Example 04:

$$F = \{ww \mid w \in \{0,1\}^*\}$$

Show that F is not regular.

What if we pick $\underline{0^p 0^p}$ as the string to prove F is not regular?

$\Sigma = \{0,1\}$

00 0101 111111

0101010 X

$s = \underline{0}^p 0^p \in F$ ✓

$|s| = 2p \geq p$

$s = xyz$ $|y| > 0$ $\underline{|xy| \leq p}$

$$s = \underbrace{0^{p-a}}_{x} \underbrace{0^{a}}_{y} \underbrace{0^{p}}_{z}$$

$a \in \mathbb{N}$ $a > 0$

$p - a + a = p$

let a be a odd number

$$xy^0z = 0^{p-a}0^{p} = 0^{2p-a} \notin F$$

Let $\underline{\underline{a}}$ be a even number

$$xy^iz = 0^{p-a}0^{ai}0^{p} = 0^{p-a+ai+p}$$

$$= 0^{2p+a(i-1)}$$

$2p+a(i-1)$ is even

because $2p$ & a is even.

Proof:

$s = \underline{0^p 1}\, \underline{0^p 1} \in F$

$|s| = 2p+2 \geq p$

Assume F is regular, then s must follow pumping lemma.

Then s can be decomposed into 3 components $s = xyz$ s.t

① $xy^i z \in F$

② $|y| > 0$

③ $|xy| \leq p$

$x = 0^{p-a}$ $y = 0^a$

By ③, y must contain only 0's (before the first 1)

Then, $xy^0z = 0^{p-a} 1 0^p 1$, $a > 0$, $p-a \neq p$

Therefore $xy^0z \notin F$, Thus F is not regular.

Ex 5!

$\Sigma = \{1\}$

$D = \{1^{n^2} \mid n \geq 0\}$ is not regular.

$n=0$	$n=1$	$n=2$	$n=3$
ε	1	1111	111111111

$s = 1^{p^2}$ $\quad |s| = p^2 \geq p$

$1^{(p+1)^2}$

$(p+1)^2 = p^2+2p+1$

$s = \underbrace{111111\ldots}_{p \text{ number of } 1\text{'s}}\ \underbrace{1\ldots\ldots\ldots 1}_{p^2-p \text{ number of } 1\text{'s}}$

$$p + p^2 - p = p^2$$

$x = \varepsilon \quad y = 1^p \quad z = 1^{p^2-p}$

$xy^2z = \varepsilon \cdot 1^p \cdot 1^p \cdot 1^{p^2-p}$

$|xy^2z| = p^2 - p + 2p = p^2 + p$ //

1. Deterministic finite automatons

2. Non-Deterministic finite automatons

3. Regular expressions.

End of Regular languages.

Next, we are going to look at a more powerful computation model.