Pseudo Scalar $I = \gamma_t \wedge \gamma_x \wedge \gamma_y \wedge \gamma_z$

$I_{xyz} = \gamma_x \wedge \gamma_y \wedge \gamma_z$

Electromagnetic Field Bi-Vector
$$\begin{aligned} F = &- E^x e^{-i(-\omega t+k_x x+k_y y+k_z z)}\gamma_t \wedge \gamma_x \\ &- E^y e^{-i(-\omega t+k_x x+k_y y+k_z z)}\gamma_t \wedge \gamma_y \\ &- E^z e^{-i(-\omega t+k_x x+k_y y+k_z z)}\gamma_t \wedge \gamma_z \\ &- B^z e^{-i(-\omega t+k_x x+k_y y+k_z z)}\gamma_x \wedge \gamma_y \\ &+ B^y e^{i(\omega t-k_x x-k_y y-k_z z)}\gamma_x \wedge \gamma_z \\ &- B^x e^{-i(-\omega t+k_x x+k_y y+k_z z)}\gamma_y \wedge \gamma_z \end{aligned}$$

Geom Derivative of Electomagnetic Field Bi-Vector

$$\begin{aligned} \boldsymbol{\nabla} F = 0 = &- i\left(E^x k_x + E^y k_y + E^z k_z\right) e^{-i(-\omega t+k_x x+k_y y+k_z z)}\gamma_t \\ &+ i\left(B^y k_z - B^z k_y - E^x \omega\right) e^{i(\omega t-k_x x-k_y y-k_z z)}\gamma_x \\ &+ i\left(-B^x k_z + B^z k_x - E^y \omega\right) e^{i(\omega t-k_x x-k_y y-k_z z)}\gamma_y \\ &+ i\left(B^x k_y - B^y k_x - E^z \omega\right) e^{i(\omega t-k_x x-k_y y-k_z z)}\gamma_z \\ &+ i\left(-B^z \omega - E^x k_y + E^y k_x\right) e^{i(\omega t-k_x x-k_y y-k_z z)}\gamma_t \wedge \gamma_x \wedge \gamma_y \\ &+ i\left(B^y \omega - E^x k_z + E^z k_x\right) e^{i(\omega t-k_x x-k_y y-k_z z)}\gamma_t \wedge \gamma_x \wedge \gamma_z \\ &+ i\left(-B^x \omega - E^y k_z + E^z k_y\right) e^{i(\omega t-k_x x-k_y y-k_z z)}\gamma_t \wedge \gamma_y \wedge \gamma_z \\ &- i\left(B^x k_x + B^y k_y + B^z k_z\right) e^{-i(-\omega t+k_x x+k_y y+k_z z)}\gamma_x \wedge \gamma_y \wedge \gamma_z \end{aligned}$$

$$\begin{aligned} (\boldsymbol{\nabla} F) / \left(i e^{iK\cdot X}\right) = 0 = &\left(-E^x k_x - E^y k_y - E^z k_z\right)\gamma_t \\ &+ \left(B^y k_z - B^z k_y - E^x \omega\right)\gamma_x \\ &+ \left(-B^x k_z + B^z k_x - E^y \omega\right)\gamma_y \\ &+ \left(B^x k_y - B^y k_x - E^z \omega\right)\gamma_z \\ &+ \left(-B^z \omega - E^x k_y + E^y k_x\right)\gamma_t \wedge \gamma_x \wedge \gamma_y \\ &+ \left(B^y \omega - E^x k_z + E^z k_x\right)\gamma_t \wedge \gamma_x \wedge \gamma_z \\ &+ \left(-B^x \omega - E^y k_z + E^z k_y\right)\gamma_t \wedge \gamma_y \wedge \gamma_z \\ &+ \left(-B^x k_x - B^y k_y - B^z k_z\right)\gamma_x \wedge \gamma_y \wedge \gamma_z \end{aligned}$$

set $e_E \cdot e_k = e_B \cdot e_k = 0$ and $e_E \cdot e_E = e_B \cdot e_B = e_k \cdot e_k = -e_t \cdot e_t = 1$

$$g = \begin{bmatrix} 1 & (e_E \cdot e_B) & 0 & 0 \\ (e_E \cdot e_B) & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$K \cdot X = -\omega t + k x_k$

$F = -B e^{-i(\omega t - k x_k)} e_E \wedge e_k + E e^{i(-\omega t + k x_k)} e_E \wedge t + (e_E \cdot e_B)\, B e^{i(-\omega t + k x_k)} e_B \wedge e_k$

$$\begin{aligned} \boldsymbol{\nabla} F = 0 = &i\left(Bk + E\omega\right) e^{i(-\omega t + k x_k)} e_E \\ &- i\left(e_E \cdot e_B\right) B k e^{-i(\omega t - k x_k)} e_B \\ &- i\left(B\omega + Ek\right) e^{-i(\omega t - k x_k)} e_E \wedge e_k \wedge t \\ &+ i\left(e_E \cdot e_B\right) B\omega e^{i(-\omega t + k x_k)} e_B \wedge e_k \wedge t \end{aligned}$$

$$\begin{aligned} (\boldsymbol{\nabla} F) / \left(i e^{iK\cdot X}\right) = 0 = &\left(Bk + E\omega\right) e_E \\ &- \left(e_E \cdot e_B\right) B k e_B \\ &+ \left(-B\omega - Ek\right) e_E \wedge e_k \wedge t \\ &+ \left(e_E \cdot e_B\right) B\omega e_B \wedge e_k \wedge t \end{aligned}$$

Previous equation requires that: $e_E \cdot e_B = 0$ if $B \neq 0$ and $k \neq 0$

$$
\begin{aligned}
(\boldsymbol{\nabla} F) / \left(i e^{i K \cdot X}\right) = 0 = & (B k + E \omega)\, e_E \\
& + (-B \omega - E k)\, e_E \wedge e_k \wedge t
\end{aligned}
$$

eq1: $B = -\dfrac{E\omega}{k}$

eq2: $B = -\dfrac{Ek}{\omega}$

eq3 = eq1-eq2: $0 = -\dfrac{E\omega}{k} + \dfrac{Ek}{\omega}$

eq3 = (eq1-eq2)/E: $0 = -\dfrac{\omega}{k} + \dfrac{k}{\omega}$

$k = \begin{bmatrix} -\omega \\ \omega \end{bmatrix}$

$B = \begin{bmatrix} -E \\ E \end{bmatrix}$