```
def Product_of_Rotors():
    Print_Function()
    (na,nb,nm,alpha,th,th_a,th_b) = symbols('n_a n_b n_m alpha theta theta_a theta_b',\
                                            real = True)
    g = [[na, 0, alpha],[0, nm, 0],[alpha, 0, nb]] #metric tensor
    """
    Values of metric tensor components
    [na,nm,nb] = [+1/-1,+1/-1,+1/-1]  alpha = ea|eb
    """
    (g3d, ea, em, eb) = Ga.build('e_a e_m e_b', g=g)
    print 'g =',g3d.g
    print r'%n_{a} = \bm{e}_{a}^{2}\;\;n_{b} = \bm{e}_{b}^{2}\;\;n_{m} = \bm{e}_{m}^{2}'+\
        r'\;\;\alpha = \bm{e}_{a}\cdot\bm{e}_{b}'
    (ca,cb,sa,sb) = symbols('c_a c_b s_a s_b',real=True)
    Ra = ca + sa*ea*em  # Rotor for ea^em plane
    Rb = cb + sb*em*eb  # Rotor for em^eb plane
    print r'%\mbox{Rotor in }\bm{e}_{a}\bm{e}_{m}\mbox{ plane } R_{a} =',Ra
    print r'%\mbox{Rotor in }\bm{e}_{m}\bm{e}_{b}\mbox{ plane } R_{b} =',Rb
    Rab = Ra*Rb  # Compound Rotor
    """
    Show that compound rotor is scalar plus bivector
    """
    print r'%R_{a}R_{b} = S+\bm{B} =', Rab
    Rab2 = Rab.get_grade(2)
    print r'%\bm{B} =',Rab2
    Rab2sq = Rab2*Rab2  # Square of compound rotor bivector part
    Ssq = (Rab.scalar())**2  # Square of compound rotor scalar part
    Bsq =  Rab2sq.scalar()
    print r'%S^{2} =',Ssq
    print r'%\bm{B}^{2} =',Bsq
    Dsq = (Ssq-Bsq).expand().simplify()
    print '%S^{2}-B^{2} =', Dsq
    Dsq = Dsq.subs(nm**2,S(1))  # (e_m)**4 = 1
    print '%S^{2}-B^{2} =', Dsq
    Cases = [S(-1),S(1)]  # -1/+1 squares for each basis vector
    print r'#Consider all combinations of $\bm{e}_{a}^{2}$, $\bm{e}_{b}^{2}$'+\
          r' and $\bm{e}_{m}^2$:'
    for Na in Cases:
        for Nb in Cases:
            for Nm in Cases:
                Ba_sq = -Na*Nm
                Bb_sq = -Nb*Nm
                if Ba_sq < 0:
                    Ca_th = cos(th_a)
                    Sa_th = sin(th_a)
                else:
                    Ca_th = cosh(th_a)
                    Sa_th = sinh(th_a)
                if Bb_sq < 0:
                    Cb_th = cos(th_b)
                    Sb_th = sin(th_b)
                else:
                    Cb_th = cosh(th_b)
                    Sb_th = sinh(th_b)
                print r'%\left [ \bm{e}_{a}^{2},\bm{e}_{b}^{2},\bm{e}_{m}^2\right ] =',\
                      [Na,Nb,Nm]
                Dsq_tmp = Dsq.subs({ca:Ca_th,sa:Sa_th,cb:Cb_th,sb:Sb_th,na:Na,nb:Nb,nm:Nm})
                print r'%S^{2}-\bm{B}^{2} =',Dsq_tmp,' =',trigsimp(Dsq_tmp)
    print r'#Thus we have shown that $R_{a}R_{b} = S+\bm{D} = e^{\bm{C}}$ where $\bm{C}$'+\
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        r' is a bivector blade.'
    return
```

Code Output:

$$g = \begin{bmatrix} n_a & 0 & \alpha \\ 0 & n_m & 0 \\ \alpha & 0 & n_b \end{bmatrix}$$

$n_a = \boldsymbol{e}_a^2 \;\; n_b = \boldsymbol{e}_b^2 \;\; n_m = \boldsymbol{e}_m^2 \;\; \alpha = \boldsymbol{e}_a \cdot \boldsymbol{e}_b$

Rotor in $\boldsymbol{e}_a\boldsymbol{e}_m$ plane $R_a = c_a + s_a e_a \wedge e_m$

Rotor in $\boldsymbol{e}_m\boldsymbol{e}_b$ plane $R_b = c_b + s_b e_m \wedge e_b$

$R_a R_b = S + \boldsymbol{B} = (\alpha n_m s_a s_b + c_a c_b) + c_b s_a e_a \wedge e_m + n_m s_a s_b e_a \wedge e_b + c_a s_b e_m \wedge e_b$

$\boldsymbol{B} = c_b s_a e_a \wedge e_m + n_m s_a s_b e_a \wedge e_b + c_a s_b e_m \wedge e_b$

$S^2 = (\alpha n_m s_a s_b + c_a c_b)^2$

$\boldsymbol{B}^2 = \alpha^2 (n_m)^2 (s_a)^2 (s_b)^2 + 2\alpha c_a c_b n_m s_a s_b - (c_a)^2 n_b n_m (s_b)^2 - (c_b)^2 n_a n_m (s_a)^2 - n_a n_b (n_m)^2 (s_a)^2 (s_b)^2$

$S^2 - B^2 = (c_a)^2 (c_b)^2 + (c_a)^2 n_b n_m (s_b)^2 + (c_b)^2 n_a n_m (s_a)^2 + n_a n_b (n_m)^2 (s_a)^2 (s_b)^2$

$S^2 - B^2 = (c_a)^2 (c_b)^2 + (c_a)^2 n_b n_m (s_b)^2 + (c_b)^2 n_a n_m (s_a)^2 + n_a n_b (s_a)^2 (s_b)^2$

Consider all combinations of $\boldsymbol{e}_a^2$, $\boldsymbol{e}_b^2$ and $\boldsymbol{e}_m^2$:

$\left[\boldsymbol{e}_a^2, \boldsymbol{e}_b^2, \boldsymbol{e}_m^2\right] = [-1, -1, -1]$

$S^2 - \boldsymbol{B}^2 = \sin^2(\theta_a)\sin^2(\theta_b) + \sin^2(\theta_a)\cos^2(\theta_b) + \sin^2(\theta_b)\cos^2(\theta_a) + \cos^2(\theta_a)\cos^2(\theta_b) = 1$

$\left[\boldsymbol{e}_a^2, \boldsymbol{e}_b^2, \boldsymbol{e}_m^2\right] = [-1, -1, 1]$

$S^2 - \boldsymbol{B}^2 = \sinh^2(\theta_a)\sinh^2(\theta_b) - \sinh^2(\theta_a)\cosh^2(\theta_b) - \sinh^2(\theta_b)\cosh^2(\theta_a) + \cosh^2(\theta_a)\cosh^2(\theta_b) = 1$

$\left[\boldsymbol{e}_a^2, \boldsymbol{e}_b^2, \boldsymbol{e}_m^2\right] = [-1, 1, -1]$

$S^2 - \boldsymbol{B}^2 = -\sin^2(\theta_a)\sinh^2(\theta_b) + \sin^2(\theta_a)\cosh^2(\theta_b) - \cos^2(\theta_a)\sinh^2(\theta_b) + \cos^2(\theta_a)\cosh^2(\theta_b) = 1$

$\left[\boldsymbol{e}_a^2, \boldsymbol{e}_b^2, \boldsymbol{e}_m^2\right] = [-1, 1, 1]$

$S^2 - \boldsymbol{B}^2 = -\sin^2(\theta_b)\sinh^2(\theta_a) + \sin^2(\theta_b)\cosh^2(\theta_a) - \cos^2(\theta_b)\sinh^2(\theta_a) + \cos^2(\theta_b)\cosh^2(\theta_a) = 1$

$\left[\boldsymbol{e}_a^2, \boldsymbol{e}_b^2, \boldsymbol{e}_m^2\right] = [1, -1, -1]$

$S^2 - \boldsymbol{B}^2 = -\sin^2(\theta_b)\sinh^2(\theta_a) + \sin^2(\theta_b)\cosh^2(\theta_a) - \cos^2(\theta_b)\sinh^2(\theta_a) + \cos^2(\theta_b)\cosh^2(\theta_a) = 1$

$\left[\boldsymbol{e}_a^2, \boldsymbol{e}_b^2, \boldsymbol{e}_m^2\right] = [1, -1, 1]$

$S^2 - \boldsymbol{B}^2 = -\sin^2(\theta_a)\sinh^2(\theta_b) + \sin^2(\theta_a)\cosh^2(\theta_b) - \cos^2(\theta_a)\sinh^2(\theta_b) + \cos^2(\theta_a)\cosh^2(\theta_b) = 1$

$\left[\boldsymbol{e}_a^2, \boldsymbol{e}_b^2, \boldsymbol{e}_m^2\right] = [1, 1, -1]$

$S^2 - \boldsymbol{B}^2 = \sinh^2(\theta_a)\sinh^2(\theta_b) - \sinh^2(\theta_a)\cosh^2(\theta_b) - \sinh^2(\theta_b)\cosh^2(\theta_a) + \cosh^2(\theta_a)\cosh^2(\theta_b) = 1$

$\left[\boldsymbol{e}_a^2, \boldsymbol{e}_b^2, \boldsymbol{e}_m^2\right] = [1, 1, 1]$

$S^2 - \boldsymbol{B}^2 = \sin^2(\theta_a)\sin^2(\theta_b) + \sin^2(\theta_a)\cos^2(\theta_b) + \sin^2(\theta_b)\cos^2(\theta_a) + \cos^2(\theta_a)\cos^2(\theta_b) = 1$

Thus we have shown that $R_a R_b = S + \boldsymbol{D} = e^{\boldsymbol{C}}$ where $\boldsymbol{C}$ is a bivector blade.