

Decidable languages related to CFG's

NC will consider following languages today -

$$A_{CFG} = \{ \langle G, w \rangle \mid G \text{ is a CFG that generates } w \}$$

$$E_{CFG} = \{ \langle G \rangle \mid G \text{ is a CFG and } L(G) = \emptyset \}$$

$$EQ_{CFG} = \{ \langle G, H \rangle \mid G, H \text{ are CFG's, } L(G) = L(H) \}$$

$A_{CFG} = \{ \langle G, w \rangle \mid G \text{ is a CFG and } w \text{ is a string and } G \text{ generates } w \}$

- go through all the possible derivations of the grammar G and check whether w is generated.
- we do not have a bound on the # of derivations.
- What if we convert G into CNF

Theorem 4.7

Acfg is decidable

- If G were in Chomsky Normal Form, any derivation of w where $|w|=n$, has $2n-1$ steps.

Proof: construct TM S as follows:

$S = "on \langle G, w \rangle$ where G is (FG & w is a string".

1. Convert G into CNF.
2. List all derivations with $2n-1$ steps, where $n=|w|$
3. If any of the derivations generates w , accept ; otherwise, reject ."

Is this an efficient algorithm?

No, we may have to look
at exponential # of derivations.

$$E_{CFG} = \{ \langle G \rangle \mid G \text{ is a CFG and } L(G) = \emptyset \}$$

Use A_{CFG} and try all possible
 $w \in \Sigma^*$

We try to check whether start variable can generate a string of terminals.

Theorem 4.8 E_{CFG} is decidable.

construct TM R for E_{CFG}' :

1. Mark all terminals in G .
2. Repeat until no new variables get marked
 - Mark A , if $A \rightarrow u_1 u_2 \dots u_k$ is a rule in G and all u_i 's are marked.
3. If the start variables is not marked, accept; otherwise, reject.

Ex: $S \rightarrow ABX \mid BAX$

$A \rightarrow AX$

$B \rightarrow BX$

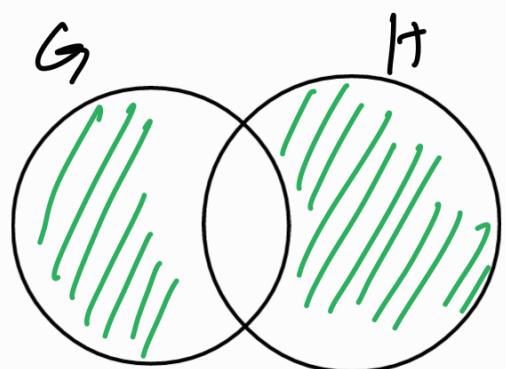
$X \rightarrow a$

$S \rightarrow ABX \quad S \rightarrow BAX \quad A \rightarrow AX \quad B \rightarrow BX$
 $X \rightarrow a$

$$EQ_{CFG} = \left\{ \langle G, H \rangle \mid G, H \text{ are CFG's and } L(G) = L(H) \right\}$$

Can we follow the idea used in
 EQ_{DFA} ?

EQ_{CFG} : Given two CFGs, are they equal?



$$Q: (G \cap \bar{H}) \cup (\bar{G} \cap H)$$

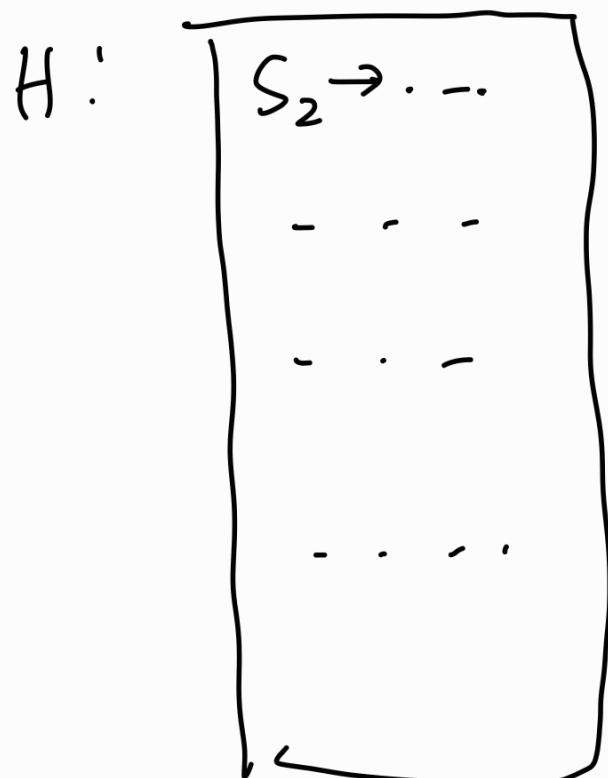
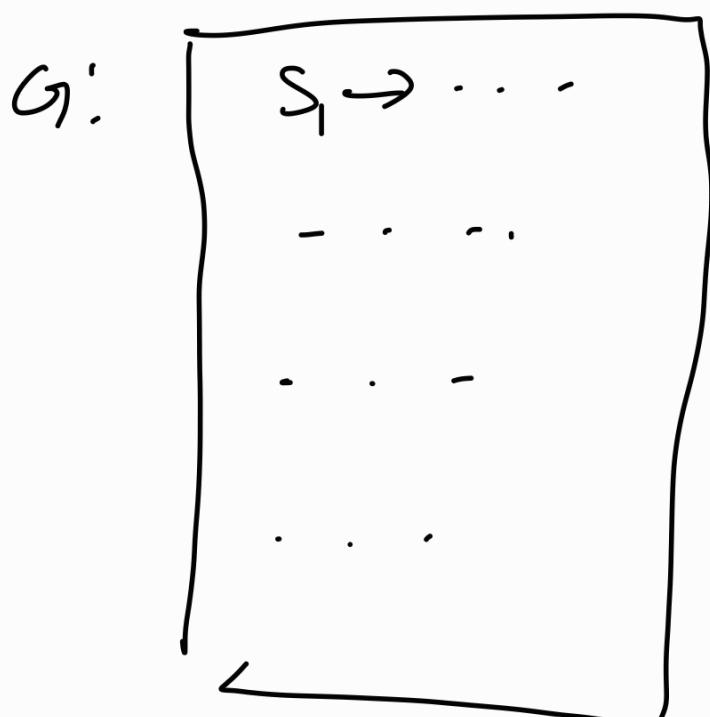
if $G = H$, then $C = \emptyset$

if $C = \emptyset$, then $G = H$

Q: Are CFG's closed under $\cup, \cap, -$ operations?

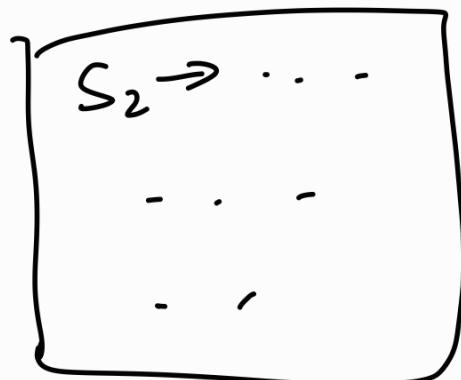
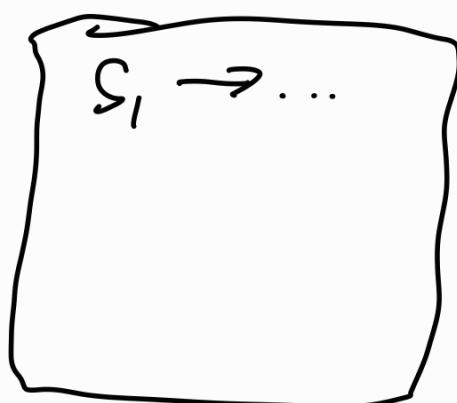
claim 1: CFL's are closed under union operation.

Proof Idea:



construct G' s.t

$$S \rightarrow S_1 \mid S_2$$



claim 2:

CFL's are not closed under
 $\cap, -$ operation.

We only need one counter example.

$$A = \{a^i b^j c^k \mid i, k, j \geq 0, i=j \text{ or } j=k\}$$

* this is a CFL

$$B = \{a^m b^m c^n \mid m, n \geq 0\}$$
 * this is
a CFL

$$C = \{a^n b^m c^m \mid m, n \geq 0\}$$
 * this is
a CFL

$$D = \{a^n b^n c^n \mid n \geq 0\}$$
 * this is
not a CFL

$$D = B \cap C$$

Claim 02: CFL's are not closed under complement ($\bar{\cdot}$) operation.

Consider CFGs B and C

Let $L = B \cap C$

$$B \cap C = \overline{\overline{B} \cup \overline{C}} \quad (\text{De morgan's law})$$

This is a counter example.

- Operator should not be closed for CFL's.

- Later we show that EQ_{CFG} is not decidable.

