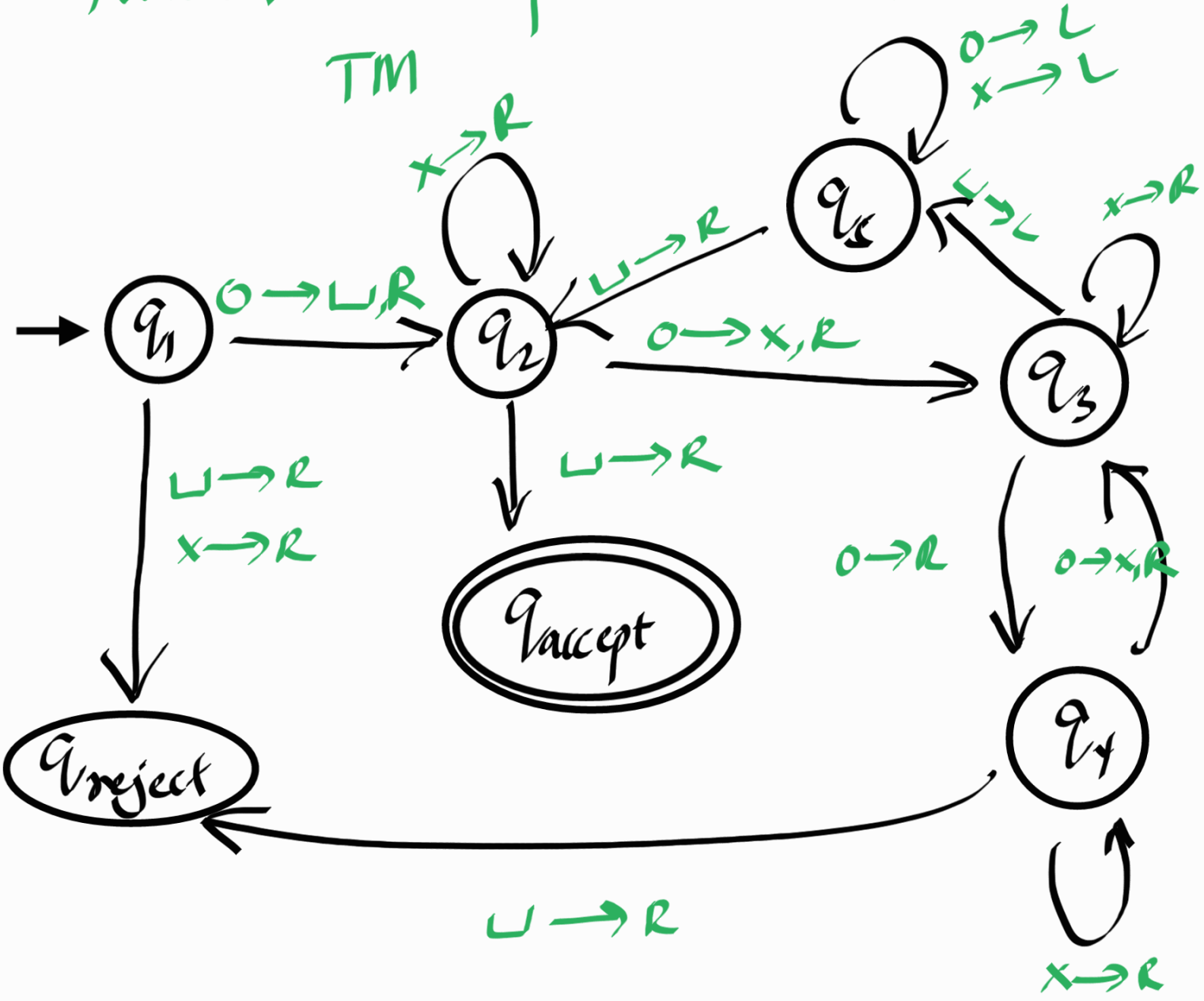
$A = \{0^{2^n} \mid n \geq 0\}$

M = "on input string w"

1. Sweep left to right across the tape, crossing off every other zero.
2. If in step 1, the tape contains a single 0, accept
3. If in stage 1, the tape contains more than single 0 and the # of 0's was odd, reject
4. Return the head to the left hand end of the tape
5. Go to stage 1.

formal description of this TM

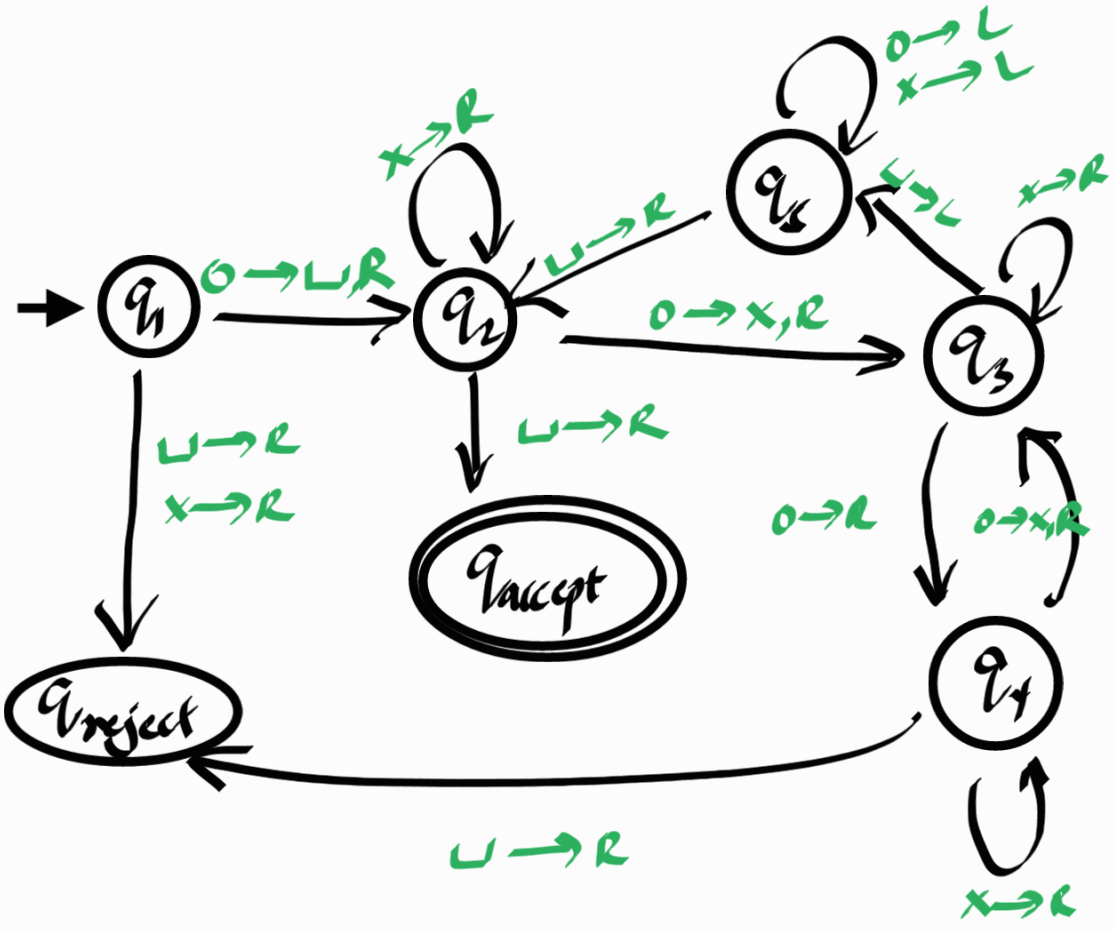


$a \longrightarrow \tau, R$ means

read a from head writ τ to stack and move head to write.

$a \rightarrow L$ means read a from head don't change tape and move left.

$a \rightarrow a, L$


q1
q2
q3
q4
qaccept
qreject
0 → ␣ R
x → R
␣ → R
0 → x, R
0 → L
x → L
␣ → L
0 → R
x → R

00

q1 00
0 0 ␣ ...
␣ q2 0
␣ 0 ␣
x q3 ␣
␣ x ␣
␣ q3 x
␣ x ␣
q5 ␣ x ␣
␣ q2 x
␣ x ␣
x q2 ␣
␣ x ␣
␣ qaccept ␣
␣ x ␣ ␣

The collection of strings that turing machine M accepts is the language of M, or the language recognized by M, denoted by L(M).

Unlike, DFA, NFA and PDAs, given an input there are 3 possibilities for a turing machine.

(i) Accept

(ii) Reject

(iii) Loop: machine simply does not halt.

given an input any of these 3 things can happen.

Def

We call a language L is Turing-Recognizable if some turing machine recognizes it.

Turing machines that halts on all inputs are called deciders.

Def

We call a Language L is Turing-Decidable or simply decidable if some turing machine decides it.

Given language L and turing machine M.

If M decides language L, then

if $x \in L$	M accepts x
if $x \notin L$	M rejects x.

If M recognizes language L, then

if $x \in L$	then	M accepts x
if $x \notin L$	then	M either rejects or loops indefinitely.

Ex: check if $p(x) = c_0x^n + c_1x^{n-1} + \cdots + c_{n-1}x + c_n$

has an integral root x_0, i.e.,

checking if there is an integer x_0 s.t $p(x_0) = 0$

Algorithm (TM)

check if x_0 is an integral root for $x_0 = 0, \pm 1, \pm 2, \pm 3, \pm 4, \cdots$,

Q1: Does this algorithm provide a recognizer?

Yes, If $x_0 = 2001$, algorithm will find it.

Q2: Does this algorithm provide a decider?

No, because if the $p(x)$ does not have an integral root this will loop forever.