

Homework-1 for CSCE 625

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Question 1. a

The symbols/literals we are using in this problem are:

- Observations we can find by drawing the tennis ball are:
O1W, O1Y, O2W, O2Y, O3W, O3Y
- Actual colors present in the boxes
C1W, C1Y, C1B, C2W, C2Y, C2B, C3W, C3Y, C3B
- Labels given to the boxes
L1W , L1Y , L1B, L2W , L2Y , L2B, L3W , L3Y , L3B
- What various observation imply:
 $c1 : O1Y \implies C1Y \vee C1B$
 $c2 : O1W \implies C1W \vee C1B$
 $c3 : O2Y \implies C2Y \vee C2B$
 $c4 : O2W \implies C2W \vee C2B$
 $c5 : O3Y \implies C3Y \vee C3B$
 $c6 : O3W \implies C3W \vee C3B$
- Information from the labels - Since the initial labels are wrong:
 $c7 : L1Y \implies \neg C1Y$
 $c8 : L1W \implies \neg C1W$
 $c9 : L1B \implies \neg C1B$
 $c10 : L2Y \implies \neg C2Y$
 $c11 : L2W \implies \neg C2W$
 $c12 : L2B \implies \neg C2B$
 $c13 : L3Y \implies \neg C3Y$
 $c14 : L3W \implies \neg C3W$
 $c15 : L3B \implies \neg C3B$
- Each box has exactly one color among Y,W and B (Y and W):
 $c16 : C1Y \vee C1W \vee C1B$
 $c17 : C2Y \vee C2W \vee C2B$
 $c18 : C3Y \vee C3W \vee C3B$

- Constraints on the number of the colors each box can have

$$\begin{aligned}
c19 : C1Y &\implies \neg C2Y \wedge \neg C3Y \\
c20 : C1W &\implies \neg C2W \wedge \neg C3W \\
c21 : C1B &\implies \neg C2B \wedge \neg C3B \\
c22 : C2Y &\implies \neg C1Y \wedge \neg C3Y \\
c23 : C2W &\implies \neg C1W \wedge \neg C3W \\
c24 : C2B &\implies \neg C1B \wedge \neg C3B \\
c25 : C3Y &\implies \neg C2Y \wedge \neg C1Y \\
c26 : C3W &\implies \neg C2W \wedge \neg C1W \\
c27 : C3B &\implies \neg C2B \wedge \neg C1B
\end{aligned}$$

Question 1. b Now lets prove by using the Natural deduction (KB) entails $C2W$

$$KB \models C2W$$

Using the constraints from previous section. Now we have observed that:

- Knowledge Base (Observed for the given scenario) we have;

$$\begin{aligned}
R1 : O1Y \\
R2 : O2W \\
R3 : O3Y \\
R4 : L1W \\
R5 : L2Y \\
R6 : L3B
\end{aligned}$$

- By applying Modus Ponens to R3 and c5 , we derive

$$R7 : C3Y \vee C3B$$

- By applying Modus Ponens to R6 and c , we derive

$$R8 : \neg C3B$$

- By resolving R7 and R8, and we have

$$R9 : C3Y$$

- By applying Modus Ponens to R9 and c25 we derive

$$R10 : \neg C1Y \wedge C2Y$$

- By applying Modus Ponens to R1 and c1, we derive

$$R11 : C1Y \vee C1B$$

- By applying And-Elimination on R10, we derive

$$R12 : \neg C1Y$$

- By resolving R11 and R12, we have

$$R13 : C1B$$

- By applying Modus Ponens to R2 and c4, we derive
 $R14 : C2W \wedge C2B$
- From R13 and c21, we have
 $R15 : \neg C2B \wedge \neg C3B$
- From R15, we have
 $R16 : \neg C2B$
- By applying resolving R14 and R16 we have
 $R17 : C2W$
 WHICH IS THE QUERY ITSELF ... PROVED

Question 1. c

- Knowledge Base (Observed for the given scenario) we have;
 $R1 : O1Y$
 $R2 : O2W$
 $R3 : O3Y$
 $R4 : L1W$
 $R5 : L2Y$
 $R6 : L3B$
- For applying the transformation of converting to CNF (Conjunctive Normal Form) on various constraints we need in solving this problem:
 - Transform c6 to CNF we get
 $R7 : \neg O3Y \vee C3Y \vee C3B$
 - Transform c1 to CNF we get
 $R8 : \neg O1Y \vee C1Y \vee C1B$
 - Transform c4 to CNF we get
 $R9 : \neg O2W \vee C2W \vee C2B$
 - Transform c15 to CNF we get
 $R10 : \neg L3B \vee \neg C3B$
 - Transform c21 to CNF we get
 $R11a : \neg C1B \vee \neg C2B$
 $R11b : \neg C1B \vee \neg C3B$
 - Transform c21 to CNF we get
 $R12a : \neg C3Y \vee \neg C2Y$
 $R12b : \neg C3Y \vee \neg C1Y$
- Solving for the query q as C2W, Getting the negation of the query
 $R13 : \neg C2W$

- Resolution Rule on R3 and R7, we have
 $R14 : C3Y \vee C3B$
- Resolution Rule on R6 and R10, we have
 $R15 : \neg C3B$
- Resolution Rule on R14 and R15, we have
 $R16 : C3Y$
- Resolution Rule on R16 and R12b, we have
 $R17 : \neg C1Y$
- Resolution Rule on R1 and R8, we have
 $R18 : C1Y \wedge C1B$
- Resolution Rule on R17 and R18, we have
 $R19 : C1B$
- Resolution Rule on R19 and R11a, we have
 $R20 : \neg C2B$
- Resolution Rule on R2 and R9, we have
 $R21 : C2W \wedge C2B$
- Resolution Rule on R20 and R21, we have
 $R22 : C2W$
- Resolution Rule on R22 and R13, we have
 $R23 : \text{Null Set}$
Thus it terminated with empty clause and hence the solution to the entailment is true.

Question 2.

The propositions we need for this question involves following literals/symbols:

Row 1: Q1A, Q1B, Q1C, Q1D

Row 2: Q2A, Q2B, Q2C, Q2D

Row 3: Q3A, Q3B, Q3C, Q3D

Row 4: Q4A, Q4B, Q4C, Q4D

Corresponding Knowledge Base we have for the 4-Queen problem is given by:

KB= Each Row has at least one queen and can be transformed into the clauses like:

$R1 : Q1A \vee Q1B \vee Q1C \vee Q1D$

$R2 : Q2A \vee Q2B \vee Q2C \vee Q2D$

$R3 : Q3A \vee Q3B \vee Q3C \vee Q3D$

$R4 : Q4A \vee Q4B \vee Q4C \vee Q4D$

Now placing one queen at most in one row and one column

$$R5 : \neg Q1A \vee (\neg Q2A \wedge \neg Q3A \wedge \neg Q4A \wedge \neg Q1B \wedge \neg Q1C \wedge \neg Q1D)$$

After distributing the dis-junction over conjunction we get:

$$R5 : (\neg Q1A \vee \neg Q2A) \wedge (\neg Q1A \vee \neg Q3A) \wedge (\neg Q1A \vee \neg Q4A) \wedge (\neg Q1A \vee \neg Q1B) \wedge (\neg Q1A \vee \neg Q1C) \wedge (\neg Q1A \vee \neg Q1D)$$

$$R6 : (\neg Q2A \vee \neg Q1A) \wedge (\neg Q2A \vee \neg Q3A) \wedge (\neg Q2A \vee \neg Q4A) \wedge (\neg Q2A \vee \neg Q2B) \wedge (\neg Q2A \vee \neg Q2C) \wedge (\neg Q2A \vee \neg Q2D)$$

$$R7 : (\neg Q3A \vee \neg Q1A) \wedge (\neg Q3A \vee \neg Q2A) \wedge (\neg Q3A \vee \neg Q4A) \wedge (\neg Q3A \vee \neg Q3B) \wedge (\neg Q3A \vee \neg Q3C) \wedge (\neg Q3A \vee \neg Q3D)$$

$$R8 : (\neg Q4A \vee \neg Q1A) \wedge (\neg Q4A \vee \neg Q2A) \wedge (\neg Q4A \vee \neg Q3A) \wedge (\neg Q4A \vee \neg Q4B) \wedge (\neg Q4A \vee \neg Q4C) \wedge (\neg Q4A \vee \neg Q4D)$$

$$R9 : (\neg Q1B \vee \neg Q2B) \wedge (\neg Q1B \vee \neg Q3B) \wedge (\neg Q1B \vee \neg Q4B) \wedge (\neg Q1B \vee \neg Q1A) \wedge (\neg Q1B \vee \neg Q1C) \wedge (\neg Q1B \vee \neg Q1D)$$

$$R10 : (\neg Q2B \vee \neg Q1B) \wedge (\neg Q2B \vee \neg Q3B) \wedge (\neg Q2B \vee \neg Q4B) \wedge (\neg Q2B \vee \neg Q2A) \wedge (\neg Q2B \vee \neg Q2C) \wedge (\neg Q2B \vee \neg Q2D)$$

$$R11 : (\neg Q3B \vee \neg Q1B) \wedge (\neg Q3B \vee \neg Q2B) \wedge (\neg Q3B \vee \neg Q4B) \wedge (\neg Q3B \vee \neg Q3A) \wedge (\neg Q3B \vee \neg Q3C) \wedge (\neg Q3B \vee \neg Q3D)$$

$$R12 : (\neg Q4B \vee \neg Q1B) \wedge (\neg Q4B \vee \neg Q2B) \wedge (\neg Q4B \vee \neg Q3B) \wedge (\neg Q4B \vee \neg Q4A) \wedge (\neg Q4B \vee \neg Q4C) \wedge (\neg Q4B \vee \neg Q4D)$$

$$R13 : (\neg Q1C \vee \neg Q2C) \wedge (\neg Q1C \vee \neg Q3C) \wedge (\neg Q1C \vee \neg Q4C) \wedge (\neg Q1C \vee \neg Q1A) \wedge (\neg Q1C \vee \neg Q1B) \wedge (\neg Q1C \vee \neg Q1D)$$

$$R14 : (\neg Q2C \vee \neg Q1C) \wedge (\neg Q2C \vee \neg Q3C) \wedge (\neg Q2C \vee \neg Q4C) \wedge (\neg Q2C \vee \neg Q2A) \wedge (\neg Q2C \vee \neg Q2B) \wedge (\neg Q2C \vee \neg Q2D)$$

$$R15 : (\neg Q3C \vee \neg Q1C) \wedge (\neg Q3C \vee \neg Q2C) \wedge (\neg Q3C \vee \neg Q4C) \wedge (\neg Q3C \vee \neg Q3A) \wedge (\neg Q3C \vee \neg Q3B) \wedge (\neg Q3C \vee \neg Q3D)$$

$$R16 : (\neg Q4C \vee \neg Q1C) \wedge (\neg Q4C \vee \neg Q2C) \wedge (\neg Q4C \vee \neg Q3C) \wedge (\neg Q4C \vee \neg Q4A) \wedge (\neg Q4C \vee \neg Q4B) \wedge (\neg Q4C \vee \neg Q4D)$$

$$R17 : (\neg Q1D \vee \neg Q2D) \wedge (\neg Q1D \vee \neg Q3D) \wedge (\neg Q1D \vee \neg Q4D) \wedge (\neg Q1D \vee \neg Q1A) \wedge (\neg Q1D \vee \neg Q1B) \wedge (\neg Q1D \vee \neg Q1C)$$

$$R18 : (\neg Q2D \vee \neg Q1D) \wedge (\neg Q2D \vee \neg Q3D) \wedge (\neg Q2D \vee \neg Q4D) \wedge (\neg Q2D \vee \neg Q2A) \wedge (\neg Q2D \vee \neg Q2B) \wedge (\neg Q2D \vee \neg Q2C)$$

$$R19 : (\neg Q3D \vee \neg Q1D) \wedge (\neg Q3D \vee \neg Q2D) \wedge (\neg Q3D \vee \neg Q4D) \wedge (\neg Q3D \vee \neg Q3A)$$

$$\neg Q3A) \wedge (\neg Q3D \vee \neg Q3B) \wedge (\neg Q3D \vee \neg Q3C)$$

$$R20 : (\neg Q4D \vee \neg Q1D) \wedge (\neg Q4D \vee \neg Q2D) \wedge (\neg Q3D \vee \neg Q4D) \wedge (\neg Q4D \vee \neg Q4A) \wedge (\neg Q4D \vee \neg Q4B) \wedge (\neg Q4D \vee \neg Q4C)$$

Now putting the constraints of the having diagonals on the queens

$$R21 : (\neg Q1A \vee \neg Q2B) \wedge (\neg Q1A \vee \neg Q3C) \wedge (\neg Q1A \vee \neg Q4D)$$

$$R22 : (\neg Q2A \vee \neg Q1B) \wedge (\neg Q2A \vee \neg Q3B) \wedge (\neg Q2A \vee \neg Q4C)$$

$$R23 : (\neg Q3A \vee \neg Q2B) \wedge (\neg Q3A \vee \neg Q4B) \wedge (\neg Q3A \vee \neg Q1C)$$

$$R24 : (\neg Q4A \vee \neg Q3B) \wedge (\neg Q4A \vee \neg Q2C) \wedge (\neg Q4A \vee \neg Q1D)$$

$$R25 : (\neg Q1B \vee \neg Q2A) \wedge (\neg Q1B \vee \neg Q2C) \wedge (\neg Q1B \vee \neg Q3D)$$

$$R26 : (\neg Q2B \vee \neg Q1A) \wedge (\neg Q2B \vee \neg Q3A) \wedge (\neg Q2B \vee \neg Q1C) \wedge (\neg Q2B \vee \neg Q3C) \wedge (\neg Q2B \vee \neg Q4D)$$

$$R27 : (\neg Q3B \vee \neg Q2A) \wedge (\neg Q3B \vee \neg Q4A) \wedge (\neg Q3B \vee \neg Q2C) \wedge (\neg Q3B \vee \neg Q4C) \wedge (\neg Q3B \vee \neg Q1D)$$

$$R28 : (\neg Q4B \vee \neg Q3A) \wedge (\neg Q4B \vee \neg Q3C) \wedge (\neg Q4B \vee \neg Q2D)$$

$$R29 : (\neg Q1C \vee \neg Q3A) \wedge (\neg Q1C \vee \neg Q2B) \wedge (\neg Q1C \vee \neg Q2D)$$

$$R30 : (\neg Q2C \vee \neg Q4A) \wedge (\neg Q2C \vee \neg Q1B) \wedge (\neg Q2C \vee \neg Q3B) \wedge (\neg Q2C \vee \neg Q1D) \wedge (\neg Q2C \vee \neg Q3D)$$

$$R31 : (\neg Q3C \vee \neg Q1A) \wedge (\neg Q3C \vee \neg Q2B) \wedge (\neg Q3C \vee \neg Q4B) \wedge (\neg Q3C \vee \neg Q2D) \wedge (\neg Q3C \vee \neg Q4D)$$

$$R32 : (\neg Q4C \vee \neg Q2A) \wedge (\neg Q4C \vee \neg Q3B) \wedge (\neg Q4C \vee \neg Q3D)$$

$$R33 : (\neg Q1D \vee \neg Q4A) \wedge (\neg Q1D \vee \neg Q3B) \wedge (\neg Q1D \vee \neg Q2C)$$

$$R34 : (\neg Q2D \vee \neg Q4B) \wedge (\neg Q2D \vee \neg Q1C) \wedge (\neg Q2D \vee \neg Q3C)$$

$$R35 : (\neg Q3D \vee \neg Q1B) \wedge (\neg Q3D \vee \neg Q2C) \wedge (\neg Q3D \vee \neg Q4C)$$

$$R36 : (\neg Q4D \vee \neg Q1A) \wedge (\neg Q4D \vee \neg Q2B) \wedge (\neg Q4D \vee \neg Q3C)$$

There are total number of 80 clauses in total without repetition. I have added the clauses which are related to same row and column in similar R_i clauses for bookkeeping purposes, so that the reference is easily made.

2 a. Now applying the simple DPLL method without any heuristics to the 4-queen problem having Knowledge Base as above

Iteration No	Comments	Q1A	Q2A	Q3A	Q4A	Q1B	Q2B	Q3B	Q4B	Q1C	Q2C	Q3C	Q4C	Q1D	Q2D	Q3D	Q4D
1	Q1A True	T	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?
2	Q2A False Constraint R5	T	F	?	?	?	?	?	?	?	?	?	?	?	?	?	?
3	Q3A False Constraint R5	T	F	F	?	?	?	?	?	?	?	?	?	?	?	?	?
4	Q4A False Constraint R5	T	F	F	F	?	?	?	?	?	?	?	?	?	?	?	?
5	Q1B False Constraint R5	T	F	F	F	F	?	?	?	?	?	?	?	?	?	?	?
6	Q2B False Constraint R21	T	F	F	F	F	F	?	?	?	?	?	?	?	?	?	?
7	Q3B No constraint	T	F	F	F	F	F	T	?	?	?	?	?	?	?	?	?
8	Q4B False Constraint R11	T	F	F	F	F	F	T	F	?	?	?	?	?	?	?	?
9	Q1C False Constraint R5	T	F	F	F	F	F	T	F	F	?	?	?	?	?	?	?
10	Q2C False Constraint R27	T	F	F	F	F	F	T	F	F	F	?	?	?	?	?	?
11	Q3C False Constraint R11	T	F	F	F	F	F	T	F	F	F	F	?	?	?	?	?
12	BackTrack to Iteration 7, by R27, R3	T	F	F	F	F	F	T	F	F	F	F	?	?	?	?	?
13	Iteration 7, Q3B as False	T	F	F	F	F	F	F	?	?	?	?	?	?	?	?	?
14	Q4B True, By R2	T	F	F	F	F	F	F	T	?	?	?	?	?	?	?	?
15	Q1C False, By R5	T	F	F	F	F	F	F	T	F	?	?	?	?	?	?	?
16	Q2C True	T	F	F	F	F	F	F	T	F	T	?	?	?	?	?	?
17	Q3C False, By R7 and R28	T	F	F	F	F	F	F	T	F	T	F	?	?	?	?	?
18	Q4C False, By R7 and R28	T	F	F	F	F	F	F	T	F	T	F	F	?	?	?	?
19	Q1D False, By R5	T	F	F	F	F	F	F	T	F	T	F	F	F	?	?	?
20	Q2D False, By R14	T	F	F	F	F	F	F	T	F	T	F	F	F	F	?	?
21	Q3D False, By R30	T	F	F	F	F	F	F	T	F	T	F	F	F	F	F	?
22	Q4D BackTrack To Iteration 16 , By R12, R4	T	F	F	F	F	F	F	T	F	T	F	F	F	F	F	F
23	Q2C False	T	F	F	F	F	F	F	T	F	F	?	?	?	?	?	?
24	Q3C False, By R7 and R28	T	F	F	F	F	F	F	T	F	T	F	?	?	?	?	?
25	BackTrack to Iteration 1, by R12, R3	T	F	F	F	F	F	F	T	F	T	F	F	?	?	?	?
26	Q1A False	F	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?
27	Q2A True	F	T	?	?	?	?	?	?	?	?	?	?	?	?	?	?
28	Q3A False Constraint R6	F	T	F	?	?	?	?	?	?	?	?	?	?	?	?	?
29	Q4A False Constraint R6	F	T	F	F	?	?	?	?	?	?	?	?	?	?	?	?
30	Q1B False Constraint R22	F	T	F	F	F	?	?	?	?	?	?	?	?	?	?	?
31	Q2B False Constraint R6	F	T	F	F	F	F	?	?	?	?	?	?	?	?	?	?
32	Q3B False Constraint R22	F	T	F	F	F	F	F	?	?	?	?	?	?	?	?	?
33	Q4B True	F	T	F	F	F	F	F	T	?	?	?	?	?	?	?	?
34	Q1C True	F	T	F	F	F	F	F	T	T	?	?	?	?	?	?	?
35	Q2C False Constraint R13	F	T	F	F	F	F	F	T	T	F	?	?	?	?	?	?
36	Q3C False Constraint R13	F	T	F	F	F	F	F	T	T	F	F	?	?	?	?	?
37	Q4C False Constraint R13	F	T	F	F	F	F	F	T	T	F	F	F	?	?	?	?
38	Q1D False Constraint R13	F	T	F	F	F	F	F	T	T	F	F	F	F	?	?	?
39	Q2D False Constraint R28	F	T	F	F	F	F	F	T	T	F	F	F	F	F	?	?
40	Q3D True	F	T	F	F	F	F	F	T	T	F	F	F	F	F	T	?
41	Q4D False Constraint R12	F	T	F	F	F	F	F	T	T	F	F	F	F	F	T	F

Table 1: DPLL algorithm on 4-Queen Problem

2 b. Now applying Pure Symbol and Unit Clause heuristics on DPLL method to the 4-queen problem having Knowledge Base as given in Q2a. Please check the Table 2 for the detailed DPLL trace with Heuristics.

3. We consider the following literals to define the propositional logic for the tic-tac-toe problem.

- Positions of player X :
 X_{11}, X_{12}, X_{13}
 X_{21}, X_{22}, X_{23}
 X_{31}, X_{32}, X_{33}

Iteration No	Comments	Q1A	Q2A	Q3A	Q4A	Q1B	Q2B	Q3B	Q4B	Q1C	Q2C	Q3C	Q4C	Q1D	Q2D	Q3D	Q4D
1	Q1A True	T	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?
2	Q2A Unit Clause : F BY R5	T	F	?	?	?	?	?	?	?	?	?	?	?	?	?	?
3	Q3A Unit Clause : F BY R5	T	F	F	?	?	?	?	?	?	?	?	?	?	?	?	?
4	Q4A Unit Clause : F BY R5	T	F	F	F	?	?	?	?	?	?	?	?	?	?	?	?
5	Q1B Unit Clause : F BY R5	T	F	F	F	F	?	?	?	?	?	?	?	?	?	?	?
6	Q2B Unit Clause : F BY R21	T	F	F	F	F	F	?	?	?	?	?	?	?	?	?	?
7	Q1C Unit Clause : F BY R5	T	F	F	F	F	F	?	?	F	?	?	?	?	?	?	?
8	Q3C Unit Clause : F BY R21	T	F	F	F	F	F	?	?	F	?	F	?	?	?	?	?
9	Q1D Unit Clause : F BY R5	T	F	F	F	F	F	?	?	F	?	F	?	F	?	?	?
10	Q4D Unit Clause : F BY R21	T	F	F	F	F	F	?	?	F	?	F	?	F	?	?	F
11	Q3B True	T	F	F	F	F	F	T	?	F	?	F	?	F	?	?	F
12	Q4B Unit Clause: F BY R11	T	F	F	F	F	F	T	F	F	?	F	?	F	?	?	F
13	Q2C Unit Clause: F BY R27	T	F	F	F	F	F	T	F	F	F	F	?	F	?	?	F
14	Q3D Unit Clause: F BY R21	T	F	F	F	F	F	T	F	F	F	F	?	F	?	F	F
15	Q4C F Backtrack To Iteration 11	T	F	F	F	F	F	T	?	F	?	F	?	F	?	?	F
16	Q3B False	T	F	F	F	F	F	F	?	F	?	F	?	F	?	?	F
17	Q4B Unit Clause: F BY R2	T	F	F	F	F	F	F	T	F	?	F	?	F	?	?	F
18	Q4C Unit Clause : F BY R12	T	F	F	F	F	F	F	T	F	?	F	F	F	?	?	F
19	Q2D Unit Clause : F BY R28	T	F	F	F	F	F	F	T	F	?	F	F	F	F	?	F
20	Q2C Unit Clause: F BY R3	T	F	F	F	F	F	F	T	F	T	F	F	F	F	?	F
21	Q3D T/F Backtrack to Iteration 1	T	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?
22	Q1A False	F	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?
23	Q2A TRUE	F	T	?	?	?	?	?	?	?	?	?	?	?	?	?	?
24	Q3A Unit Clause : F BY R6	F	T	F	?	?	?	?	?	?	?	?	?	?	?	?	?
25	Q4A Unit Clause : F BY R6	F	T	F	F	?	?	?	?	?	?	?	?	?	?	?	?
26	Q1B Unit Clause : F BY R22	F	T	F	F	F	?	?	?	?	?	?	?	?	?	?	?
27	Q2B Unit Clause : F BY R6	F	T	F	F	F	F	?	?	?	?	?	?	?	?	?	?
28	Q3B Unit Clause : F BY 22	F	T	F	F	F	F	F	?	?	?	?	?	?	?	?	?
29	Q2C Unit Clause : F BY R6	F	T	F	F	F	F	F	?	?	F	?	?	?	?	?	?
30	Q4C Unit Clause : F BY R22	F	T	F	F	F	F	F	?	?	F	?	F	?	?	?	?
31	Q2D Unit Clause : F BY R6	F	T	F	F	F	F	F	?	?	F	?	F	?	F	?	?
32	Q4B True	F	T	F	F	F	F	F	T	?	F	?	F	?	F	?	?
33	Q3C Unit Clause	F	T	F	F	F	F	F	T	?	F	F	F	?	F	?	?
34	Q4D Unit Clause	F	T	F	F	F	F	F	T	?	F	F	F	?	F	?	F
35	Q1C True	F	T	F	F	F	F	F	T	T	F	F	F	?	F	?	F
36	Q1D Pure Symbol : F	F	T	F	F	F	F	F	T	T	F	F	F	F	F	?	F
37	Q3D Pure Symbol : T	F	T	F	F	F	F	F	T	T	F	F	F	F	F	T	F

Table 2: DPLL algorithm with Heuristics on 4-Queen Problem

- Positions of player O :

O_{11} , O_{12} , O_{13}
 O_{21} , O_{22} , O_{23}
 O_{31} , O_{32} , O_{33}

- Void positions ?:

?11, ?12, ?13
?21, ?22, ?23
?31, ?32, ?33

- Move positions:

moveX11, moveX12, moveX13
moveX21, moveX22, moveX23

moveX31, moveX32, moveX33

- canWin positions:

canWinX11, canWinX12, canWinX13
canWinX21, canWinX22, canWinX23
canWinX31, canWinX32, canWinX33
canWinX

canWinO11, canWinO12, canWinO13
canWinO21, canWinO22, canWinO23
canWinO31, canWinO32, canWinO33
canWinO

- forcedMove positions:

forcedMoveX11, forcedMoveX12, forcedMoveX13
forcedMoveX21, forcedMoveX22, forcedMoveX23
forcedMoveX31, forcedMoveX32, forcedMoveX33

Knowledge Base (KB):

- We need at least one move by X :

$c1 : \text{moveX11} \vee \text{moveX12} \vee \text{moveX13} \vee \text{moveX21} \vee \text{moveX22} \vee \text{moveX23} \vee \text{moveX31} \vee \text{moveX32} \vee \text{moveX33}$

- canWin moves for X :

Horizontal pattern:

$c2 : X11 \wedge X12 \wedge ?13 \implies \text{canWinX13}$
 $c3 : X21 \wedge X22 \wedge ?23 \implies \text{canWinX23}$
 $c4 : X31 \wedge X32 \wedge ?33 \implies \text{canWinX33}$

$c5 : X11 \wedge ?12 \wedge X13 \implies \text{canWinX12}$
 $c6 : X21 \wedge ?22 \wedge X23 \implies \text{canWinX22}$
 $c7 : X31 \wedge ?32 \wedge X33 \implies \text{canWinX32}$

$c8 : ?11 \wedge X12 \wedge X13 \implies \text{canWinX11}$
 $c9 : ?21 \wedge X22 \wedge X23 \implies \text{canWinX21}$
 $c10 : ?31 \wedge X32 \wedge X33 \implies \text{canWinX31}$

Vertical pattern:

$$\begin{aligned}
c11 : X11 \wedge X21 \wedge ?31 &\implies \text{canWin}X31 \\
c12 : X12 \wedge X22 \wedge ?23 &\implies \text{canWin}X23 \\
c13 : X13 \wedge X23 \wedge ?33 &\implies \text{canWin}X33
\end{aligned}$$

$$\begin{aligned}
c14 : X11 \wedge ?21 \wedge X31 &\implies \text{canWin}X21 \\
c15 : X12 \wedge ?22 \wedge X23 &\implies \text{canWin}X22 \\
c16 : X13 \wedge ?23 \wedge X33 &\implies \text{canWin}X23
\end{aligned}$$

$$\begin{aligned}
c17 : ?11 \wedge X21 \wedge X31 &\implies \text{canWin}X11 \\
c18 : ?12 \wedge X22 \wedge X23 &\implies \text{canWin}X12 \\
c19 : ?13 \wedge X23 \wedge X33 &\implies \text{canWin}X13
\end{aligned}$$

Diagonal pattern:

$$\begin{aligned}
c20 : X11 \wedge X22 \wedge ?33 &\implies \text{canWin}X33 \\
c21 : X13 \wedge X22 \wedge ?31 &\implies \text{canWin}X31
\end{aligned}$$

$$\begin{aligned}
c22 : X11 \wedge ?22 \wedge X33 &\implies \text{canWin}X22 \\
c23 : X13 \wedge ?22 \wedge X31 &\implies \text{canWin}X22
\end{aligned}$$

$$\begin{aligned}
c24 : ?11 \wedge X22 \wedge X33 &\implies \text{canWin}X11 \\
c25 : ?13 \wedge X22 \wedge X31 &\implies \text{canWin}X13
\end{aligned}$$

$$\begin{aligned}
c26 : &\text{canWin}X11 \vee \text{canWin}X12 \vee \text{canWin}X13 \\
&\vee \text{canWin}X21 \vee \text{canWin}X22 \vee \text{canWin}X23 \\
&\vee \text{canWin}X31 \vee \text{canWin}X32 \vee \text{canWin}X33 \implies \text{canWin}X
\end{aligned}$$

- canWin moves for O:

Horizontal pattern:

$$\begin{aligned}
c27 : O11 \wedge O12 \wedge ?13 &\implies \text{canWin}O13 \\
c28 : O21 \wedge O22 \wedge ?23 &\implies \text{canWin}O23 \\
c29 : O31 \wedge O32 \wedge ?33 &\implies \text{canWin}O33
\end{aligned}$$

$$\begin{aligned}
c30 : O11 \wedge ?12 \wedge O13 &\implies \text{canWin}O12 \\
c31 : O21 \wedge ?22 \wedge O23 &\implies \text{canWin}O22 \\
c32 : O31 \wedge ?32 \wedge O33 &\implies \text{canWin}O32
\end{aligned}$$

$$\begin{aligned}
c33 : ?11 \wedge O12 \wedge O13 &\implies \text{canWin}O11 \\
c34 : ?21 \wedge O22 \wedge O23 &\implies \text{canWin}O21 \\
c35 : ?31 \wedge O32 \wedge O33 &\implies \text{canWin}O31
\end{aligned}$$

Vertical pattern:

$$\begin{aligned} c36 : O11 \wedge O21 \wedge ?31 &\implies \text{canWin}O31 \\ c37 : O12 \wedge O22 \wedge ?23 &\implies \text{canWin}O23 \\ c38 : O13 \wedge O23 \wedge ?33 &\implies \text{canWin}O33 \end{aligned}$$

$$\begin{aligned} c39 : O11 \wedge ?21 \wedge O31 &\implies \text{canWin}O21 \\ c40 : O12 \wedge ?22 \wedge O23 &\implies \text{canWin}O22 \\ c41 : O13 \wedge ?23 \wedge O33 &\implies \text{canWin}O23 \end{aligned}$$

$$\begin{aligned} c42 : ?11 \wedge O21 \wedge O31 &\implies \text{canWin}O11 \\ c43 : ?12 \wedge O22 \wedge O23 &\implies \text{canWin}O12 \\ c44 : ?13 \wedge O23 \wedge O33 &\implies \text{canWin}O13 \end{aligned}$$

Diagonal pattern:

$$\begin{aligned} c45 : O11 \wedge O22 \wedge ?33 &\implies \text{canWin}O33 \\ c46 : O13 \wedge O22 \wedge ?31 &\implies \text{canWin}O31 \end{aligned}$$

$$\begin{aligned} c47 : O11 \wedge ?22 \wedge O33 &\implies \text{canWin}O22 \\ c48 : O13 \wedge ?22 \wedge O31 &\implies \text{canWin}O22 \end{aligned}$$

$$\begin{aligned} c49 : ?11 \wedge X22 \wedge X33 &\implies \text{canWin}O11 \\ c50 : ?13 \wedge X22 \wedge X31 &\implies \text{canWin}O13 \end{aligned}$$

$$\begin{aligned} c51 : &\text{canWin}O11 \vee \text{canWin}O12 \vee \text{canWin}O13 \\ &\vee \text{canWin}O21 \vee \text{canWin}O22 \vee \text{canWin}O23 \\ &\vee \text{canWin}O31 \vee \text{canWin}O32 \vee \text{canWin}O33 \implies \text{canWin}O \end{aligned}$$

- Conditions for winning moves by X :

$$\begin{aligned} c52 : \text{canWin}X11 &\implies \text{move}X11 \\ c53 : \text{canWin}X12 &\implies \text{move}X12 \\ c54 : \text{canWin}X13 &\implies \text{move}X13 \\ c55 : \text{canWin}X21 &\implies \text{move}X21 \\ c56 : \text{canWin}X22 &\implies \text{move}X22 \\ c57 : \text{canWin}X23 &\implies \text{move}X23 \\ c58 : \text{canWin}X31 &\implies \text{move}X31 \\ c59 : \text{canWin}X32 &\implies \text{move}X32 \\ c60 : \text{canWin}X33 &\implies \text{move}X33 \end{aligned}$$

- Conditions for forcedMoves by X :

$c61 : \text{canWin}O11 \implies \text{forcedMove}X11$
 $c62 : \text{canWin}O12 \implies \text{forcedMove}X12$
 $c63 : \text{canWin}O13 \implies \text{forcedMove}X13$
 $c64 : \text{canWin}O21 \implies \text{forcedMove}X21$
 $c65 : \text{canWin}O22 \implies \text{forcedMove}X22$
 $c66 : \text{canWin}O23 \implies \text{forcedMove}X23$
 $c67 : \text{canWin}O31 \implies \text{forcedMove}X31$
 $c68 : \text{canWin}O32 \implies \text{forcedMove}X32$
 $c69 : \text{canWin}O33 \implies \text{forcedMove}X33$

$c70 : \neg \text{canWin}X \wedge \text{forcedMove}X11 \implies \text{move}X11$
 $c71 : \neg \text{canWin}X \wedge \text{forcedMove}X12 \implies \text{move}X12$
 $c72 : \neg \text{canWin}X \wedge \text{forcedMove}X13 \implies \text{move}X13$
 $c73 : \neg \text{canWin}X \wedge \text{forcedMove}X21 \implies \text{move}X21$
 $c74 : \neg \text{canWin}X \wedge \text{forcedMove}X22 \implies \text{move}X22$
 $c75 : \neg \text{canWin}X \wedge \text{forcedMove}X23 \implies \text{move}X23$
 $c76 : \neg \text{canWin}X \wedge \text{forcedMove}X31 \implies \text{move}X31$
 $c77 : \neg \text{canWin}X \wedge \text{forcedMove}X32 \implies \text{move}X32$
 $c78 : \neg \text{canWin}X \wedge \text{forcedMove}X33 \implies \text{move}X33$