Introduction to Recommender Systems

CSCI 347

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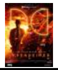
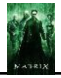
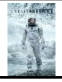
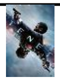
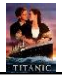
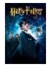
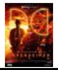
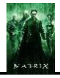
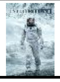
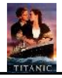
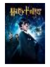
What is a recommender system?

- “There is an extensive class of Web applications that involve predicting user responses to options. Such a facility is called a recommendation system” — Mining Massive Data Sets, Ch. 9
- Ex: Netflix: Predicting user preference for movies in order to make the best recommendation for movies to watch.

						
User 1	5	-	1	-	1	-
User 2	-	5	1	3	4	-
User 3	5	-	2	4	-	1
User 4	2	5	3	2	1	-
User 5	3	-	5	5	1	5
User 6	2	3	4	4	4	2
User 7	-	-	3	3	3	-

HOW DO USERS BENEFIT FROM A RECOMMENDER SYSTEM?

- Discover new items of interest
- Narrow down set of choices

						
User 1	5	-	1	-	1	-
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User 3	5	-	2	4	-	1
User 4	2	5	3	2	1	-
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User 6	2	3	4	4	4	2
User 7	-	-	3	3	3	-

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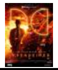
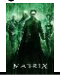
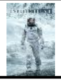
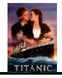
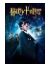
Goals of recommender system

- Predict whether a use will interact with an item (binary or unary data)
  - Will a user click on a webpage?
  - Will a user purchase an item?
  - Will a user play a particular item?
- Predict user rating of an item
  - What rating will a user give a particular movie?
  - What items will be most preferred by the user?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User 5	3	-	5	5	1	5
User 6	2	3	4	4	4	2
User 7	-	-	3	3	3	-

Goals of a recommender system

- In 2006, Netflix announced a challenge: beat their rating prediction algorithm by 10% in rating prediction accuracy and take home $1 million!
- A lot of rapid advances in rating prediction techniques made as a result
  - An important piece of the winning solution (in 2009) was a matrix decomposition-based approach
- Focus on rating prediction has shifted in recent years.
  - Implicit Feedback Dominance: Modern systems (e.g., YouTube, Amazon, Spotify) rely on implicit feedback (clicks, views, purchases) rather than explicit ratings, as users rarely provide star ratings. This aligns with unary data, where only positive interactions are recorded.
  - Contextual and Sequential Data: Modern systems incorporate context (e.g., time, location, device) and sequential patterns (e.g., what a user watched before), which go beyond static rating prediction.

CHALLENGES OF RECOMMENDATION

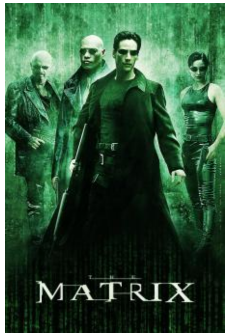
- Large data set
- Sparsity of the data
- User bias
- Discrete/binary/unary data
- Time-evolving data

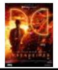
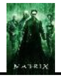
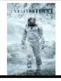
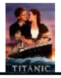
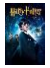
COMMON APPROACHES TO RECOMMENDATION

- COMMON APPROACHES TO RECOMMENDATION
- Content-based recommendation
  - Recommends items based on their content (features) and a user's past preferences.
- Collaborative Filtering (Recommends items based on patterns of user-item interactions, leveraging the "wisdom of the crowd.")
  - User-based
  - Item-based
- Hybrid

K-NN approach to recommendation

$user_3 \quad \vec{?}$

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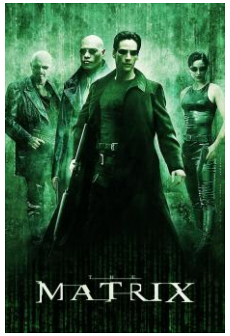
User 4	2	5	3	2	1	-
User 5	3	-	5	5	1	5
User 6	5	5	1	2	4	2
User 7	-	-	3	3	3	-

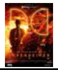
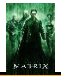
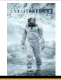
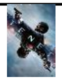
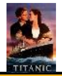
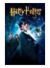
K-NN approach to recommendation

$user_3$ $\vec{?}$

$k = 2$ nearest neighbors

$user_1$ $user_6$

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User 4	2	5	3	2	1	-
User 5	3	-	5	5	1	5
User 6	5	5	1	2	4	2
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K-NN approach to recommendation

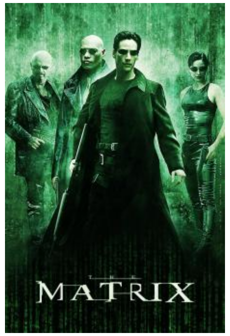
$user_3$

$k = 2$ nearest neighbors

$user_1$ $user_6$

4

5



$predicted\ rating = 4$

User 1	5	4	1	-	4	-
User 2	-	5	1	3	4	-
User 3	5	-	2	4	-	1
User 4	2	5	3	2	1	-
User 5	3	-	5	5	1	5
User 6	5	5	1	2	4	2
User 7	-	-	3	3	3	-

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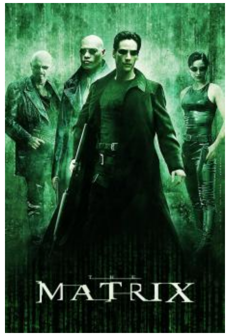
K-NN approach to recommendation

$user_3$

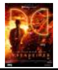
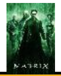
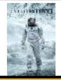
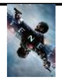
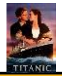
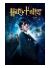
$k = 2$ nearest neighbors

$user_1$ $\xrightarrow{4}$ 

$user_6$ $\xrightarrow{5}$

$\rightarrow$ $predicted\ rating = 4$

The Problem is, how do we find the nearest neighbors? What distance metric should we use?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User 5	3	-	5	5	1	5
User 6	5	5	1	2	4	2
User 7	-	-	3	3	3	-

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K-NN APPROACH TO RECOMMENDATION

Pearson correlation coefficient: A common distance measure

$$Pearson(u,v) = \frac{\sum_{i \in \mathcal{C}} (r_{ui} - \overline{r_u})(r_{vi} - \overline{r_v})}{\sqrt{\sum_{i \in \mathcal{C}} (r_{ui} - \overline{r_u})^2} \sqrt{\sum_{i \in \mathcal{C}} (r_{vi} - \overline{r_v})^2}}$$

$\mathcal{C}$ is the set of co-rated items between u and v

K-NN APPROACH TO RECOMMENDATION

Pearson correlation coefficient

$$Pearson(u,v) = \frac{\sum_{i \in \mathcal{C}} (r_{ui} - \bar{r_u})(r_{vi} - \bar{r_v})}{\sqrt{\sum_{i \in \mathcal{C}} (r_{ui} - \bar{r_u})^2} \sqrt{\sum_{i \in \mathcal{C}} (r_{vi} - \bar{r_v})^2}}$$

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	5	-	1	4	3	-	-
User 2	-	4	-	2	2	5	-
User 3	-	1	2	5	-	3	5
User 4	4	4	-	3	-	4	3
User 5	1	-	5	5	-	5	5

$$\sigma_{12} = \frac{1}{n-1} \sum_{i=1}^{n} (x_{i1} - \hat{\mu}_1)(x_{i2} - \hat{\mu}_2)$$

$$\rho_{12} = \frac{\hat{\sigma}_{12}}{\hat{\sigma}_1 \hat{\sigma}_2} = \frac{\frac{1}{n-1} \sum_{i=1}^{n} (x_{i1} - \hat{\mu}_1)(x_{i2} - \hat{\mu}_2)}{\sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_{i1} - \hat{\mu}_1)^2} \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_{i2} - \hat{\mu}_2)^2}}$$

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K-NN APPROACH TO RECOMMENDATION

Pearson correlation coefficient

$$Pearson(u,v) = \frac{\sum_{i \in \mathcal{C}} (r_{ui} - \overline{r_u})(r_{vi} - \overline{r_v})}{\sqrt{\sum_{i \in \mathcal{C}} (r_{ui} - \overline{r_u})^2} \sqrt{\sum_{i \in \mathcal{C}} (r_{vi} - \overline{r_v})^2}}$$

$$Pearson(3,4) = \frac{\sum_{i \in \mathcal{C}} (r_{3i} - \overline{r_3})(r_{4i} - \overline{r_4})}{\sqrt{\sum_{i \in \mathcal{C}} (r_{3i} - \overline{r_3})^2} \sqrt{\sum_{i \in \mathcal{C}} (r_{4i} - \overline{r_4})^2}}$$

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	5	-	1	4	3	-	-
User 2	-	4	-	2	2	5	-
User 3	-	1	2	5	-	3	5
User 4	4	4	-	3	-	4	3
User 5	1	-	5	5	-	5	5

$$\mathcal{C} = \{2,4,6,7\}$$

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K-NN APPROACH TO RECOMMENDATION

Pearson correlation coefficient

$$Pearson(u,v) = \frac{\sum_{i \in \mathcal{C}} (r_{ui} - \overline{r_u})(r_{vi} - \overline{r_v})}{\sqrt{\sum_{i \in \mathcal{C}} (r_{ui} - \overline{r_u})^2} \sqrt{\sum_{i \in \mathcal{C}} (r_{vi} - \overline{r_v})^2}}$$

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User 4	4	4	-	3	-	4	3
User 5	1	-	5	5	-	5	5

$$Pearson(3,4) = \frac{\sum_{i \in \mathcal{C}} (r_{3i} - \overline{r_3})(r_{4i} - \overline{r_4})}{\sqrt{\sum_{i \in \mathcal{C}} (r_{3i} - \overline{r_3})^2} \sqrt{\sum_{i \in \mathcal{C}} (r_{4i} - \overline{r_4})^2}}$$

$$\mathcal{C} = \{2,4,6,7\}$$

$$\bar{r}_3 = \frac{1+5+3+5}{4} = \frac{14}{4} = 3.5$$

$$\bar{r}_4 = \frac{4+3+4+3}{4} = \frac{14}{4} = 3.5$$

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K-NN APPROACH TO RECOMMENDATION

Pearson correlation coefficient

$$Pearson(u,v) = \frac{\sum_{i\in\mathcal{C}}(r_{ui}-\bar{r_u})(r_{vi}-\bar{r_v})}{\sqrt{\sum_{i\in\mathcal{C}}(r_{ui}-\bar{r_u})^2}\sqrt{\sum_{i\in\mathcal{C}}(r_{vi}-\bar{r_v})^2}}$$

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User 3	-	1	2	5	-	3	5
User 4	4	4	-	3	-	4	3
User 5	1	-	5	5	-	5	5

$$\mathcal{C} = \{2,4,6,7\}$$

$$\bar{r}_3 = 3.5 \qquad \bar{r}_4 = 3.5$$

$$Pearson(3,4) = \frac{(1-3.5)(4-3.5)+(5-3.5)(3-3.5)+(3-3.5)(4-3.5)+(5-3.5)(3-3.5)}{\sqrt{(1-3.5)^2+(5-3.5)^2+(3-3.5)^2+(5-3.5)^2}\sqrt{(4-3.5)^2+(3-3.5)^2+(4-3.5)^2+(3-3.5)^2}}$$

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K-NN APPROACH TO RECOMMENDATION

Pearson correlation coefficient

$$Pearson(u,v) = \frac{\sum_{i \in \mathcal{C}} (r_{ui} - \bar{r_u})(r_{vi} - \bar{r_v})}{\sqrt{\sum_{i \in \mathcal{C}} (r_{ui} - \bar{r_u})^2} \sqrt{\sum_{i \in \mathcal{C}} (r_{vi} - \bar{r_v})^2}}$$

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User 3	-	1	2	5	-	3	5
User 4	4	4	-	3	-	4	3
User 5	1	-	5	5	-	5	5

$$\mathcal{C} = \{2,4,6,7\}$$

$$\bar{r}_3 = 3.5 \qquad \bar{r}_4 = 3.5$$

$$Pearson(3,4) = -\frac{3}{\sqrt{11}\sqrt{1}} = -0.9045$$

K-NN approach to recommendation

Aspect	Pearson correlation	Euclidean distance
Measures	Similarity of pattern	Similarity of pattern
Scale invariant?	Yes	No
Handles rating bias	Yes	No
Good with sparse data?	Yes	Needs normalization

- Some users are harsh (always rate 1-3), others are generous (always 4-5).
- Pearson correlation centers the ratings (i.e., subtracts the mean), so it focuses on how users rate items relative to their own average.
  - Ex: User 1: [5,4,3]
  - Use 2: [3,2,1]
  - This is the same pattern but in different scale
  - Pearson Correlation = 1
  - Euclidean distance = large (they look far apart)
- Direction vs. Distance
  - Pearson focuses on the direction of ratings — are they going up/down in sync? Euclidean is about the magnitude — how close are the raw numbers.

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K-NN APPROACH TO RECOMMENDATION

Suppose we use the Pearson's coefficient as a similarity measure

We can predict the user rating as follows:

$$\hat{r}_{u,i} = \frac{\sum_{v \in N_u} sim(u,v) \cdot r_{v,i}}{\sum_{v \in N_u} |sim(u,v)|}$$

where N_u is the set of k closest users who rated item i.

K-NN APPROACH TO RECOMMENDATION: WHICH DISTANCE TO USE?

Another common distance measure: adjusted cosine similarity

- $Pearson(u,v) = \frac{\sum_{i \in \mathcal{C}}(r_{ui}-\overline{r_u})(r_{vi}-\overline{r_v})}{\sqrt{\sum_{i \in \mathcal{C}}(r_{ui}-\overline{r_u})^2}\sqrt{\sum_{i \in \mathcal{C}}(r_{vi}-\overline{r_v})^2}}$

- $cosine(i,j) = \frac{\sum_{u \in \mathcal{U}}(r_{ui})(r_{uj})}{\sqrt{\sum_{u \in \mathcal{U}}(r_{ui})^2}\sqrt{\sum_{u \in \mathcal{U}}(r_{uj})^2}}$

- $adjustedcosine(u,v) = \frac{\sum_{u \in \mathcal{U}}(r_{ui}-\overline{r_u})(r_{uj}-\overline{r_u})}{\sqrt{\sum_{u \in \mathcal{U}}(r_{ui}-\overline{r_u})^2}\sqrt{\sum_{u \in \mathcal{U}}(r_{uj}-\overline{r_u})^2}}$

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	5	-	1	4	3	-	-
User 2	-	4	-	2	2	5	-
User 3	-	1	2	5	-	3	5
User 4	4	4	-	3	-	4	3
User 5	1	-	5	5	-	5	5

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User 3	-	1	2	5	-	3	5
User 4	4	4	-	3	-	4	3
User 5	1	-	5	5	-	5	5

- $cosine(i,j) = \frac{\sum_{u\in\mathcal{U}}(r_{ui})(r_{uj})}{\sqrt{\sum_{u\in\mathcal{U}}(r_{ui})^2}\sqrt{\sum_{u\in\mathcal{U}}(r_{uj})^2}}$

- $adjustedcosine(u,v) = \frac{\sum_{u\in\mathcal{U}}(r_{ui}-\overline{r_u})(r_{uj}-\overline{r_u})}{\sqrt{\sum_{u\in\mathcal{U}}(r_{ui}-\overline{r_u})^2}\sqrt{\sum_{u\in\mathcal{U}}(r_{uj}-\overline{r_u})^2}}$

- $\mathcal{U} = \{1,3,5\}$

EVALUATION OF RATING PREDICTION PERFORMANCE

How do we evaluate the results of rating prediction?

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	5	-	1	4	3	-	-
User 2	-	4	-	2	2	5	-
User 3	-	1	2	5	-	3	5
User 4	4	4	-	3	-	4	3
User 5	1	-	5	5	-	5	5

Test set

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	5	-	1	4	3	-	-
User 2	-	4	-	2	2	5	-
User 3	-	1	2	5	-	3	5
User 4	4	4	-	3	-	4	3
User 5	1	-	5	5	-	5	5

Training set

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	-	-	1	4	3	-	-
User 2	-	4	-	-	2	5	-
User 3	-	1	2	5	-	3	-
User 4	4	-	-	3	-	4	3
User 5	1	-	5	5	-	-	5

EVALUATION OF RATING PREDICTION PERFORMANCE

How do we evaluate the results of rating prediction?

Use a method like K-NN to predict rating in the test set.

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	5	-	1	4	3	-	-
User 2	-	4	-	2	2	5	-
User 3	-	1	2	5	-	3	5
User 4	4	4	-	3	-	4	3
User 5	1	-	5	5	-	5	5

Test set

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	5	-	1	4	3	-	-
User 2	-	4	-	2	2	5	-
User 3	-	1	2	5	-	3	5
User 4	4	4	-	3	-	4	3
User 5	1	-	5	5	-	5	5

Training set

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	4.5	-	1	4	3	-	-
User 2	-	4	-	3	2	5	-
User 3	-	1	2	5	-	3	4
User 4	4	2	-	3	-	4	3
User 5	1	-	5	5	-	3.7	5

EVALUATION OF RATING PREDICTION PERFORMANCE

How do we evaluate the results of rating prediction?

- Use a method like K-NN to predict rating in the test set.
- Common evaluation metrics:
  - Mean absolute error (MAE)
  - Root Mean Squared Error

Test set

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	5	-	1	4	3	-	-
User 2	-	4	-	2	2	5	-
User 3	-	1	2	5	-	3	5
User 4	4	4	-	3	-	4	3
User 5	1	-	5	5	-	5	5

Training set

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	4.5	-	1	4	3	-	-
User 2	-	4	-	3	2	5	-
User 3	-	1	2	5	-	3	4
User 4	4	2	-	3	-	4	3
User 5	1	-	5	5	-	3.7	5

EVALUATION OF RATING PREDICTION PERFORMANCE

How do we evaluate the results of rating prediction?

- Use a method like K-NN to predict rating in the test set.
- Common evaluation metrics:
  - Mean absolute error (MAE)
  - Root Mean Squared Error
- $MAE = \frac{1}{|TestSet|} \sum_{r_{ui} \in TestSet} |r_{ui} - \hat{r}_{ui}|$

Test set

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	5	-	1	4	3	-	-
User 2	-	4	-	2	2	5	-
User 3	-	1	2	5	-	3	5
User 4	4	4	-	3	-	4	3
User 5	1	-	5	5	-	5	5

Training set

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	4.5	-	1	4	3	-	-
User 2	-	4	-	3	2	5	-
User 3	-	1	2	5	-	3	4
User 4	4	2	-	3	-	4	3
User 5	1	-	5	5	-	3.7	5

EVALUATION OF RATING PREDICTION PERFORMANCE

How do we evaluate the results of rating prediction?

- Use a method like K-NN to predict rating in the test set.
- Common evaluation metrics:
    - Mean absolute error (MAE)
    - Root Mean Squared Error

- $MAE = \frac{1}{|TestSet|} \sum_{r_{ui} \in TestSet} |r_{ui} - \hat{r}_{ui}|$
- $MAE = \frac{1}{5}(|5 - 4.5| + |4 - 2| + |2 - 3| + |5 - 3.7| + |5 - 4|)$
- $= \frac{1}{5} 5.8 = 1.16$

Test set

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	5	-	1	4	3	-	-
User 2	-	4	-	2	2	5	-
User 3	-	1	2	5	-	3	5
User 4	4	4	-	3	-	4	3
User 5	1	-	5	5	-	5	5

Training set

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	4.5	-	1	4	3	-	-
User 2	-	4	-	3	2	5	-
User 3	-	1	2	5	-	3	4
User 4	4	2	-	3	-	4	3
User 5	1	-	5	5	-	3.7	5

EVALUATION OF RATING PREDICTION PERFORMANCE

How do we evaluate the results of rating prediction?

- Use a method like K-NN to predict rating in the test set.
- Common evaluation metrics:
  - Mean absolute error (MAE)
  - Root Mean Squared Error
- $MAE = 1.16$
- $RMSE = \sqrt{\frac{1}{|TestSet|} \sum_{r_{ui} \in TestSet} (r_{ui} - \hat{r}_{ui})^2}$

Test set

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	5	-	1	4	3	-	-
User 2	-	4	-	2	2	5	-
User 3	-	1	2	5	-	3	5
User 4	4	4	-	3	-	4	3
User 5	1	-	5	5	-	5	5

Training set

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	4.5	-	1	4	3	-	-
User 2	-	4	-	3	2	5	-
User 3	-	1	2	5	-	3	4
User 4	4	2	-	3	-	4	3
User 5	1	-	5	5	-	3.7	5

EVALUATION OF RATING PREDICTION PERFORMANCE

How do we evaluate the results of rating prediction?

- Use a method like K-NN to predict rating in the test set.
- Common evaluation metrics:
  - Mean absolute error (MAE)
  - Root Mean Squared Error

- $MAE = 1.16$
- $RMSE = \sqrt{\frac{1}{|TestSet|}\sum_{r_{ui}\in TestSet}(r_{ui} - \hat{r}_{ui})^2}$
- $RMSE$

$$= \sqrt{\frac{1}{5}\left(\left((5-4.5)^2 + (4-2)^2 + (2-3)^2 + (5-3.7)^2 + (5-4)^2\right)\right)}$$

$$= \sqrt{1.588} = 1.260$$

Test set

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	5	-	1	4	3	-	-
User 2	-	4	-	2	2	5	-
User 3	-	1	2	5	-	3	5
User 4	4	4	-	3	-	4	3
User 5	1	-	5	5	-	5	5

Training set

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	4.5	-	1	4	3	-	-
User 2	-	4	-	3	2	5	-
User 3	-	1	2	5	-	3	4
User 4	4	2	-	3	-	4	3
User 5	1	-	5	5	-	3.7	5

EVALUATION OF RATING PREDICTION PERFORMANCE

How do we evaluate the results of rating prediction?

- Use a method like K-NN to predict rating in the test set.
- Common evaluation metrics:
  - Mean absolute error (MAE)
  - Root Mean Squared Error
- $MAE = \frac{1}{|TestSet|} \sum_{r_{ui} \in TestSet} |r_{ui} - \hat{r}_{ui}| \rightarrow 1.16$
- $RMSE = \sqrt{\frac{1}{|TestSet|} \sum_{r_{ui} \in TestSet} (r_{ui} - \hat{r}_{ui})^2} \rightarrow 1.260$

Test set

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	5	-	1	4	3	-	-
User 2	-	4	-	2	2	5	-
User 3	-	1	2	5	-	3	5
User 4	4	4	-	3	-	4	3
User 5	1	-	5	5	-	5	5

Training set

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	4.5	-	1	4	3	-	-
User 2	-	4	-	3	2	5	-
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User 5	1	-	5	5	-	3.7	5

EVALUATION OF RATING PREDICTION PERFORMANCE

How do we evaluate the results of rating prediction?

- Use a method like K-NN to predict rating in the test set.
- Common evaluation metrics:
  - Mean absolute error (MAE)
  - Root Mean Squared Error
- MAE Treats all errors equally (linear).
- $RMSE$ Penalizes larger errors more heavily (quadratic).

Test set

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	5	-	1	4	3	-	-
User 2	-	4	-	2	2	5	-
User 3	-	1	2	5	-	3	5
User 4	4	4	-	3	-	4	3
User 5	1	-	5	5	-	5	5

Training set

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
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User 4	4	2	-	3	-	4	3
User 5	1	-	5	5	-	3.7	5

EVALUATION OF RATING PREDICTION PERFORMANCE

How do we evaluate the results of rating prediction?

- Use a method like K-NN to predict rating in the test set.
- Common evaluation metrics:
    - Mean absolute error (MAE)
    - Root Mean Squared Error
    - More recently, focus on "Top-N" performance rather than pure rating prediction accuracy

- $MAE = \frac{1}{|TestSet|} \sum_{r_{ui} \in TestSet} |r_{ui} - \hat{r}_{ui}| \rightarrow 1.16$
- $RMSE = \sqrt{\frac{1}{|TestSet|} \sum_{r_{ui} \in TestSet} (r_{ui} - \hat{r}_{ui})^2} \rightarrow 1.260$

Test set

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
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User 5	1	-	5	5	-	3.7	5

EVALUATION OF RECOMMENDATION PERFORMANCE

How do we evaluate the results of rating prediction?

- How do we evaluate recommendation algorithms?
  - For each user (in test set), recommend a (ranked) list of N items
  - Compare the list to actual user interactions

	Recommended List of 3 movies	Movies which user enjoyed (rating >= 3)
User 1	1,2,3	1,4,5
User 2	2,4,5	6,2
User 3	1,4,6	4,6,7
User 4	3,4,5	1,2,4,6,7

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7
User 1	5	-	1	4	3	-	-
User 2	-	4	-	2	2	5	-
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User 3	1,4,6	4,6,7
User 4	3,4,5	1,2,4,6,7

- $precision = \frac{|Recommended \cap Enjoyed|}{|Recommended|}$

- $recall = \frac{|Recommended \cap Enjoyed|}{|Enjoyed|}$

EVALUATION OF RECOMMENDATION PERFORMANCE

How do we evaluate the results of rating prediction?

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  - For each user (in test set), recommend a (ranked) list of N items
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User 4	3,4,5	1,2,4,6,7

- $user\ 1\text{: } precision@3 = \frac{|Recommended \cap Enjoyed|}{|Recommended|} = \frac{|\{1\}|}{3} = \frac{1}{3}$

EVALUATION OF RECOMMENDATION PERFORMANCE

How do we evaluate the results of rating prediction?

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  - For each user (in test set), recommend a (ranked) list of N items
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- $user\ 1:\ precision@3 = \frac{|Recommended \cap Enjoyed|}{|Recommended|} = \frac{|\{1\}|}{3} = \frac{1}{3}$
- $user\ 4: precision@3 = \frac{|Recommended \cap Enjoyed|}{|Recommended|} = \frac{|\{4\}|}{3} = \frac{1}{3}$

EVALUATION OF RECOMMENDATION PERFORMANCE

How do we evaluate the results of rating prediction?

- How do we evaluate recommendation algorithms?
  - For each user (in test set), recommend a (ranked) list of N items
  - Compare the list to actual user interactions

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User 2	2,4,5	6,2
User 3	1,4,6	4,6,7
User 4	3,4,5	1,2,4,6,7

- $user\ 1:\ precision@3 = \frac{|Recommended \cap Enjoyed|}{|Recommended|} = \frac{|\{1\}|}{3} = \frac{1}{3}$
- $user\ 4: precision@3 = \frac{|Recommended \cap Enjoyed|}{|Recommended|} = \frac{|\{4\}|}{3} = \frac{1}{3}$
- $user\ 3: precision@3 = \frac{|Recommended \cap Enjoyed|}{|Recommended|} = \frac{|\{4,6\}|}{3} = \frac{2}{3}$

EVALUATION OF RECOMMENDATION PERFORMANCE

How do we evaluate the results of rating prediction?

- How do we evaluate recommendation algorithms?
    - For each user (in test set), recommend a (ranked) list of N items
    - Compare the list to actual user interactions

	Recommended List of 3 movies	Movies which user enjoyed (rating >= 3)
User 1	1,2,3	1,4,5
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User 3	1,4,6	4,6,7
User 4	3,4,5	1,2,4,6,7

- $user\ 1{:}\, recall@3 = \frac{|Recommended \cap Enjoyed|}{|Enjoyed|} = \frac{|\{1\}|}{\{1,4,5\}} = \frac{1}{3}$
- $user\ 1{:}\ precision@3 = \frac{|Recommended \cap Enjoyed|}{|Recommended|} = \frac{|\{1\}|}{3} = \frac{1}{3}$

EVALUATION OF RECOMMENDATION PERFORMANCE

How do we evaluate the results of rating prediction?

- How do we evaluate recommendation algorithms?
  - For each user (in test set), recommend a (ranked) list of N items
  - Compare the list to actual user interactions

	Recommended List of 3 movies	Movies which user enjoyed (rating >= 3)
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User 2	2,4,5	6,2
User 3	1,4,6	4,6,7
User 4	3,4,5	1,2,4,6,7

- $user\ 4: recall@3 = \frac{|Recommended \cap Enjoyed|}{|Enjoyed|} = \frac{|\{1\}|}{\{1,2,4,6,7\}} = \frac{1}{5}$
- $user\ 4:\ precision@3 = \frac{|Recommended \cap Enjoyed|}{|Recommended|} = \frac{|\{1\}|}{3} = \frac{1}{3}$

EVALUATION OF RECOMMENDATION PERFORMANCE

How do we evaluate the results of rating prediction?

- How do we evaluate recommendation algorithms?
  - For each user (in test set), recommend a (ranked) list of N items
  - Compare the list to actual user interactions
- How to aggregate user results?
  - Take the average precision/recall over entire test set of users.

	Recommended List of 3 movies	Movies which user enjoyed (rating >= 3)
User 1	1,2,3	1,4,5
User 2	2,4,5	6,2
User 3	1,4,6	4,6,7
User 4	3,4,5	1,2,4,6,7

- $user\ 1{:}\ precision@3 = \frac{1}{3};\ user\ 2{:}\ precision@3 = \frac{1}{3};\ user\ 3{:}\ precision@3 = \frac{2}{3};\ user\ 4{:}\ precision@3 = \frac{1}{3};$
- $average\ precision\ @3 = \frac{1}{4}\left(\frac{1}{3} + \frac{1}{3} + \frac{2}{3} + \frac{1}{3}\right) = 0.4167$

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