Example 04:

$$F = \{ww \mid w \in \{0,1\}^*\}$$

Show that F is not regular.

What if we pick $\underline{0^p 0^p}$ as the string to prove F is not regular?

$\Sigma = \{0,1\}$

00 0101 111111

010101 X

$s = \underline{\underline{0}}^p 0^p \in F$ ✓

$|s| = 2p \geq p$

$s = xyz$ $|y| > 0$ $\underline{|xy| \leq p}$

$s = \underbrace{0^{p-a}}_{x} \underbrace{0^{a}}_{y} \underbrace{0^{p}}_{z}$ $a \in \mathbb{N}$ $a > 0$

$p - a + a = p$

let a be a odd number

$$xy^0z = 0^{p-a} 0^{p} = 0^{2p-a} \notin F$$

Let $\underline{\underline{a}}$ be a even number

$$xy^iz = 0^{p-a} 0^{ai} 0^{p} = 0^{p-a+ai+p}$$

$$= 0^{2p+a(i-1)}$$

$2p+a(i-1)$ is even

because $2p$ & a is even.

Proof:

$S = \underline{0^p 1}\, \underline{0^p 1} \in F$

$|S| = 2p+2 \geq p$

Assume F is regular, then s must follow pumping lemma.

Then s can be decomposed into 3 components $s = xyz$ s.t

① $xy^i z \in F$

② $|y| > 0$

③ $|xy| \leq p$

$x = 0^{p-a}$ $y = 0^a$

By ③, y must contain only 0's (before the first 1)

Then, $xy^0 z = 0^{p-a} 1 0^p 1$, $a > 0$, $p-a \neq p$

Therefore $xy^0 z \notin F$, Thus F is not regular.

Ex 5!

$\Sigma = \{1\}$

$D = \{ 1^{n^2} \mid n \geq 0 \}$ is not regular.

$n=0$	$n=1$	$n=2$	$n=3$
ε	1	1111	111111111

$s = 1^{p^2}$ $\quad |s| = p^2 \geq p$

$1^{(p+1)^2}$

$(p+1)^2 = p^2 + 2p + 1$

$s = \underbrace{111111\ldots}_{p \text{ number of 1's}} \underbrace{1\ldots\ldots\ldots1}_{p^2-p \text{ number of 1's}}$

$p + p^2 - p = p^2$

$x = \varepsilon \quad y = 1^p \quad z = 1^{p^2-p}$

$xy^2z = \varepsilon \cdot 1^p \cdot 1^p \cdot 1^{p^2-p}$

$|xy^2z| = p^2 - p + 2p = p^2 + p$ //

Proof:

Assume D is regular.

Since D is regular D should follow Pumping lemma.

Suppose pumping length of D is p.

Select $s = 1^{p^2}$, $|s| = p^2 \geq p$

Since D is regular and $|s| \geq p$, s should be able to decompose into 3 strings $s = xyz$ s.t

1. $xy^iz \in D$ for $i \geq 0$
2. $|y| > 0$
3. $|xy| \leq p$

By ③, $|xy| \leq p$, therefor $|y| \leq p$

By ② $|y| > 0$ or $|y| \geq 1$

- we have

$$|xy^2z| = |xyz| + |y| = p^2 + |y|$$

$$p^2 < |xy^2z| = p^2 + |y| \leq p^2 + p \leq p^2 + 2p + 1$$

$$p^2 < |xy^2z| < (p+1)^2$$

Basically this means even if we allocate max number of 1's to y, we still cannot generate the next string $1^{(p+1)^2}$, where $|1^{(p+1)^2}| = (p+1)^2$.

Therefore, $xy^2z \notin D$, a contradiction.

$\therefore$ D is not regular.

1. Deterministic finite automatons

2. Non-Deterministic finite automatons

3. Regular expressions.

End of Regular languages.

Next, we are going to look at a more powerful computation model.