$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$Av = \lambda v$$

$$Av - \lambda v = 0$$

$$(A - \lambda) v = 0$$

$$\det(A - \lambda \cdot I) = 0$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

$$A-\lambda I = \begin{pmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{pmatrix}$$

$$\det(A-\lambda I) = 0$$

$$(2-\lambda)^2 - 1 = 0$$

$$(2-\lambda-1)(2-\lambda+1) = 0$$

$$(1-\lambda)(3-\lambda) = 0$$

$$\lambda = \underline{1} \quad \text{or} \quad \lambda = 3$$

$$A v_1 = \lambda v_1$$

$$(A-\lambda I)X=0$$ To find eigen vectors.

$$\left[\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}-\begin{pmatrix}\lambda & 0\\ 0 & \lambda\end{pmatrix}\right]X=0$$

$$\begin{bmatrix}2-\lambda & 1\\ 1 & 2-\lambda\end{bmatrix}X=0$$

when $\lambda=1$, we have

$$\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}x_1\\ y_1\end{pmatrix}=0$$

using row reduction

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = 0$$

$$x_1 + y_1 = 0$$

$$x_1 = -y_1$$

$$x = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} c \\ -c \end{bmatrix} = c \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

eigenvector

when $\lambda = 3$

$$\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = 0$$

using row reduction

$$\begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = 0$$

$$-x_2 + y_2 = 0$$

$$x_2 = y_2$$

$$x = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} d \\ d \end{pmatrix} = d \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

eigenvector.