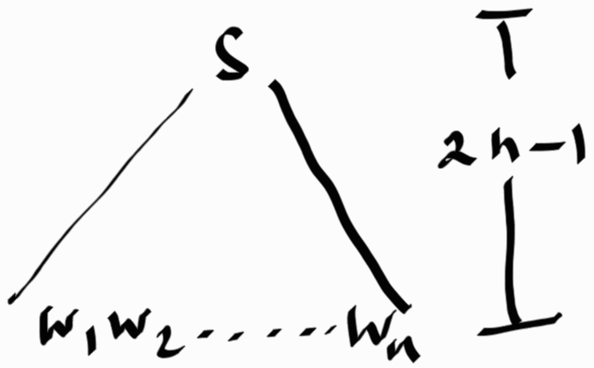
$A_{CFG} = \{ \langle G, w \rangle \mid G \text{ is a CFG and } G \text{ generates } w \}$

- Solution

$G \longrightarrow$ convert $\underline{G \text{ to } CNF}$

Theorem 4.7

$s = w_1 w_2 \cdots\cdots w_n$




$O(|G|^{2n-1})$

G is a CFG in CNF

This is too costly.

- Solution 02
  - Dynamic programming solution.

If $A \rightarrow BC$ is a rule

$B \overset{*}{\Rightarrow} X$ and $C \overset{*}{\Rightarrow} Y$

$A \overset{*}{\Rightarrow} XY$

- Define a table[i, j] — which stores the collection of variables that generates the substring $w_i w_{i+1} \cdots w_j$

- Idea is simillar to matrix multiplication.

$\underbrace{w_i w_{i+1} \cdots w_k}$ | $\underbrace{w_{k+1} \cdots w_j}$

in the table[i, k] you have the variable B

in the table you have variable C

Then If I have a rule $A \rightarrow BC$

then I can add A to table[i, j]

Refer the handout for algorithm.

## Example

$w = a\,a\,b\,c\,c$

$G$ : $S \longrightarrow AB$

$A \longrightarrow AB \mid AA \mid a$

$B \longrightarrow BC \mid b$

$C \longrightarrow c$

$w_1 \mid w_2\, w_3$

$a \mid a\, b$

$A \quad A, S$

$? \longrightarrow AA$

$? \longrightarrow AS$

$w_1 w_2 \mid w_3$

$a\, a \mid b$

$A \quad B$

$? \longrightarrow AB$

$S, A$

| i \ j | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | A | A | A, S | | |
| 2 | – | A | A, S | S, A | |
| 3 | – | – | B | B | B |
| 4 | – | – | – | C | $\emptyset$ |
| 5 | – | – | – | – | C |

$w_2\, w_3\, w_4$

$a\; b\; c$

$a \mid b\, c$

$A \quad B$

$? \longrightarrow AB$

$S, A$

$w_2 w_3 \mid w_4$

$a\, b \mid c$

$A, S \mid C$

$? \longrightarrow A\;C$

$\longrightarrow S\;C$

$w_3\, w_4\, w_5$

$b\; c\; c$

$w_3 \mid w_4\, w_5$

$b \mid c\; c$

$B \quad \emptyset$

$w_3 w_4 \mid w_5$

$b\,c \mid c$

$B \mid C$

$B \longrightarrow B\,C$

G: $S \to AB$

$A \to AB \mid AA \mid a$

$B \to BC \mid b$

$C \to c$

$w = aabcc$

| i \ j | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | A | A | A, S | A, S | |
| 2 | — | A | A, S | S, A | S, A |
| 3 | — | — | B. | B | B |
| 4 | — | — | — | C | ∅ |
| 5 | — | — | — | — | c |

$w_2\, w_3\, w_4\, w_5$

a b c c

a | b c c

A B

→ AB

a b | c c

A, S ∅

a a b c

a | a b c

A S, A

→ AS

→ AA

a a | b c

A B

→ AB

a a b | c

A, S C

→ AC

→ SC

a b c | c

S, A C

→ SC

→ AC

$G$ : $S \longrightarrow AB$

$A \longrightarrow AB \mid AA \mid a$

$B \longrightarrow BC \mid b$

$C \longrightarrow c$

$w = aabcc$

| i \ j | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | A | A | A, S | A, S | A, S |
| 2 | — | A | A, S | S, A | S, A |
| 3 | — | — | B. | B | B |
| 4 | — | — | — | C | $\emptyset$ |
| 5 | — | — | — | — | C |

a a b c c

a | a b c c
A   S, A
$\rightarrow$ AS
$\rightarrow$ AA

a a | b c c
A   B
$\rightarrow$ AB

a a b | c c
A, S   $\emptyset$

a a b c | c
A, S | C
? $\longrightarrow$ SC
? $\longrightarrow$ AC

In the table[1, n], if we can find the starting variable, then $w = w_1 w_2 \cdots w_n$ can be generated by G.