Complexity Aversion in Asset Choice

1 Model Set Up

- An individual i has k units of capital.
- She has access to a set A of finitely-many assets from a distribution of assets. Each asset a has a random payoff $v_a \sim F(a)$ per unit of time.
  - She knows $F(a)$ $\forall a \in A$
- She chooses an allocation $x \in \Phi(A) \subseteq \Delta(A)$ to maximize expected utility

$$EU_i = \int_{A}^{...} \int u\left(\sum_{a \in A} v_a x_a\right) dF(a_1)...dF(a_{|A|}) - g(|A_i|) \tag{1}$$

where g is a disutility from allocation and $A_i = \{a \in A : x_a \neq 0\}$.

- i's problem is thus:

$$\max_{x \in \Phi(A)} \int_{A}^{...} \int u\left(\sum_{a \in A} v_a x_a\right) dF(a_1)...dF(a_{|A|}) - g(|A_i|) \tag{2}$$

- if the individual is **complexity averse**, then we have that:
  1. $g(\cdot) > 0$ for $|A_i| > 1$
     - Ex: $g(\cdot) = \theta(|A_i|)^2$ where θ is the *complexity aversion coefficient*.
  2. $\Phi(A) \subseteq \Delta(A)$ is restricted to a subset of simple allocations (with $\Phi(A) \subseteq \Phi(A')$ if $A \subseteq A'$).
     - Ex: a "simple" allocation could only allow for numbers with one digit after the decimal.
- Suppose she (costlessly) learns (i.e., $\bar{v}_{new}(h)$) about a new asset and can choose to add this a_{new} to A_i so that $A'_i = \{A_i, a_{new}\}$.
  - if she is complexity averse, adding this a_{new} will incur a disutility of allocation. Thus, she will add a_{new} if, given $\Phi(A')$:

$$\int_{A'}^{...} \int u\left(\sum_{a \in A'} v_a x_a\right) dF(a_1)...dF(a_{|A'|}) - g(|A'_i|) > \int_{A}^{...} \int u\left(\sum_{a \in A} v_a x_a\right) dF(a_1)...dF(a_{|A|}) - g(|A_i|)$$

2 Predictions

Complexity aversion will lead to:

1. rigid capital allocations. I.e., small changes in payoff distributions will lead to no changes in hour allocations.
2. smaller menus A_i, implying people take on extra risk.
   (a) This effect will be greater for people with less capital.
3. less take-up (undervaluation) of profitable opportunities.

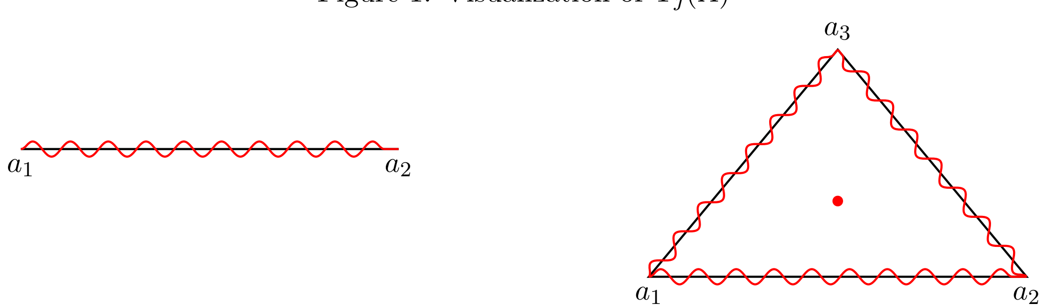
3 Simple Example

Consider two risk-averse investors, i and i, both with Bernoulli utility $u(c) = \log(c+1)$, and the same set of asset options A with $|A| = 3$. For each asset, let $F(a)$ be an independent Bernoulli random variable scaled by v_a. Investors receive return v_a with probability p_a and 0 otherwise. The expected return for investing fraction x_a in asset a is $v_a \cdot x_a \cdot p_a$.

Let i be *not complexity averse*; $g_i(|A_i|) = 0$ and $\Phi_i(A) = \Delta(A)$. Let j be *complexity averse*; $g_j(|A_j|) = \theta_j(|A_j| - 1)^2$ with $\theta_j > 0$ and $\Phi_j(A) \subset \Delta(A)$. In particular, we restrict $\Phi_j(A)$ so that j can either:

1. perfectly optimize over any 2 assets ($|A_j| = 2$) or
2. evenly allocate over > 2 assets ($|A_j| > 2$).

Figure 1: Visualization of $\Phi_j(A)$



Notes: The line on the left represents j's options when $|A| = 2$ and the triangle on the right represents j's options when $|A| = 3$

i, then, solves the following problem:

$$\max_{x \in \Delta(A)} \int_{A}^{\dots} \int log \left(\sum_{a \in A} v_a x_a + 1 \right) dF(a_1) \dots dF(a_3)$$

and j solves:

$$\max_{x \in \Phi_j(A)} \int_A \dots \int log \left(\sum_{a \in A} v_a x_a + 1 \right) dF(a_1) \dots dF(a_3) - g(|A_j|)$$

Clearly, i chooses $|A_i| = 3$ and perfectly allocates hours to the different job types. Assuming for simplicity that at most one asset pays off in a period (mutually exclusive hits), we get

$$x_k = \frac{p_k}{\sum_{a \in A} p_a} \left(1 + \sum_{a \in A} \frac{1}{v_a} \right) - \frac{1}{v_k}$$

However, unless we have $v = (v_1, v_2, v_3) =$ and $p = (p_1, p_2, p_3) =$, this allocation isn't feasible for j. Let $\bar{x}$ represent the optimal allocation over the set of size 2 menus. So, j chooses $x_j = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ if

$$\int_A \dots \int log \left(\sum_{a \in A} \frac{1}{3} v_a + 1 \right) dF(a_1) \dots dF(a_3) - g(3) \geq \int_A \dots \int log \left(\sum_{a \in A} v_a \bar{x_a} + 1 \right) dF(a_1) \dots dF(a_3) - g(2)$$

$$\implies \int_A \dots \int log \left(\frac{1}{3} v_a + 1 \right) dF(a_1) \dots dF(a_3) - \int_A \dots \int log \left(\sum_{a \in A} v_a \bar{x_a} + 1 \right) dF(a_1) \dots dF(a_3) \geq g(3) - g(2)$$

and some allocation over two assets otherwise. I run simulations to show this result in a setting with 6 assets[1]. Figure 2 and Figure 5 show how individuals allocate assets over varying levels of complexity aversion (θ).

4 Experiment Design: IRR + Strategy method

4.1 Game 1: IRR

From a set of assets, menus of varying sizes are randomly presented to respondents. Respondents rate/rank these menus, knowing that menu rating will inform a machine learning model that will match them to menus of assets for the second game.

4.2 Game 2: Strategy method

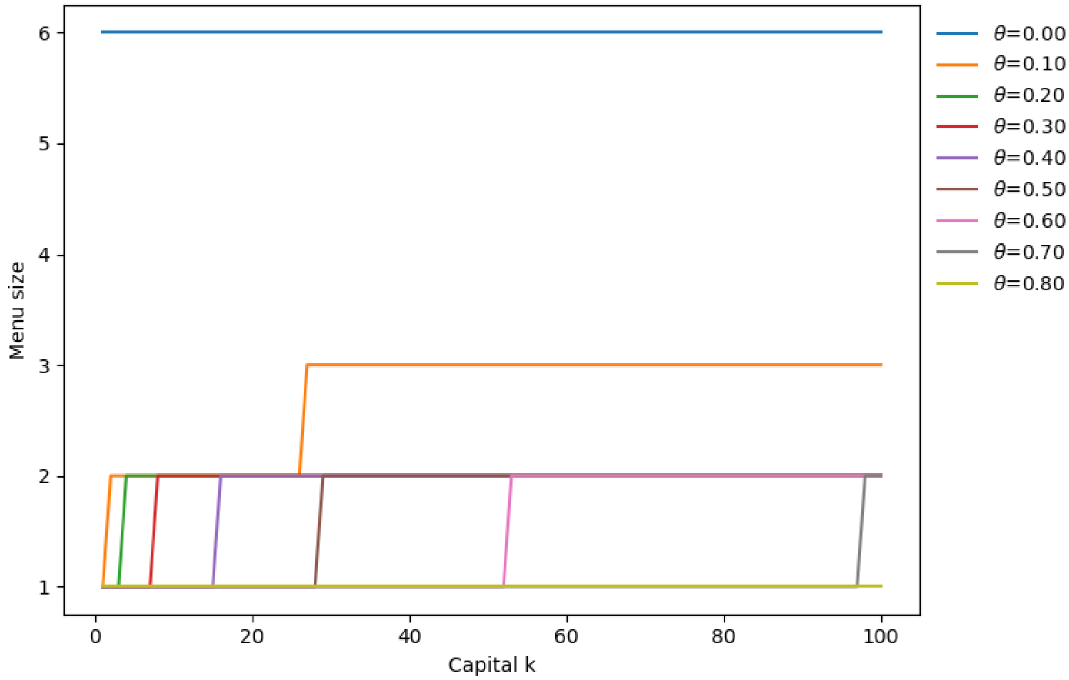
Based on their responses to game 1, respondents are presented with several menus. Respondents also have randomly determined amounts of "capital." For each menu, respondents allocate their capital to the different assets. For a subset of menus, another asset will be presented and the respondent will have the option to add it to the menu and allocate capital to it. Menus will include dominating/dominated strategies to test whether people who display more complexity aversion are more likely to choose dominated strategies.

One menu and the respective allocation is chosen at random and implemented. All participants are given investment advice based on the results of both games.

[1]Details of the simulation are in Appendix A.2

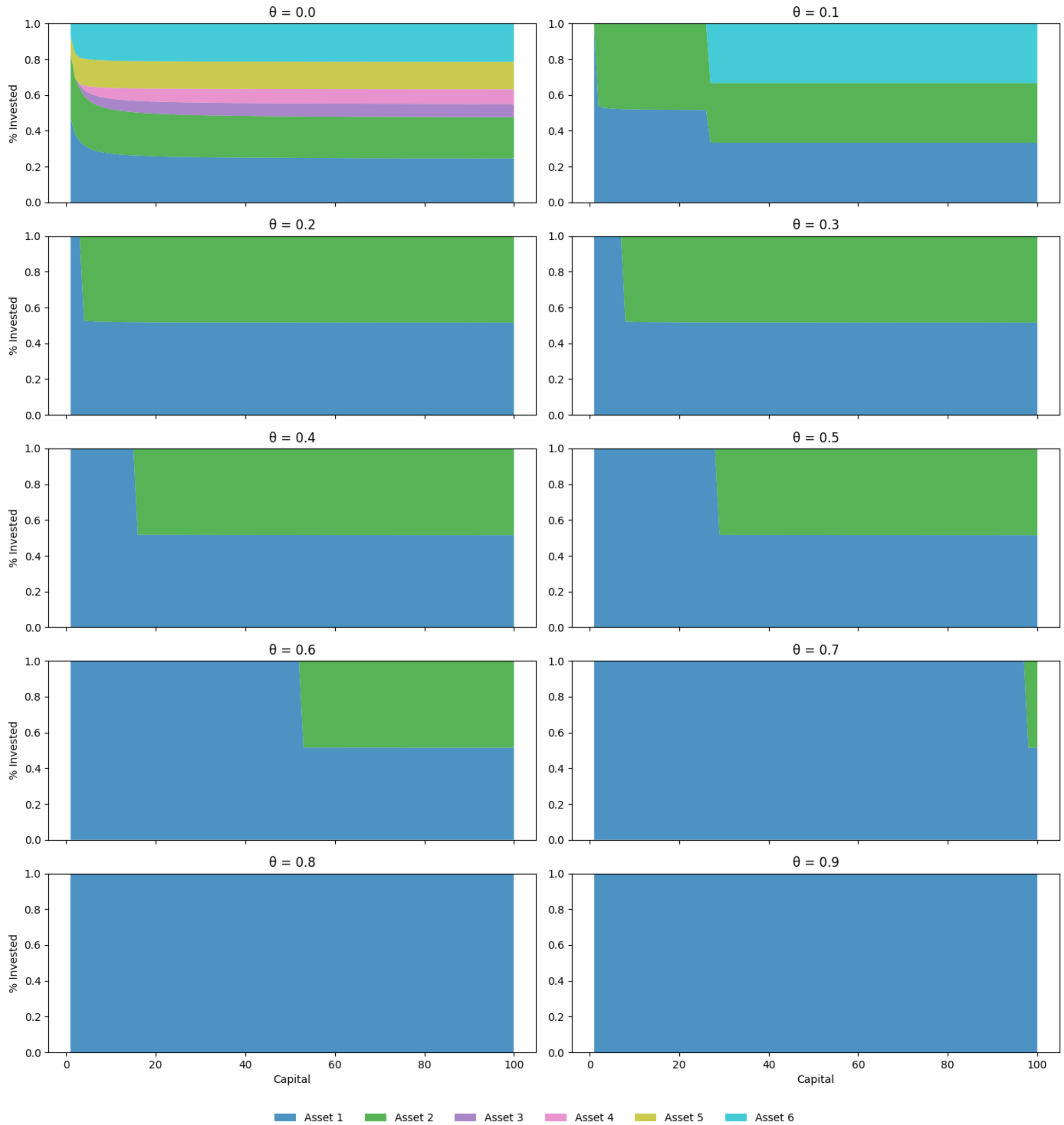
5 Figures

Figure 2: Menu size by θ



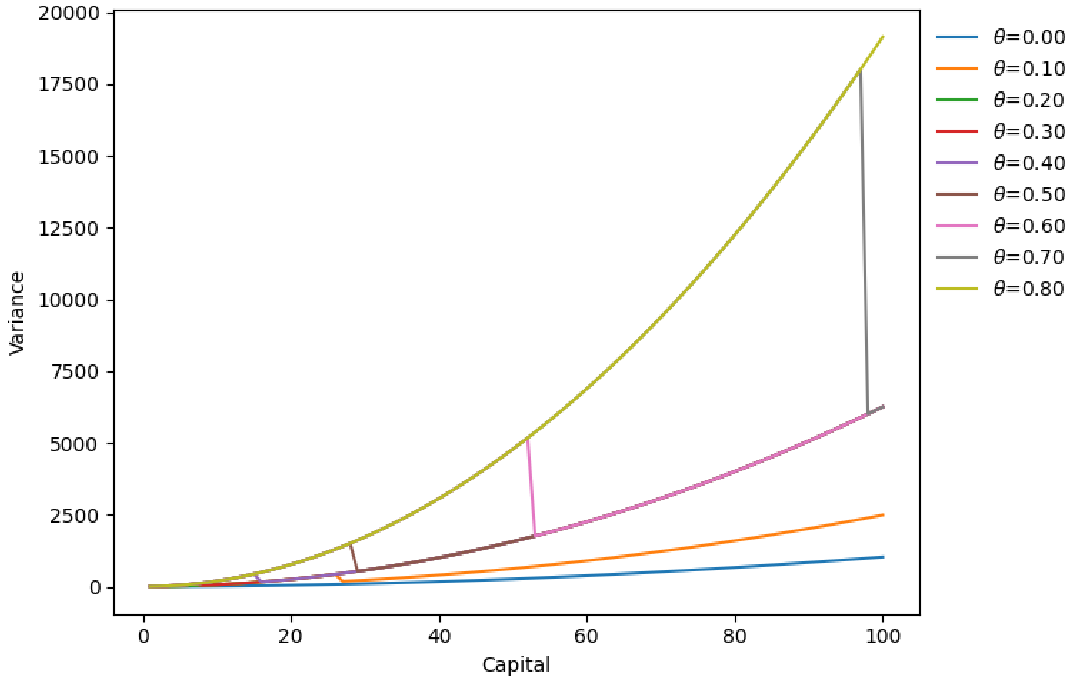
Note: This figure displays the optimal menu size by capital allowance for different levels of θ. There are six total assets. Asset payoffs and Bernoulli distributions are drawn from i.i.d uniform distributions.

Figure 3: Capital allocations by θ



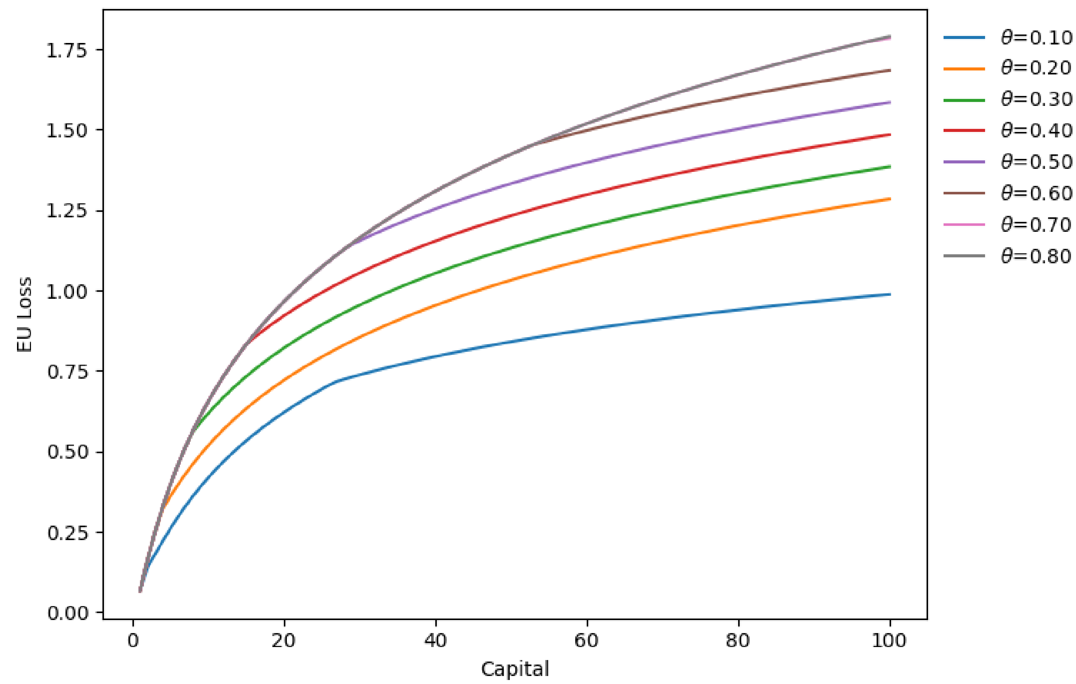
Note: This figure displays the optimal allocations by capital allowance for different levels of θ. There are six total assets. Asset payoffs and Bernoulli distributions are drawn from i.i.d uniform distributions.

Figure 4: Wealth variance by θ and k



Note: This figure displays the variance implied by the optimal allocations by capital allowance for different levels of θ. Vertical lines occur when menus increase in size, implying a drop from the higher curve to the lower curve. There are six total assets. Asset payoffs and Bernoulli distributions are drawn from i.i.d uniform distributions.

Figure 5: Missed opportunitie bys θ and k



Note: This figure displays the difference in expected utility between a complexity-neutral agent and a complexity-averse agent (for varying levels of θ) by capital allowance. There are six total assets. Asset payoffs and Bernoulli distributions are drawn from i.i.d uniform distributions.

A.1 Appendix: Derivations

A.1.1 Closed form: mutually exclusive payoff case

Assume at most one asset pays off in a period (mutually exclusive hits). Equivalently, with probability p_k the outcome is $1 + v_k x_k$ (asset k pays), and with probability $1 - \sum_k p_k$ the outcome is 1 (no asset pays). The objective (for investor a, not complexity averse) is

$$\max_{x \in \Delta^3} \sum_{k=1}^{3} p_k \log(1 + v_k x_k)$$

(the constant term $(1 - \sum_k p_k) \log 1$ drops out).

For any active asset k (i.e. $x_k > 0$) the first-order condition is

$$\frac{\partial}{\partial x_k} \sum_j p_j \log(1 + v_j x_j) = \frac{p_k v_k}{1 + v_k x_k} = \lambda,$$

where λ is the Lagrange multiplier for $\sum_k x_k = 1$. Hence

$$x_k = \frac{p_k}{\lambda} - \frac{1}{v_k}, \qquad k \in S,$$

where S denotes the active set. Summing these equations over $k \in S$ and using $\sum_{k \in S} x_k = 1$ gives

$$\frac{\sum_{k \in S} p_k}{\lambda} - \sum_{k \in S} \frac{1}{v_k} = 1 \quad \Rightarrow \quad \lambda = \frac{\sum_{k \in S} p_k}{1 + \sum_{k \in S} \frac{1}{v_k}}$$

and therefore, for $k \in S$,

$$\boxed{x_k = \frac{p_k}{\sum_{j \in S} p_j} \Big(1 + \sum_{j \in S} \tfrac{1}{v_j}\Big) - \frac{1}{v_k}.}$$

Procedure to obtain the optimizer: enumerate candidate active sets S (here: $\{1, 2, 3\}$, all 2-subsets, singletons). For each S compute λ by the boxed formula and the corresponding x_k. Accept the candidate S if all computed $x_k \geq 0$ for $k \in S$ (and $x_k = 0$ for $k \notin S$). One of these feasible candidates is the global maximizer because the problem is concave.

A.2 Appendix: Simulations

We simulate portfolio allocation choices under log-utility preferences with convex complexity costs.

Model setup. There are $n = 6$ assets, each with an associated payoff v_i and success probability p_i. Payoffs v_i are drawn independently from a uniform distribution on $[1, 5]$. Probabilities p_i are drawn independently from a uniform distribution on $[0.05, 0.2]$. An investor chooses an allocation vector (shares across assets), given a capital level k. Expected log-utility is defined as:

$$EU(x) = \sum_{i=1}^{n} p_i \log(1 + v_i x_i k) + \Big(1 - \sum_{i=1}^{n} p_i\Big) \log(1).$$

When the investor is not complexity-averse ($\theta = 0$), agents can freely choose any portfolio $x \in \Phi(A) = k * \Delta(A)$. However, when the investor is complexity-averse ($\theta > 0$), the feasible set shrinks to a restricted subset $\Phi(A) \subseteq k * \Delta(A)$. The effective utility is also reduced by a cost term:

$$g(|A_i|, \theta) = \theta(|A_i| - 1)^2,$$

where $|A_i|$ denotes the number of distinct assets chosen with positive allocation. This penalty captures aversion to using menus of greater size. The overall objective is then:

$$\max_{x \in \Phi(A)} \; EU(x) - g(|A_i|, \theta).$$

Implementation of the feasible set restriction. Let $\Delta(A) = \{x \in \mathbb{R}^n_+ : \sum_{i=1}^n x_i = 1\}$ denote the full simplex. When $\theta = 0$ (no complexity aversion), the agent solves

$$\max_{x \in \Delta(A)} \mathbb{E}[\log(1 + \sum_i v_i x_i k \cdot \mathbf{1}\{\text{asset } i \text{ hits}\})],$$

implemented via sequential least squares quadratic programming with linear equality and box constraints.

When $\theta > 0$, the feasible set is a proper subset $\Phi(A) \subset \Delta(A)$ constructed as follows. We enumerate all menus (subsets) of cardinality $r = 1, \ldots, n$. For a given menu of size r, if $r \leq 2$ we allow continuous optimization of weights on that menu; if $r > 2$ we constrain to the equal-weight allocation $x_i = 1/r$ on the menu. For each menu, we compute the expected log utility and subtract the complexity cost $g(r, \theta) = \theta(r-1)^2$. We then select the menu, A_i, and its within-menu allocation, x_i, that maximizes utility net of the complexity cost.

Procedure. For each specification of θ and capital k, we solve the optimization numerically to obtain the optimal allocation x^*. By varying θ and k, we trace out how complexity aversion alters risk-taking, menu size, and responsiveness to payoff perturbations. These simulations generate the predictions and figures reported in the main text.