Estimate of Earth's Orbital Velocity

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1 Introduction

Using the formula for orbital velocity and Kepler's Third Law, I estimate the orbital velocity of the Earth around the Sun to be about 30 km/s. This theoretical estimate agrees with the numerical velocity pulled using SPICE.

2 Orbital Velocity

We use the orbital velocity equation in Carroll and Ostlie (equation 2.36, page 48).

$$v^2 = G(m_1 + m_2)\left(\frac{2}{r} - \frac{1}{a}\right)$$

We make a couple simplifying assumptions because this is a loose estimate. First, we assume that a circular orbit for the Earth around the Sun is good enough. This means, we assume the semi-major axis is the radius of the circular orbit, i.e $r = a$. Second, we use the fact that the mass of the Sun is much greater than the mass of the Earth. In other words, $M_\odot = m_1 >> m_2$. Therefore

$$v^2 = \frac{GM_\odot}{r}$$

Knowing that Kepler's third law reduces to unity when using AU, we derive the value of G. Kepler's third law is

$$P^2 = \frac{4pi^2a^3}{G(m_1 + m_2)}$$

where P is the orbital period and a is the semi-major axis. Using AU, we know the equation is

$$P^2 = a^3$$

which implies

$$1 = \frac{4pi^2}{G}$$

canceling the mass of the sun because it defines one solar mass. By dimensional analysis, this 1 has units $yr^2 M_\odot AU^{-3}$ Rearranging terms, we see

$$G = 4\pi^2$$

with units $AU^3 M_\odot^{-1} yr^{-2}$.

Using this value for G in the orbital velocity equation and $r = 1AU$, we see

$$v^2 = \frac{GM_\odot}{r} = 4\pi^2$$

This means that $v \sim 6$AU/yr. Referencing that $1AU \sim 1.5 * 10^1 1m$ and $1yr \sim 3 * 10^7 s$, we conclude that

$$v \sim 6AU/yr \sim 30000m/s \sim 30m/s$$