Practical undecidability problems.

Post correspondence problem (PCP)

- you are given some domino cards, with string written below and above the mid-line at each card.

$$\left[\frac{abc}{ab}\right]$$

- stack cards one by one, so that the strings above match strings below.
- You can use the same card any number of times including zero times.
- You can think of this as having infinite copies of each domino.

Ex:

card1	card2	card3	card4
b	a	ca	abc
ca	ab	a	c

2	1	3	2	4
a	b	ca	a	abc
ab	ca	a	ab	c

Ex 2'

$\frac{aaa}{aa}$ $\frac{abb}{bba}$ $\frac{aa}{aaa}$

1   2   3

1   3   3   2   1

$\frac{aaa}{aa}$ $\frac{aa}{aaa}$ $\frac{aa}{aaa}$ $\frac{abb}{bba}$ $\frac{aaa}{aa}$

aa abb aaa

aa abb aaa

Ex 3:

$\dfrac{bb}{b}$	$\dfrac{bba}{aa}$	$\dfrac{abbb}{baaa}$
1	2	3

no solution

Theorem PCP is undecidable.

Proof is too long

Another practical problem that is undecidable.

1. Write the shortest program to solve a problem.

Mapping Reducibility

- Map Reductions $\leq$, α, $\leq_m$

Definitions

A function $f: \Sigma^* \rightarrow \Sigma^*$ is a computable function if some TM M, on every input w, halts with $f(w)$ on its tape.

A TM computes a function by starting with the input to the function on the tape and halting with the output of the function on the tape.

Ex: Arithmetic operations on integers are computable functions.

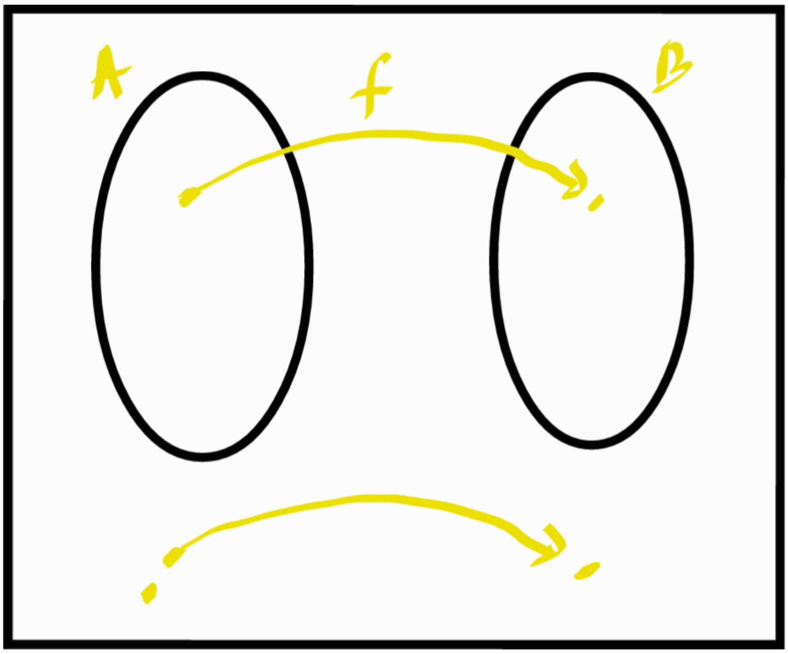
Definition

Mapping Reducibility

A language A is mapping reducible to language B, written as $A \le_m B$, If there is a computable function $f: \Sigma^* \longrightarrow \Sigma^*$, where every w

$$w \in A \iff f(w) \in B$$

The function f is calle the reduction from A to B.



All previous reductions are map reductions.

* In a map reduction, you solve an instance of A by calling a solution to B only once

Theorem 5.22

If $A \leq_m B$ and B is decidable, then A is decidable.

Corollary 5.23

If $A \leq_m B$, and A is undecidable, then B is undecidable.

Theorem 5.28

If $A \leq_m B$ and B is Turing-recognizable, then A is Turing-Recognizable.