02/03/2025

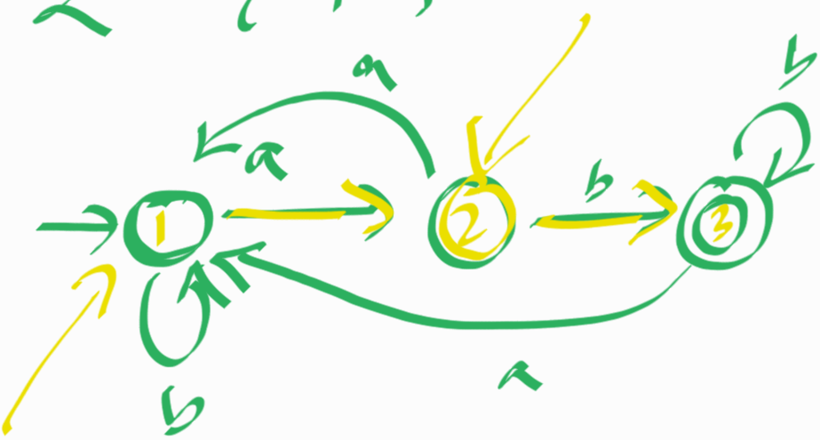
Today's goal is to understand
why NFA = DFA

But first, let us do some examples.

1. Design a DFA that accepts following language.

$C = \{w \mid w$ has an odd number of a's, and ends with b$\}$

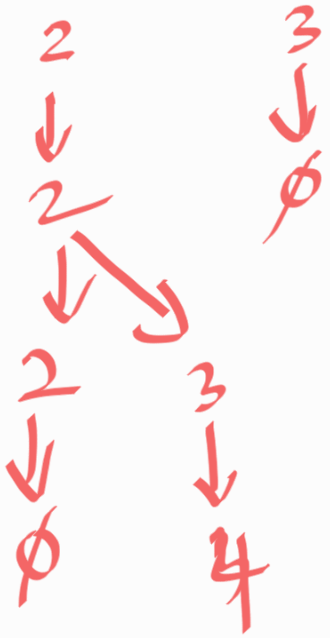
$\Sigma = \{a, b\}$



2. Design a NFA for

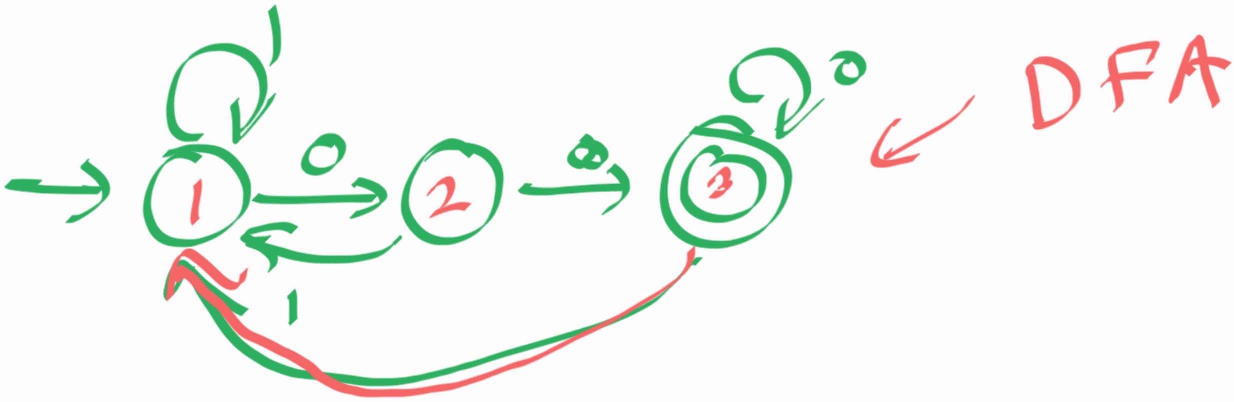
$D = \{a^+ b^+ a^+\}$ $\quad \Sigma = \{a, b\}$

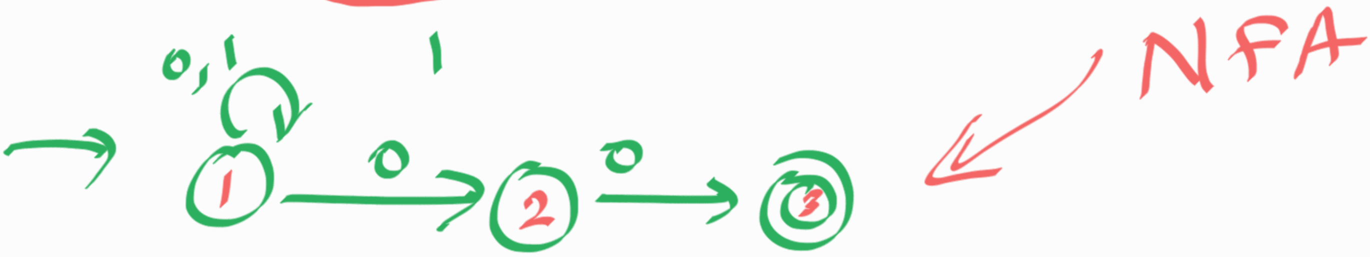
a a b b a a b



3. Design an NFA for

$E = \{ w \mid w \text{ ends with } 00 \}$ $\Sigma = \{0,1\}$





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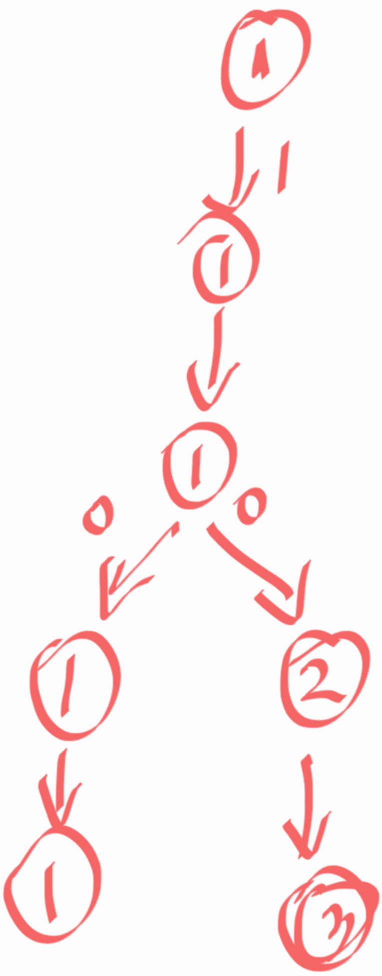

Two machines are equivalent if they accept the same language.

Theorem 1.39

Every NFA has an equivalent DFA.

Proof: let $N = (Q, \Sigma, \delta, q_0, F)$ be the NFA recognizing A.

we'll construct a DFA M, $M = (Q', \Sigma, \delta', q_0', F)$ such that M recognizes A.

What should this machine do?

a) No ε- transitions in N

1. $Q' = P(Q)$

2. $\delta'(R, a) = \bigcup_{r \in R} \delta(r, a)$, $R \in Q'$, $a \in \Sigma$

3. $q'_0 = q_0$

4. $F' = \{R \in Q' \mid R$ contains an accept state of $N\}$

What about ε-arrows?

We define $E(R)$ for $R \subseteq Q$ as the collection of states that we can reach from R by going along ε arrows, including members of R themselves.

With this updated notation the δ function should be redefined.

$$\delta'(R,a) = \{q \in Q \mid q \in E(\delta(r,a)) \text{ for some } r \in R\}$$

Also, slight change in the starting state as well, since there could be ε transitions on starting state

$$q_0 = E(\{q_0\})$$

NFA N:

Example without ε-transitions

DFA M:

Corollary 1.40

A language is regular iff some NFA accepts it.

Theorem 1.45

The class of regular languages is closed under the union operation.

Proof

Proof idea

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1

Let $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognizes A_2

we construct the NFA N,
$N = (Q, \Sigma, \delta, q_0, F)$ from N_1 & N_2

1. $Q = \{q_0\} \cup Q_1 \cup Q_2$
2. $q_0 \in Q$ is the new starting state
3. $F = F_1 \cup F_2$
4. $$\delta(q,a) = \begin{cases} \delta_1(q,a) & \text{if } q \in Q_1 \\ \delta_2(q,a) & \text{if } q \in Q_2 \\ \{q_1, q_2\} & q = q_0 \text{ and } a = \varepsilon \\ \phi & , q = q_0 \text{ and } a \neq \varepsilon \end{cases}$$