick
a string s in A of length
at least p such that it
violates the pumping lemma.

