$$\nabla = e_u \left(\frac{4v^2 + 1}{4u^2 + 4v^2 + 1} \frac{\partial}{\partial u} - \frac{4uv}{4u^2 + 4v^2 + 1} \frac{\partial}{\partial v} \right) + e_v \left(-\frac{4uv}{4u^2 + 4v^2 + 1} \frac{\partial}{\partial u} + \frac{4u^2 + 1}{4u^2 + 4v^2 + 1} \frac{\partial}{\partial v} \right)$$

$$g = \begin{bmatrix} 4u^2 + 1 & 4uv \\ 4uv & 4v^2 + 1 \end{bmatrix}$$

$$\boldsymbol{e}_u \cdot \boldsymbol{\nabla} = \frac{\partial}{\partial u}$$

$$oldsymbol{e}_v\cdotoldsymbol{
abla}=rac{\partial}{\partial v}$$

$$\partial_u e_u = \frac{4u}{4u^2 + 4v^2 + 1} e_u + \frac{4v}{4u^2 + 4v^2 + 1} e_v$$

$$\partial_v \boldsymbol{e}_u = 0$$

$$\partial_u \boldsymbol{e}_v = 0$$

$$\partial_v \mathbf{e}_v = \frac{4u}{4u^2 + 4v^2 + 1} \mathbf{e}_u + \frac{4v}{4u^2 + 4v^2 + 1} \mathbf{e}_v$$

$$f = (v+1)\,\boldsymbol{e}_u + u^2\boldsymbol{e}_v$$

$$\nabla f = \frac{4u(uv + v + 1)}{4u^2 + 4v^2 + 1} + \frac{1}{4u^2 + 4v^2 + 1} \left(-4u^3 - 4u^2 + 8uv^2 + 2u + 4v^2 + 4v - 1 \right) e_u \wedge e_v$$

$$\nabla \cdot f = \frac{4u\left(uv + v + 1\right)}{4u^2 + 4v^2 + 1}$$

$$\nabla \wedge f = \frac{1}{4u^2 + 4v^2 + 1} \left(-4u^3 - 4u^2 + 8uv^2 + 2u + 4v^2 + 4v - 1 \right) e_u \wedge e_v$$

$$((\boldsymbol{e}_{u} + \boldsymbol{e}_{v}) \cdot \nabla)f = \frac{1}{4u^{2} + 4v^{2} + 1} \left(4u^{3} + 4u^{2} + 4uv + 4u + 4v^{2} + 1\right) \boldsymbol{e}_{u} + \frac{1}{4u^{2} + 4v^{2} + 1} \left(8u^{3} + 4u^{2}v + 8uv^{2} + 2u + 4v^{2} + 4v\right) \boldsymbol{e}_{v}$$

$$\nabla = e_r \frac{1}{4r^2 + 1} \frac{\partial}{\partial r} + e_{\theta} r^{-2} \frac{\partial}{\partial \theta}$$

$$g = \begin{bmatrix} 4r^2 + 1 & 0\\ 0 & r^2 \end{bmatrix}$$

$$oldsymbol{e}_u\cdotoldsymbol{
abla}=rac{\partial}{\partial r}$$

$$oldsymbol{e}_v\cdotoldsymbol{
abla}=rac{\partial}{\partial heta}$$

$$\partial_u \boldsymbol{e}_u = \frac{4r}{4r^2 + 1} \boldsymbol{e}_r$$

$$\partial_v oldsymbol{e}_u = rac{1}{r} oldsymbol{e}_ heta$$

$$\partial_u oldsymbol{e}_v = rac{1}{r} oldsymbol{e}_{ heta}$$

$$\partial_v oldsymbol{e}_v = -rac{r}{4r^2+1}oldsymbol{e}_r$$

$$f = (r+1)\mathbf{e}_r + r^2\mathbf{e}_\theta$$

$$\nabla f = \frac{12r^3 + 8r^2 + 2r + 1}{r(4r^2 + 1)} + \frac{4r}{4r^2 + 1}e_r \wedge e_\theta$$

$$\nabla \cdot f = \frac{12r^3 + 8r^2 + 2r + 1}{r(4r^2 + 1)}$$

$$\nabla \wedge f = \frac{4r}{4r^2 + 1} e_r \wedge e_\theta$$

$$((\boldsymbol{e}_r + \boldsymbol{e}_\theta) \cdot \boldsymbol{\nabla})f = \frac{1}{4r^2 + 1} \left(-r^3 + 8r^2 + 4r + 1 \right) \boldsymbol{e}_r + \left(3r + 1 + \frac{1}{r} \right) \boldsymbol{e}_\theta$$