Recap

DFA/NFA : models for devices with limited memory

PDA : Devices that has unlimited stack.

These two models have limitations.

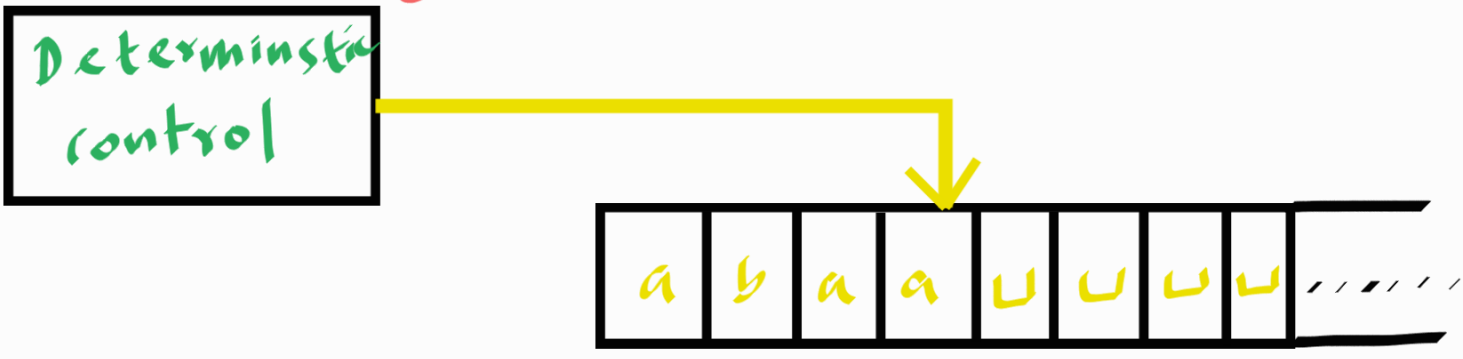
These models are too restrictive.

$B = \{a^n b^n \mid n \geq 0\}$

$A = \{a^n b^n c^n \mid n \geq 0\}$

Model for general computers.

- has unlimited and unrestricted memory.



Infinite tape.

1. Initially tape contains the input, and blank every where else.

2. Machine can store information on tape.

3. To read information from the tape, machine can move its head back over it.

4. Machine computes until it produces an output.
   - accept / reject.

Ex: $A = \{0^{2^n} \mid n \geq 0\}$

$0 \in A$

$00 \in A$

$000 \notin A$

$0000 \in A$

⇕ general programming language

C, C++, Java, ...

test if # of 0's (x) $= 2^n$

Algo(x)

x % 2 == 1, No

x % 2 == 0, recurse on $x/2$

Any function that can be computed by an intuitive algorithm can be computed by an Turing machine.

intuitive algorithm

1. A step-by-step finite process
2. Using discrete, finite symbols
3. Running in finite time.

Differences between finite automata and Turing Machines.

1. Turing Machine can both write on tape and read from tape.

2. Read-write head can move both left and right.

3. Tape is infinite.

4. Special states for accepting and rejecting take effect immediately.

Def

A turing Machine is a 7-tuple, $(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$, where

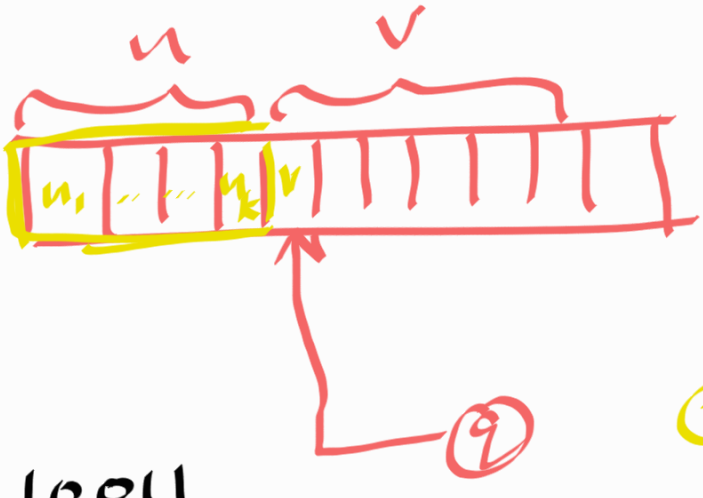
1. Q is the set of states.
2. Σ is the input alphabet
3. Γ is the stack alphabet, where $\sqcup \in \Gamma$ and $\Sigma \subseteq \Gamma$ ← $\sqcup$ blank symbol
4. $\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \times \{L, R\}$
5. $q_0 \in Q$ is the start state
6. $q_{accept} \in Q$ is the accept state
7. $q_{reject} \in Q$ is the reject state

$q_{accept} \neq q_{reject}$

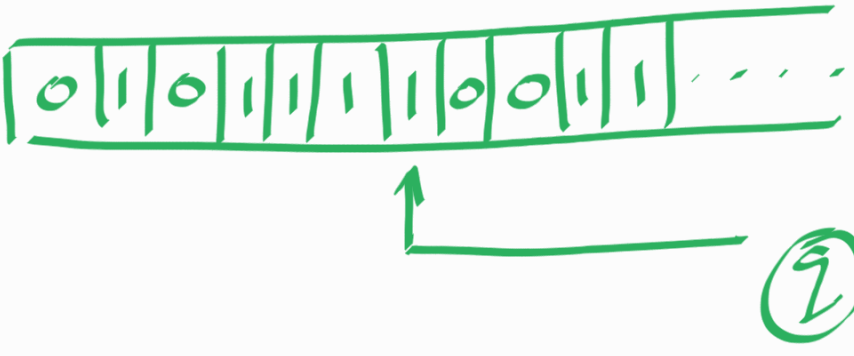
How to represent the state of the machine (configuration)

we have 3 items.

(i) current state
(ii) tape content
(iii) current head location



we have 01011q10011



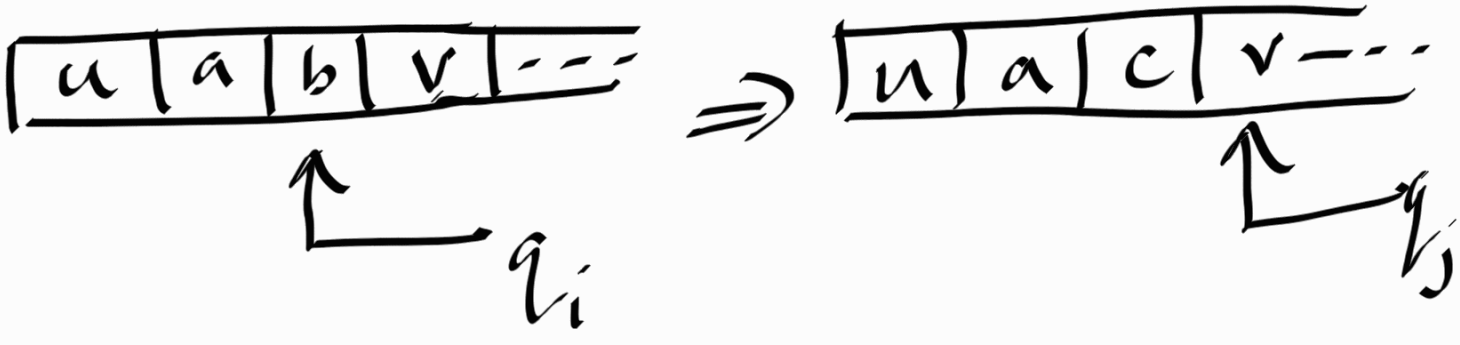
we say configuration C_1 yields configuration C_2 if the turing machine can legally go from C_1 to C_2 in a single step.

If we have $\delta(q_i, b) = (q_j, c, L)$

$u a q_i b v$ yields $u q_j a c v$

If we have $\delta(q_i, b) = (q_j, c, R)$

$u a q_i b v$ yields $u a c q_j v$



start configuration

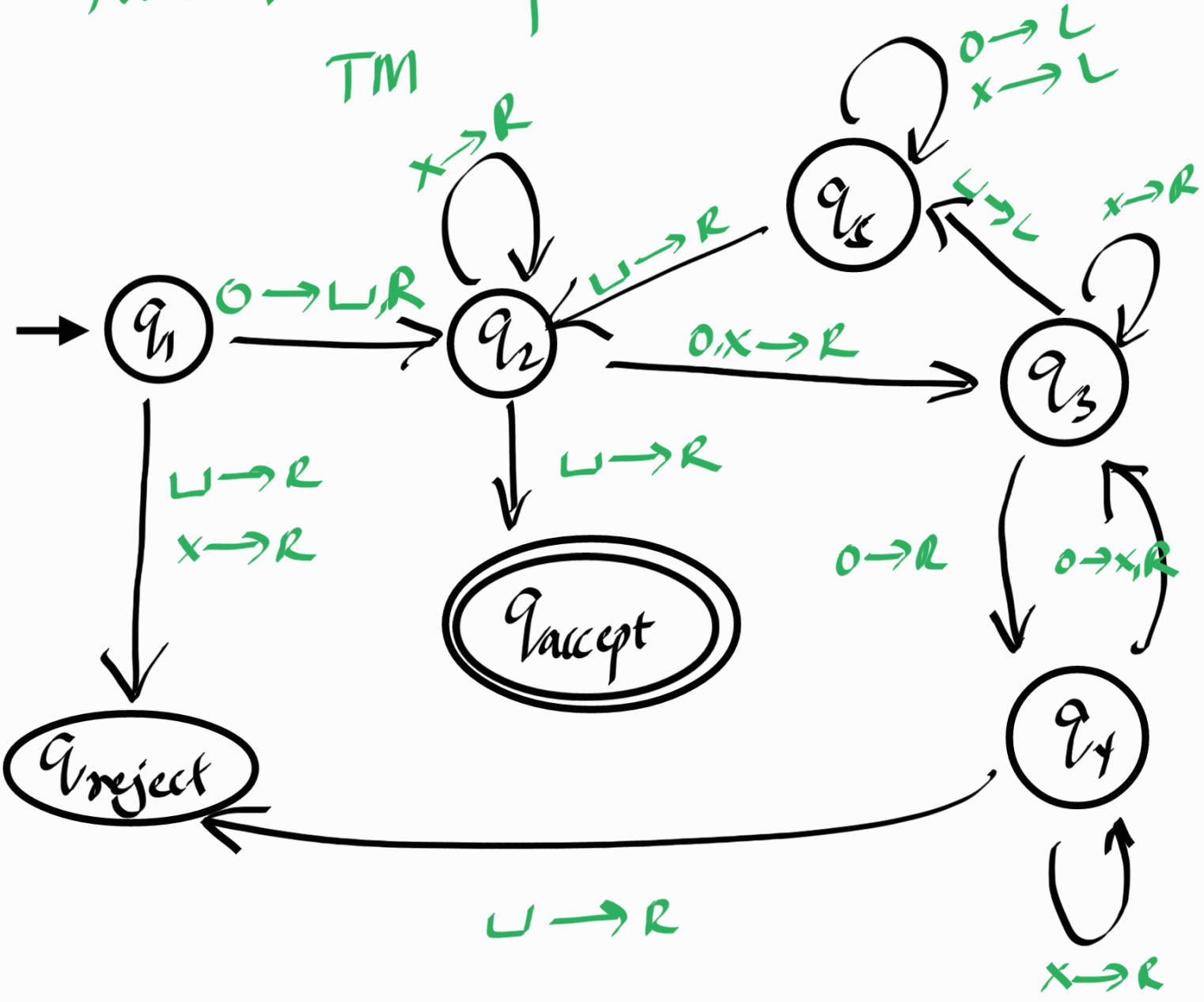
$q_0 w$

$A = \{0^{2^n} \mid n \geq 0\}$

M = "on input string w"

1. Sweep left to right across the tape, crossing off every other zero.
2. If in stage 1, the tape contains a single 0, accept
3. If in stage 1, the tape contains more than single 0 and the # of 0's was odd, reject
4. Return the head to the left hand end of the tape
5. Go to stage 1.

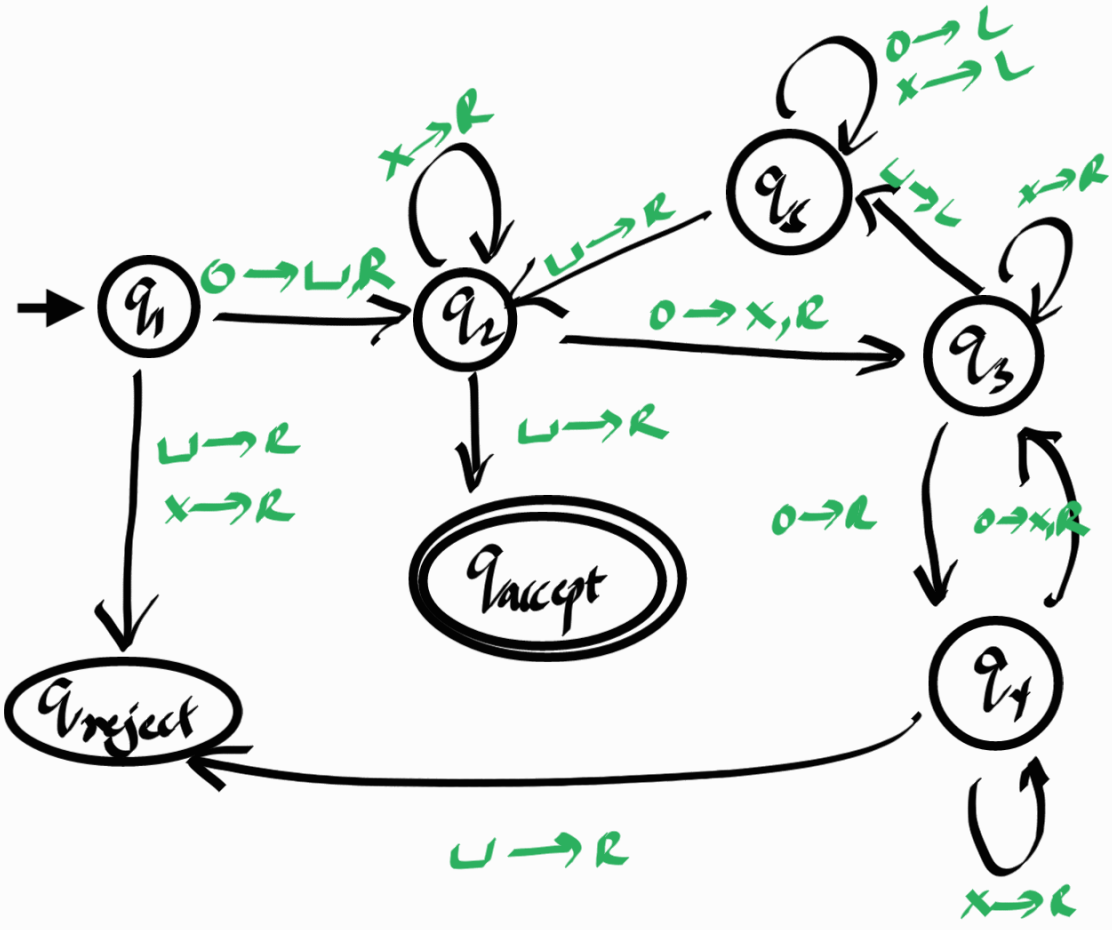
formal description of this TM



$a \longrightarrow \tau, R$ means

read a from head writ τ to stack and move head to write.

$a \rightarrow L$ means read a from head don't change tape and move left.


q1
q2
q3
q4
q5
q6
qaccept
qreject
0 → ␣R
␣ → R
x → R
0 → x,R
0 → L
x → L
L → L
0 → R
0 → x,R