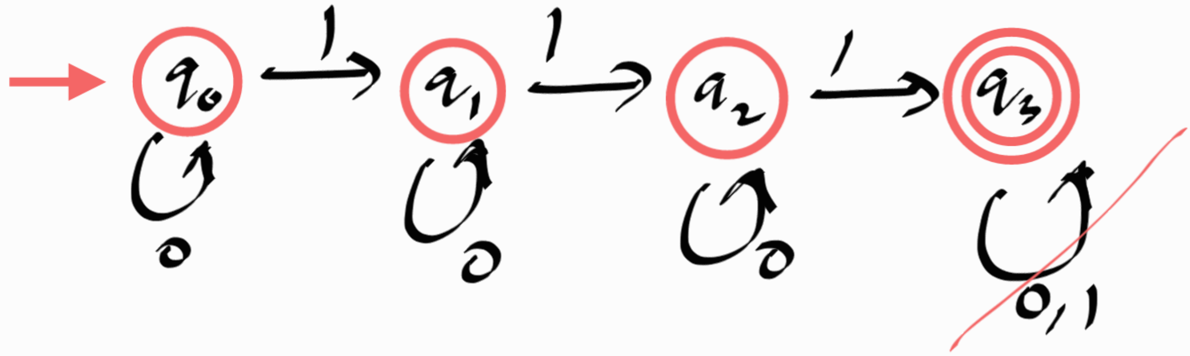
01/31/2025

clarifications.

Q1: $A = \{w \mid w \text{ contains at least } 3 \text{ 1s}\}$ $\quad \Sigma = \{0,1\}$



111 ✓

111111 × ← not accepted

What happens if we do not have this transition?

Q2: $A \circ B = \{xy \mid x \in A, y \in B\}$

How important is the order xy?

Let $A = \{0^*\}$ $B = \{1^*\}$

00001111, $001111 \in A \circ B$

$111111 \in A \circ B$ why 0^0

But what happens

$11110000 \notin A \circ B$

$1111000 \in B \circ A$

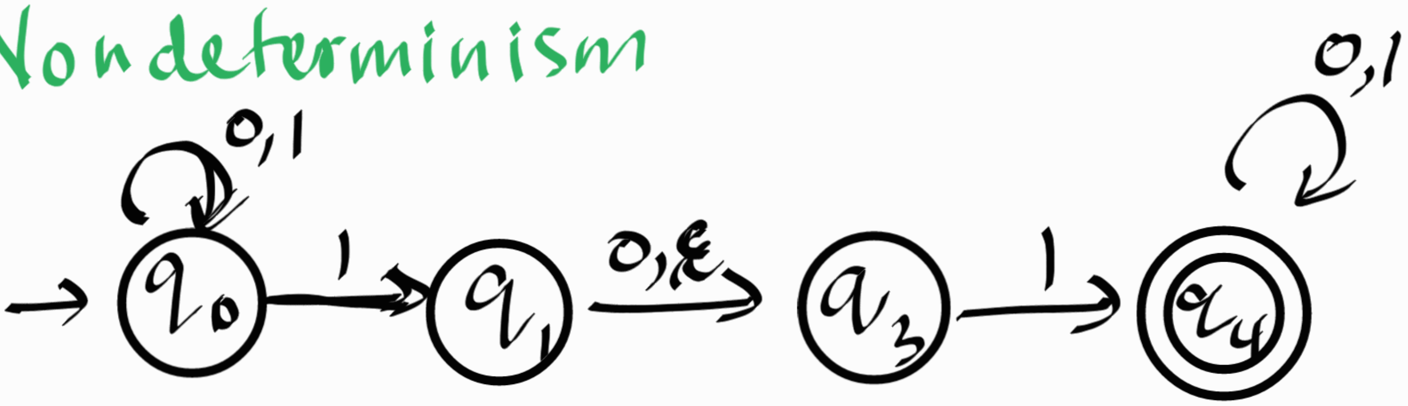
$A \circ B \neq B \circ A$ when $A \neq B$

Let's do some examples

Next week

Deterministic – Given the current state and the input, we know exactly which state to reach to.

Nondeterminism

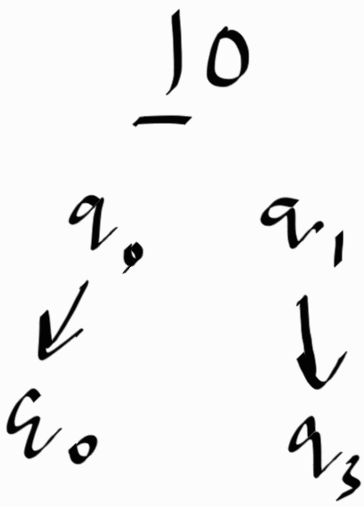


Language?

$(0|1)^* \, 1 \, (0|\varepsilon) \, 1 \, (0|1)^*$

$(0|1)^* \, 1 \, 0 \, 1 \, (0|1)^*$

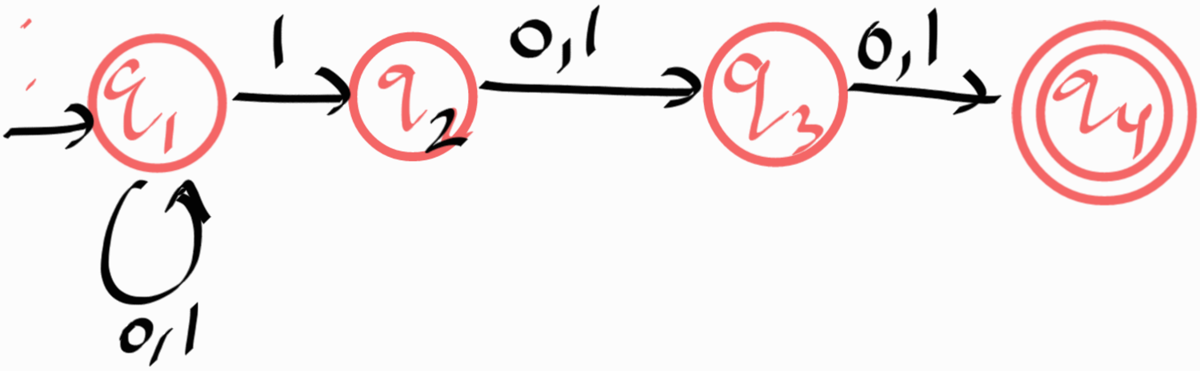
$(0|1)^* \, 1 \, 1 \, (0|1)^*$



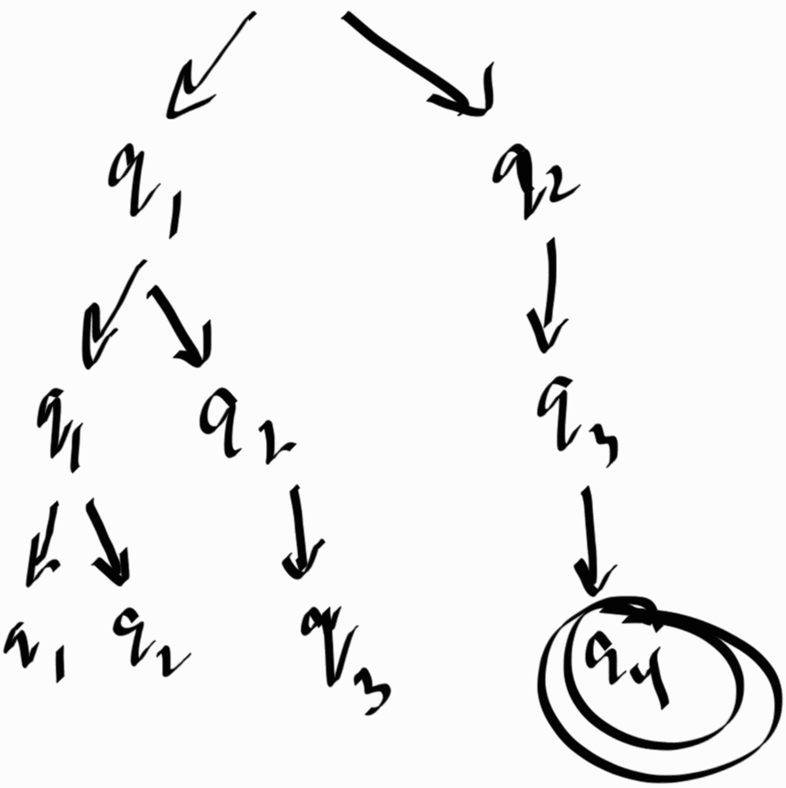
How does NFA compute?

- If any of the accept state can be reached by reading the input x in any way, we say NFA accepts x.

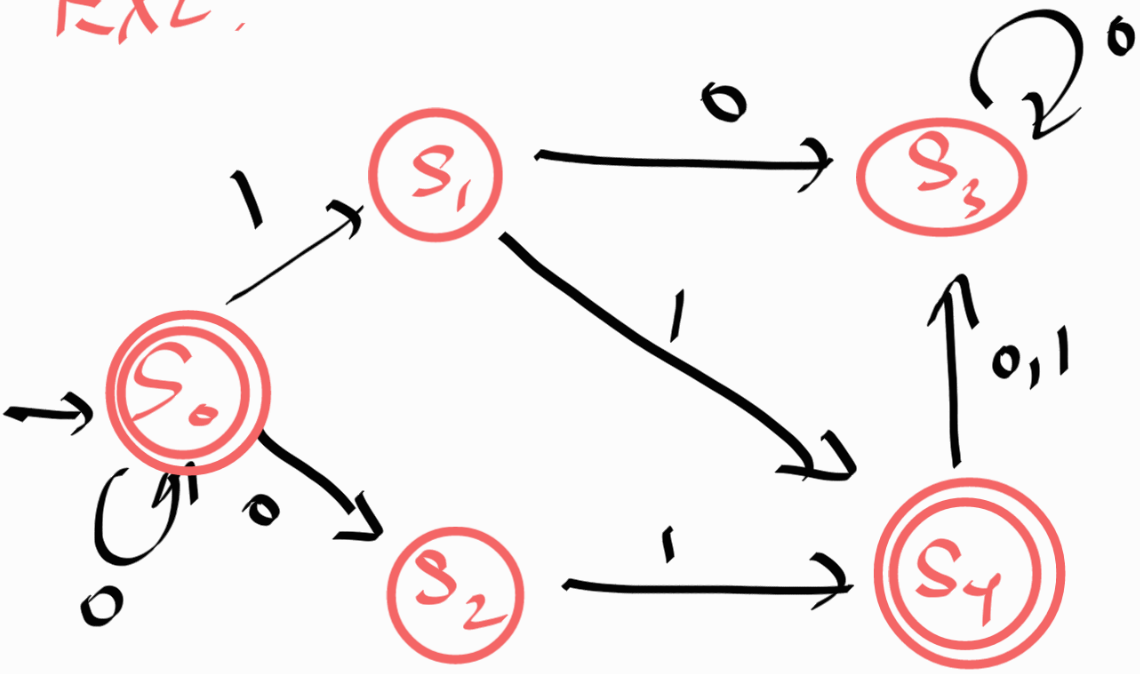
Ex:



1 1 1



Ex2:



0^* ✓

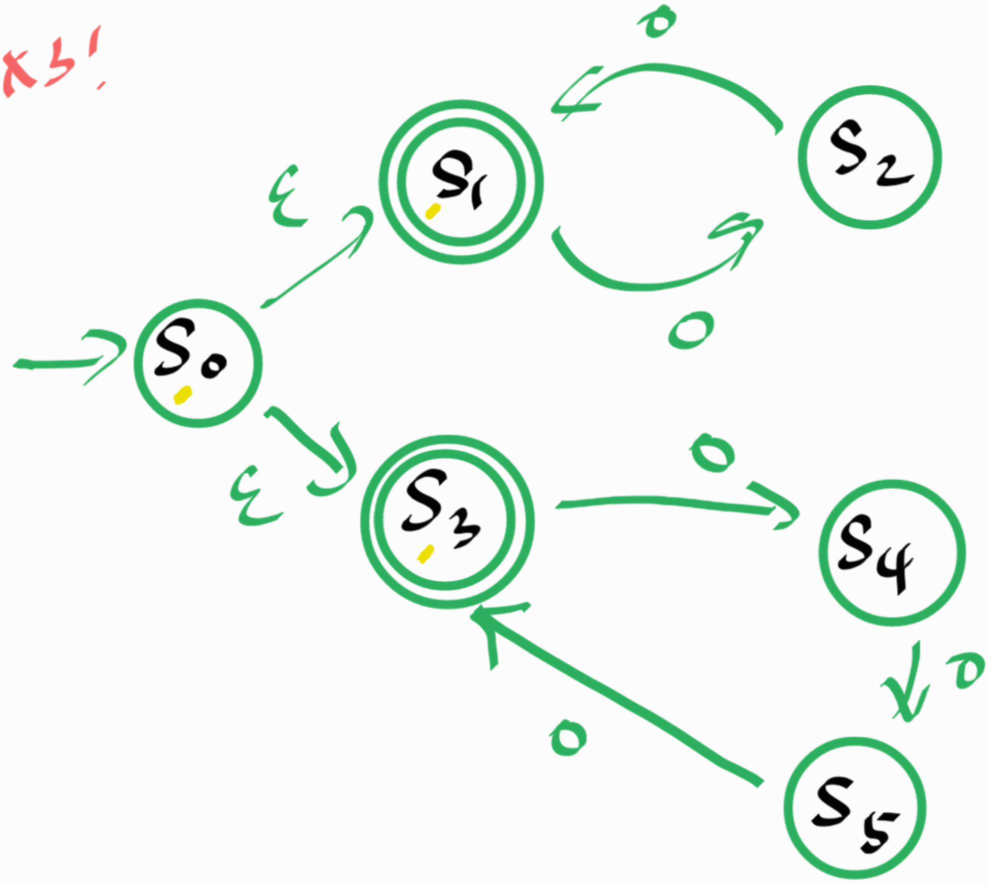
$0^* 1 0$ ✗

$0^* 1 1$ ✓

$0^* 1 1 (0|1)$ ✗

$0^* 0 1$ ✓

Ex 3!



$\Sigma = \{0, 1\}$

$\underline{1}001$

$\underline{0}\,\underline{0}\,\underline{0}$

S_0 · X

$S_1 \to S_2 \to S_1 \to S_2$ X

$S_3 \to S_4 \to S_5 \to S_3$ ✓

Proper definition of NFA

is a 5-tuple

$(Q, \Sigma, \delta, q_0, F)$, where

01. Q is the set of states.
02. Σ is a finite alphabet.
03. $\delta : Q \times \Sigma_\varepsilon \longrightarrow P(Q)$

$Q \times \Sigma \to Q$

$$\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$$

$\delta(q_0, 0)$

$P(Q)$ = power set of Q

04. $q_0 \in Q$ is the starting state.
05. $F \subseteq Q$ is the accepting state.

Ex: $\delta(q_1, \varepsilon) = \{q_2, q_3, q_4\}$

$\delta(q_2, 1) = \{q_4\}$

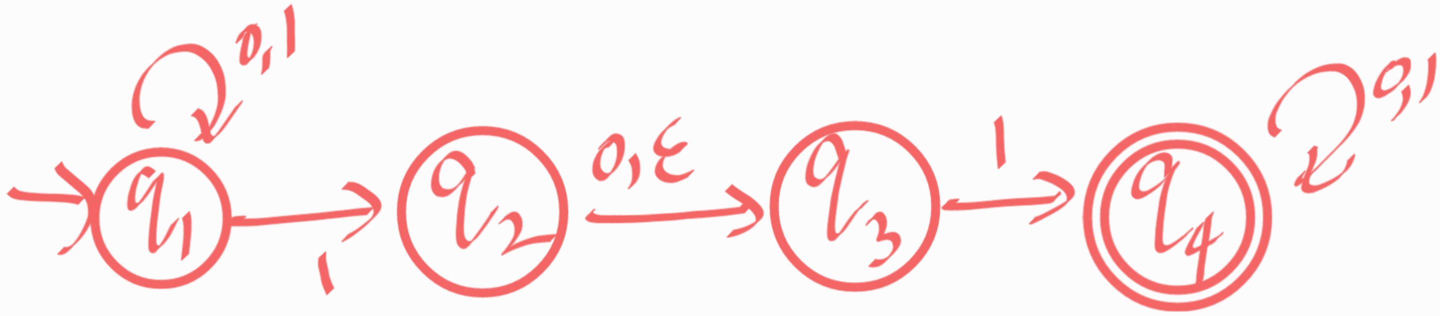
Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA, and $w = \langle w_1, w_2, w_3, \dots, w_n \rangle$, $w_i \in \Sigma$

N accepts w if

$\exists\, r_0, r_1, r_2, \dots, r_n \in Q$ s.t

1. $r_0 = q_0$
2. $r_{i+1} \in \delta(r_i, w_{i+1})$, $\forall i \in [0 \dots n-1]$
3. $r_n \in F$

TWO MACHINES ARE EQUIVALENT if they recognize the same language



1. $Q = \{q_1, q_2, q_3, q_4\}$
2. $\Sigma = \{0, 1\}$, $\Sigma_\varepsilon = \{0, 1, \varepsilon\}$
3. δ is given as follows

$\{\} = \emptyset$ ✓

	ε	0	1
q_1	$\emptyset$	$\{q_1\}$	$\{q_1, q_2\}$
q_2	$\{q_3\}$	$\{q_3\}$	$\emptyset$
q_3	$\emptyset$	$\emptyset$	$\{q_4\}$
q_4	$\emptyset$	$\{q_4\}$	$\{q_4\}$

4. $q_1 \in Q$ is the start state.
5. $\{q_4\} \subseteq Q$ is the accepting states