Reg No.:_______________ Name:__________________________

02000MAT206052101

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth Semester B.Tech Degree Examination July 2021 (2019 Scheme)

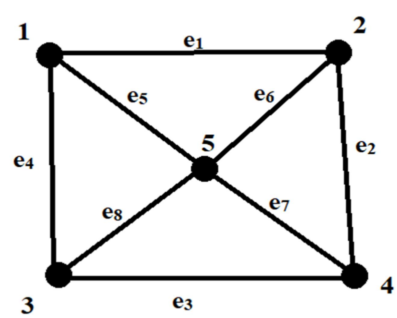
Course Code: MAT206
Course Name: GRAPH THEORY

Max. Marks: 100 Duration: 3 Hours

PART A

(Answer all questions; each question carries 3 marks) Marks

1	What is the maximum number of edges in a simple graph with n vertices? Justify your answer.	3
2	There are 25 telephones in Metropolis. Is it possible to connect them with wires so that each telephone is connected with exactly 7 others? Why?	3
3	Show that all vertices of an Euler graph G are of even degree	3
4	Explain strongly connected and weakly connected graphs with the help of examples.	3
5	Prove that a connected graph G with n vertices and n-1 edges is a tree.	3
6	How many labelled trees are there with n vertices? Draw all labelled trees with 3 vertices.	3
7	Define planar graphs. Is K_4, the complete graph with 4 vertices, a planar graph? Justify.	3
8	Define fundamental circuits and fundamental cut-sets.	3
9	Construct the adjacency matrix and incidence matrix of the graph .	3



10	Define chromatic number. What is the chromatic number of a tree with two or more vertices?	3

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11 a) Define complete graph and complete bipartite graph. Draw a graph which is a complete graph as well as a complete bipartite graph. 7

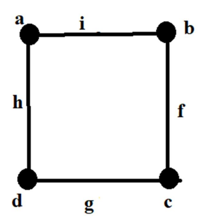
b) Explain walks, paths and circuits with the help of examples. 7

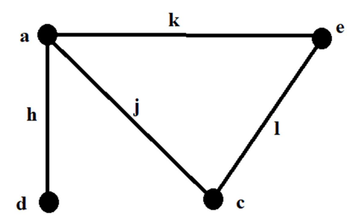
12 a) Define isolated vertex, pendant vertex, even vertex and odd vertex. Draw a graph that contains all the above. 7

b) Prove that simple graph with n vertices and k components can have at most (n-k)(n-k+1)/2 edges. 7

Module -2

13 a) 9





Find the union, intersection and ring sum of the above graphs.

b) State travelling salesman problem. How it is related to Hamiltonian circuits? 5

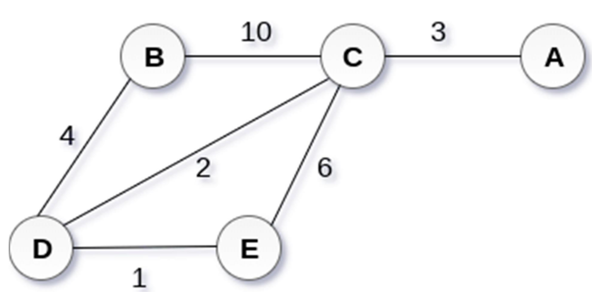
14 a) Prove that in a complete graph with n vertices there are (n-1)/2 edge disjoint Hamiltonian circuits, if n is an odd number and $n \geq 3$. 7

b) For which values of m, n is the complete graph $K_{m,n}$ an Euler graph ? Justify your answer. 7

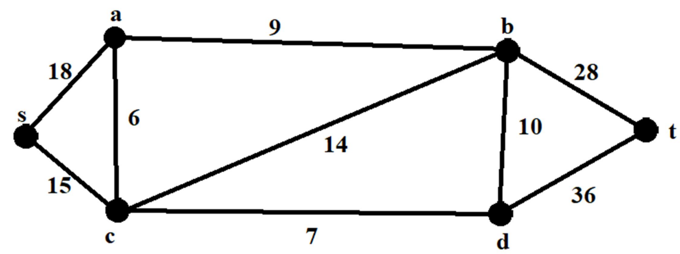
Module -3

15 a) Prove that a binary tree with n vertices has (n+1)/2 pendant vertices. 7

b) Using Prims algorithm, find a minimal spanning tree for the following graph. 7



16 a) Write down Dijkstra's algorithm and use it to find the shortest path from s to t. 9

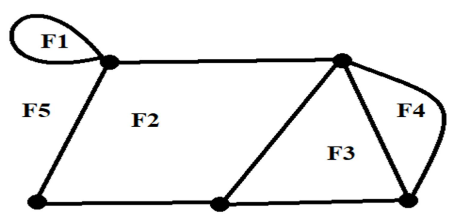


b) Prove that every tree has either one or two centers. 5

Module -4

17 a) Define cut-set. Prove that every circuit in G has an even number of edges in common with any cut-set. 8

b) Construct the geometric dual of the graph below 6



18 a) Prove that a connected planar graph with n vertices and e edges has e-n+2 regions. 9

b) Let G be a connected graph and e an edge of G. Show that e is a cut-edge if and only if e belongs to every spanning tree. 5

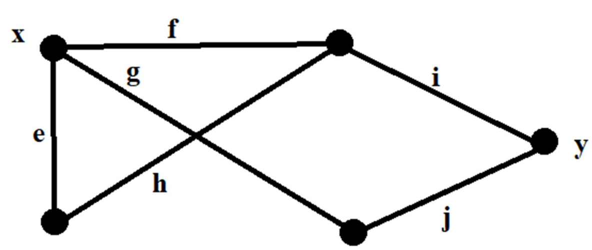
Module -5

19 a) Explain *four colour problem* using the concept of chromatic number. 5

b) Let B and A be the circuit matrix and the incidence matrix of a graph G which is free from loops, whose columns are arranged using the same order of edges. Show that $AB^T = BA^T = 0 \pmod 2$. 9

20 a) Show that chromatic polynomial of a tree with n vertices is $P_n(\lambda) = \lambda(\lambda-1)^{n-1}$ 7

b) Define path matrix of a graph. Find the path matrix P(x, y) for the graph below. 7



*****