Question: If I want to prove a language A is not regular what can we do?

Now, let's see whether there are languages we cannot recognize by a NFA/DFA.

1.4 Non-Regular languages

$A = \{w \mid w \text{ has more 0's than 1's}\}$

not regular

$B = \{a^n b^n \mid n \geq 0\}$

$C = \{w \mid w \text{ has equal number of 0's and 1's}\}$

$D = \{w \mid w \text{ has an equal number of 01 and 10 as substrings}\}$ regular

In A machine needs to keep track of # of 0's or 1's. Depending on the string this value could be different.

Hence, a machine with limited memory cannot do this.

However, just because a language appears to require unbounded number of memory does not mean that the language is not regular.

$C = \{ w \mid w \text{ has an equal \# of } 0\text{'s and } 1\text{'s} \}$

$D = \{ w \mid w \text{ has equal \# of occurrences of } 01 \text{ and } 10 \}$

Sometimes our intuition is misleading

Let's look at how we can show a particular language is not regular.

Pumping Lemma

- Basically this theorem states that regular languages have a special property.
- If we can show that a language does not have this property, then the language is not regular.

* The basic idea of this property is that all strings in a regular language can be "pumped" if they are at least as long as a certain special value, called the <u>pumping length</u>.

Pumping Lemma

If A is a regular language, then there is a number p (the pumping length) where if s is any string in A of length at least p, then s may be divided into three pieces, $s = xyz$ satisfying the following conditions:

1. for each $i \geq 0$, $xy^iz \in A$
2. $|y| > 0$, and
3. $|xy| \leq p$

* $|s|$ represents the length of the string
* y^i means i copies of string y.

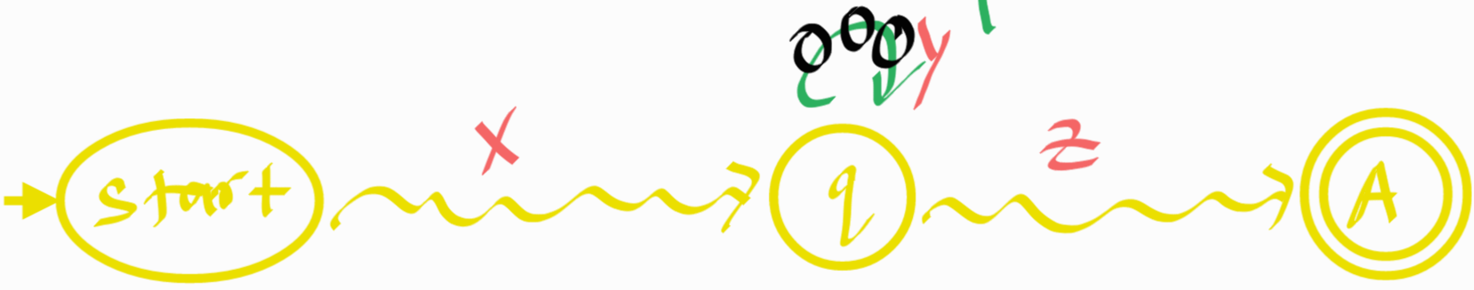
Proof idea:

Pick p to be # of states in the DFA that recognizes the regular language A.

Then pick any string s in A of length at least p.

* If no such strings exist, then the claim is vacuously true.

Since the size of s is larger than p, then in its execution path to an accepting state, we must contain a repeated state.



$s = xyz$ $\quad xz$

Since this path leads to a accepting state, this should accept $xy, \underline{\underline{xyz}}, xy^2z, xy^3z, \ldots\ldots, xy^iz$.

This is the basic idea.