Review 01/17/2025

Sets: A set is an undored collection of distinct objects.

$V = \{A, E, I, O, U\}$

bits $= \{0, 1\}$

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

The size of set S as $|S|$, denotes how many element we have in the set.

$|V| = 5$

$A \in V \leftarrow$ A exists in V.

$B \notin V \leftarrow$ B does not exist in V.

Subset

A set A is a subset of set B, if every element in set A appears in set B.

We denote this $A \subseteq B$

If A is not a subset of B, we denote this

$$A \not\subseteq B$$

$A = \{2, 5\}$ $B = \{1, 2, 3, 4, 5\}$

$$A \subseteq B$$

$$B \not\subseteq A$$

$$D = \{1, 6\}$$

$$D \not\subseteq B$$

Question: If $|A| = n$, how many subsets does A have?

2^n

Ex: $A = \{1, 2, 3\}$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

$|P(A)| = 8 = 2^3$

Def: A is a proper subset of B, if $A \subseteq B$ and $|A| \neq |B|$, denoted as $A \subset B$ or $A \subsetneq B$

$A = \{1, 2\}$ $\quad$ $B = \{1, 2, 3, 4\}$

$A \subset B$ $\quad$ $D = \{1, 2, 3, 4\}$

$D \subseteq B$

$D \not\subseteq B$

Question

$$(X \subseteq Y) \wedge (Y \subseteq X) \Rightarrow (X = Y)$$

$a, b \in \mathbb{R}$

$$(a \leq b \wedge b \leq a) \Rightarrow a = b$$

Multisets: A set that allows duplicate elements.

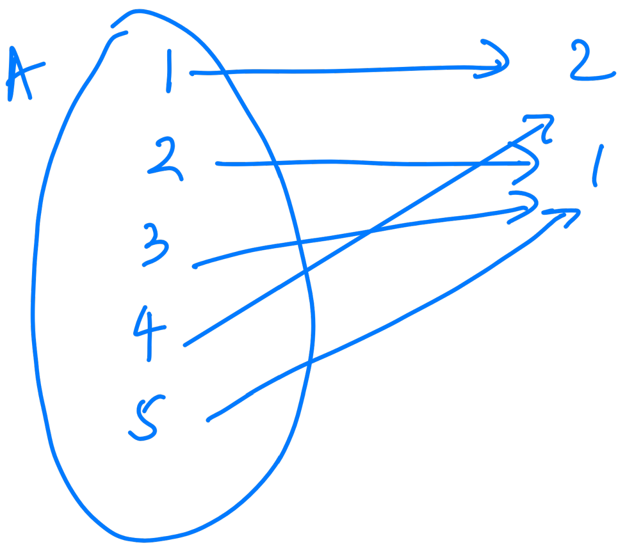
$M = \{1, 1, 2, 3, 4, 4, 5\}$ ← Multiset

not a set

A multiset M is an ordered pair (A, m), where A is the underlying set, and m is a function such that $m: A \to \mathbb{Z}^+$ that denotes the multiplicity.

$$M = \{1, 1, 2, 3, 4, 4, 5\}$$

$$A = \{1, 2, 3, 4, 5\}$$



$m(1) = 2$

$m(2) = 1$

$m(3) = 1$

$m(4) = 2$

$m(5) = 1$

Some important sets

$\mathbb{Z}$ - The set of all integers

$$\mathbb{Z} = \{\ldots\ldots, -2, -1, 0, 1, 2, \ldots\ldots\}$$

$\mathbb{N}$ - The set of natural numbers

$$\mathbb{N} = \{1, 2, 3, 4, \ldots\ldots\}$$

$$\mathbb{Z}^{+} = \{1, 2, 3, 4, \ldots\ldots\}$$

$$\mathbb{Z}^{-} = \{\ldots\ldots, -2, -1\}$$

$$\mathbb{Z} = \mathbb{Z}^{+} \cup \mathbb{Z}^{-} \cup \{0\}$$

$\mathbb{R}$ = The set of all real numbers

$\mathbb{Z}^{\geq 0}$ = All the integers ≥ 0

$\mathbb{Q}$ = The set of all rational numbers

The set builder notation

$S = \{ x : \text{rule about } x \}$

Ex:

$\mathbb{Q} = \{ a : (p, q \in \mathbb{Z}) \wedge (q \neq 0) \wedge (a = p/q) \}$

$EVENS = \{ e \mid (e \in \mathbb{Z}) \wedge (e \bmod 2 = 0) \}$

Set operations

1. Union ($\cup$)
2. Intersection ($\cap$)
3. Complement ($\overline{}$)
4. Exclusive or ($\oplus$)

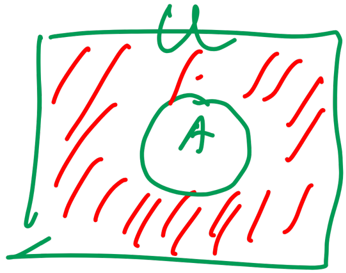
$A = \{1, 3, 5, 6\}$ $\quad$ $B = \{2, 4, 6, 8\}$

$\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$

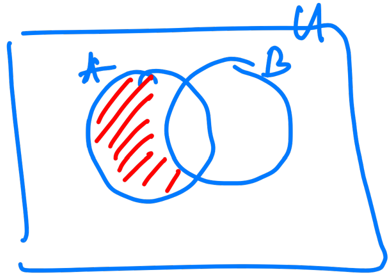
$A \cap B = \{6\}$

$\overline{A} = \{2, 4\ 7, 8\}$

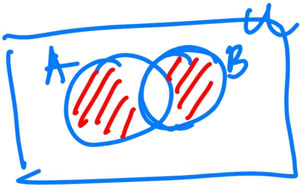


Note! $A \cup \overline{A} = \mathcal{U}$

$A - B = A \setminus B = \{1, 3, 5\}$



$A \oplus B = \{1, 3, 5, 2, 4, 8\}$



Sequences and tuples

- A sequence is list of ordered elements

$$\langle 1,5,6,7\rangle = (1,5,6,7)$$

$$\langle 1,5,6,7\rangle \neq \langle 1,6,5,7\rangle$$

- finite sequences are also called tuples
- 2-tuple are called pair

$(a,b), \langle a,b\rangle$

Ex: A Graph $G=(V,E)$

- V is a set of vertices
- E is a set of edges

Cartesian product of two Sets

The carttesian product of set A,B is denoted by $A \times B$,

$$A \times B = \{ \langle a, b \rangle : a \in A \wedge b \in B \}$$

contains all ordered pairs where first element comes from A, and second element comes from B.

$$A = \{1, 2, 3\} \qquad B = \{\bigstar, \Delta\}$$

$$A \times B = \{ \langle 1, \bigstar \rangle, \langle 1, \Delta \rangle, \langle 2, \bigstar \rangle, \langle 2, \Delta \rangle, \langle 3, \bigstar \rangle, \langle 3, \Delta \rangle \}$$

$$A \times B \neq B \times A \quad (A \neq B)$$