x = absolute foot position in x

z_0 = hip offset in z

z = absolute z foot position

L_1 = hip length

L_2 = knee length

x_0 = hip offset in x

Inverse Kinematics (geometric)

$$x - x_0 = -L_1 \sin(\theta_1) + L_2 \cos(\theta_1 + \theta_2) \quad — (1)$$

$$z - z_0 = -L_1 \cos(\theta_1) - L_2 \sin(\theta_1 + \theta_2) \quad — (2)$$

$\theta_1 + \theta_2 = \varphi$, $x - x_0 = x_{local}$, $z - z_0 = z_{local}$.

$$x_{local} = -L_1 \sin(\theta_1) + L_2 \cos(\varphi) \quad — (3)$$

$$z_{local} = -L_1 \cos(\theta_1) - L_2 \sin(\varphi) \quad — (4)$$

$$x_{local} + L_1 \sin(\theta_1) = L_2 \cos(\varphi) \quad — (3')$$

$$z_{local} + L_1 \cos(\theta_1) = -L_2 \sin(\varphi) \quad — (4')$$

$\sin^2\theta_1 + \cos^2\theta = 1$

$$(x_{local} + L_1 \sin(\theta_1))^2 + (z_{local} + L_1 \cos(\theta_1))^2 = (L_2 \cos\varphi)^2 + (-L_2 \sin\varphi)^2$$

$L_1^2 \sin^2\theta_1$ $\quad + L_1^2 \cos^2\theta_1$

$$x_{local}^2 + 2x_{local} L_1 \sin(\theta_1) + L_1^2 \sin^2\theta_1 + z_{local}^2 + 2z_{local} L_1 \cos(\theta_1) + L_1^2 \cos^2\theta_1 = L_2^2$$

$\rightarrow L_1^2$

$$\therefore \underbrace{x_{local}^2 + z_{local}^2}_{r^2} + 2L_1(x_{local} \sin(\theta_1) + z_{local} \cos(\theta_1)) = L_2^2 - L_1^2$$

$$x_{local} \sin\theta_1 + z_{local} \cos\theta_1 = \frac{L_2^2 - L_1^2 - r^2}{2(L_1)} = K$$

using trig methods,

big identity

$\therefore x_{local} \sin\theta_1 + z_{local}\cos\theta_1 = r\sin(\theta_1+\varphi)$

$\varphi = \text{arctan2}\left(\frac{z_{local}}{x_{local}}\right)$

$r = \sqrt{x_{local}^2 + z_{local}^2}$

$r \sin(\theta_1 + \varphi) = \frac{L_2^2 - L_1^2 - r^2}{2(L_1)}$

$\sin(\theta_1+\varphi) = \frac{L_2^2 - L_1^2 - r^2}{2(L_1)(r)}$

$\theta_1 + \varphi = \sin^{-1}\left(\frac{L_2^2 - L_1^2 - r^2}{2(L_1)(r)}\right)$

$\therefore \theta_1 = \sin^{-1}\left(\frac{L_2^2 - L_1^2 - r^2}{2(L_1)(r)}\right) - \text{arctan2}\left(\frac{z_{local}}{x_{local}}\right)$

using (3'), (4')

$\frac{\cancel{-L_2}\sin\varphi}{\cancel{L_2}\cos\varphi} = \frac{z_{local} + L_1\cos(\theta_1)}{x_{local} + L_1\sin(\theta_1)}$

$\varphi = \tan^{-1}\left(\frac{-z_{local} + L_1\cos(\theta_1)}{x_{local} + L_1\sin(\theta_1)}\right)$

$\theta_1 + \theta_2 =$

$\therefore \theta_2 = \tan^{-1}\left(\frac{-z_{local} + L_1\cos(\theta_1)}{x_{local} + L_1\sin(\theta_1)}\right) - \theta_1$