09/02/2025

Finite Automaton

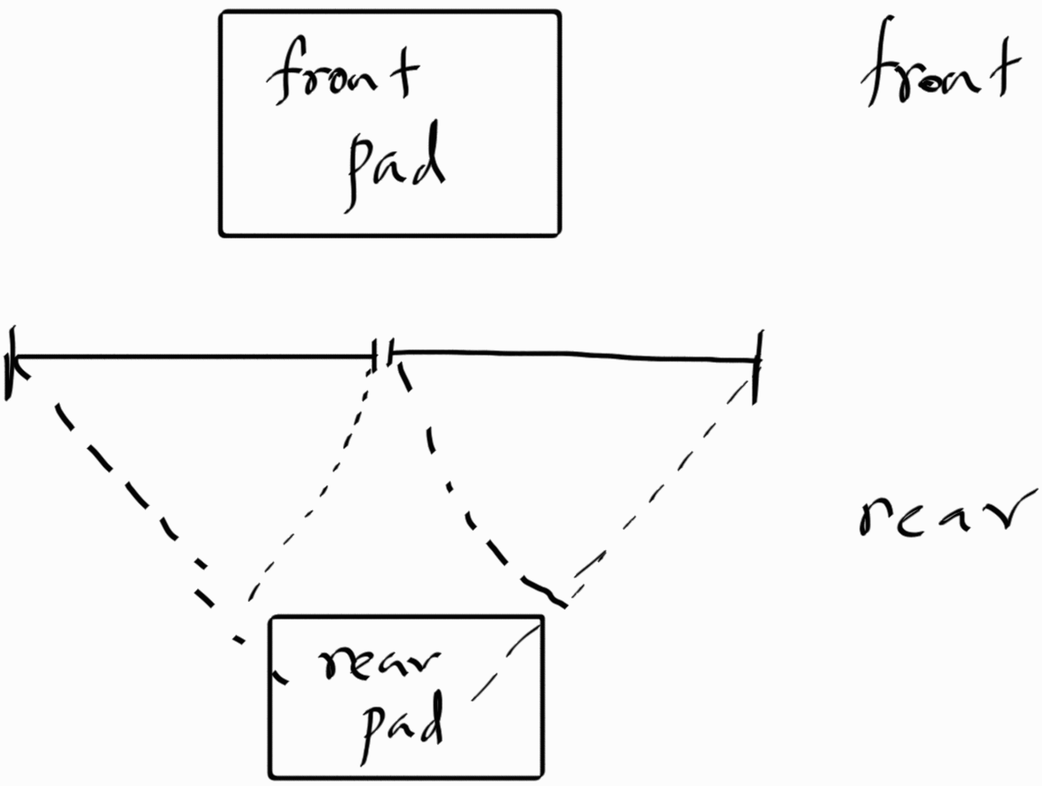
What is a computational model?

Computational model is an idealized version of a computer.

- computation models are accurate in someways and not in others.

Finite Automatons

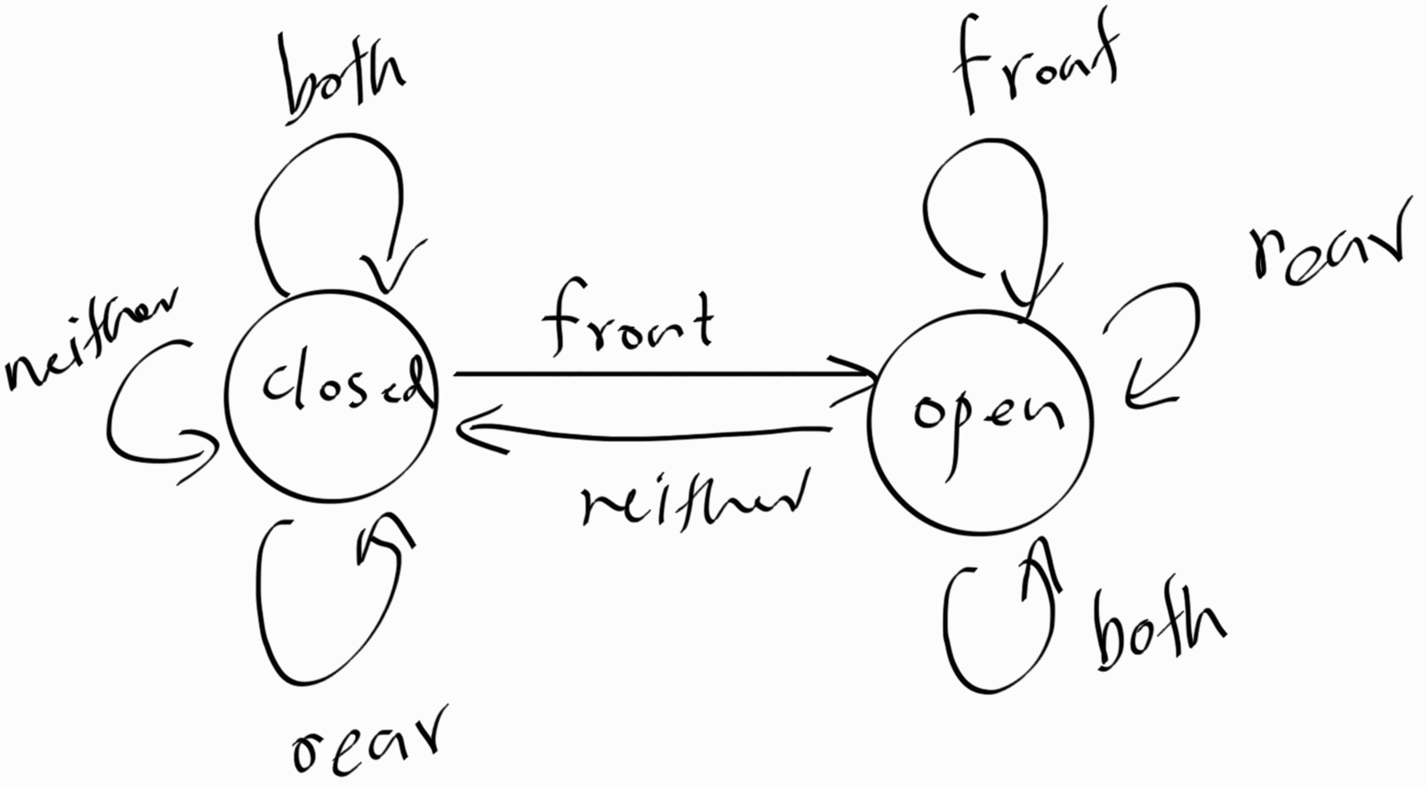
- A model to understand computers with extremely limited amount of memory.



controller is either closed or open,

There are 4 possible inputs

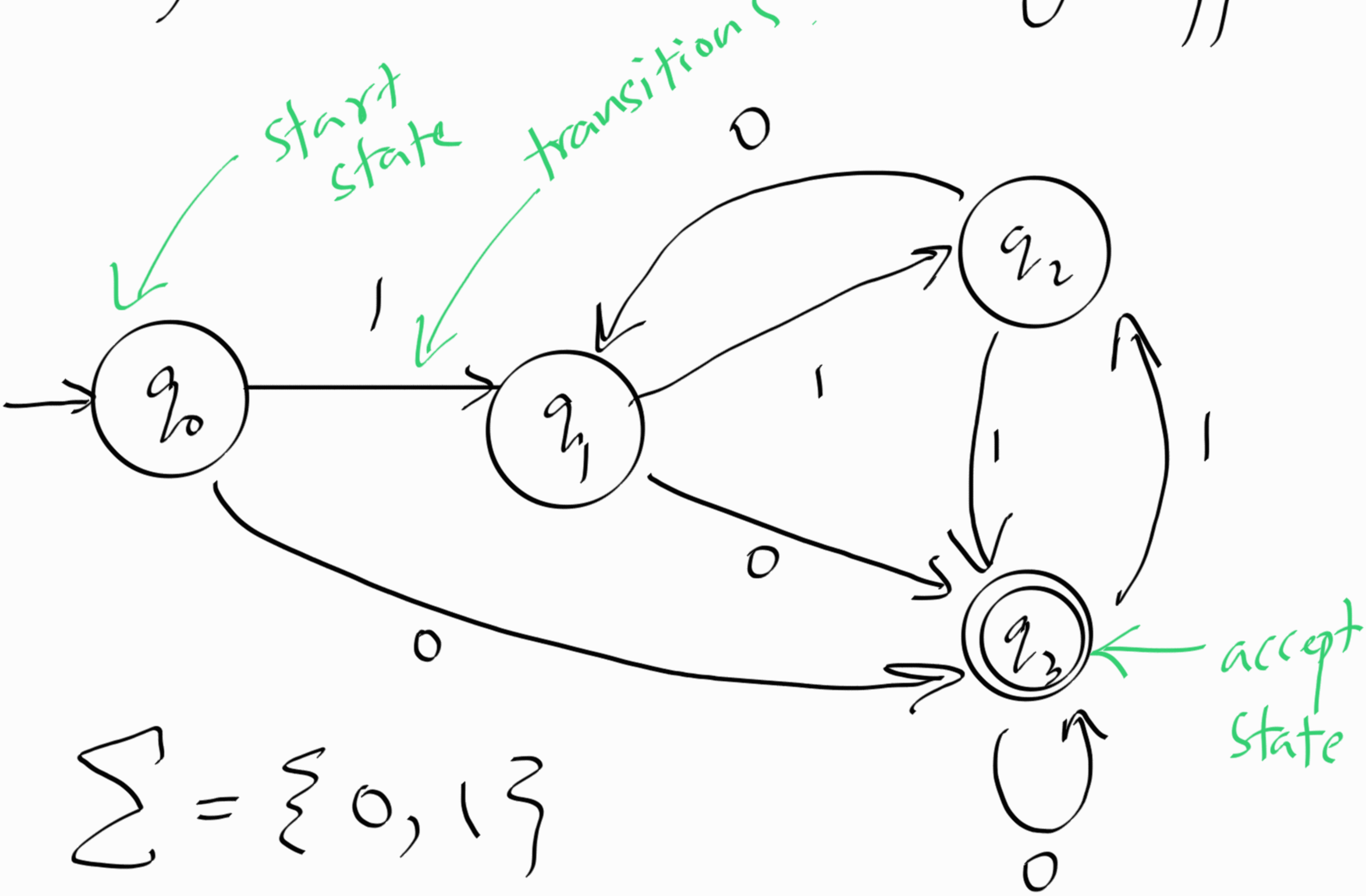
1. Front
2. Rear
3. Both
4. Neither



memory of this machine is 1 bit.

Let's look at this in a mathematical perspective.

Let's abstract this concept of finite automaton, without any reference to any application,



$\Sigma = \{0, 1\}$

$\boxed{10 \text{ accepted by } M}$

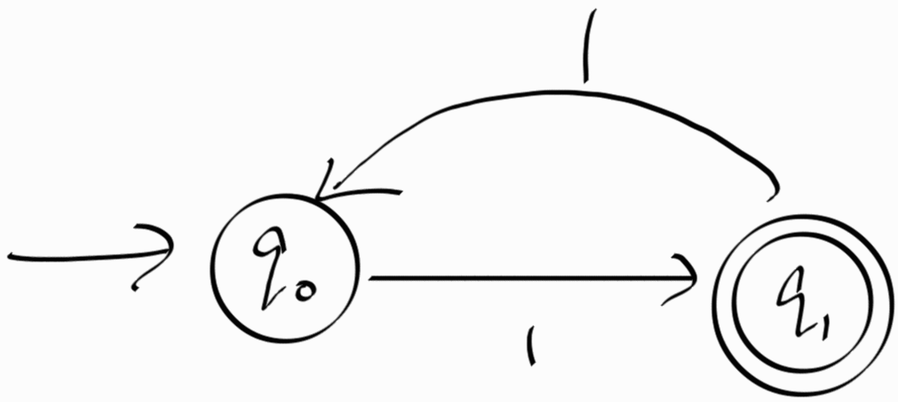
M

101 is not accepted by M

Ex: Design a finite automaton which accepts

$A = \{ w \mid \text{the length of } w \text{ is odd} \}$

$\Sigma = \{1\}$



1111

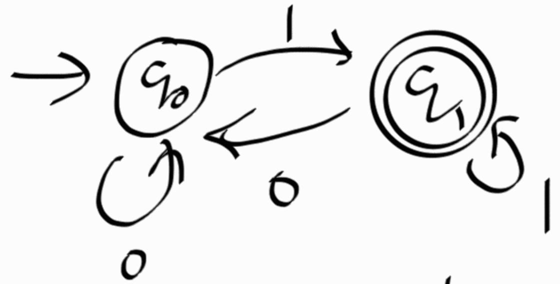
111

state that represents we have processed odd # characters up to now

- States are nodes, some states are accepting states
- arrows represent transitions.
- inputs are symbols available in the alphabet.

Definition

Ex:



- A finite state automaton is a 5-tuple: $(Q, \Sigma, \delta, q_0, F)$

  - Q is the set of states

    $Q = \{q_0, q_1\}$

  - Σ is the alphabet.

    $\Sigma = \{0, 1\}$

  - $\delta : Q \times \Sigma \longrightarrow Q$

  - $q_0 \in Q$ is the start state

  - $F \subseteq Q$ is the set of accepting states.

    $F = \{q_1\}$

    $\{q_1\} \subseteq \{q_1, q_2\}$

If A is the set of strings that is accepted by the machine M, then

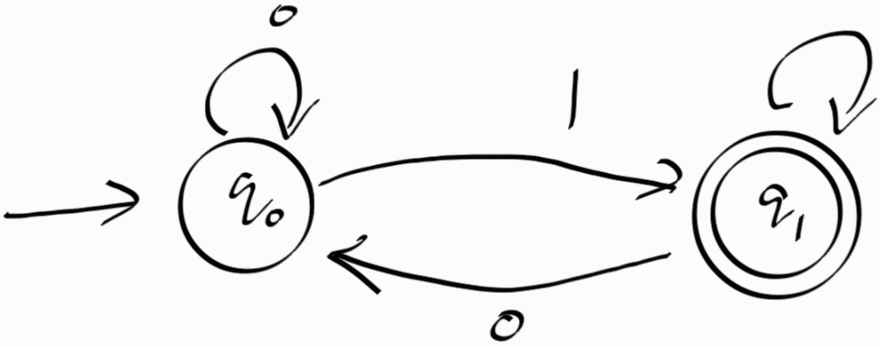
$$L(M) = A$$

M recognizes A.
M accepts A.

Formally, given $M = (Q, \Sigma, \delta, q_0, F)$ and $w = w_1 w_2 w_3 \cdots w_n$ be a string over alphabet Σ, M accepts w if there is a sequence of states $r_0, r_1, r_2, \ldots, r_n$ such that

1. $r_0 = q_0$
2. $\delta(r_i, w_{i+1}) = r_{i+1}$; $\forall i \in \{0, 1, 2, \ldots, n-1\}$
3. $r_n \in F$

$$r_0 \xrightarrow{w_1} r_1 \xrightarrow{w_2} r_2 \xrightarrow{w_3} \cdots\cdots \xrightarrow{w_n} r_n$$



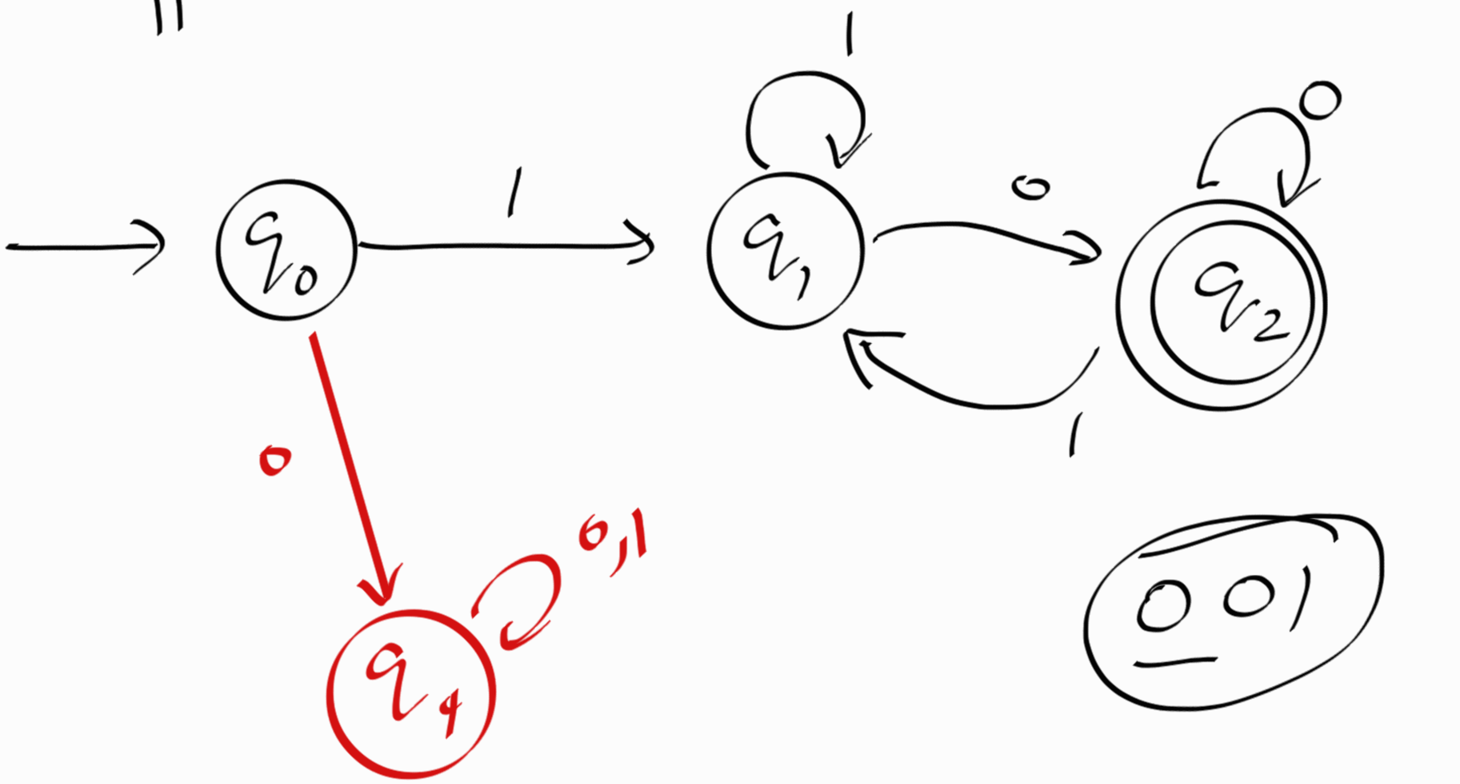
0011 ✓

$$q_0 \xrightarrow{0} q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1$$

010 ×

$$q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{0} \underline{q_0}$$

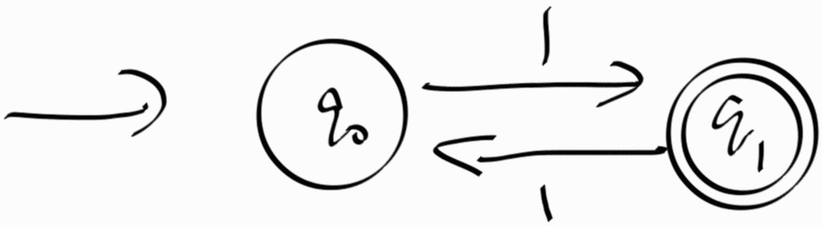
Suppose you have the following



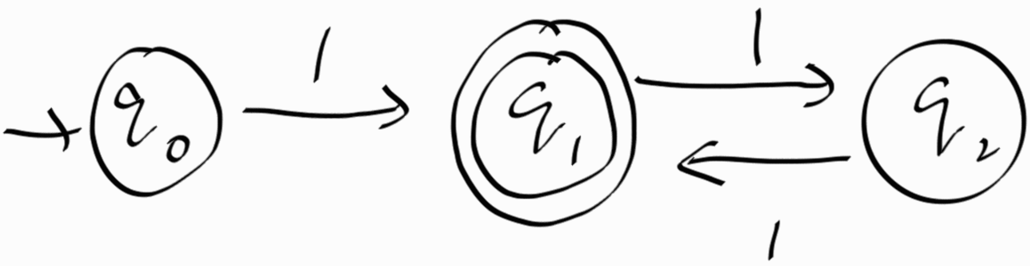
Question: Is q_4 needed?

Yes, we need this formally but since this transition moves to a state that will not lead to an accepting state we could drop it.

$\Sigma = \{1\}$



M_1



M_2

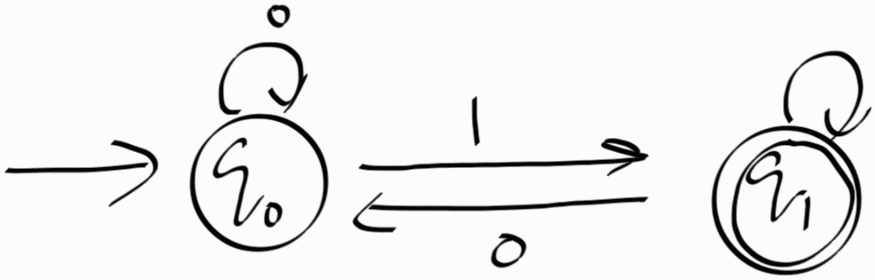
Do these machines accept the same language?

<u>Yes</u>

$M_1 = M_2$

$L(M_1) = L(M_2)$

$M_1 = M_2 \Longleftrightarrow L(M_1) = L(M_2)$



$\Sigma = \{0, 1\}$

1. $Q = \{q_0, q_1\}$
2. $\Sigma = \{0, 1\}$
3. δ is described as follows:

	0	1
q_0	q_0	q_1
q_1	q_0	q_1

4. q_0 is the start state
5. $F = \{q_1\}$

Let us try to identify languages accepted by the following machines.

EX1:

M_1

1

11

111111

101

$$L(M_1) = \{1^n \mid n \geq 0\}$$

kleen star

$$L(M_1) = \{1^*\}$$