Electromagnetic Cloaking using Meta-materials

Aditya Krishna Rao,
Deptt. of Electrical Engineering, IIT Kanpur
Email: adikrao@iitk.ac.in

I. INTRODUCTION

Electromagnetic cloaking is a phenomenon that has historically gained interest in literature and science, often referred to as "invisibility". From providing a feasible scientific explanation in "The Invisible Man" By H.G. Wells to the Invisibility Cloak in Happy Potter series, the idea is gaining momentum in the scientific community. Cloaking has major applications beyond fantasy, from stealth technology in contemporary military vehicles to electromagnetic shielding in deep space exploration objects.

Several methods have been introduced to attain partial/ complete invisibility for several types of objects. These can be broadly classified into the following: Scattering Method, Coordinate Transformation Method Linear Transmission Method.

As a part of this project, we focus on the **Coordinate Transformation Method**. This method has a sound mathematical foundation and only attains limitations in the physical realisation of those results. First, we develop a generalised mathematical algorithm to attain the idea of electromagnetic cloaking for various geometrical structures. We then illustrate the same using a few simple structures for physical realisation like the following Annular coordinate transformation, Concentric meta-material for Cloaking.

II. COORDINATE TRANSFORMATION AND INVARIANCE OF MAXWELL'S EQUATION

Transformation Optics is a powerful tool manipulate flow of electromagnetic waves and design various architecture. To execute electromagnetic cloaking, we first try to understand underlying principles of transformational optics.

A. Coordinate Transformation

Let the following equations define any general transformation between two sets of coordinate systems $\{x_i\}$ and $\{x_i'\}$,

$$x_1' = U(x_1, x_2, x_3) \tag{1}$$
$$x_2' = V(x_1, x_2, x_3) \tag{2}$$
$$x_3' = W(x_1, x_2, x_3) \tag{3}$$

We define a Jacobian matrix for this transformation given by

$$J = \begin{bmatrix} \frac{h_1'}{h_1}\frac{\partial x_1'}{\partial x_1} & \frac{h_1'}{h_2}\frac{\partial x_1'}{\partial x_2} & \frac{h_1'}{h_3}\frac{\partial x_1'}{\partial x_3} \\ \frac{h_2'}{h_1}\frac{\partial x_2'}{\partial x_1} & \frac{h_2'}{h_2}\frac{\partial x_2'}{\partial x_2} & \frac{h_2'}{h_3}\frac{\partial x_2'}{\partial x_3} \\ \frac{h_3'}{h_1}\frac{\partial x_3'}{\partial x_1} & \frac{h_3'}{h_2}\frac{\partial x_3'}{\partial x_2} & \frac{h_3'}{h_3}\frac{\partial x_3'}{\partial x_3} \end{bmatrix} \tag{4}$$

The above expression is in generalised form for transformation between either of cartesian, cylindrical and spherical coordinates. Table (I) lists appropriate values of h/h.

Table I

Coordinates	h_1	h_2	h_3
(x,y,s)	1	1	1
(ρ, θ, s)	1	ρ	1
(r, θ, ϕ)	1	1	$r \sin\theta$

Under these transformation, the Maxwell's equation transform in an organised manner.

B. Invariance of Maxwell Equations

Under the above transformation, the Maxwell equations take a similar form given as in normal coordinates. A vector field ($\vec{F}$) and an operator/tensor ($\hat{O}$) transforms as following,

$$\vec{F}'(x') = (J^T)^{-1} F\vec{(x)} \tag{5}$$
$$\hat{O}'(x') = \frac{J\hat{O}J^T}{det(J)} \tag{6}$$

A detailed derivation can be found here. Thus we can write permittivity and permeability tensors as following,

$$\epsilon' = \frac{J\epsilon J^T}{det(J)} \tag{7}$$
$$\mu' = \frac{J\mu J^T}{det(J)} \tag{8}$$

In a nutshell, shifting of all points in real space according to the coordinate transformation is equivalent to substituting isotropic medium ϵ and μ with ϵ' and μ'. We will illustrate this shortly using an example.

III. CYLINDRICAL TRANSFORMATION AND CLOAKING

Consider a circular region of radius a. We wish to transform and shift all points inside the circle outside a. Consider another circle of radius b as in Fig. (1) and the following transformation in cylindrical coordinates.

$$\rho' = a + \frac{b-a}{b}\rho \; ; \; \rho < b \tag{9}$$
$$\theta' = \theta \tag{10}$$
$$s' = s \tag{11}$$

The above transformation effectively squeezes every point inside $\rho < b$ to the annulus between $a < \rho < b$ without affecting anything outside. Fig. (2) shows transformation to

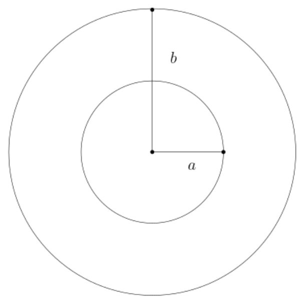
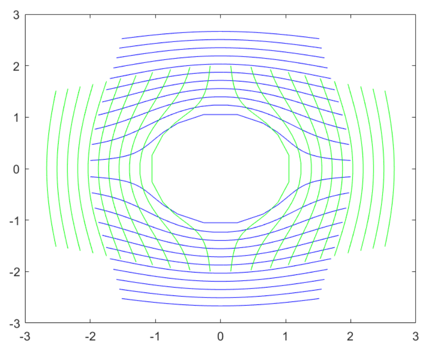


Figure 1. Cylinder and Annulus



a bunch of parallel lines under this. Thus, a parallel beam of light in $\epsilon = 1$ and $\mu = 1$ will **bend** according to this transformation if substituted by the transformed ϵ' and μ' tensors leaving a void. Since, points at $\rho > b$ remains untransformed, the parallel beam will traverse unaffected. On computing the Jacobian matrix, the transformed permittivity and permeability come out as,

$$\epsilon_r = \mu_r = \frac{r' - a}{r'} \tag{12}$$

$$\epsilon_\theta = \mu_\theta = \frac{r'}{r' - a} \tag{13}$$

$$\epsilon_z = \mu_z = \left(\frac{b}{b - a}\right)^2 \frac{r' - a}{r'} \tag{14}$$

A material having these permittivity tensor will effectively bend light around the circular object to achieve perfect cloaking. An observer on the receiving end will see a beam of perfectly parallel wave-fronts effectively cloaking the object completely independent of the material type present inside $\rho < a$.

A. *Simulation Results*

The following simulation on *COMSOL Multiphysics 6.1* illustrates the same. The material permittivity and permeability of the cloak were accurately modelled using continuous functions instead of using standard materials in library at $f = 19.21$ Ghz.

Remark: An important observation is that the above transformation is independent of frequency of source. The same cloak works for the entire electromagnetic spectrum given

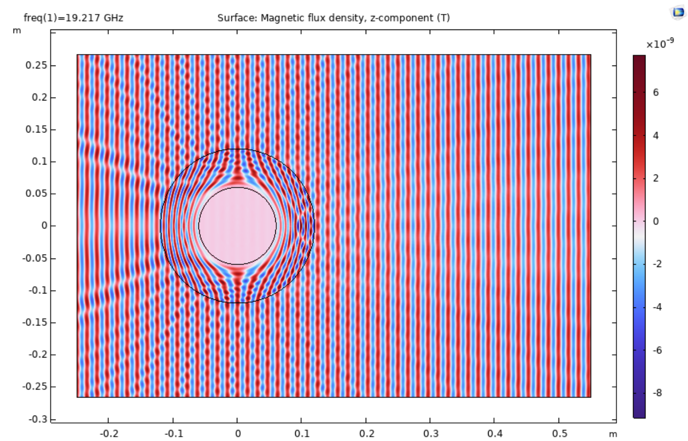


Figure 2. Perfect Cloaking for conducting core

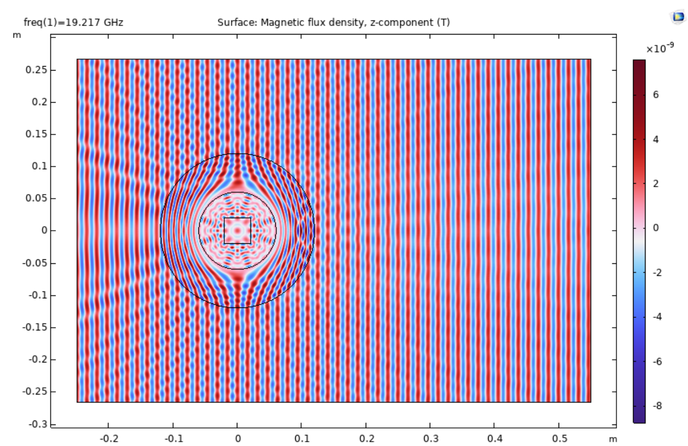


Figure 3. Perfect Cloaking for Silicon-Air core

its material properties remains unaffected by all frequencies (an idealistic assumption). The following simulation were for $f = 9.4993$ Ghz.

The above method works perfectly in achieving electromagnetic cloaking. For materials with different shapes, we need to find an appropriate coordinate transformation that may even be a piece-wise continuous function. The underlying idea is that as long as a connected region in R^3 remains a connected region on transformation, we can use that transformation. But an issue with this method is the resulting permittivity and permeability tensors are anisotropic and spatially varying. Such materials don't exist naturally and need to designed artificially. And hence we need meta-structures.

IV. Anisotropic behaviour from Isotropic materials

To realise the above permittivity tensor given by (12)-(13), we need a radially and azimuthally varying permittivity. Consider the following linear bi-layer structure with permittivities ϵ_1, ϵ_2 and a relative thickness of η.

The effective permittivity in the two directions come out to

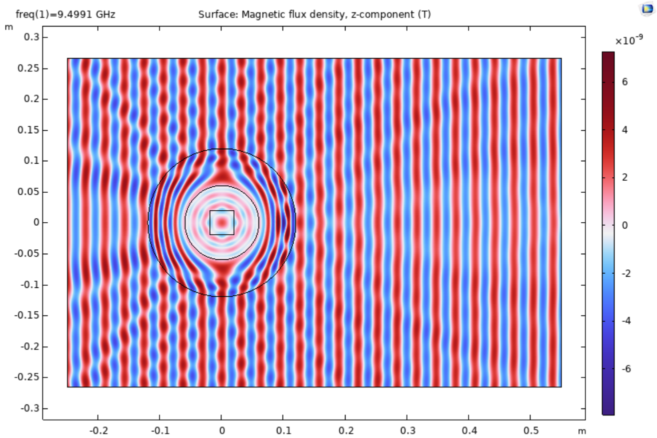


Figure 4. Perfect Cloaking at f=9.4993 Ghz

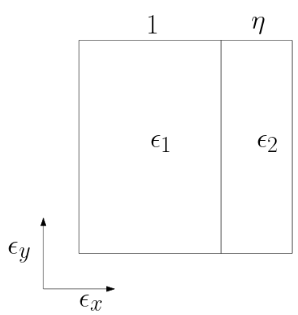


Figure 5. Bi-layer Dielectric

be

$$\frac{1+\eta}{\epsilon_x} = \frac{1}{\epsilon_1} + \frac{\eta}{\epsilon_2} \tag{15}$$

$$\epsilon_y = \frac{\epsilon_1 + \eta\epsilon_2}{1+\eta} \tag{16}$$

If we reconstruct this bi-layer in a circular geometry, we get $\epsilon_r = \epsilon_x$ and $\epsilon_\theta = \epsilon_y$ (Fig.(6)). The assumption taken here is that the thickness of bi-layer is much smaller than the radius of curvature.

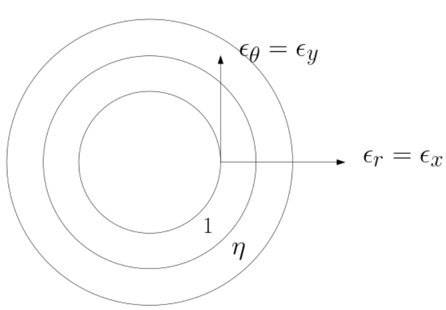


Figure 6. Concentric Meta-structure

A. Simulation Results

To realise this, we take an inner circle of radius $a = 60mm$ and aouter circle of radius $b = 120mm$. We discretise the width of $60mm$ into 600 layers each of thickness $0.1mm$ and compute the desired $\{\epsilon_\theta, \epsilon_y\}$ for each layer using the transformed permittivity tensor (12)-(14). For each layer, we model the desired $\{\epsilon_\theta, \epsilon_r\}$ using the above bi-layer decomposition and find the necessary $\{\epsilon_1, \epsilon_2\}$ for some η. We choose $\eta = 1$ throughout for simplicity of the construction. The effective meta-structure thus computed has alternating $\{\epsilon_1, \epsilon_2\}$ with the former varying from $1.4e-6$ to 0.5345 and the later varying from 8 to 7.4645. The resulting simulation are shown below. We do see scattering to some extent but realise a parallel

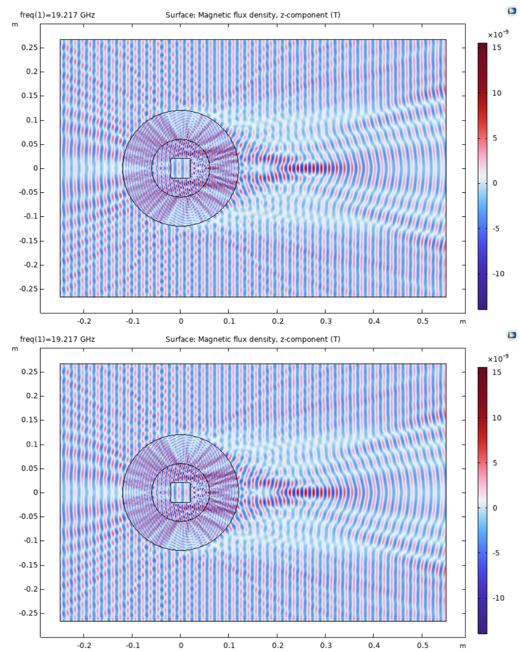


Figure 7. Concentric Meta-structure Simulation

wavefront at sufficient. The more discretisation is achieved the better will the cloak mimic an ideal structure.

B. Varying thickness instead of permittivity

In the above section, we had a range of ϵ that we realised having constant thickness. Alternatively we can choose a specific $\{\epsilon_1, \epsilon_2\}$ and instead vary thickness. Since (15)-(16) are not simultaneously solvable for η, we take 3-layer meta structure whose effective permittivity can be given by

$$\frac{1+a+b}{\epsilon_x} = \frac{1}{\epsilon_1} + \frac{a}{\epsilon_2} + \frac{b}{\epsilon_3} \tag{17}$$

$$\epsilon_y = \frac{\epsilon_1 + a\epsilon_2 + b\epsilon_3}{1+a+b} \tag{18}$$

V. Complex Transformation

In the previous sections, we saw how coordinate system transformation gives rise to anisotropic permittivity and permeability tensors. Simultaneously, realising both of them is a challenging task as we have to make several assumption to enforce such a material. But complex transformation allows us to solve this problem.

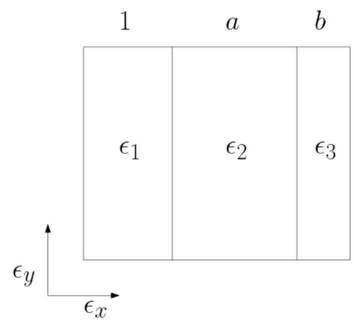


Figure 8. 3-layer Meta-structure

In R^2 space, let us work in Complex coordinates instead of Cartesian coordinates. Every point is represented as $z = x + iy = (x, y)$. Consequently, we can define a few operators as following,

$$2\partial z = \partial x - i\partial y \tag{19}$$

$$2\partial z^* = \partial x + i\partial y \tag{20}$$

$$4\partial z\partial z^* = \partial^2 x + \partial^2 y \tag{21}$$

where (21) represents the Laplacian operator and the wave equation can be written as,

$$(4\partial z\partial z^* + n^2 k_o^2)\psi = 0 \tag{22}$$

where $\psi(z)$ is the electric field. We transform these coordinates to a new complex coordinates using $w(z) = u(x, y) + iv(x, y)$. One can easily show the following transformation of Laplacian.

$$4\partial z\partial z^* = 4\partial w\partial w^* \left|\frac{dw}{dz}\right|^2 \tag{23}$$

Thus, the wave equation transforms to,

$$(4\partial w\partial w^* + n'^2 k_o^2)\psi = 0 \tag{24}$$

where $n' = \frac{n}{|dw/dz|}$. Thus, a refractive index profile of

$$n = \left|\frac{dw}{dz}\right| \tag{25}$$

gives us the desired result. Note all we have to care now is about the effective refractive index which can be done by purely concentrating on epsilon tensor.

A. Simulation Results

We take the following transformation into consideration.

$$w(z) = z + \frac{a^2}{z}$$

Fig. (9) shows the above transformation.

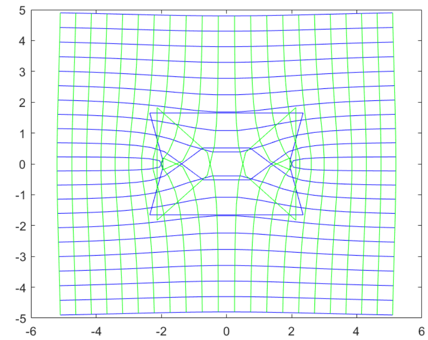


Figure 9. Complex Transformation

On simulating for this transformation, we get the following results.

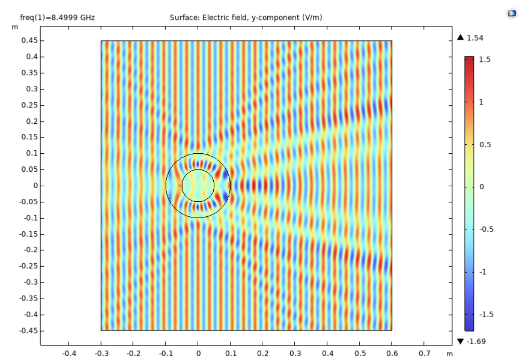


Figure 10. Cloaking due Complex Transformation

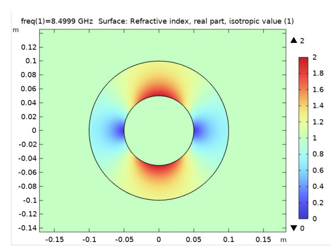


Figure 11. $n(z) = \left|1 - \frac{a^2}{z^2}\right|$

VI. Conclusion

We saw Transformation Optics can play a powerful method in designing efficient cloaking materials. Challenges regarding realisation of such complex material were addressed using some simple meta-structure topologies. The idea of achieving anisotropic behaviour from isotropic material will always remains an interesting problem. Further, we saw how complex transformation gives us a simpler way to design cloaking devices.

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