02/03/2025

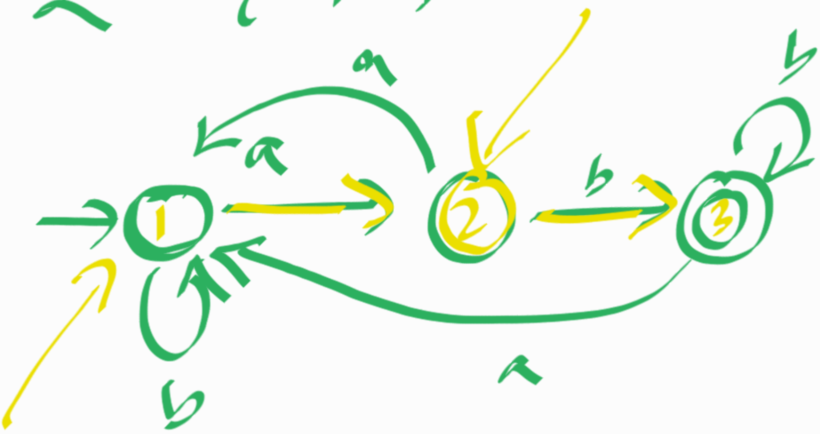
Today's goal is to understand why NFA = DFA

But first, let us do some examples.

1. Design a DFA that accepts following language.

$C = \{ w \mid w$ has an odd number of a's, and ends with b $\}$

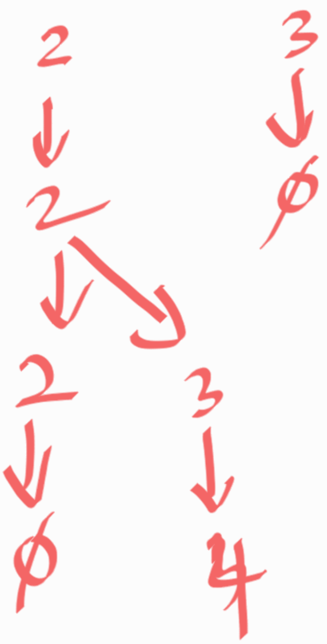
$\Sigma = \{a, b\}$



2. Design a NFA for

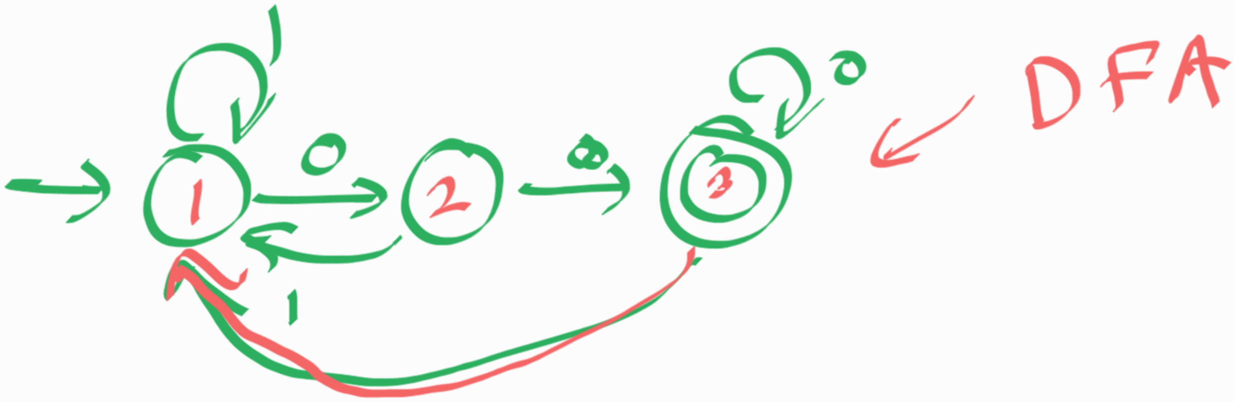
$D = \{a^+ b^+ a^+\}$ $\Sigma = \{a, b\}$

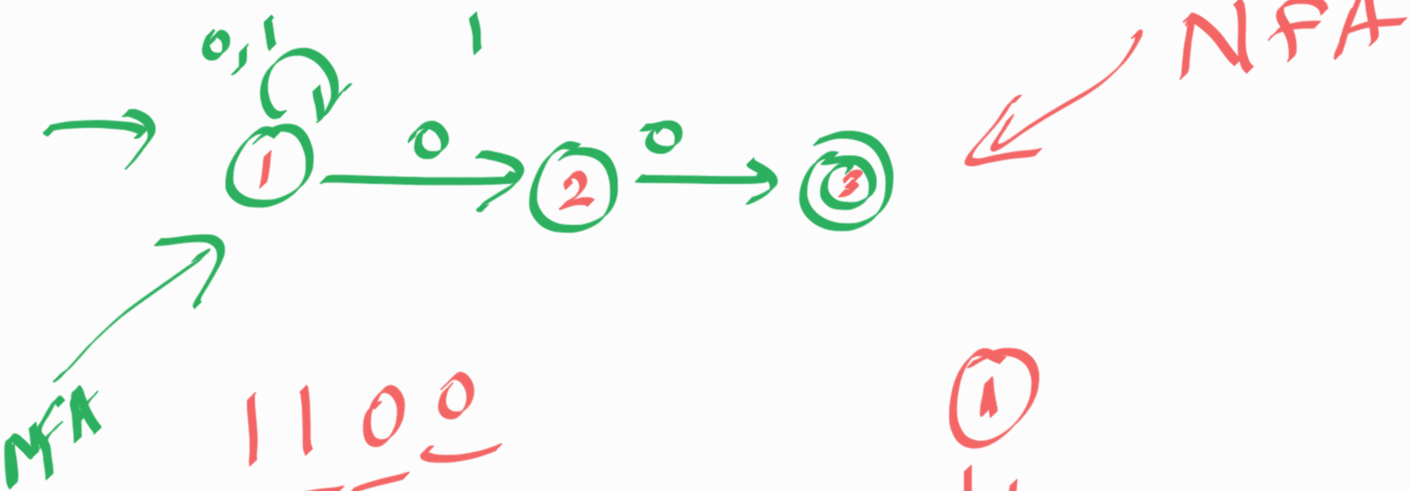
a a b b a a b



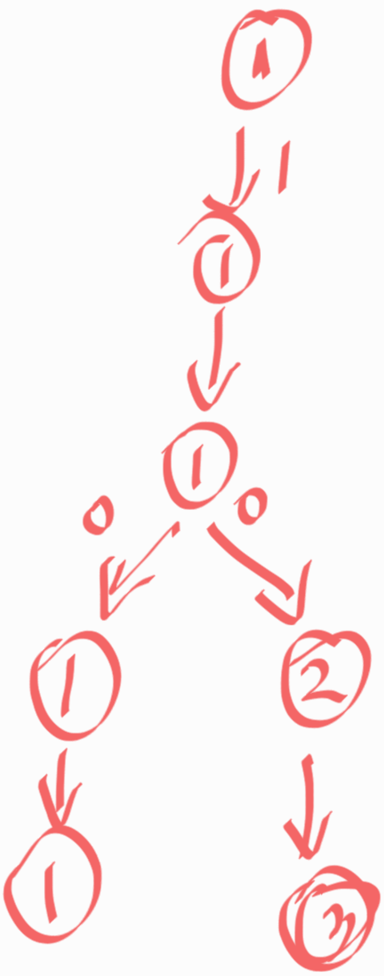
3. Design an NFA for

$E = \{ w \mid w \text{ ends with } 00 \}$ $\Sigma = \{0,1\}$





1100



Two machines are equivalent if they accept the same language.

Theorem 1.39

Every NFA has an equivalent DFA.

Proof: let $N = (Q, \Sigma, \delta, q_0, F)$ be the NFA recognizing A.

we'll construct a DFA M, $M = (Q', \Sigma, \delta', q_0', F)$ such that M recognizes A.

What should this machine do?

a) No ε- transitions in N

1. $Q' = P(Q)$
2. $\delta'(R, a) = \bigcup_{r \in R} \delta(r, a)$, $R \in Q'$, $a \in \Sigma$
3. $q_0' = q_0$
4. $F' = \{R \in Q' \mid R$ contains an accept state of $N\}$

The basic idea for using the power set is to simulate every possible execution path that can take place in an NFA.

What about ε_ arrows?

We define $E(R)$ for $R \subseteq Q$ as the collection of states that we can reach from R by going along ε arrows, including members of R themselves.

$E(R)$ function is called ε-closure

with this updated notation the δ function should be redefined.

$$\delta'(R, a) = \{q \in Q \mid q \in E(\delta(r,a)) \text{ for some } r \in R\}$$

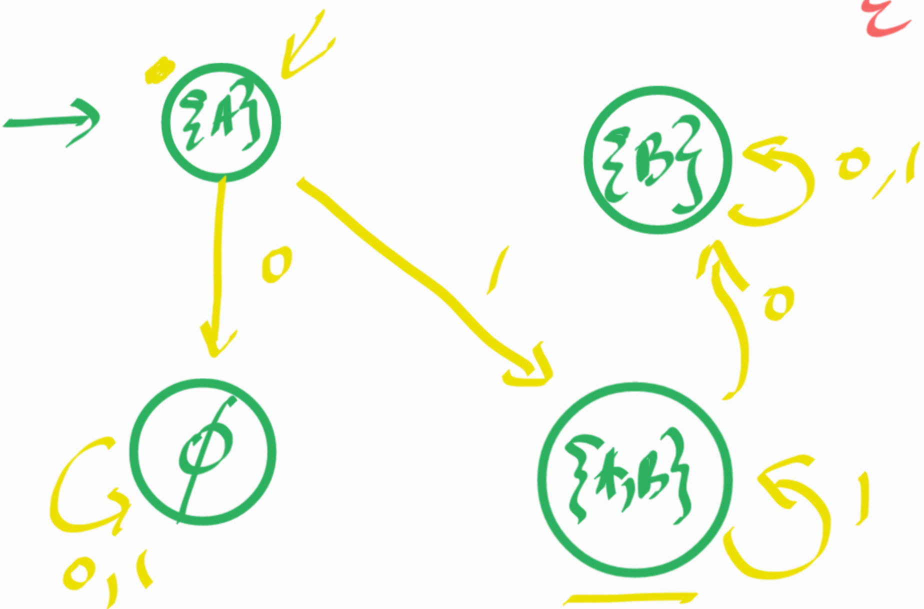
Also, slight change in the starting state as well, since there could be ε transitions on starting state

$$q_0 = E(\{q_0\})$$

NFA N:

Example without ε-transitions

DFA M:



$\delta'(\phi, 0)$

$\{\} \cup \{B\} = \{B\}$

$\delta'(\{A\}, 0) = \delta(A, 0) = \phi$

$\delta'(\{A\}, 1) = \delta(A, 1) = \{A, B\}$

$\delta'(\{B\}, 0) = \delta(B, 0) = \{B\}$

$\delta'(\{B\}, 1) = \delta(B, 1) = \{B\}$

$\delta'(\{A, B\}, 0) = \delta(A, 0) \cup \delta(B, 0)$

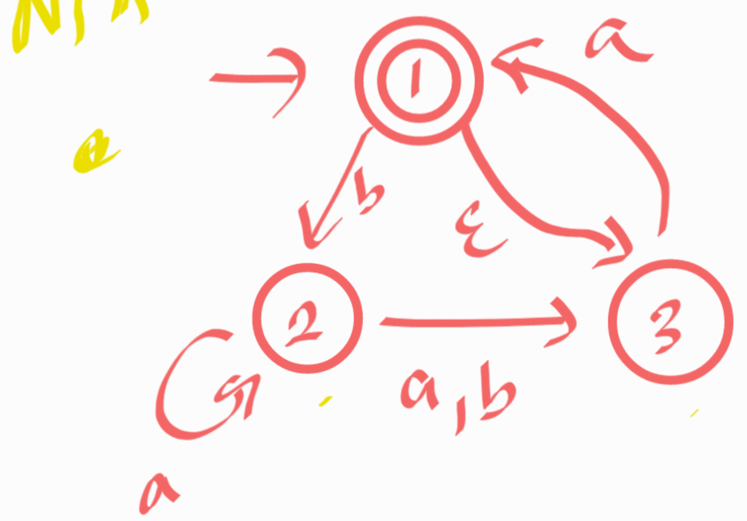
$\phi \cup \{B\} = \{B\}$

$= \{B\}$

$\delta'(\{A, B\}, 1) = \delta(A, 1) \cup \delta(B, 1)$ $\rightarrow \{A, B\} \cup \{B\} = \{A, B\}$

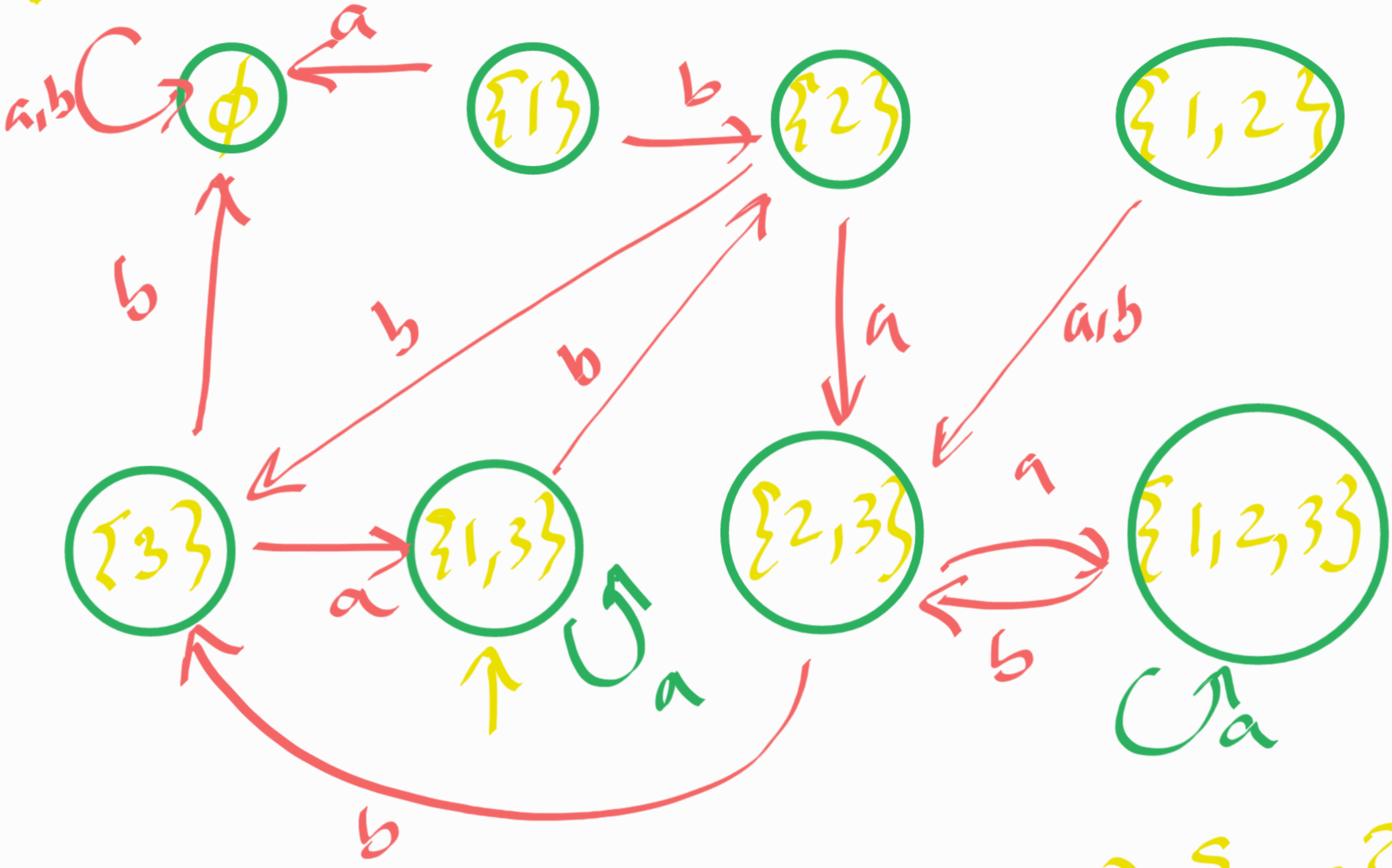
Let's try to understand E(R)

NFA



$E(\{q_0\}) = E(\{1\})$
$= \{1,3\}$

DFA



$E(\{1\}) = \{1,3\}$ $E(\{2,3\}) = \{2,3\}$

$E(\{1,2\}) = \{1,3,2\}$

$$\delta(R, a) = \{q \in Q \mid E(\delta(r,a)) \text{ for } r \in R\}$$

$$R = \{1, 3\}$$

$$E(\delta(1,a)) = E(\phi) = \phi$$

$$E(\delta(3,a)) = E(\{1\}) = \{1,3\}$$

$$\phi \cup \{1,3\}$$

$$\delta(\{1,2,3\}, a) \Rightarrow$$

$$E(\delta(1,a)) = E\{\phi\} = \phi$$

$$E(\delta(2,a)) = \{2\}$$

$$E(\delta(3,a)) = \{1,3\}$$

$$\phi \cup \{2\} \cup \{1,3\}$$

$$\{1,2,3\}$$

→ go to lec 09 notes.

Corollary 1.40

A language is regular iff some NFA accepts it.

Theorem 1.45

The class of regular languages is closed under the union operation.

Proof

Proof idea

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1

Let $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognizes A_2

we construct the NFA N,
$N = (Q, \Sigma, \delta, q_0, F)$ from N_1 & N_2

1. $Q = \{q_0\} \cup Q_1 \cup Q_2$
2. $q_0 \in Q$ is the new starting state
3. $F = F_1 \cup F_2$
4. $$\delta(q,a) = \begin{cases} \delta_1(q,a) & \text{if } q \in Q_1 \\ \delta_2(q,a) & \text{if } q \in Q_2 \\ \{q_1, q_2\} & q = q_0 \text{ and } a = \varepsilon \\ \phi & , q = q_0 \text{ and } a \neq \varepsilon \end{cases}$$