Title

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INTRODUCTION

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MAGNETIC FLUCTUATION INDUCED DOUBLE SUPERCONDUCTING TRANSITIONS IN UPt_3

The observation of two superconducting transitions in zero applied field by specific heat measurements[citation needed] has made UPt_3 the most widely studied heavy fermion superconductor. The material shows signatures of antiferromagnetic correlations in neutron scattering studies [citation needed]. The ordering in UPt_3 is characterized by the wave vectors $\mathbf{Q}_1=\frac{1}{2}\mathbf{a}^*$, $\mathbf{Q}_2=\frac{1}{2}(\mathbf{b}^*\text{-}\mathbf{a}^*)$, $\mathbf{Q}_3=\text{-}\frac{1}{2}\mathbf{b}^*$. However signatures of antiferromagnetic order are not seen in other studies like muon spin relaxation (μsr), specific heat, and magnetization[citation needed]. The starting point for a phenomenological model to account for the double superconducting transition is the Ginzburg-Landau functional for the E representations of D_{6h} in zero field [?][?][?] check references and see if i remove one.

$$f_{sc} = \alpha\eta_i\eta_i^* + \beta_1\left(\eta_i\eta_i^*\right)^2 + \beta_2|\eta_i\eta_i^*|^2 \tag{1}$$

To the free energy the coupling to antiferromagnetism is added. The magnetic order is described by $\mathbf{M} = \mathrm{m}_1\mathbf{M}_1 + \mathrm{m}_2\mathbf{M}_2 + \mathrm{m}_3\mathbf{M}_3$, where $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{M}_3$ order with wave vector $\mathbf{Q}_1$, $\mathbf{Q}_2$, $\mathbf{Q}_3$. The superconducting order parameter couples to translationally invariant product representation of (m_1,m_2,m_3). The coupling is given as [?]

$$\begin{aligned} f_{sc-m} =& K_1(m_1^2+m_2^2+m_3^2)(|\eta_1|^2+|\eta_2|^2) \\ &+K_2\left[(2m_1^2-m_2^2-m_3^2)(|\eta_1|^2-|\eta_2|^2)+\sqrt{3}(m_3^2-m_2^2)(\eta_1\eta_2^*+\eta_2\eta_1^*)\right] \end{aligned} \tag{2}$$

It is assumed in these models that there is a single Q magnetic order with $m_1 = \mathbf{M}$ and $m_2 = m_3 = 0$. Thus the first term of (2) just modifies α while the second term leads to a symmetry breaking field term.

$$f_{SBF} = \gamma M^2(|\eta_1|^2-|\eta_2|^2) \tag{3}$$

This symmetry breaking allows us the possibility of two transition where with the choice of $\beta_2 > 0$, the systems first transitions into the real A phase and then later into the time reversal broken B phase at lower temperature. However the experimental signatures of such a symmetry breaking field seem absent [citation needed](include triple Q order). This raises the question of whether there is an alternate mechanismalso what about 1d accidental degeneracy to generate these two transitions in zero field without the presence of a symmetry breaking field. We here show that coupling the superconducting order parameter to magnetic fluctuations allows us the possibility of two transitions

GINZBURG - LANDAU WITH MAGNETIC FLUCTUATION

We here assume that there is no symmetry breaking field and hence there is no single Q which orders. The superconductor is coupled to antiferromagnetic fluctuations and these fluctuations energetically favor a different ground state as favored by the true superconducting state. Thus it is these fluctuations which provide an intrinsic mechanism for two transitions in UPt_3. We start with the partition function of the system which includes f_{sc} and f_{sc-m}.

$$\begin{aligned} \mathcal{Z} &= \int D\eta_i Dm_i e^{-\beta H} \\ \beta H[\eta_i, m_j] &= \int d^3x\left(\alpha\eta_i\eta_i^* + \beta_1\left(\eta_i\eta_i^*\right)^2 + \beta_2|\eta_i\eta_i^*|^2 + Am_j^2 + K_1(m_1^2+m_2^2+m_3^2)(|\eta_1|^2+|\eta_2|^2)\right. \\ &\left.+K_2\left[(2m_1^2-m_2^2-m_3^2)(|\eta_1|^2-|\eta_2|^2)+\sqrt{3}(m_3^2-m_2^2)(\eta_1\eta_2^*+\eta_1^*\eta_2)\right]\right) \end{aligned} \tag{4}$$

We see that our magnetic fluctuations m_i are diagonal in the x basis and and are uniform i.e. no $m_i(x)m_i(x')$ terms appear and hence we will be able to integrate these fluctuations (i.e do the Gaussian integral for each $\int Dm_i$ separately) out to give us a new effective free energy. see if i can write this better

$$\begin{aligned}
\mathcal{Z}_m &= \int dm_1\, exp\left(-m_1^2\left(A + K_1(|\eta_1|^2+|\eta_2|^2) + 2K_2(|\eta_1|^2-|\eta_2|^2)\right)\right)\int dm_2\, exp\left(-m_2^2\left(A+K_1(|\eta_1|^2+|\eta_2|^2)\right.\right.\\
&\left.\left.-K_2(|\eta_1|^2-|\eta_2|^2)-\sqrt{3}K_2(\eta_1\eta_2^*+\eta_1^*\eta_2)\right)\right)\int dm_3\, exp\left(-m_3^2\left(A+K_1(|\eta_1|^2+|\eta_2|^2)-K_2(|\eta_1|^2-|\eta_2|^2)\right.\right.\\
&\left.\left.+\sqrt{3}K_2(\eta_1\eta_2^*+\eta_1^*\eta_2)\right)\right)\\
&= \sqrt{\frac{\pi}{A+K_1(|\eta_1|^2+|\eta_2|^2)+2K_2(|\eta_1|^2-|\eta_2|^2)}}\sqrt{\frac{\pi}{A+K_1(|\eta_1|^2+|\eta_2|^2)-K_2(|\eta_1|^2-|\eta_2|^2)-\sqrt{3}K_2(\eta_1\eta_2^*+\eta_1^*\eta_2)}}\\
&\times\sqrt{\frac{\pi}{A+K_1(|\eta_1|^2+|\eta_2|^2)-K_2(|\eta_1|^2-|\eta_2|^2)+\sqrt{3}K_2(\eta_1\eta_2^*+\eta_1^*\eta_2)}}\\
&= e^{\frac{1}{2}\ln\left(\frac{\pi}{A+K_1(|\eta_1|^2+|\eta_2|^2)+2K_2(|\eta_1|^2-|\eta_2|^2)}\right)}e^{\frac{1}{2}\ln\left(\frac{\pi}{A+K_1(|\eta_1|^2+|\eta_2|^2)-K_2(|\eta_1|^2-|\eta_2|^2)-\sqrt{3}K_2(\eta_1\eta_2^*+\eta_1^*\eta_2)}\right)}\\
&\times e^{\frac{1}{2}\ln\left(\frac{\pi}{A+K_1(|\eta_1|^2+|\eta_2|^2)-K_2(|\eta_1|^2-|\eta_2|^2)+\sqrt{3}K_2(\eta_1\eta_2^*+\eta_1^*\eta_2)}\right)}\\
&= e^{\left(\frac{3}{2}\ln(\pi)-\frac{3}{2}\ln(A)\right)}e^{-\frac{1}{2}\ln\left(1+\frac{K_1}{A}(|\eta_1|^2+|\eta_2|^2)+2\frac{K_2}{A}(|\eta_1|^2-|\eta_2|^2)\right)}e^{-\frac{1}{2}\ln\left(1+\frac{K_1}{A}(|\eta_1|^2+|\eta_2|^2)-\frac{K_2}{A}(|\eta_1|^2-|\eta_2|^2)-\sqrt{3}\frac{K_2}{A}(\eta_1\eta_2^*+\eta_1^*\eta_2)\right)}\\
&\times e^{-\frac{1}{2}\ln\left(1+\frac{K_1}{A}(|\eta_1|^2+|\eta_2|^2)-\frac{K_2}{A}(|\eta_1|^2-|\eta_2|^2)+\sqrt{3}\frac{K_2}{A}(\eta_1\eta_2^*+\eta_1^*\eta_2)\right)} \qquad (5)
\end{aligned}$$

We will now expand the logarithmic terms which are dependent on η and then we will get a effective free energy.

$$\begin{aligned}
\beta H &= \int d^3x f\\
f &= f_{sc} + \frac{1}{2}\left(\ln\left(1+\frac{K_1}{A}(|\eta_1|^2+|\eta_2|^2)+2\frac{K_2}{A}(|\eta_1|^2-|\eta_2|^2)\right)\right.\\
&+\ln\left(1+\frac{K_1}{A}(|\eta_1|^2+|\eta_2|^2)-\frac{K_2}{A}(|\eta_1|^2-|\eta_2|^2)-\sqrt{3}\frac{K_2}{A}(\eta_1\eta_2^*+\eta_1^*\eta_2)\right)\\
&\left.+\ln\left(1+\frac{K_1}{A}(|\eta_1|^2+|\eta_2|^2)-\frac{K_2}{A}(|\eta_1|^2-|\eta_2|^2)+\sqrt{3}\frac{K_2}{A}(\eta_1\eta_2^*+\eta_1^*\eta_2)\right)\right) \qquad (6)
\end{aligned}$$

We now expand this to order η^4 and throw away the constant terms.

$$\begin{aligned}
f &= f_{sc} + \frac{1}{2}\left(\frac{K_1}{A}(|\eta_1|^2+|\eta_2|^2)+2\frac{K_2}{A}(|\eta_1|^2-|\eta_2|^2)\right) - \frac{1}{4}\left(\frac{K_1}{A}(|\eta_1|^2+|\eta_2|^2)+2\frac{K_2}{A}(|\eta_1|^2-|\eta_2|^2)\right)^2\\
&+\frac{1}{2}\left(\frac{K_1}{A}(|\eta_1|^2+|\eta_2|^2)-\frac{K_2}{A}(|\eta_1|^2-|\eta_2|^2)-\sqrt{3}\frac{K_2}{A}(\eta_1\eta_2^*+\eta_1^*\eta_2)\right)-\frac{1}{4}\left(\frac{K_1}{A}(|\eta_1|^2+|\eta_2|^2)-\frac{K_2}{A}(|\eta_1|^2-|\eta_2|^2)\right.\\
&\left.-\sqrt{3}\frac{K_2}{A}(\eta_1\eta_2^*+\eta_1^*\eta_2)\right)^2+\frac{1}{2}\left(\frac{K_1}{A}(|\eta_1|^2+|\eta_2|^2)-\frac{K_2}{A}(|\eta_1|^2-|\eta_2|^2)+\sqrt{3}\frac{K_2}{A}(\eta_1\eta_2^*+\eta_1^*\eta_2)\right)\\
&-\frac{1}{4}\left(\frac{K_1}{A}(|\eta_1|^2+|\eta_2|^2)-\frac{K_2}{A}(|\eta_1|^2-|\eta_2|^2)+\sqrt{3}\frac{K_2}{A}(\eta_1\eta_2^*+\eta_1^*\eta_2)\right)^2+\mathcal{O}(\eta^6) \qquad (7)
\end{aligned}$$

After further simplification we get the following effective free energy

$$f = f_{sc} + \frac{3}{2}\frac{K_1}{A}(|\eta_1|^2+|\eta_2|^2) - \frac{3}{4}\left(\frac{K_1}{A}\right)^2(|\eta_1|^2+|\eta_2|^2)^2 - \frac{6}{4}\left(\frac{K_2}{A}\right)^2(|\eta_1|^2-|\eta_2|^2) \qquad (8)$$

Thus we see that we get the following effective free energy

$$f_{ef} = \left(\alpha + \frac{3}{2}\frac{K_1}{A}\right)(|\eta_1|^2+|\eta_2|^2) + \left(\beta_1 - \frac{3}{4}\left(\frac{K_1}{A}\right)^2\right)(|\eta_1|^2+|\eta_2|^2)^2 + \left(\beta_2 - \frac{6}{4}\left(\frac{K_2}{A}\right)^2\right)(|\eta_1|^2-|\eta_2|^2) \qquad (9)$$

It is clear the magnetic fluctuations can change the sign of the β_2 term which will allow the possibility of a different ground state. The principal idea is that that at higher temperature the coefficient is ($|\eta_1|^2 - |\eta_2|^2 < 0$) and the real state is energetically favored. While at lower temperature the true superconducting ground state exists with that coefficient can become positive and transition in the the broken time reversal state.

SPIN FLUCTUATION FEEDBACK EFFECT

We see that in magnetic fluctuations can be treated by phenomenological approach, where the fluctuations favor as different ground state as compared to the true superconducting ground state. We now apply the same phenomenological method to superfluid He3 to capture the celebrated microscopic Spin-Fluctuation feedback effect. The anisoptropic superfluid phases of liquid He3 was the first example of unconventional pairing. Due the repulsion between the atoms it was realized that pairing would take place in a non zero angular momentum channel. It was seen that the cooper pair formed in superfluid He3 had $L = 1$ and $S = 1$, i.e. it was a p-wave spin triplet. The order parameter $d_{i\alpha}$ is therefore a 3×3 matrix with each element being complex. The i index is the spin index and the α is the spin index. The pairing in this material is mediated by spin fluctuations which arises from the virtual spin polarization of the material(these are also know as virtual paramagnons). It is this spin polarization which favors spin triplet pairing and suppresses spin singlet pairing. The phase diagram of ^{3}He Note that here unlike (5), we do not have a coupling that is diagonal. To be able to integrate out each of the magnetic fluctuations we use the following identity

$$\int_{-\infty}^{\infty} e^{-\frac{1}{2}\sum_{i,j=1}^{n} A_{ij}x_i x_j} d^n x = \sqrt{\frac{(2\pi)^n}{DetA}} \tag{10}$$

where for us $x_i = m_x, m_y, m_z$ and from the form of the F_{sc-m} coupling we see that the matrix A_{ij} for us will be the following

$$A_{ij} = \begin{pmatrix} A_1 + A_2 d_{ab}d^*_{ab} + K_1 d_{xc}d^*_{xc} & \frac{1}{2}K_1\left(d_{xd}d^*_{yd} + d^*_{xe}d_{ye}\right) & \frac{1}{2}K_1\left(d_{xf}d^*_{zf} + d^*_{xg}d_{zg}\right) \\ \frac{1}{2}K_1\left(d_{xh}d^*_{yh} + d^*_{xi}d_{yi}\right) & A_1 + A_2 d_{jk}d^*_{jk} + K_1 d_{yl}d^*_{yl} & \frac{1}{2}K_1\left(d_{ym}d^*_{zm} + d^*_{yn}d_{zn}\right) \\ \frac{1}{2}K_1\left(d_{xo}d^*_{zo} + d^*_{xp}d_{pn}\right) & \frac{1}{2}K_1\left(d_{yq}d^*_{zq} + d^*_{yr}d_{zr}\right) & A_1 + A_2 d_{st}d^*_{st} + K_1 d_{xu}d^*_{xu} \end{pmatrix} \tag{11}$$

where the repeated indices are summed over. The determinant of this matrix is then, given as

$$DetA = \tag{12}$$