$$\boldsymbol{\nabla} = \boldsymbol{e}_u \left(\frac{4v^2+1}{4u^2+4v^2+1} \frac{\partial}{\partial u} - \frac{4uv}{4u^2+4v^2+1} \frac{\partial}{\partial v} \right) + \boldsymbol{e}_v \left(-\frac{4uv}{4u^2+4v^2+1} \frac{\partial}{\partial u} + \frac{4u^2+1}{4u^2+4v^2+1} \frac{\partial}{\partial v} \right)$$

$$g = \begin{bmatrix} 4u^2+1 & 4uv \\ 4uv & 4v^2+1 \end{bmatrix}$$

$$\boldsymbol{e}_u \cdot \boldsymbol{\nabla} = \frac{\partial}{\partial u}$$

$$\boldsymbol{e}_v \cdot \boldsymbol{\nabla} = \frac{\partial}{\partial v}$$

$$\partial_u \boldsymbol{e}_u = \frac{4u}{4u^2+4v^2+1} \boldsymbol{e}_u + \frac{4v}{4u^2+4v^2+1} \boldsymbol{e}_v$$

$$\partial_v \boldsymbol{e}_u = 0$$

$$\partial_u \boldsymbol{e}_v = 0$$

$$\partial_v \boldsymbol{e}_v = \frac{4u}{4u^2+4v^2+1} \boldsymbol{e}_u + \frac{4v}{4u^2+4v^2+1} \boldsymbol{e}_v$$

$$f = (v+1)\, \boldsymbol{e}_u + u^2 \boldsymbol{e}_v$$

$$\boldsymbol{\nabla} f = \frac{4u\,(uv+v+1)}{4u^2+4v^2+1} + \frac{1}{4u^2+4v^2+1} \left(-4u^3-4u^2+8uv^2+2u+4v^2+4v-1\right) \boldsymbol{e}_u \wedge \boldsymbol{e}_v$$

$$\boldsymbol{\nabla} \cdot f = \frac{4u\,(uv+v+1)}{4u^2+4v^2+1}$$

$$\boldsymbol{\nabla} \wedge f = \frac{1}{4u^2+4v^2+1} \left(-4u^3-4u^2+8uv^2+2u+4v^2+4v-1\right) \boldsymbol{e}_u \wedge \boldsymbol{e}_v$$

$$((\boldsymbol{e}_u + \boldsymbol{e}_v) \cdot \boldsymbol{\nabla}) f = \frac{1}{4u^2+4v^2+1} \left(4u^3+4u^2+4uv+4u+4v^2+1\right) \boldsymbol{e}_u + \frac{1}{4u^2+4v^2+1} \left(8u^3+4u^2v+8uv^2+2u+4v^2+4v\right) \boldsymbol{e}_v$$

$$\boldsymbol{\nabla} = \boldsymbol{e}_r \frac{1}{4r^2+1} \frac{\partial}{\partial r} + \boldsymbol{e}_\theta r^{-2} \frac{\partial}{\partial \theta}$$

$$g = \begin{bmatrix} 4r^2+1 & 0 \\ 0 & r^2 \end{bmatrix}$$

$$\boldsymbol{e}_u \cdot \boldsymbol{\nabla} = \frac{\partial}{\partial r}$$

$$\boldsymbol{e}_v \cdot \boldsymbol{\nabla} = \frac{\partial}{\partial \theta}$$

$$\partial_u \boldsymbol{e}_u = \frac{4r}{4r^2+1} \boldsymbol{e}_r$$

$$\partial_v \boldsymbol{e}_u = \frac{1}{r} \boldsymbol{e}_\theta$$

$$\partial_u \boldsymbol{e}_v = \frac{1}{r} \boldsymbol{e}_\theta$$

$$\partial_v \boldsymbol{e}_v = -\frac{r}{4r^2+1}\boldsymbol{e}_r$$

$$f = (r+1)\,\boldsymbol{e}_r + r^2 \boldsymbol{e}_\theta$$

$$\boldsymbol{\nabla} f = \frac{12r^3+8r^2+2r+1}{r\,(4r^2+1)} + \frac{4r}{4r^2+1}\boldsymbol{e}_r \wedge \boldsymbol{e}_\theta$$

$$\boldsymbol{\nabla} \cdot f = \frac{12r^3+8r^2+2r+1}{r\,(4r^2+1)}$$

$$\boldsymbol{\nabla} \wedge f = \frac{4r}{4r^2+1}\boldsymbol{e}_r \wedge \boldsymbol{e}_\theta$$

$$((\boldsymbol{e}_r + \boldsymbol{e}_\theta)\cdot\boldsymbol{\nabla})f = \frac{1}{4r^2+1}\left(-r^3+8r^2+4r+1\right)\boldsymbol{e}_r + \left(3r+1+\frac{1}{r}\right)\boldsymbol{e}_\theta$$