CSCE 629-601 Homework 8
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1. Exercise 29.1-4, on Page 857

Change the Given Linear Program to standard form

Given: Minimize:

$$2x_1 + 7x_2 + x_3 \tag{1}$$

Subject to

$$\begin{aligned} x_1 - x_3 &= 7 \\ 3x_1 + x_2 &\geq 24 \\ x_2 &\geq 0 \\ x_3 &\leq 0 \end{aligned}$$

The given Linear program can be converted to following standard form using the rules presented in the book, like min to max and then all the equalities to be in less than equal to form. Giving the direct equations here as writing in latex is so troublesome.

And we change the variables here:

$$x_3 = x_6 - x_7 \tag{2}$$

$$x_1 = x_4 - x_5 \tag{3}$$

Maximize:

$$-2x_1 - 7x_2 + x_6 - x_7 \tag{4}$$

Subject to

$$\begin{aligned} x_4 - x_5 - x_6 + x_7 &\leq 7 \\ -x_4 + x_5 + x_6 - x_7 &\leq -7 \\ -3x_4 + 3x_5 - x_2 &\geq -24 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7 &\geq 0 \end{aligned}$$

2. Exercise 29.1-5 , Textbook page 858.

Change the Given Linear Program to Slack form

Given: Maximize:

$$2x_1 - 6x_3 \tag{5}$$

Subject to

$$\begin{aligned} x_1 + x_2 - x_3 &\leq 7 \\ 3x_1 - x_2 &\geq 8 \\ -x_1 + 2x_2 + 2x_3 &\geq 0 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

The given Linear program can be converted to following Slack form using the rules presented in the book, like min to max and then all the equalities to be in less than equal to form. Giving the direct equations here as writing in latex is so troublesome.

The Slack Form is :

$$Z = 2x_1 - 6x_3 \tag{6}$$

Subject to

$$\begin{aligned} x_4 &= 7 - x_1 - x_2 + x_3 \\ x_5 &= -8 + 3x_1 - x_2 \\ x_6 &= 0 + 2x_2 + 2x_3 - x_1 \\ x_1, x_2, x_3&, x_4, x_5, x_6 \geq 0 \end{aligned}$$

We could have skipped the last line as it is implicitly implied from the slack form.
Basic variables are x_4, x_5, x_6
Non-Basic variables are x_1, x_2, x_3

3. Exercise 29.2-1 , Textbook page 863.

INPUT: Given a weighted directed graph $G = (V, E)$ and equations from the text book.
OUTPUT: Convert the single pair shortest path problem into standard form linear program.

Solution $(a.)$ **:** We have the equations from the textbook as:
Maximize:

$$dt \tag{7}$$

Subject to

$$dv \leq du + w(u, v) \tag{8}$$

$$ds = 0 \tag{9}$$

Now we have this linear program and we want to change the same into the standard form. First we will change the equation 1. As we have two variables and constant value.

$$dv - du \leq w(u, v) \text{ for each edge } (u, v) \in E$$

Now since the weights of the edges can also be negative, so we have can have distances as negative as well. Hence we have to change variables so that all the variables will have positive values/greater than zero values. So the equation we get is:

$$dv' - dv'' - du' - du'' \leq w(u, v) \text{ for each edge } (u, v) \in E$$

Also, since the final value can also be negative so we have to change the first equation as well.

$$Z = dt' - dt''$$

Now coming to the Equation 3. we have an equality sign and we have to change the same to less than or equal to.

$$\begin{aligned} ds &\leq 0 \\ -ds &\leq 0 \end{aligned}$$

So putting all these Equations into one standard form we get:
Maximize:

$$Z = dt' - dt'' \tag{10}$$

Subject to

$$dv' - dv'' - du' - du'' \leq w(u, v) \text{ for each edge } (u, v) \in E \tag{11}$$

$$ds \leq 0 \tag{12}$$

$$-ds \leq 0 \tag{13}$$

$$dv', dv'', du', du'', ds, dt', dt'' \geq 0 \tag{14}$$

Hence we have our standard form

4. Exercise 29.2-7 , Textbook page 864.

Minimum Cost Multi Commodity Flow Problem into the Linear program.
Here we have to minimize the cost of the flow which is also feasible. Since when we are putting the constraints, then the minimize cost by using the constraints.
The equations can be put like this:
Minimize

$$Z = \sum_{u,v \in V} \left(a\left(u, v\right) \sum_{i=1}^{k} f_i\left(u, v\right) \right) \tag{15}$$

Subject to

$$\sum_{i=1}^{k} f_i\left(u, v\right) \leq c\left(u, v\right) \text{ for each } u, v \in V \tag{16}$$

$$\sum_{v \in V} f_{i,s_i,v} - \sum_{v \in V} f_{i,v,s_i} \leq d_i \text{ for each } i = 1, 2, 3, \ldots, k \tag{17}$$

$$-\sum_{v \in V} f_{i,s_i,v} + \sum_{v \in V} f_{i,v,s_i} \leq -d_i \text{ for each } i = 1, 2, 3, \ldots, k \tag{18}$$

$$\sum_{v \in V} f_i\left(u, v\right) - \sum_{v \in V} f_i\left(u, v\right) \leq 0 \text{ for each } u \in V - \{s_i, t_i\} \text{ and for each } i = 1, 2, 3, \ldots, k \tag{19}$$

$$-\sum_{v \in V} f_i\left(u, v\right) + \sum_{v \in V} f_i\left(u, v\right) \leq 0 \text{ for each } u \in V - \{s_i, t_i\} \text{ and for each } i = 1, 2, 3, \ldots, k \tag{20}$$

$$f_i\left(u, v\right) \geq 0 \text{ for each } u, v \in V \text{ and for each } i = 1, 2, 3, \ldots, k \tag{21}$$

Hence we have the formulated the solution in terms of Linear Program.