Computational Complexity

1. We learnt automaton.

- DFA
  - Regular language
  - Regular Expressions
- PDA
  - CFLs
  - CFG
- Turing Machines

- Decidable problem
- Undecidable problems.

Decidable problems

Definition 7.1

Let M be a deterministic Turing machine that halts on all inputs. The running time or time complexity of M is the function $f : \mathbb{N} \longrightarrow \mathbb{N}$, where $f(n)$ is the maximum number of steps that M uses on any input of length n. If $f(n)$ is the running time of M, we say that M runs in time $f(n)$ and M is an $f(n)$ time Turing Machine.

$$f(n) = n^3 + 5n + 3$$

$$f_1(n) = 2^n + 212 n \log n + 5$$

Definition

Let f and g be functions, $f, g : \mathbb{N} \longrightarrow \mathbb{R}^{+}$. we say that $f(n) = O(g(n))$

$$\exists c > 0, n_0 \geq 0 ; \forall n \geq n_0 : f(n) \leq c \cdot g(n)$$

when $f(n) = O(g(n))$ we say that $g(n)$ is an upper bound for $f(n)$

or

$g(n)$ is an asymptotic upper bound on $f(n)$

or

f is less than or equal to g if we disregard the differency up to a constant factor.

$f(n) = 2n^2 + n + 100$

$g(n) = n^2$

$f(n) = O(n^2)$

$f(n) = 5n^3 + 2n^2 + 11n + 5$

$g(n) = n^3$

$f(n) = O(n^3)$

Pick $c = 23$

$23n^3 \geq 5n^3 + 2n^2 + 11n + 5$

for $n \geq 1$

Little-o-notation

Let f and g be functions,
$f, g : \mathbb{N} \longrightarrow \mathbb{R}^+$, say $f(n) = o(g(n))$,
if

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$$

In other words, $f(n) = o\,g(n)$ means for any constant $c > 0$ a number n_0 exists s.t
$f(n) < g(n) \quad \forall n \geq n_0$

$$f(n) = 2n^2 + 3n + 100$$

$$f(n) = O(n^2)$$ ✓ Big-O

$$f(n) = o(n^2)$$ ✗ little-o

$$\lim_{n\to\infty} \frac{2n^2+3n+100}{n^2} = \lim_{n\to\infty} 2 + \cancel{\frac{3}{n}}^{\,0} + \cancel{\frac{100}{n^2}}^{\,0}$$

$$f(n) = O(n^3)$$

- CFG
  - CFG for a given language
  - CFG → PDA
  - Chomsky Normal form
  - PDAs
  - Pumping lemma
- TM
  - decidability
  - Undecidability

Prove 2-PALINDROME = $\{\langle M\rangle : L(M)$ contains exactly two palindromes$\}$ is undecidable.

$A_{TM} \leq$ 2-PALINDROME

$\langle M, w\rangle$ $\quad\quad$ $\langle M'\rangle$

Assume 2-PALINDROME is decidable and the decider is R.

M accepts w $\Rightarrow L(M') = \{001100, 11011\}$

M does not accept w $\Rightarrow L(M') = \{001100\}$

M' = on input $\langle x\rangle$

1. if $x = 001100$, accept

2. if $x = 11011$

Run M on w and accept x if M accepts w

S is the decider for A_{TM}

S = on <M, w>

1. construct M'

2. Run R on <M'>

if R accepts, accept

R rejects, reject