$$(u,v) \rightarrow (r,\theta,\phi) = [1,u,v]$$

Unit Sphere Manifold:

$$g = \begin{bmatrix}1 & 0\\0 & \sin^{2}{\left (u \right)}\end{bmatrix}$$

$$a = a^{u} e_{u} + a^{v} e_{v}$$

$$f = f^{u} e_{u} + f^{v} e_{v}$$

$$\nabla = e_{u} \frac{\partial}{\partial u} + e_{v} \frac{1}{\sin^{2}{\left (u \right)}} \frac{\partial}{\partial v}$$

$$a \cdot \nabla = a^{u} \frac{\partial}{\partial u} + a^{v} \frac{\partial}{\partial v}$$

$$\left (a\cdot \nabla \right) \boldsymbol{e}_{u} = \frac{a^{v}}{\tan{\left (u \right)}} e_{v}$$

$$\left (a\cdot \nabla \right) \boldsymbol{e}_{v} = - \frac{a^{v}}{2} \sin{\left (2 u \right)} e_{u} + \frac{a^{u}}{\tan{\left (u \right)}} e_{v}$$

$$\left (a\cdot \nabla \right) f = \left (a^{u} \partial_{u} f^{u} - \frac{a^{v} f^{v}}{2} \sin{\left (2 u \right)} + a^{v} \partial_{v} f^{u}\right) e_{u} + \left (\frac{a^{u} f^{v}}{\tan{\left (u \right)}} + a^{u} \partial_{u} f^{v} + \frac{a^{v} f^{u}}{\tan{\left (u \right)}} + a^{v} \partial_{v} f^{v}\right) e_{v}$$

Tensors on the Unit Sphere

$$V = a_{1}^{u} V_{u} + a_{1}^{v} V_{v}$$

$$T = T_{uu} a_{1}^{u} a_{2}^{u} + T_{uv} a_{1}^{u} a_{2}^{v} + T_{vu} a_{1}^{v} a_{2}^{u} + T_{vv} a_{1}^{v} a_{2}^{v}$$

Tensor Contraction

$$T[1,2] = \left(a_{1}^{u}\right)^{2} \partial^{2}_{u} T_{uu} + \frac{\left(a_{1}^{u}\right)^{2} \partial^{2}_{v} T_{uu}}{\sin^{2}{\left (u \right)}} + a_{1}^{u} a_{1}^{v} \partial^{2}_{u} T_{uv} + a_{1}^{u} a_{1}^{v} \partial^{2}_{u} T_{vu} + \frac{a_{1}^{u} a_{1}^{v} \partial^{2}_{v} T_{uv}}{\sin^{2}{\left (u \right)}} + \frac{a_{1}^{u} a_{1}^{v} \partial^{2}_{v} T_{vu}}{\sin^{2}{\left (u \right)}} + \left(a_{1}^{v}\right)^{2} \partial^{2}_{u} T_{vv} + \frac{\left(a_{1}^{v}\right)^{2} \partial^{2}_{v} T_{vv}}{\sin^{2}{\left (u \right)}}$$

Tensor Evaluation

$$T(a,b) = a^{u} b^{u} T_{uu} + a^{u} b^{v} T_{uv} + a^{v} b^{u} T_{vu} + a^{v} b^{v} T_{vv}$$

$$T(a,b+c) = a^{u} b^{u} T_{uu} + a^{u} b^{v} T_{uv} + a^{u} c^{u} T_{uu} + a^{u} c^{v} T_{uv} + a^{v} b^{u} T_{vu} + a^{v} b^{v} T_{vv} + a^{v} c^{u} T_{vu} + a^{v} c^{v} T_{vv}$$

$$T(a,\alpha b) = \alpha a^{u} b^{u} T_{uu} + \alpha a^{u} b^{v} T_{uv} + \alpha a^{v} b^{u} T_{vu} + \alpha a^{v} b^{v} T_{vv}$$

Geometric Derivative With Respect To Slot

$$\nabla_{a_1} T = \left (a_{1}^{u} a_{2}^{u} \partial_{u} T_{uu} + a_{1}^{u} a_{2}^{v} \partial_{u} T_{uv} + a_{2}^{u} a_{1}^{v} \partial_{u} T_{vu} + a_{1}^{v} a_{2}^{v} \partial_{u} T_{vv}\right) e_{u} + \frac{1}{\sin^{2}{\left (u \right)}} \left (a_{1}^{u} a_{2}^{u} \partial_{v} T_{uu} + a_{1}^{u} a_{2}^{v} \partial_{v} T_{uv} + a_{2}^{u} a_{1}^{v} \partial_{v} T_{vu} + a_{1}^{v} a_{2}^{v} \partial_{v} T_{vv}\right) e_{v}$$

$$\nabla_{a_2} T = \left (a_{1}^{u} a_{2}^{u} \partial_{u} T_{uu} + a_{1}^{u} a_{2}^{v} \partial_{u} T_{uv} + a_{2}^{u} a_{1}^{v} \partial_{u} T_{vu} + a_{1}^{v} a_{2}^{v} \partial_{u} T_{vv}\right) e_{u} + \frac{1}{\sin^{2}{\left (u \right)}} \left (a_{1}^{u} a_{2}^{u} \partial_{v} T_{uu} + a_{1}^{u} a_{2}^{v} \partial_{v} T_{uv} + a_{2}^{u} a_{1}^{v} \partial_{v} T_{vu} + a_{1}^{v} a_{2}^{v} \partial_{v} T_{vv}\right) e_{v}$$

Covariant Derivatives

$$\begin{aligned} \mathcal{D}V = & \partial_{u} V_{u} a_{1}^{u} a_{2}^{u} \\ & + \partial_{v} V_{u} a_{1}^{u} a_{2}^{v} \\ & + \partial_{u} V_{v} a_{1}^{v} a_{2}^{u} \\ & + \partial_{v} V_{v} a_{1}^{v} a_{2}^{v} \end{aligned}$$

$$\begin{aligned} \mathcal{D}T = & \partial_{u} T_{uu} a_{1}^{u} a_{2}^{u} a_{3}^{u} + \partial_{v} T_{uu} a_{1}^{u} a_{2}^{u} a_{3}^{v} + \partial_{u} T_{uv} a_{1}^{u} a_{2}^{v} a_{3}^{u} \\ & + \partial_{v} T_{uv} a_{1}^{u} a_{2}^{v} a_{3}^{v} + \partial_{u} T_{vu} a_{1}^{v} a_{2}^{u} a_{3}^{u} + \partial_{v} T_{vu} a_{1}^{v} a_{2}^{u} a_{3}^{v} \\ & + \partial_{u} T_{vv} a_{1}^{v} a_{2}^{v} a_{3}^{u} + \partial_{v} T_{vv} a_{1}^{v} a_{2}^{v} a_{3}^{v} \end{aligned}$$

$$\mathcal{D}T[1,3](a) = (a^u)^2 a_2^u \partial_u^3 T_{uu} + \frac{(a^u)^2 a_2^u \partial_u \partial_v^2 T_{uu}}{\sin^2(u)} + (a^u)^2 a_2^v \partial_u^2 \partial_v T_{uu} + \frac{(a^u)^2 a_2^v \partial_v^3 T_{uu}}{\sin^2(u)} + a^u a_2^u a^v \partial_u^3 T_{uv} + a^u a_2^u a^v \partial_u^3 T_{vu} + \frac{a^u a_2^u a^v \partial_u \partial_v^2 T_{uv}}{\sin^2(u)} + \frac{a^u a_2^u a^v \partial_u \partial_v^2 T_{vu}}{\sin^2(u)} + a^u a^v a_2^v \partial_u^2 \partial_v T_{uv} + a^u a^v a_2^v \partial_u^2 \partial_v T_{vu} + \frac{a^u a^v a_2^v \partial_v^3 T_{uv}}{\sin^2(u)} + \frac{a^u a^v a_2^v \partial_v^3 T_{vu}}{\sin^2(u)} + a_2^u (a^v)^2 \partial_u^3 T_{v}$$

1-D Manifold On Unit Sphere:

$$\nabla = e_s \frac{1}{\sin^2(u^s)(\partial_s v^s)^2 + (\partial_s u^s)^2} \frac{\partial}{\partial s}$$

$$\nabla g = \frac{\partial_s g}{\sin^2(u^s)(\partial_s v^s)^2 + (\partial_s u^s)^2} e_s$$

$$\nabla \cdot \boldsymbol{h} = \frac{1}{\sin^2(u^s)(\partial_s v^s)^2 + (\partial_s u^s)^2} \left(\left(\sin^2(u^s)(\partial_s v^s)^2 + (\partial_s u^s)^2 \right) \partial_s h^s + \left(\sin^2(u^s) \partial_s v^s \partial_s^2 v^s + \frac{\partial_s u^s}{2} \sin(2u^s)(\partial_s v^s)^2 + \partial_s u^s \partial_s^2 u^s \right) h^s \right)$$