Multi-winner Social Choice with Incomplete Preferences Analysis

Adi Levi

February 16, 2023

<u>Article</u>- Multi-winner Social Choice with Incomplete Preferences
<u>Authors</u>- Tyler Lu, University of Toronto, and Craig Boutilier, University of Toronto.
<u>Published</u>- Twenty-Third International Joint Conference on Artificial Intelligence. 2013

1 Definitions

N - set of n agents , A - set of alternatives of size m
Preference Profiles- The collection of votes $v = (v_1, \dots, v_n) \in \Gamma^n$
Positional scoring function- $\alpha_i(a) = m - v_i(a)$, Borda scoring rule for candidate a and voter i.
Partial Preferences profile- a collection of partial ranking of each voter $p = \langle p_1, \dots, p_n \rangle$
Completions set of p_i- $C(p_i)$, the set of votes that extends p_i. $C(p) = C(p_1)x \dots xC(p_n)$
For a selection of a set of options $\bar{a} \subseteq A$ s.t. $|\bar{a}| \leq K$:

1. **The score of a K-set:** $S(\bar{a}, v) = \sum_{i \in N} max_{a \in \bar{a}} m - v_i(a) = \sum_{i \in N} S_i(\bar{a})$
2. **The score of an optimal K-set:** $\bar{a}^*_v = argmax_{|\bar{a}| \leq K}(S(\bar{a}, v))$
3. **Pairwise max regret:** $PMR(\bar{a}, \bar{w}, p) = max_{v \in C(P)} S(\bar{w}, v) - S(\bar{a}, v)$
   Is the maximum difference (worst-case loss) under all possible realizations of voter preferences, by offering $\bar{a}$ rather than $\bar{w}$. Can be decomposable: $PMR(\bar{a}, \bar{w}, p_i) = max_{v \in C(P_i)} S_i(\bar{w}, v_i) - S_i(\bar{a}, v_i)$
4. **Max regret:** $MR(\bar{a}, p) = max_{|\bar{w}| \leq K} PMR(\bar{a}, \bar{w}, p)$ is the worst-case loss relative to the optimal K-set $\bar{a}$ under all preference realizations
5. **Minimax regret:** $MMR(p) = min_{|\bar{a}| \leq K} MR(\bar{a}, p)$
6. **Minimax optimal set under partial information:** $\bar{a}^*_p = argmin_{|\bar{a}| \leq K} MR(\bar{a}, p)$
7. **Undominated elements set:** $u_i(\bar{a}) = \left\{ a \in \bar{a} : \neg \exists \, a' \in \bar{a} \;\; s.t \;\; a' \succ_i a \right\}$
   Set of elements from $\bar{a}$ that are not dominated by any element in $\bar{a}$.
8. $B^{'}_i(a, w) = \{B \in A/\bar{a} : b \not\succ_i w \;\; and \;\; \forall a \in u_i(\bar{a}), a \not\succ_i b\}$
   The gap between candidate w and the set $\bar{a}$

Additional option problem - Given a partial profile p and a fixed set $\bar{a}$ of $k-1$ options, which addition to the set will minimize the $MR(\bar{a}, p)$ under the PR/limited choice model.

1. **Conditional PMR:** $PMR(a, w, p|\bar{a}) = PMR(\bar{a} \cup a, \bar{a} \cup w, v) = max_{v \in C(p)} S(\bar{a} \cup w, v) - S(\bar{a} \cup a, v)$
   Given $\bar{a}$ set, a is a proposed additional option, and w is an adversarial witness.
2. **Conditional Max regret:** $MR(a, p|\bar{a}) = max_{w \in A} PMR(a, w, p|\bar{a})$
3. **Conditional Minimax regret:** $MMR(p|\bar{a}) = min_{a \in A} MR(a, p|\bar{a})$
4. **Conditional Minimax optimal set under partial information:** $a^*_{\bar{a},p} \in argmin_{a \in A} MR(a, p|\bar{a})$
5. **The minimal gap between a and b:** $T_i(a, b) = \{b' : a \succ_i b' \succ_i b\}$

2 Topic

The research presented in the paper aims to tackle the Multi-winner problem of finding the best group of K items that best represents the social objective under the constraints of incomplete voter preferences. Specifically, they address the Proportional representation[1] problem and use the Borda scoring function.
The minimax regret criterion[2] is adapted to achieve a robust K-slate that balances individual preferences and overall societal goals.
The authors acknowledge that finding the minimax optimal slate is NP-hard for large slate[3], and therefore develop a greedy robust optimization algorithm with minimax regret, which provides an effective solution in polynomial time ($O(nm^3K^2)$).
However, since the solution is not optimal, they couple their adapted CSS elicitation heuristic with the greedy method to provide an approximation of the optimal slate selection.
The efficiently computable $MMR(p|\bar{a}])$ serves as a proxy for MMR(p) which provides the approximation. The factor of approximation is $1 - \frac{1}{e^{\frac{1}{a}}}$, which depends on the value of a.

3 Main Claim

The greedy optimization algorithm provides an approximation of the optimal slate selection with a factor of $1 - \frac{1}{e^{\frac{1}{a}}}$ when coupled with CSS elicitation heuristic.

4 Main Results

Result 1: The greedy robust optimization algorithm with minimax regret, provides an effective solution in polynomial time $O(nm^3K^2)$ where K is the slate size, m is the number of candidates and n is the number of voters. Instead of computing the max regret $MR(\bar{a}, p)$ for each size K slate $\bar{a}$ and then selecting the slate that minimizes regret (costs $O(nm^{2K+1}K^2)$), we are building the slate $\bar{a}$ by iterations until we get a slate of size K. At each of iteration $k \leqslant K$, we add the option with the least max regret given the prior items, to the slate.

Result 2 - Proposition 7: Let v be an (unknown) complete preference profile and let $m_k \leq S(\bar{a}_{k-1} \cup \{a_k^*\}, v) - S(\bar{a}_{k-1}, v)$ where $\bar{a}_{k-1}$ consists of the first $k-1$ greedily selected items.
If $MR(a_K^*, p|a_{K-1}^*) \leq \frac{m_k}{1-a}$ for all $k \leq K$, where $\alpha \geq 1$ then the greedy-MMR set $\bar{a_K}$ is within a factor of $1 - \frac{1}{e^{\frac{1}{\alpha}}}$ of the optimal (full-information) option slate.
Note: The marginal value of the kth item added to the slate by the full information greedy algorithm, denoted by m_k, is a lower bound on the improvement in the value of the slate achieved by adding that item.
This means that the greedy-MMR algorithm provides an approximation of the optimal slate selection, with a factor of approximation being $1 - \frac{1}{e^{\frac{1}{\alpha}}}$, which depends on the value of α. The closer α is to 1, the closer the approximation is to the optimal solution.

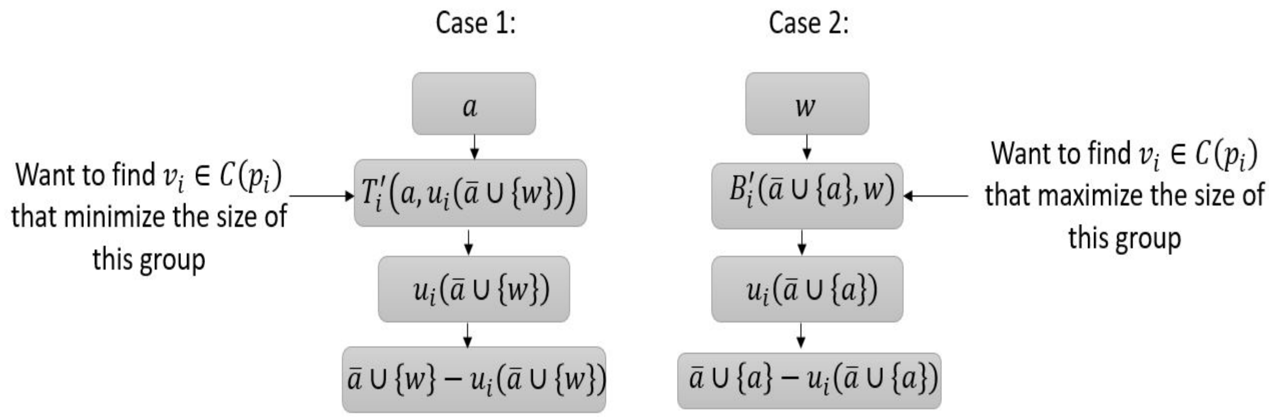
[1]**Proportional representation-** is a system used to select a representative committee, where the parties receive a number of seats in the committee that is proportional to the number of votes they received. This aims to ensure that the distribution of the views in a society is accurately reflected and that minority views are represented.

[2]**Minimax regret**- decision-making criterion when there is preference uncertainty that achieves a robust slate of options, as it balances between individual preferences and the overall societal goals. It selects the option with the smallest maximum regret, calculated as the difference between the best option not chosen and the next best option chosen.

[3]According to "Robust Approximation and Incremental Elicitation in Voting Protocols" by Tyler Lu and Craig Boutilier

5 Table

Here we can observe the Adversarial completions of p_i regret $PMR(a, w, p_i|\bar{a})$ for both cases in the greedy algorithm.



It is important to understand this since it will determine which candidate will be added to the slate $\bar{a}$.

6 Relevant Cited Paper

The paper "Robust Approximation and Incremental Elicitation in Voting Protocols" by Tyler Lu and Craig Boutilier from the University of Toronto, which proposes the use of minimax regret to measure the robustness of proposed winners and demonstrates its practical effects on real-world data sets, was cited 11 times in our paper. our paper adapts the minimax regret-based approach of this paper (for single-winner problems) to the problem of choosing a slate of K options. Furthermore, our paper adapts the solution heuristic of this paper to multi-winner choice in order to solve the problem of elicitation.

7 Relevant Paper Citing Ours

This paper "Sample Complexity for Winner Prediction in Elections" looks at the issue of figuring out who will win an election based on a limited number of voters. It gives a mathematical calculation of the smallest number of voters needed to correctly predict the election result, considering things like the number of people running, how voters are choosing, and how confident the prediction needs to be. The paper cites our paper in order to give an example of a problem with incomplete information settings, with our paper's solution being proposed- multi-winner.

8 Project Plan

In the next part, we are implementing the Greedy and Optimal algorithms mentioned in the paper. We will compare the two in terms of running times and performance (MMR) as a function of the slate size - K and number of candidates - m

Furthermore, we will apply a random elicitation strategy and show that Greedy MR is non-increasing. We are planning to run the models on the Sushi dataset that consists of 5000 complete user preference rankings over 10 varieties of sushi.
Another code we used to inspect the effect of m on the runtime of the greedy algorithm is the Sushi dataset that consists of 5000 partial user preference rankings over 100 varieties of sushi.

In order to create partial information preferences, we will randomly choose a pairwise comparison for each voter. We will later compare our results to the findings in the article.