Homework-2 for CSCE 625

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Question 1.
Translation of the sentences into First Order Logic.

- Tomatoes are either fruit or vegetable:

  $\forall_t \, Tomato(t) \Rightarrow Category(t, Fruit) \vee Category(t, Veggie)$

- Some Mushrooms are poisonous.

  $\exists_m \, Mushroom(m) \wedge Poisonous(m)$

- Define Triangle. (Using the mathematical terms to define the functions here

  $\forall_{x,y,z} \, Line(x) \wedge Line(y) \wedge Line(z) \wedge Greater(x, 0) \wedge Greater(y, 0) \wedge Greater(z, 0) \wedge Greater(Add(x, y), z) \wedge Greater(Add(x, z), y) \wedge Greater(Add(y, z), x) \wedge Connected(x, y) \wedge Connected(y, z) \wedge Connected(z, x) \Rightarrow Triangle(x, y, z)$

- A plant can only produce seeds after it has been pollinated.

  $\forall_p \, Plant(p) \wedge Pollinated(p) \Rightarrow Produces(p, seeds)$

- John's favorite movies are any movie by Stephen King except Cujo.

  $\forall_m \, MovieBy(SK, m) \wedge \neg(m = Cujo) \Rightarrow Favorite(m, John)$

- The winner of a football game is the team that has the most points at the end.

  $\forall_f \, Game(f) \wedge (\exists_t \, Greater(Score(t, Game(f)), Score(Opponent(t), Game(f))) \Rightarrow Winner(Game(f), t)$

- The warning light of a Ford Exporer will be on when its gas tank is more then 90% empty.

  $\forall_v \, Vehicle(v) \wedge Model(v, FE) \wedge GasLevel(v, LessThan(10\%)) \Rightarrow WarningLight(v, ON)$

- Al and Bob bought their computers from the same manufacturer.

  $\exists_m \, Bought(Al, comp1, m) \wedge Bought(Bob, comp2, m)$

- All laptops sold by Dell in 2012 have at least 4 Gigs of Memory.

  $\forall_l \, Laptop(l) \wedge Sold(l, Dell) \wedge SoldIn(l, 2012) \Rightarrow Greater(Memory(l), 4GB)$

Question 2.

The given FOL sentence is:
$\forall_x P(x) \Rightarrow [\forall_y P(y) \Rightarrow P(f(x,y))] \ \wedge \ [\neg\forall_y Q(x,y) \Rightarrow P(y)]$

Steps Involved:

Removing Implications, we get
$\equiv \ \forall_x \neg P(x) \vee \ [\,[\forall_y \neg P(y) \vee P(f(x,y))] \ \wedge \ [\neg\forall_y \neg Q(x,y) \vee P(y)]\,]$

Moving $\neg$ Inwards we get.
$\equiv \ \forall_x \neg P(x) \vee \ [\,[\forall_y \neg P(y) \vee P(f(x,y))] \ \wedge \ [\exists_y Q(x,y) \wedge \neg P(y)]\,]$

Standardize the Variables
$\equiv \ \forall_x \neg P(x) \vee \ [\,[\forall_y \neg P(y) \vee P(f(x,y))] \ \wedge \ [\exists_z Q(x,z) \wedge \neg P(z)]\,]$

Skolemize
$\equiv \ \forall_x \neg P(x) \vee \ [\,[\forall_y \neg P(y) \vee P(f(x,y))] \ \wedge \ [Q(x,F(x)) \wedge \neg P(F(x))]\,]$

Drop Universal Quantifiers
$\equiv \ \neg P(x) \vee \ [\,[\,\neg P(y) \vee P(f(x,y))] \ \wedge \ [Q(x,F(x)) \wedge \neg P(F(x))]\,]$

Distribute $\vee$ over $\wedge$:
$\equiv \ [\neg P(x) \vee \neg P(y) \vee P(f(x,y))] \wedge [\neg P(x) \vee Q(x,F(x))] \wedge [\neg P(x) \vee \neg P(F(x))]$

Question 3. a Translation of the Situation to the First Order Logic: Knowledge Base:

1. $Pompeian(Marcus)$
2. $\forall_p\ Pompeian(p) \Rightarrow Roman(p)$
3. $Ruler(Ceaser)$
4. a. $\forall_p\ Roman(p) \wedge \neg Loyal(p, Ceaser) \Rightarrow Hate(p, Ceaser)$
   b. $\forall_p\ Roman(p) \wedge \neg Hate(p, Ceaser) \Rightarrow Loyal(p, Ceaser)$
5. $\forall_p\ \exists_x\ Loyal(p, x)$
6. $\exists_{p,r}\ \neg Loyal(p, r) \wedge Ruler(r) \wedge TryAssassinate(r, p)$
7. $TryAssassinate(Ceaser, Marcus)$

Question 3. b & c

Query is $Hates(Marcus, Ceaser)$

Proof by Natural deduction:

8. Existential Instantiation on 6, $\theta = \{p/Marcus, r/Ceaser\}$:
   $\neg Loyal(Marcus, Ceaser) \wedge Ruler(Ceaser) \wedge TryAssassinate(Ceaser, Marcus)$

9. And Elimination of 8 using 3 and 7:
   $\neg Loyal(Marcus, Ceaser)$

10. General Modus Ponens on 1 and 2, $\theta = \{p/Marcus\}$:
    $Roman(Marcus)$

11. General Modus Ponens on 4.a and 10, $\theta = \{p/Marcus\}$:
    $Hate(Marcus, Ceaser)$

Question 4a. We use the following binary predicates to formulate Sammy's Sport Shop problem.

- $Observation(b, c)$ refers to the fact that a ball of color $c \in \{W, Y\}$ was observed from box $b \in \{1, 2, 3\}$
- $Labelled(b, c)$ refers to the fact that incorrect label of box $b \in \{1, 2, 3\}$ is $c \in \{W, Y, B\}$
- $Contains(b, c)$ means that the actual contents of box $b \in \{1, 2, 3\}$ is balls of type $c \in \{W, Y, B\}$
- $\forall c_1, c_2 \in \{W, Y, B\}$ $equalColor(c_1, c_2)$ returns 1 if $c_1 == c_2$, 0 otherwise.
- $\forall b_1, b_2 \in \{1, 2, 3\}$ $equalBox(c_1, c_2)$ returns 1 if $b_1 == b_2$, 0 otherwise.

Knowledge Base:

- Constraints on observations and labels and contents:
  1. $\forall b\ Observation(b, Y) \vee Observation(b, W)$
  2. $\forall b\ Labelled(b, W) \vee Labelled(b, Y) \vee Labelled(b, B)$
  3. $\forall b\ Contains(b, W) \vee Contains(b, Y) \vee Contains(b, B)$

- **Uniqueness constraint on the actual contents:**
  4a. $\forall b, c1, c2\ \neg[Contains(b, c_1) \wedge Contains(b, c_2) \wedge \neg equalColor(c_1, c_2)]$
  4b. $\forall b1, b2, c\ \neg[Contains(b1, c) \wedge Contains(b2, c) \wedge \neg equalBox(b_1, b_2)]$

- **Implications from labels:**
  5. $\forall b, c\ Labelled(b, c) \Rightarrow \neg Contains(b, c)$

- **Implications from observation:**
  6a. $\forall b, c\ Observation(b, Y) \Rightarrow Contains(b, W) \vee Contains(b, B)$
  6b. $\forall b, c\ Observation(b, W) \Rightarrow Contains(b, Y) \vee Contains(b, B)$

- **Initial Facts:**
  7. $Observation(1, Y)$
  8. $Observation(2, W)$

9. $Observation(3, Y)$
10. $Labelled(1, W)$
11. $Labelled(2, Y)$
12. $Labelled(3, B)$

Question 4b. Natural Deduction:

- General Modus Ponens on **5 and 10,** $\theta = \{b/1, c/W\}$**:**
  13. $\neg Contains(1, W)$
- **General Modus Ponens on 5 and 11,** $\theta = \{b/2, c/Y\}$**:**
  14. $\neg Contains(2, Y)$
- **General Modus Ponens on 5 and 12,** $\theta = \{b/3, c/B\}$**:**
  15. $\neg Contains(3, B)$
- **General Modus Ponens on 6b and 8,** $\theta = \{b/2\}$**:**
  16. $Contains(2, W) \vee Contains(2, B)$
- **General Modus Ponens on 6b and 7,** $\theta = \{b/1\}$**:**
  17. $Contains(1, Y) \vee Contains(1, B)$
- **General Modus Ponens on 6b and 9,** $\theta = \{b/3\}$**:**
  18. $Contains(3, Y) \vee Contains(3, B)$
- **By resolution on 18 using 15, we get:**
  19. $Contains(3, Y)$
- **General Modus Ponens on 4b ,** $\theta = \{b_1/1, b_2/3, c = Y\}$:
  20. $\neg Contains(1, Y) \vee \neg Contains(3, Y) \vee equalBox(1, 3)$
- **By resolution on 20**
  21. $\neg Contains(1, Y) \vee \neg Contains(3, Y)$
- **By resolution on 21 using 19 we get**
  22. $\neg Contains(1, Y)$
- **Resolution on 3 using 13,22 with** $\theta = \{b/1\}$:
  23. $Contains(1, B)$
- **General Modus Ponens on 4b ,** $\theta = \{b_1/1, b_2/2, c = B\}$:
  24. $\neg Contains(1, B) \vee \neg Contains(2, B) \vee equalBox(1, 2)$
- **By resolution on 24**
  25. $\neg Contains(1, B) \vee \neg Contains(2, B)$
- **By resolution on 25 using 23 we get**
  26. $\neg Contains(2, B)$
- **Resolution on 3 using 14,26 with** $\theta = \{b/2\}$:
  27. $Contains(2, W)$

Question 5. We use the following predicates to formulate the tic-tac-toe problem:

1. $canWin(P, i, j)$ means player P can win the game if he makes a move in row i, column j.
2. $forcedMove(X, i, j)$ means X is forced to move in row i, column j to block O from winning.
3. $Move(X, i, j)$ means the best move for X is to move in row i, column j.
4. $TwoInaRow(P, R)$ means that the row R has two pieces placed by player P
5. $TwoInaColumn(P, C)$ means that the column C has two pieces placed by player P
6. $TwoInaDiagonal(P, R, C)$ means that the diagonal (either main diagonal or off-diagonal) containing the element (R, C) has two pieces placed by player P
7. $equal(a, b)$ means a, b are equal.
8. $unique(a, b, c) \Leftrightarrow \neg equal(a, b) \wedge \neg equal(b, c)$ means a, b, c are unique.
9. $Place(x, i, j)$ refers to an xX, O placed in row i, column j.
10. $Blank(i, j) \Leftrightarrow \neg Place(X, i, j) \wedge \neg Place(O, i, j)$ refers to a blank space in row i, column j.
11. $isOffDiag(R_1, C_1, R_2, C_2, R_3, C_3) \Leftrightarrow unique(R_1, R_2, R_3) \wedge unique(C_1, C_2, C_3) \wedge$ $[\,\text{equal}(\text{R}_1, C_1, 2) \wedge equal(AbsDiff(R_2, C_2), 2) \wedge equal(AbsDiff(R_3, C_3), 2)] \vee$ $[equal(R_2, C_2, 2) \wedge equal(AbsDiff(R_1, C_1), 2) \wedge equal(AbsDiff(R_3, C_3), 2)] \vee$ $[equal(R_3, C_3, 2) \wedge equal(AbsDiff(R_1, C_1), 2) \wedge equal(AbsDiff(R_2, C_2), 2)]$
12. $isDiag(R_1, C_1, R_2, C_2, R_3, C_3) \Leftrightarrow isMainDiag(R_1, C_1, R_2, C_2, R_3, C_3) \vee isOffDiag(R_1, C_1, R_2, C_2, R_3, C_3)$
13. $isMainDiag(R_1, C_1, R_2, C_2, R_3, C_3) \Leftrightarrow unique(R_1, R_2, R_3) \wedge unique(C_1, C_2, C_3) \wedge$ $[equal(R_1, C_1) \wedge equal(R_2, C_2) \wedge equal(R_3, C_3)]$

Knowledge Base:

A. Sentences to get the concepts:

1. $\forall P, R \;\; \{\exists C_1, C_2, C_3 \;\; Place(P, R, C_1) \wedge Place(P, R, C_2) \wedge Blank(R, C_3) \wedge unique(C_1, C_2, C_3)\} \Rightarrow TwoInaRow(P, R)$

2. $\forall P, C \;\; \{\exists R_1, R_2, R_3 \;\; Place(P, R_1, C) \wedge Place(P, R_2, C) \wedge Blank(R_3, C) \wedge unique(R_1, R_2, R_3)\} \Rightarrow TwoInaCol(P, C)$

3. $\forall P, R, C \quad \{\exists R_1, C_1, R_2, C_2 \quad Place(P, R_1, C_1) \wedge Place(P, R_2, C_2) \wedge Blank(R, C) \wedge isDiag(R, C, R_1, C_1, R_2, C_2)\} \Rightarrow TwoInaDiag(P, R, C)$

B. Conditions for $canWin$:

4. $\forall P, R, C \;\; TwoInaRow(P, R) \vee TwoInaCol(P, C) \vee TwoInaDiag(P, R, C) \Rightarrow canWin(P, R, C)$

5. $\exists R, C \quad canWin(X, R, C) \wedge canWinX$

C. Conditions for $forcedMove$:

6. $\forall R, C \quad \neg canWinX \wedge canWin(O, R, C) \Rightarrow forcedMove(X, R, C)$

D. Conditions for $move$:

7. $\forall R, C \quad canWin(X, R, C) \vee forcedMove(X, R, C) \Rightarrow Move(X, R, C)$