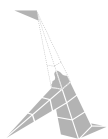


FOUNDATION



Cost of misusing types

```
case class Country(value: String)  
  
val UK      = Country("United Kingdom")  
val France  = Country("France")  
val Germany = Country("Germany")
```



Cost of misusing types

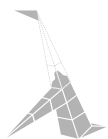
```
case class Country(value: String)

val UK      = Country("United Kingdom")
val France  = Country("France")
val Germany = Country("Germany")
```

```
def getCurrency(country: Country): String = ???
```

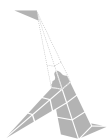
such as

```
getCurrency(Country("United Kingdom")) == "GBP"
getCurrency(Country("France"))         == "EUR"
getCurrency(Country("Germany"))        == "EUR"
```



Cost of misusing types

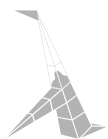
```
def getCurrency(country: Country): Option[String] =  
  country.value match {  
    case "United Kingdom"    => Some("GBP")  
    case "France" | "Germany" => Some("EUR")  
    case _                   => None  
  }
```



Cost of misusing types

```
def getCurrency(country: Country): Option[String] =  
  country.value match {  
    case "United Kingdom"    => Some("GBP")  
    case "France" | "Germany" => Some("EUR")  
    case _                   => None  
  }
```

```
getCurrency(Country("UK"))  
// res0: Option[String] = None  
getCurrency(Country("GBR"))  
// res1: Option[String] = None  
getCurrency(Country("Royaume-Uni"))  
// res2: Option[String] = None
```



Cost of misusing types

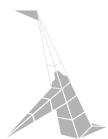
```
sealed trait Country
```

```
object Country {  
  case object UnitedKingdom extends Country  
  case object France        extends Country  
  case object Germany       extends Country  
}
```

```
import Country._
```

```
def getCurrency(country: Country): String =  
  country match {  
    case UnitedKingdom => "GBP"  
    case France | Germany => "EUR"  
  }
```

```
def parseCountry(country: String): Option[Country] = ???
```



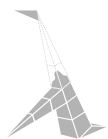
Cost of misusing types

```
sealed trait Country
object Country {
  case object UnitedKingdom extends Country
  case object France        extends Country
  case object Germany       extends Country
}
```

```
sealed trait Currency
object Currency {
  case object BritishPound extends Currency
  case object Euro         extends Currency
}
```

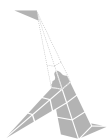
```
import Country._, Currency._

def getCurrency(country: Country): Currency =
  country match {
    case UnitedKingdom => BritishPound
    case France | Germany => Euro
  }
```



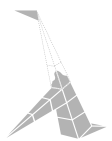
Plan

- What is the cost of misusing types
- How to use ADTs to encode data
- Learn how to measure impact of types and tests
- Explore relationship between types, algebra and logic

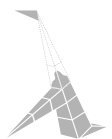


Exercise 1: Misused types

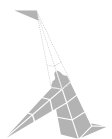
`exercises.types.TypeExercises.scala`



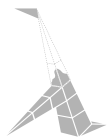
Type should **exactly** fit business requirements



**Imprecise data lead to errors
and misleading documentation**

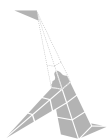


How should we encode data?



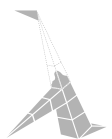
Algebraic Data Type (ADT)

- OR, a ConfigValue is
 - a String OR
 - an Int OR
 - Empty
- AND, a User is
 - an userId AND
 - a name AND
 - an address



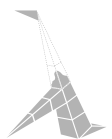
OR

- a Boolean is true **OR** false
- an Int is a -10 **OR** 0 **OR** 1 **OR** ...
- a DayOfTheWeek is Monday **OR** Tuesday **OR** Wednesday **OR** ...
- an Option is a Some **OR** a None
- a Json is a JsonNumber **OR** a JsonString **OR** a JsonArray **OR** a JsonObject **OR** ...



How should we encode OR?

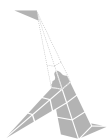
A ConfigValue is a String OR an Int OR Empty



OR Encoding

```
sealed trait ConfigValue

object ConfigValue {
  case class ConfigString(value: String) extends ConfigValue
  case class ConfigNumber(value: Double) extends ConfigValue
  case object ConfigEmpty extends ConfigValue
}
```



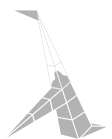
OR Encoding

```
sealed trait ConfigValue

object ConfigValue {
  case class ConfigString(value: String) extends ConfigValue
  case class ConfigNumber(value: Double) extends ConfigValue
  case object ConfigEmpty extends ConfigValue
}
```

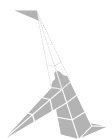
In Scala 3

```
enum ConfigValue {
  case ConfigString(value: String)
  case ConfigNumber(value: Double)
  case ConfigEmpty
}
```



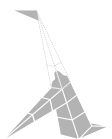
AND

- a User is a userId **AND** a name **AND** an address
- a ZonedDateTime is a dateTime **AND** a timeZone
- a NonEmptyList is a head **AND** a tail
- a Tuple2 is a _1 **AND** a _2



How should we encode AND?

A User is a `userId` AND a `name` AND an `address`



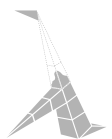
AND Encoding

```
import java.util.UUID

case class User(userId: UUID, name: String, address: Address)

case class Address(streetNumber: Int, streetName: String, postCode: String)
```

```
User(UUID.randomUUID(), "John Doe", Address(108, "Cannon Street", "EC4N 6EU"))
// res5: User = User(
//   f61b09b4-5895-4526-a062-f7eb69cf69,
//   "John Doe",
//   Address(108, "Cannon Street", "EC4N 6EU")
// )
```

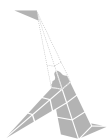


Algebraic data types mix AND and OR

```
sealed trait Role

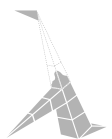
object Role {
  case class Reader(accountId: Long, premiumUser: Boolean) extends Role
  case class Editor(accountId: Long, favoriteFont: Option[String]) extends Role
  case object Admin extends Role
}

case class User(userId: Long, role: Role, address: Option[Address])
```



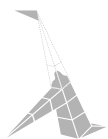
Exercise 2: Data Encoding

`exercises.types.TypeExercises.scala`

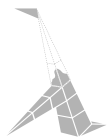


Case class and sealed trait map exactly to business language AND and OR

Nested AND and OR form what is called Algebraic Data Types (ADTs)



ADT allow to encode data precisely



How can we compare two encodings?

```
def getCurrency(country: String): Option[String]
```

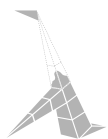
Is it better to reduce input or reduce output?

```
def getCurrency(country: Country): String
```

```
def getCurrency(country: String): Option[Currency]
```

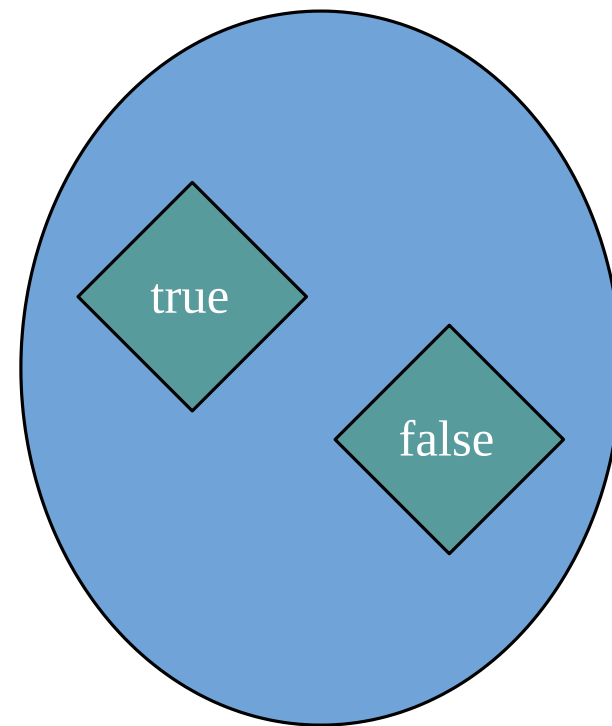
How much better it is to reduce both?

```
def getCurrency(country: Country): Currency
```

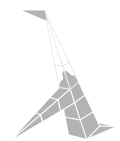
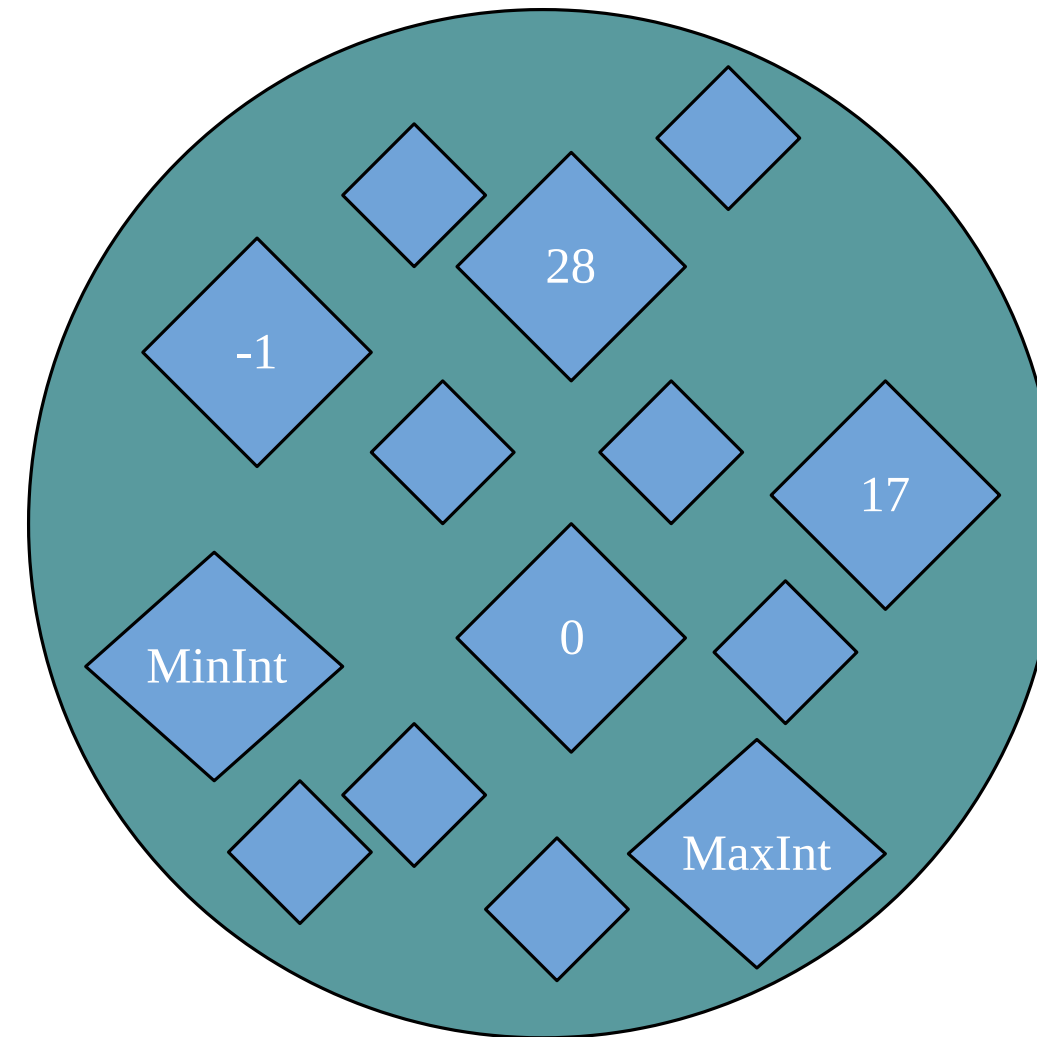


Type is a set

Boolean

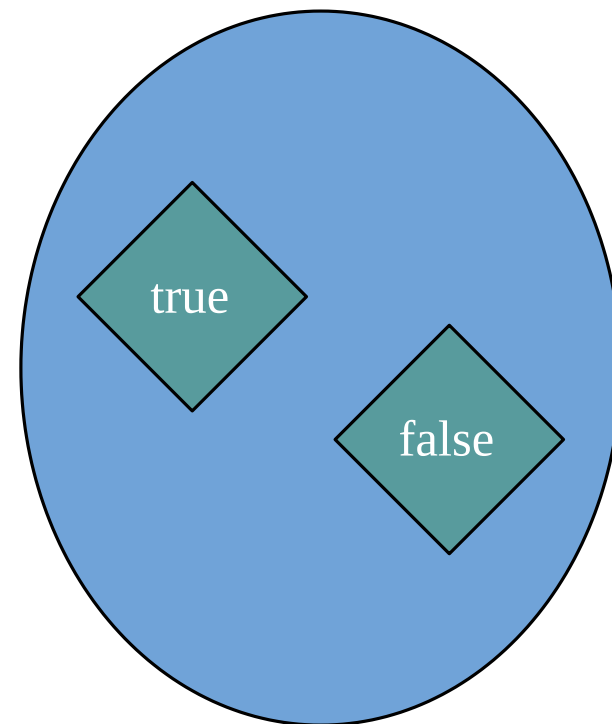


Int

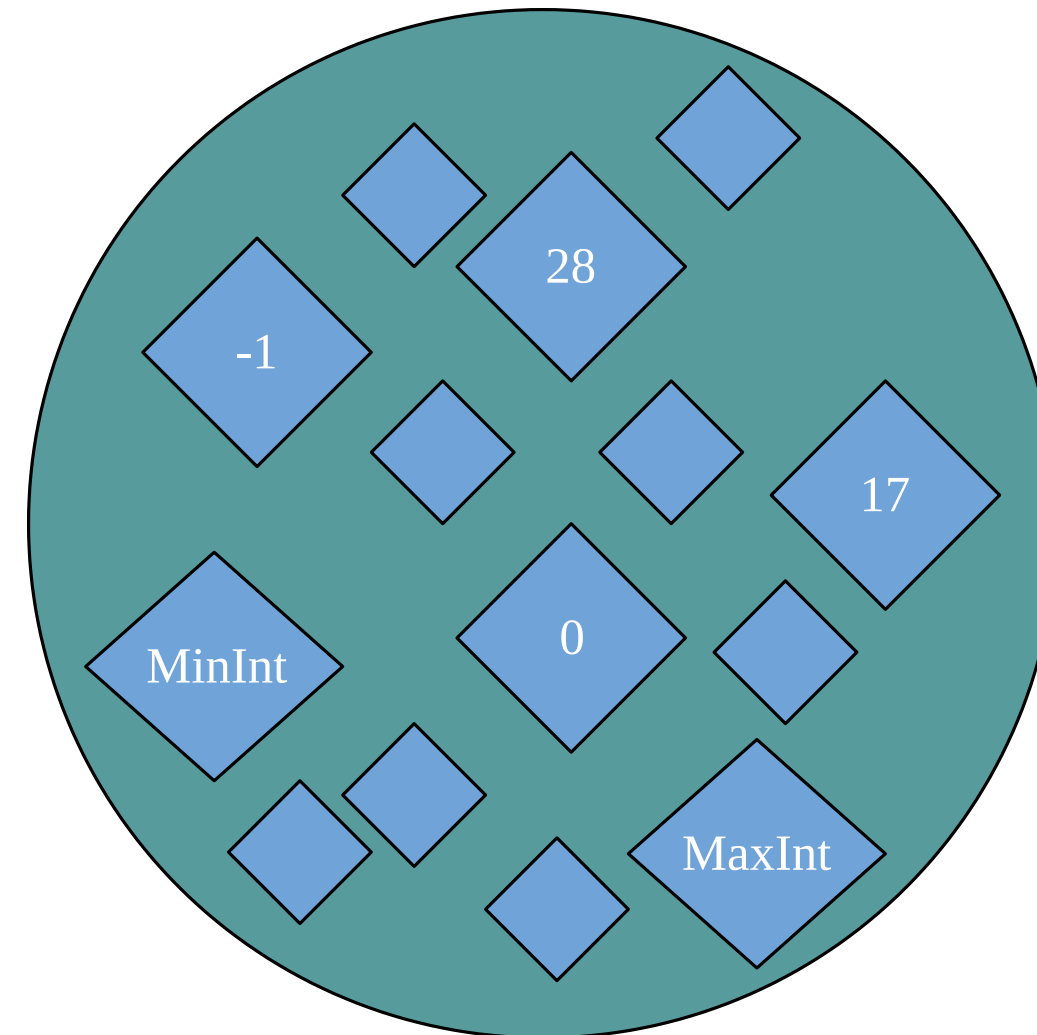


Cardinality

$$|\text{Boolean}| = 2$$

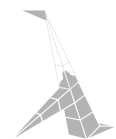


$$|\text{Int}| = 2^{32}$$



Exercise 3: Cardinality

`exercises.types.TypeExercises.scala`



Sealed trait

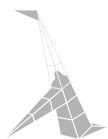
```
sealed trait IntOrBoolean

case class AnInt(value: Int) extends IntOrBoolean
case class ABoolean(value: Boolean) extends IntOrBoolean
```

```
AnInt(Int.MinValue) // ~ -2 billion
...
AnInt(0)
AnInt(1)
...
AnInt(Int.MaxValue) // ~ +2 billion

ABoolean(false)
ABoolean(true)
```

```
|IntOrBoolean| = |AnInt| + |ABoolean|
               = |Int|  + |Boolean|
```



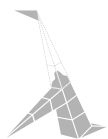
Case class

```
case class IntAndBoolean(i: Int, b: Boolean)
```

```
IntAndBoolean(Int.MinValue, false) // ~ -2 billion
...
IntAndBoolean(0, false)
IntAndBoolean(1, false)
...
IntAndBoolean(Int.MaxValue, false) // ~ +2 billion

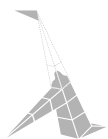
IntAndBoolean(Int.MinValue, true) // ~ -2 billion
...
IntAndBoolean(0, true)
IntAndBoolean(1, true)
...
IntAndBoolean(Int.MaxValue, true) // ~ +2 billion
```

```
|IntAndBoolean| = |Int| * |Boolean|
```



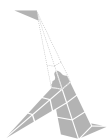
A sealed trait is called a **sum** type

A case class is called a **product** type



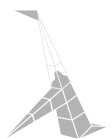
$$|A \text{ OR } B \text{ OR } C| = |A| + |B| + |C|$$

$$|A \text{ AND } B \text{ AND } C| = |A| * |B| * |C|$$

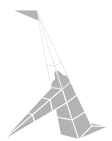
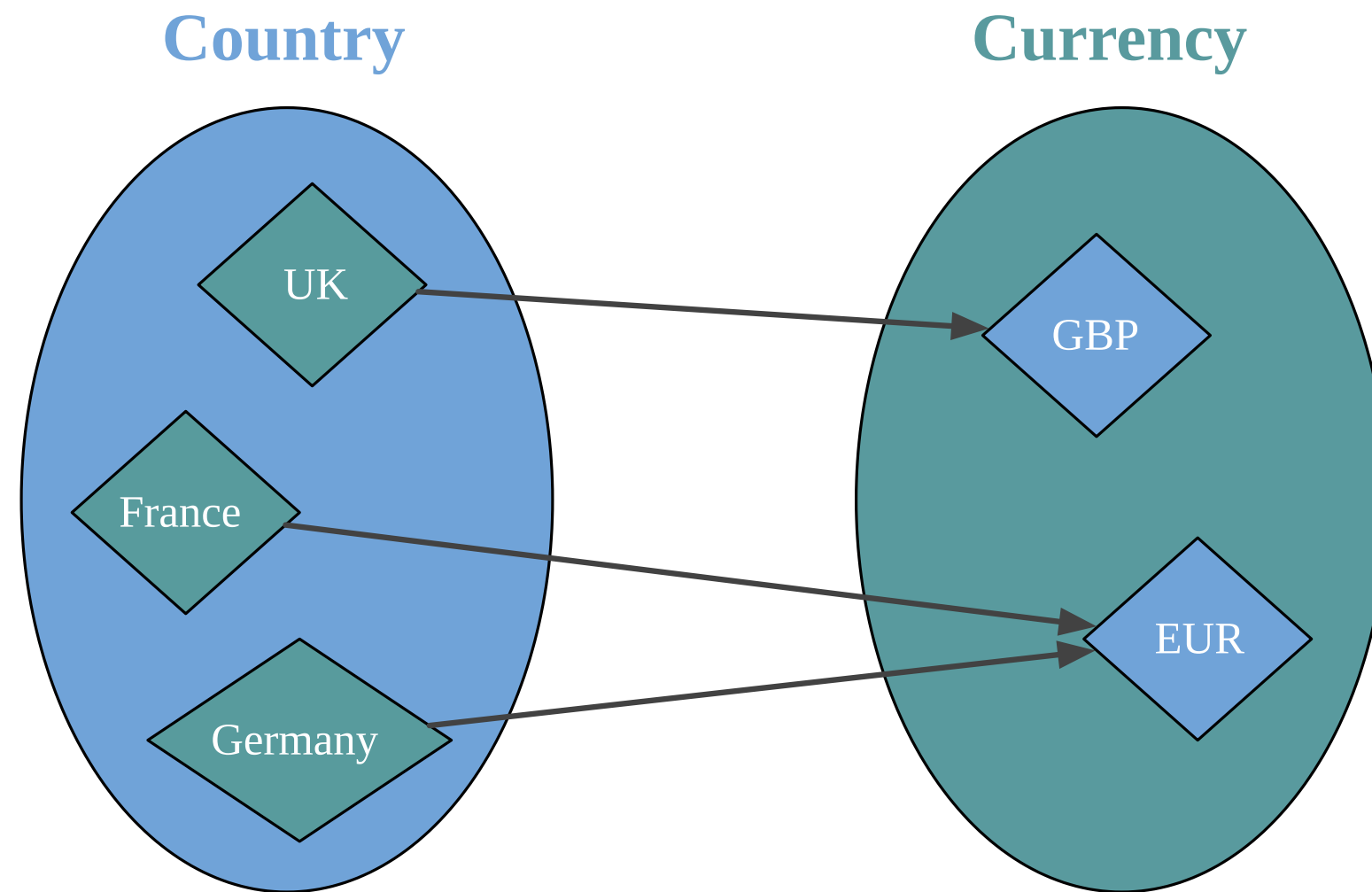


Finish Exercise 3

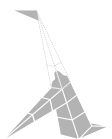
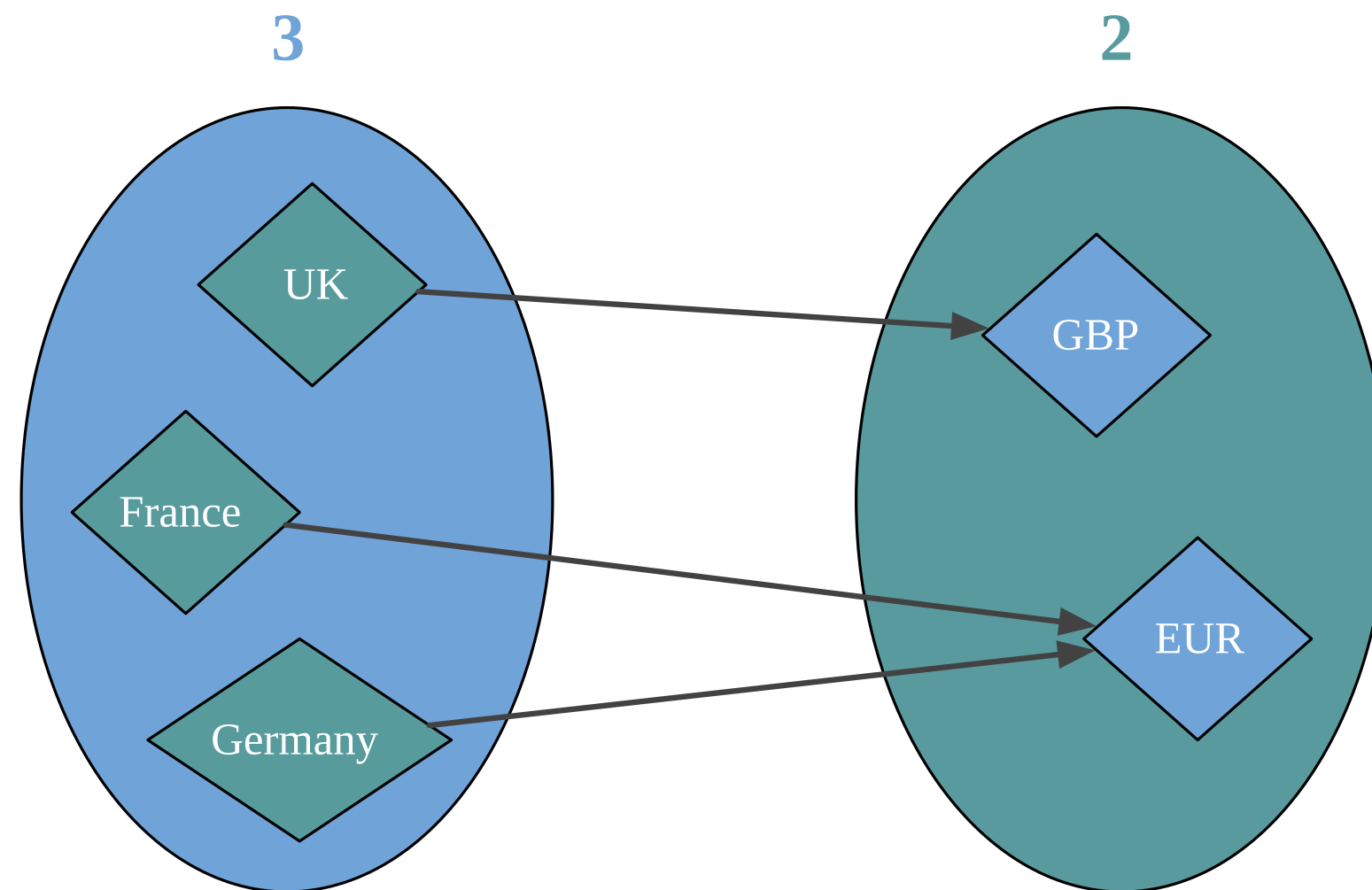
`exercises.types.TypeExercises.scala`



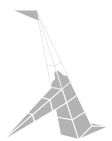
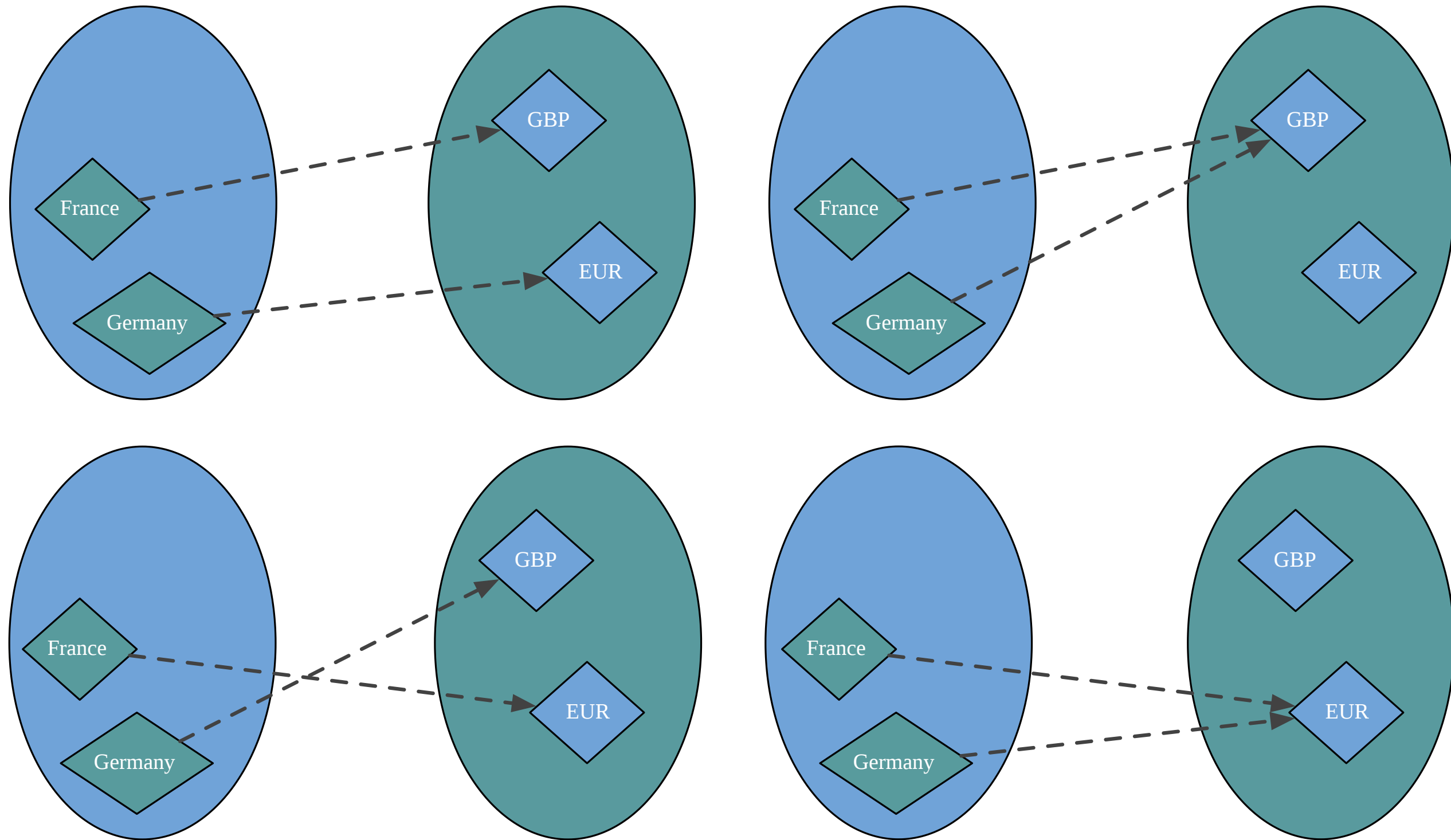
|Country => Currency|



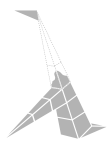
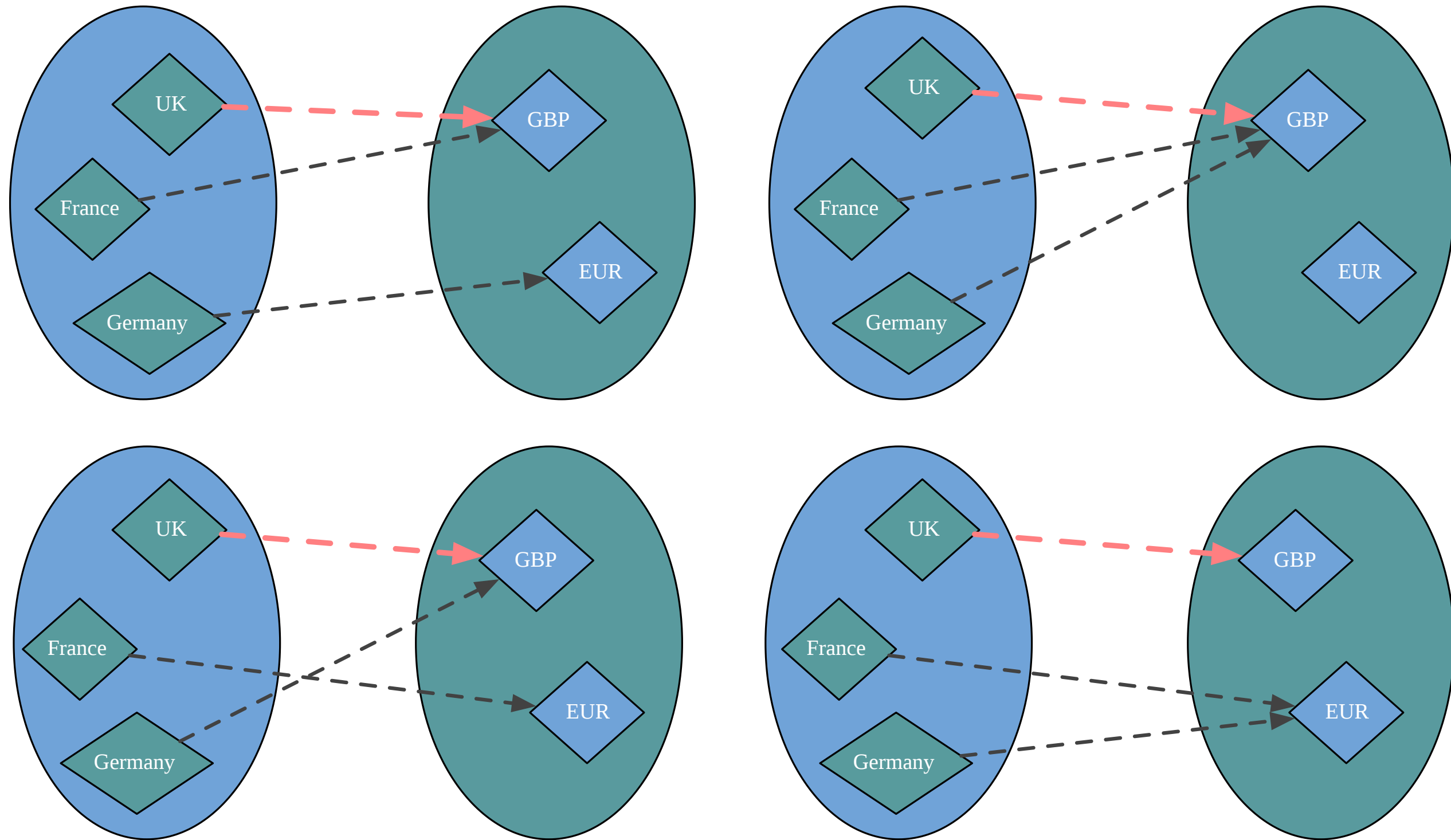
| 3 => 2 |



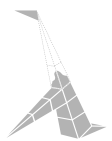
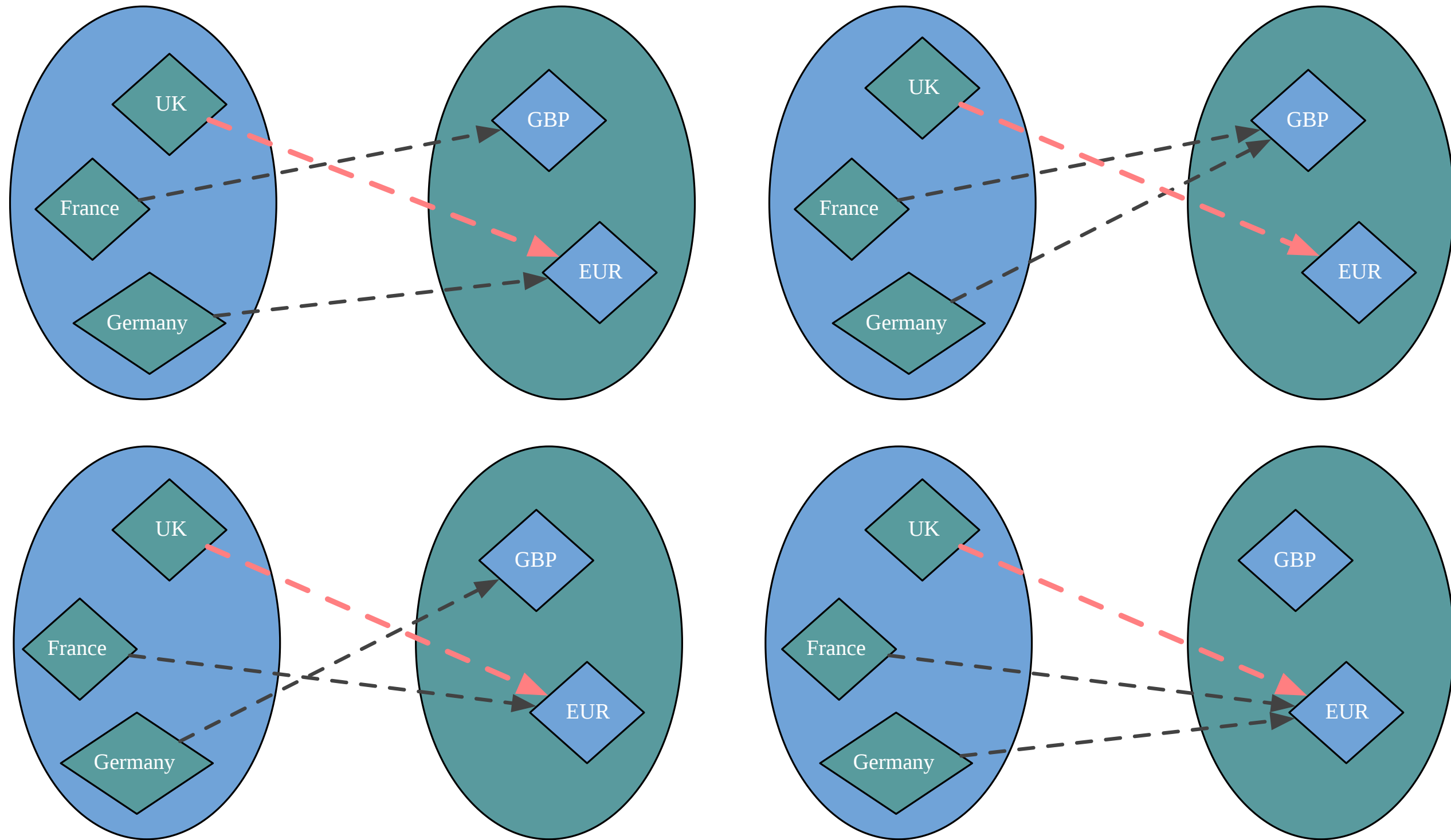
$$|2 \Rightarrow 2| = 4$$



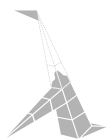
| 3 => 2 |



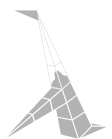
$$|3 \Rightarrow 2| = |2 \Rightarrow 2| + |2 \Rightarrow 2|$$



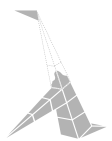
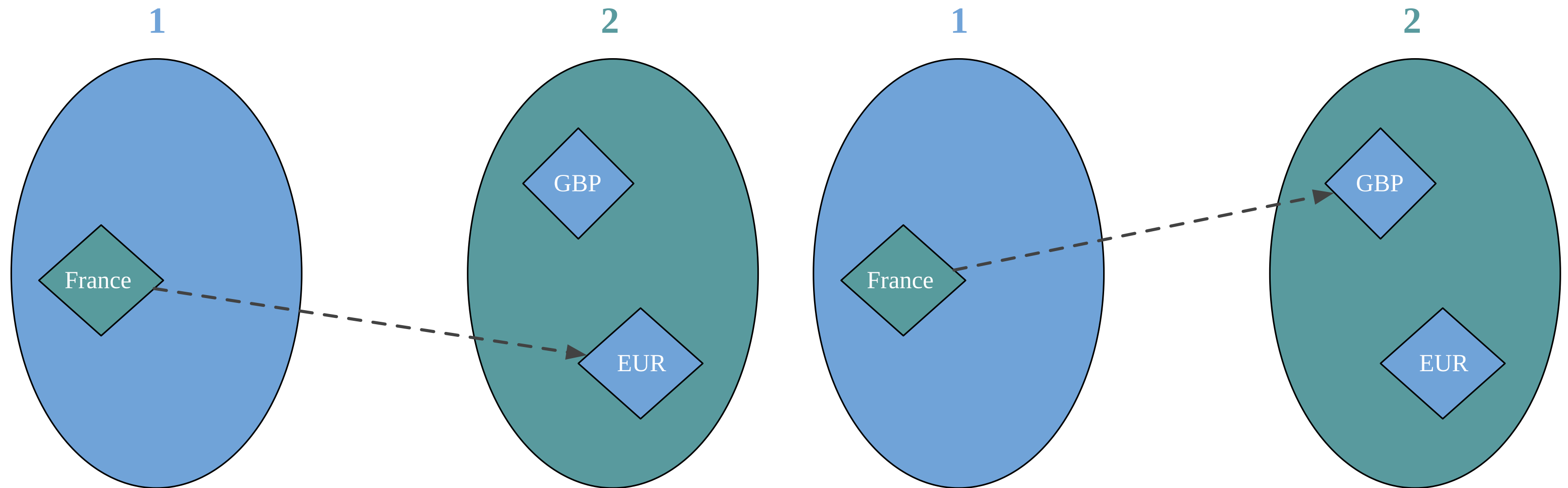
$$|3 \Rightarrow 2| = 2 * |2 \Rightarrow 2|$$



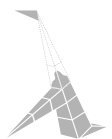
$$\begin{aligned} |3 \Rightarrow 2| &= 2 * |2 \Rightarrow 2| \\ &= 2 * 2 * |1 \Rightarrow 2| \end{aligned}$$



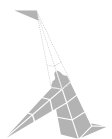
$$|1 \Rightarrow 2| = 2$$



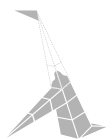
$$\begin{aligned} |3 \Rightarrow 2| &= 2 * |2 \Rightarrow 2| \\ &= 2 * 2 * |1 \Rightarrow 2| \\ &= 2 * 2 * 2 \end{aligned}$$



$$\begin{aligned} |3 \Rightarrow 2| &= 2 * |2 \Rightarrow 2| \\ &= 2 * 2 * |1 \Rightarrow 2| \\ &= 2 * 2 * 2 \\ &= 2 ^ 3 \end{aligned}$$

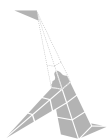


$$|A \Rightarrow B| = |B| \wedge |A|$$

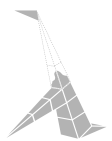
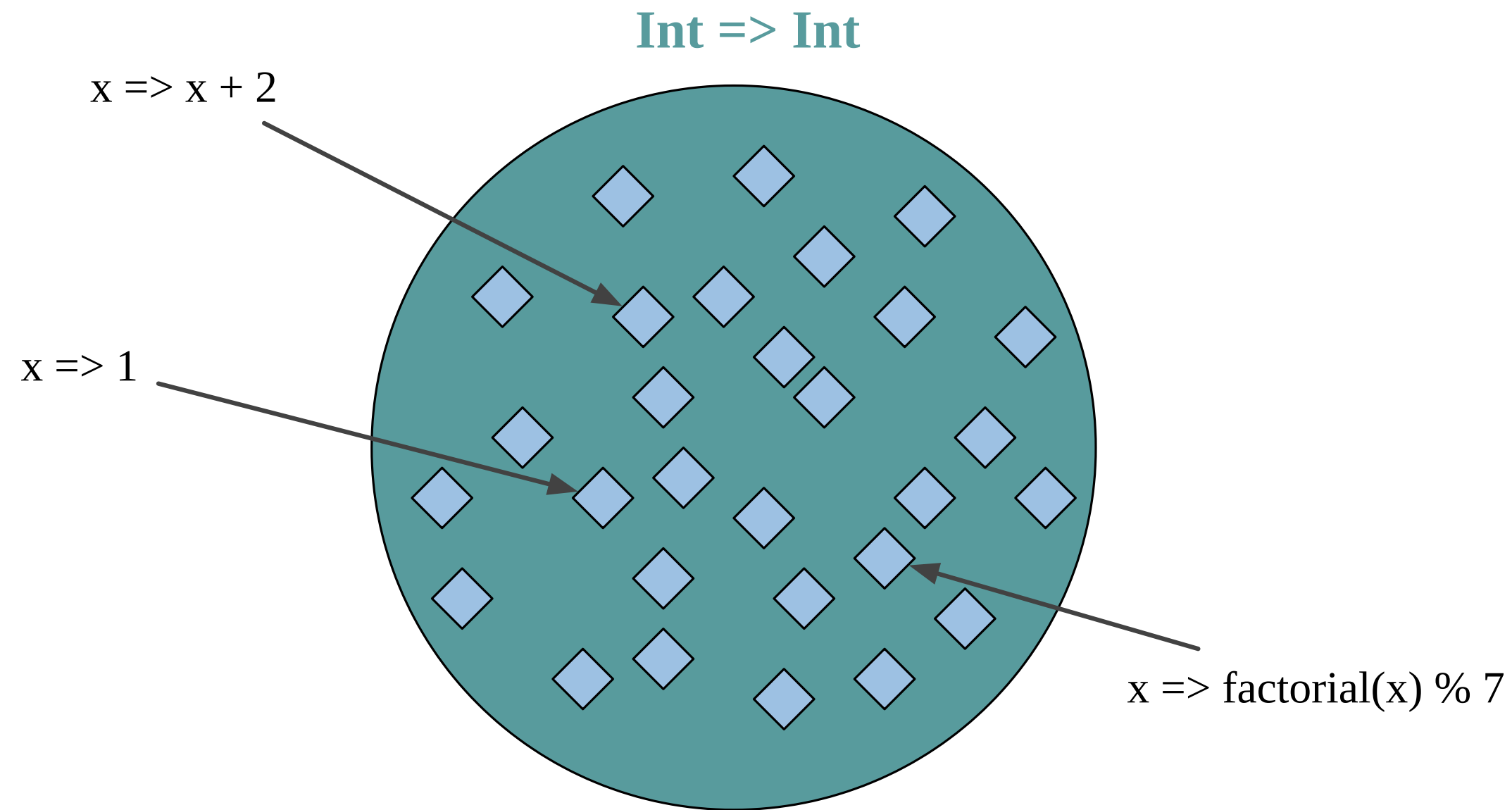


Exercise 4: Parametricity

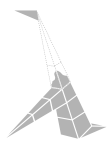
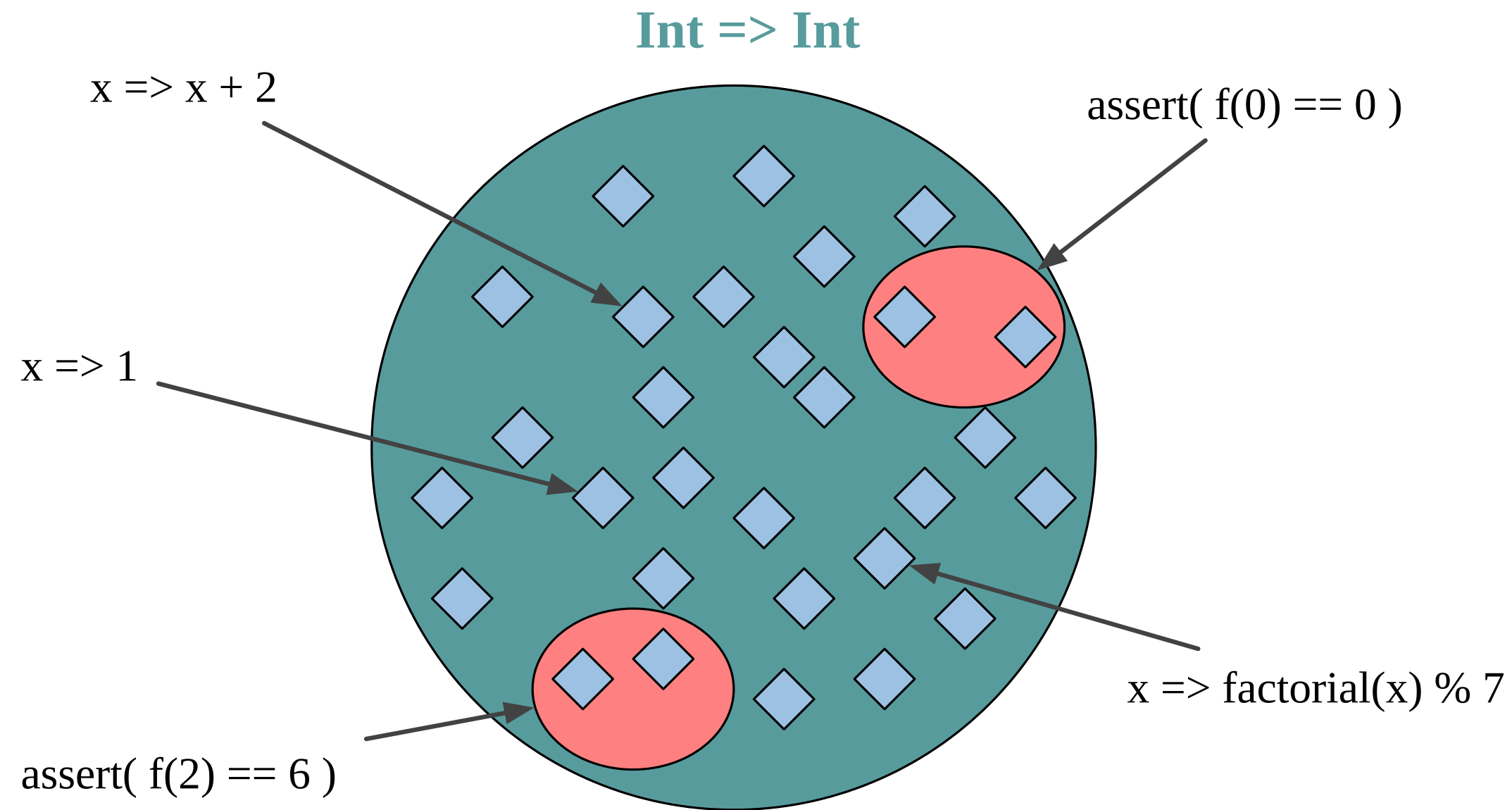
`exercises.types.TypeExercises.scala`



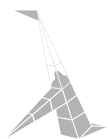
Functions are sets!

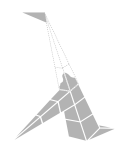


Unit tests



Valid Implementation Count (VIC)

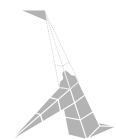
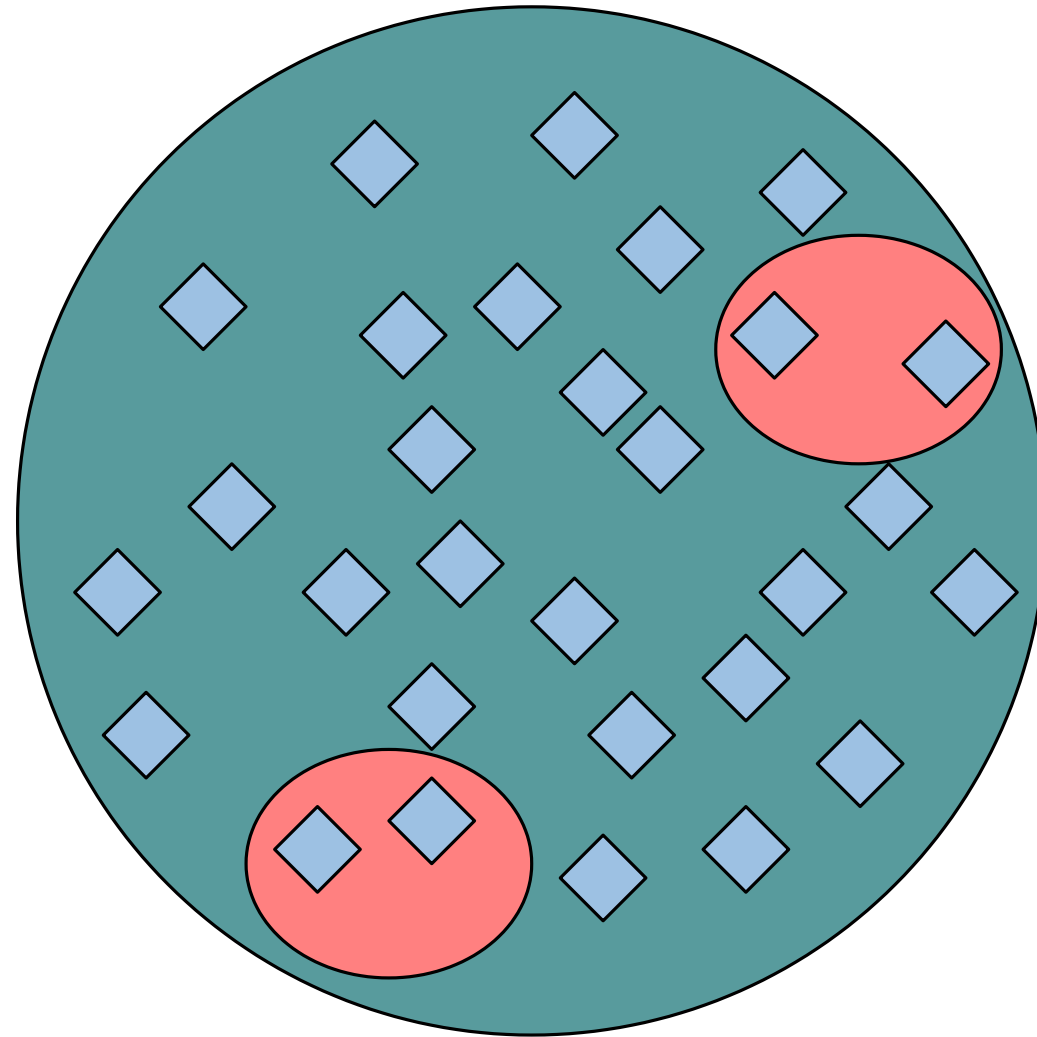




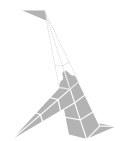
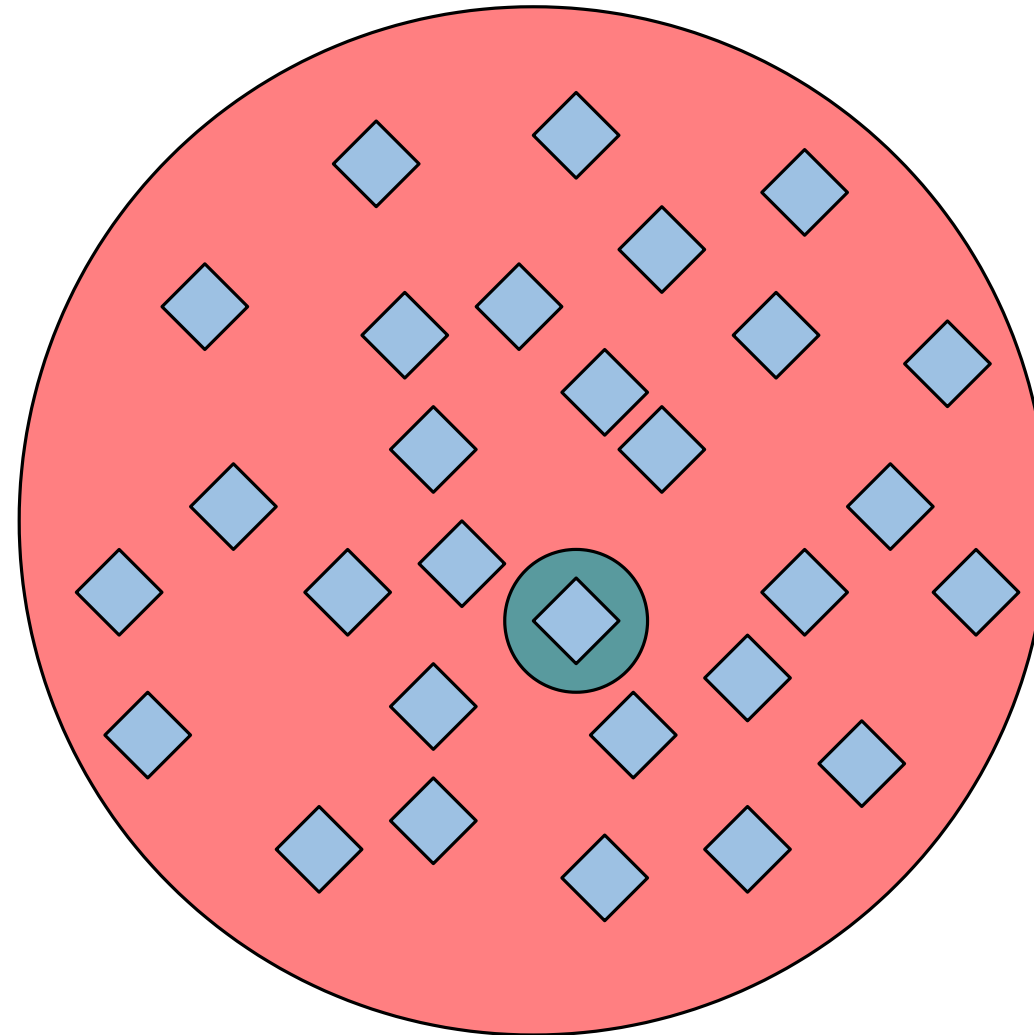
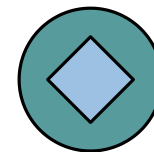
$$|A \Rightarrow B|$$

???

$VIC = \text{blue diamond} - \text{red circle with blue diamond}$

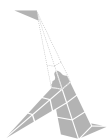


$$\text{VIC}(f) = 1$$



Exercise 5: Tests

`exercises.types.TypeExercises.scala`



Unit Test

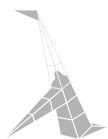
```
sealed trait Country
object Country {
  case object UnitedKingdom extends Country
  case object France         extends Country
  case object Germany        extends Country
}

sealed trait Currency
object Currency {
  case object GBP extends Currency
  case object EUR extends Currency
}

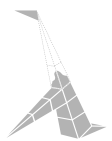
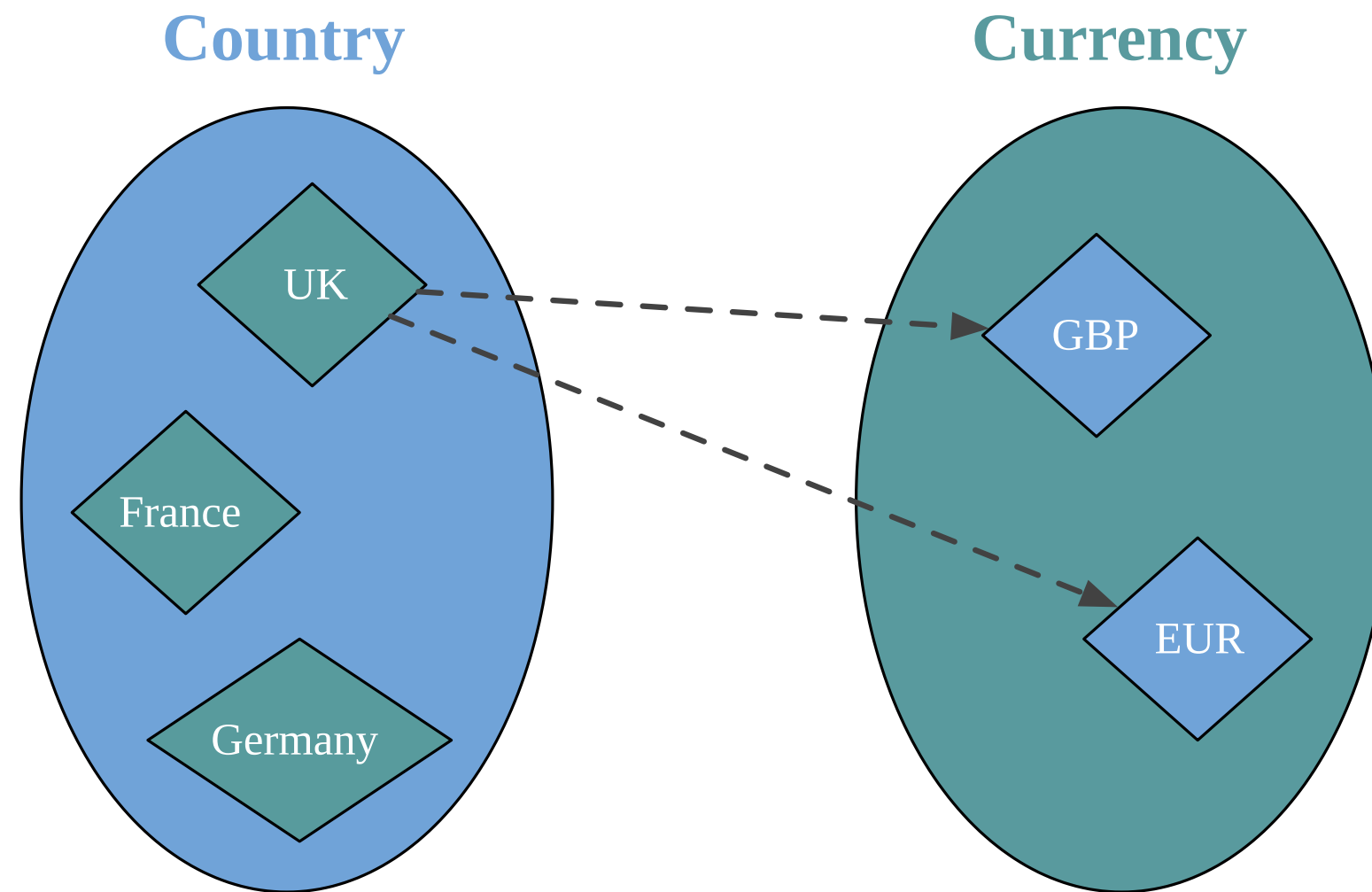
def getCurrency(country: Country): Currency = ???
```

such as

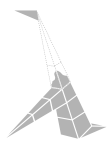
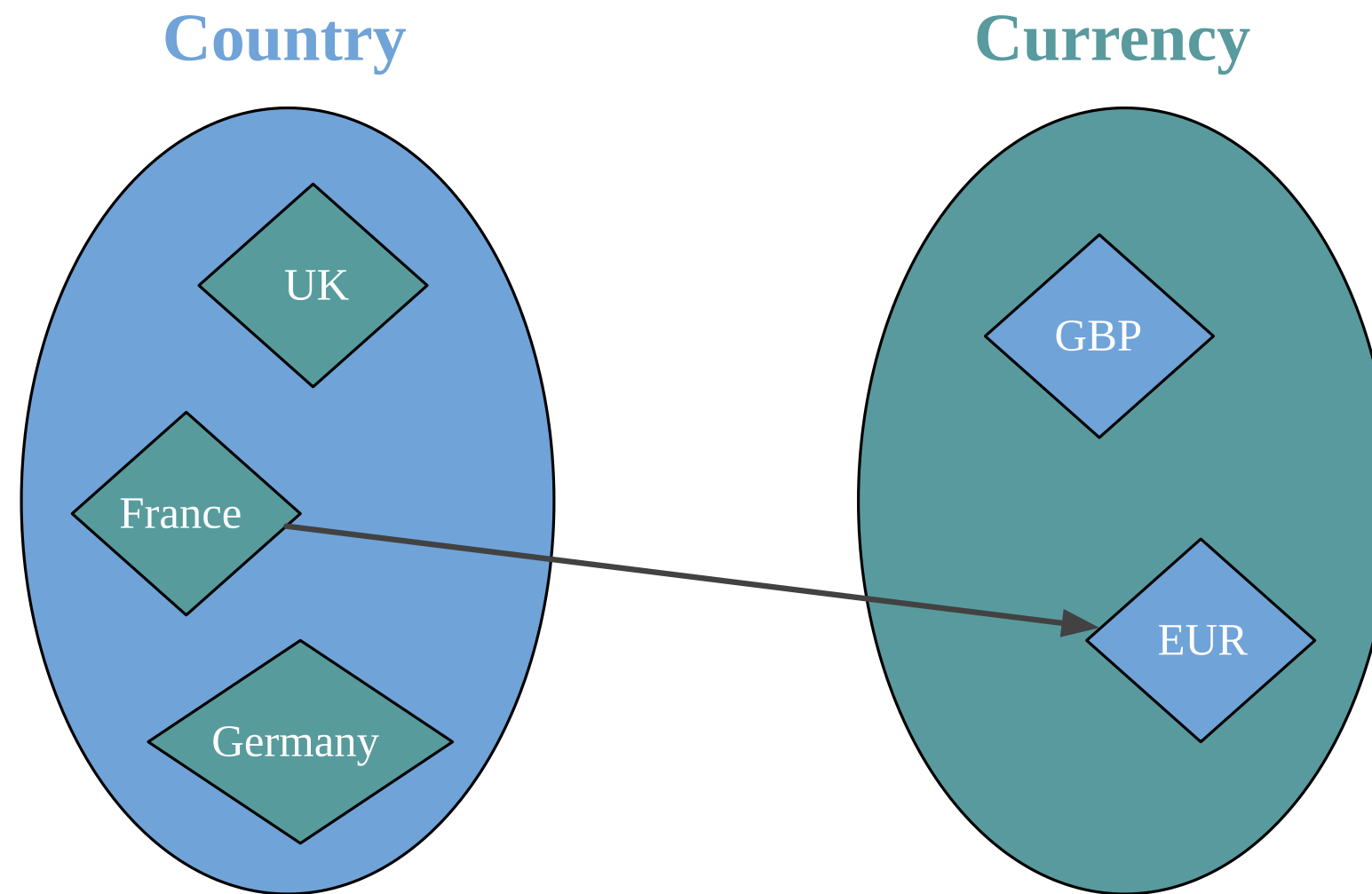
```
assert(getCurrency(France) == EUR)
```



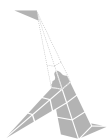
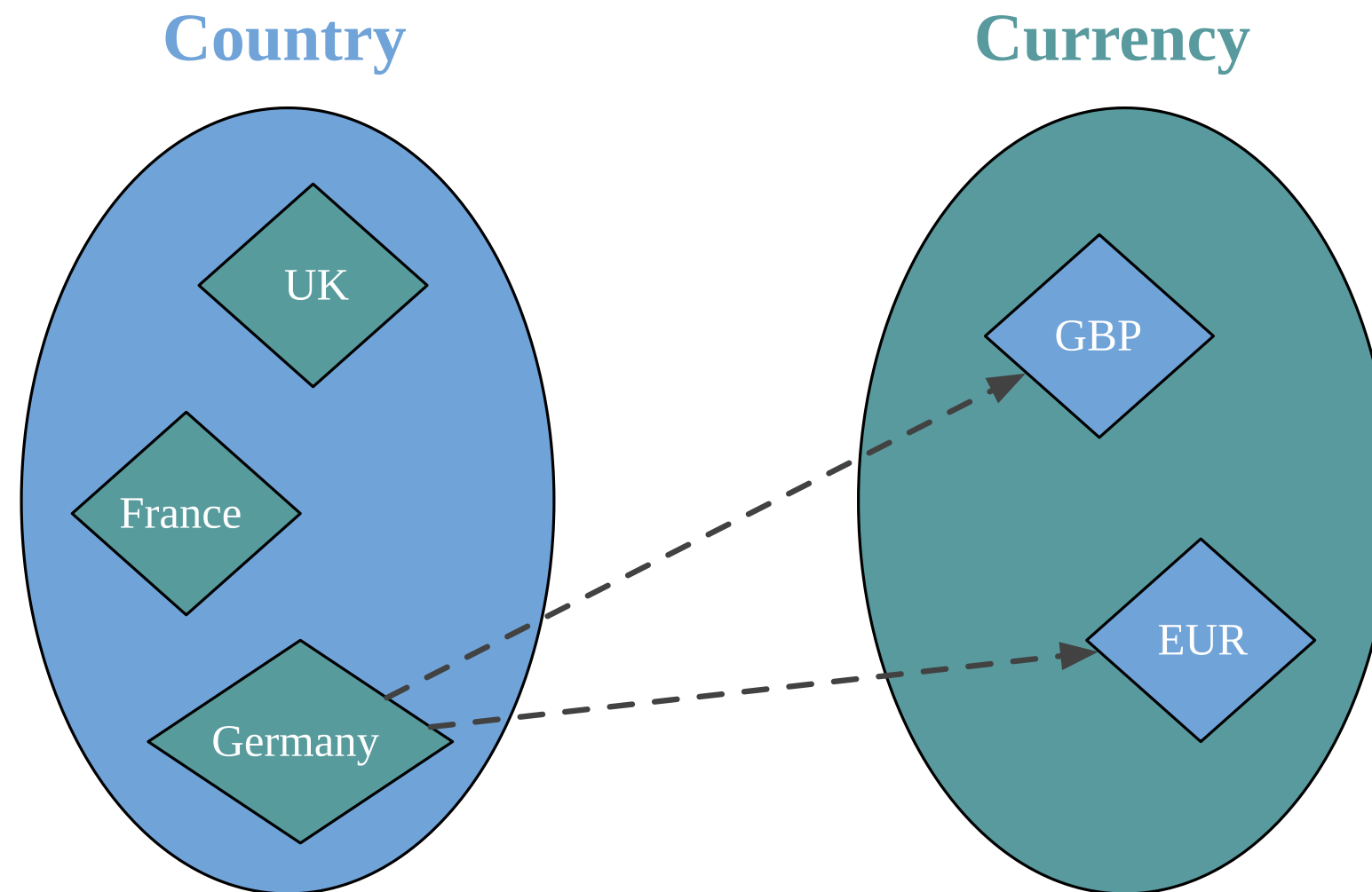
$VIC(\text{getCurrency}) = 2 * \dots$



$VIC(\text{getCurrency}) = 2 * 1 * \dots$

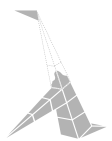


$$VIC(\text{getCurrency}) = 2 * 1 * 2$$



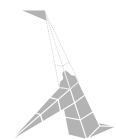
$$VIC(f: A \Rightarrow B) = |B| \wedge (|A| - n)$$

where n is the number of unit tests



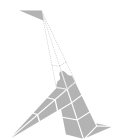
Exercise 6: Type Algebra

`exercises.types.TypeExercises.scala`



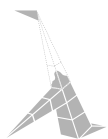
Type Algebra

Type	Algebra
Nothing	0
Unit	1
Either[A, B]	$A + B$
(A, B)	$A * B$
$A \Rightarrow B$	$B \wedge A$
Isomorphism	$A == B$



Curry–Howard isomorphism

[Propositions as types](#) from Philip Wadler

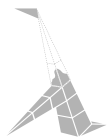


Type Algebra Logic

Type	Algebra	Logic
Nothing	0	\perp
Unit	1	\top
Either[A, B]	$A + B$	$A \vee B$
(A, B)	$A * B$	$A \wedge B$
$A \Rightarrow B$	$B \wedge A$	$A \rightarrow B$
Isomorphism	$A == B$	$A \Leftrightarrow B$

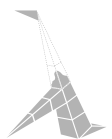


```
Either[A, Nothing] == A
```

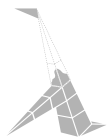


`Either[A, Nothing] == A`

$A \vee \perp \Leftrightarrow A$

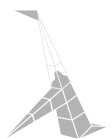


`(A, Nothing) == Nothing`



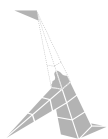
$(A, \text{Nothing}) == \text{Nothing}$

$A \wedge \perp \Leftrightarrow \perp$



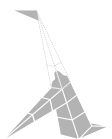
Find the representation that makes sense to you

```
Either[Int, String] => Boolean    <==>    (Int => Boolean, String => Boolean)
```



Find the representation that makes sense to you

```
Either[A, B] => C  <==>  (A => C, B => C)
```



Find the representation that makes sense to you

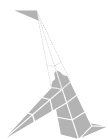
$$\text{Either}[A, B] \Rightarrow C \iff (A \Rightarrow C, B \Rightarrow C)$$

Algebra

$$\begin{aligned}\text{Either}[A, B] \Rightarrow C &= C \wedge (A + B) \\ &= C \wedge A * C \wedge B \\ &= (A \Rightarrow C, B \Rightarrow C)\end{aligned}$$

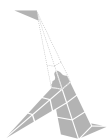
Logic

$$\begin{aligned}\text{Either}[A, B] \Rightarrow C &= (A \vee B) \rightarrow C \\ &= (A \rightarrow C) \wedge (B \rightarrow C) \\ &= (A \Rightarrow C, B \Rightarrow C)\end{aligned}$$

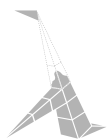


Summary

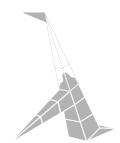
- Cardinality of types matters
- Unit tests offer almost no benefit in term of correctness
- $VIC(f: A \Rightarrow B) = |B| \wedge (|A| - n)$
- Two techniques to achieve correctness
 - Property based testing
 - Parametric polymorphism



All dynamic languages are static languages with a **single** type

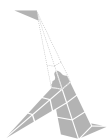


Any



Any => Any

```
def inc(value: Any): Any = value match {  
  case x: Int    => x + 1  
  case x: Double => x + 1  
  case x: Char   => x.toString + "1"  
  case x: String => x + "1"  
}
```

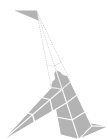


Any => Any

```
def inc(value: Any): Any = value match {  
  case x: Int    => x + 1  
  case x: Double => x + 1  
  case x: Char   => x.toString + "1"  
  case x: String => x + "1"  
}
```

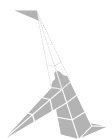
```
inc(5)  
// res7: Any = 6  
inc(10.3)  
// res8: Any = 11.3  
inc('c')  
// res9: Any = "c1"
```

```
inc(java.time.Instant.ofEpochMilli(0))  
// scala.MatchError: 1970-01-01T00:00:00Z (of class java.time.Instant)  
//    at repl.Session$App6.inc(4-Type.html:218)  
//    at repl.Session$App6$$anonfun$11.apply(4-Type.html:242)
```



$$VIC(Any \Rightarrow Any) = |Any| \wedge (|Any| - n)$$

where n is the number of unit tests



Resources and further study

- [Programming with Algebra](#): property based testing with storage
- [Much Ado About Testing](#): property based testing best practices and pitfalls
- [Choosing properties for property-based testing](#)
- [Property-Based Testing in a Screencast Editor](#)
- [Property-Based Testing The Ugly Parts: Case Studies from Komposition](#)
- [Types vs Tests](#)
- [Counting type inhabitants](#)
- [Thinking with types](#): type, algebra, logic
- [Propositions as types](#): Curry–Howard isomorphism

