

Reg No.: _____

Name: _____

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIFTH SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2017**

Course Code: CS309

Course Name: GRAPH THEORY AND COMBINATORICS (CS)

Max. Marks: 100

Duration: 3 Hours

PART A*Answer all questions, each carries 3 marks.*

Marks

1) Consider a graph G with 4 vertices: v₁, v₂, v₃ and v₄ and the degrees of vertices are 3, 5, 2 and 1 respectively. Is it possible to construct such a graph G? If not, why?

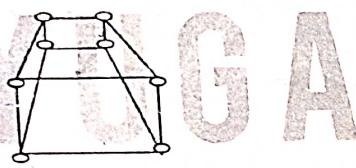
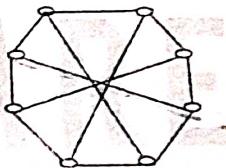
2) Draw a disconnected simple graph G₁ with 10 vertices and 4 components and also calculate the maximum number of edges possible in G₁.

3) State Dirac's theorem for hamiltonicity and why it is not a necessary condition for a simple graph to have a Hamiltonian circuit.

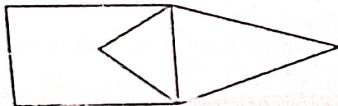
4) Differentiate between symmetric and asymmetric digraphs with examples and draw a complete symmetric digraph of four vertices.

PART B*Answer any two full questions, each carries 9 marks.*

- 5) a) What are the basic conditions to be satisfied for two graphs to be isomorphic? (6)
 Are the two graphs below isomorphic? Explain with valid reasons



- b) Write any two applications of graphs with sufficient explanation (3)
 c) Consider the graph G given below: (4)



- d) Define Euler graph. Is G an Euler? If yes, write an Euler line from G. (5)
 e) What is the necessary and sufficient condition for a graph to be Euler? And also, prove it. (5)
 f) a) Define Hamiltonian circuits and paths with examples. Find out the number of edge-disjoint Hamiltonian circuits possible in a complete graph with five vertices (5)
 b) State Travelling-Salesman Problem and how TSP solution is related with Hamiltonian Circuits? (4)

PART C*Answer all questions, each carries 3 marks.*

- g) List down any two properties of trees and also prove the theorem: A graph is a tree if and only if it is a minimally connected. (3)

E
Reg No.: _____

E5840



APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIFTH SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: CS309

Course Name: GRAPH THEORY AND COMBINATORICS

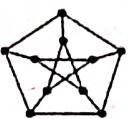
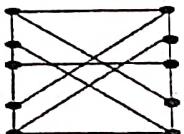
Max. Marks: 100

Duration: 3 Hours

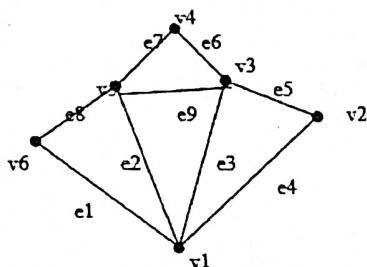
PART A

Answer all questions, each carries 3 marks

- 1 Define isomorphism between two graphs. Are the following graphs are isomorphic to each other? Justify your answer. (3)



- 2 For the following graph, find the shortest path between from v_1 to v_4 . Also find a Euler circuit. (3)



- 3 Define the following with example. (3)
i) Isomorphic digraph ii) Complete symmetric digraph
- 4 Define Hamiltonian graph. Find an example of a non-Hamiltonian graph with a Hamiltonian path. (3)

PART B

Answer any two full questions, each carries 9 marks

- 5 a) For a Eulerian graph G, prove the following properties. (6)
i) The degree of each vertex of G is even. ii) G is an edge-disjoint union of cycles.
- b) Discuss the Konigsberg Bridge problem. Is there any solution to the problem? (3)
Justify your answer.
- 6 a) Prove that a simple graph with n vertices must be connected, if it has more than $(n-1)(n-2)/2$ edges. (6)
- b) 19 students in a nursery school play a game each day, where they hold hands to form a circle. For how many days can they do this, with no students holding hands with the same playmates more than once? Substantiate your answer with graph theoretic concepts. (3)

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIFTH SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: CS309

Course Name: GRAPH THEORY AND COMBINATORICS

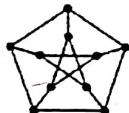
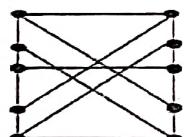
Max. Marks: 100

Duration: 3 Hours

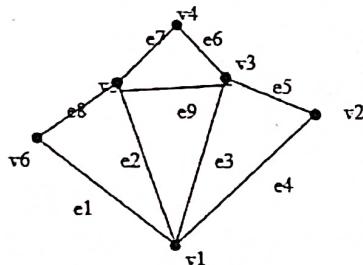
PART A

Answer all questions, each carries 3 marks

- 1 Define isomorphism between two graphs. Are the following graphs isomorphic to each other? Justify your answer. (3)



- 2 For the following graph, find the shortest path from v1 to v4. Also find a Euler circuit. (3)



- 3 Define the following with example. (3)
i) Isomorphic graph ii) Complete symmetric digraph
- 4 Define Hamiltonian graph. Find an example of a non-Hamiltonian graph with a Hamiltonian path. (3)

PART B

Answer any two full questions, each carries 9 marks

- 5 a) For a Eulerian graph G, prove the following properties. (6)
i) The degree of each vertex of G is even. ii) G is an edge-disjoint union of cycles.
b) Discuss the Konigsberg Bridge problem. Is there any solution to the problem? (3)
Justify your answer.
- 6 a) Prove that a simple graph with n vertices must be connected, if it has more than $(n-1)(n-2)/2$ edges. (6)
b) 19 students in a nursery school play a game each day, where they hold hands to form a circle. For how many days can they do this, with no students holding hands with the same playmates more than once? Substantiate your answer with graph theoretic concepts. (3)

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
V SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018

Course Code: CS309

Course Name: GRAPH THEORY AND COMBINATORICS

Duration: 3 Hours

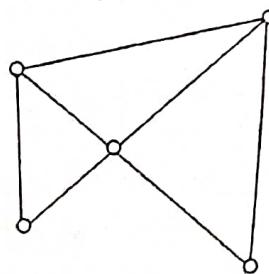
Max. Marks: 100

PART A

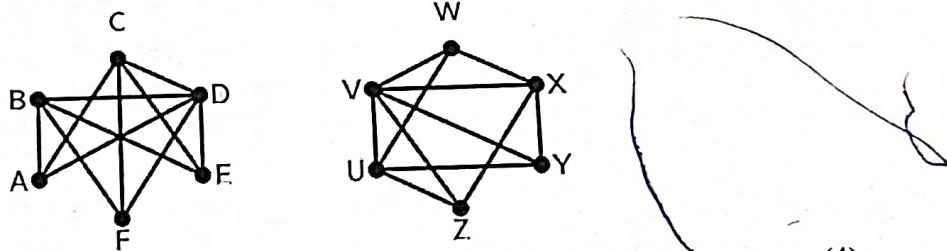
Marks

Answer all questions, each carries 3 marks.

- 1 Prove that the number of vertices of odd degree in a graph is always even (3)
- 2 Show that in a simple graph with n vertices, the maximum number of edges is $\frac{n(n-1)}{2}$ and the maximum degree of any vertex is $n-1$. (3)
- 3 Differentiate between complete symmetric and complete asymmetric graph with an example each. (3)
- 4 State Dirac's Theorem and check its applicability in the following graph, G (3)

**PART B***Answer any two full questions, each carries 9 marks.*

- 5 a) Define isomorphism between graphs? Are the two graphs below isomorphic? (5)
 Justify



- b) Consider a complete graph G with 11 vertices.
 1. Find the maximum number of edges possible in G.
 2. Find the number of edge-disjoint Hamiltonian circuits in G.
- 6 a) A connected graph G is an Euler graph if and only if all vertices of G are of even degree. Prove the statement. (6)
- b) There are 37 telephones in the city of Istanbul, Turkey. Is it possible to connect them with wires so that each telephone is connected with exactly 7 others? Substantiate your answer with graph concepts. (3)
- 7 a) Give any two applications of graphs. Explain. (2)

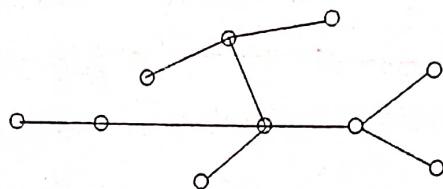
- b) Define Hamiltonian circuit. Give an example. What general class of graphs is guaranteed to have a Hamiltonian circuit? Also draw a graph that has a Hamiltonian path but not a Hamiltonian circuit. (4)
- c) Prove that if a connected graph G is decomposed into two subgraphs g_1 and g_2 , there must be at least one vertex common between g_1 and g_2 . (3)

PART C

Answer all questions, each carries 3 marks.

8 Prove that the distance between vertices of a connected graph is a metric. (3)

9 i) Find the eccentricity of all vertices in G given below and also mark the center of G (3)



ii) Find the number of possible labelled trees that can be constructed with 50 vertices.

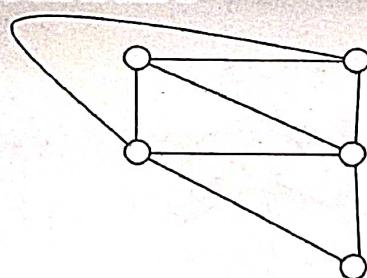
10 Draw the two simplest non-planar graphs and also mention their properties. (3)

11 What is the necessary and sufficient condition for two graphs to be duals of each other? Prove. (3)

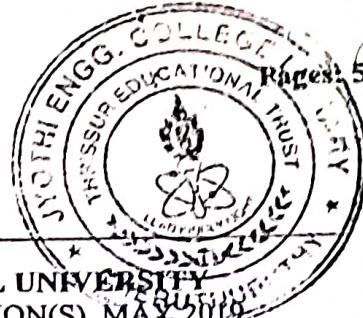
PART D

Answer any two full questions, each carries 9 marks.

12 a) Draw the geometric dual (G^*) of G given and also write about the relationship between a planar graph G and its dual G^* (6)



- 13 b) Define rooted binary tree with an example (3)
- a) Find the number of edges and vertices of a graph G if its rank and nullity are 6 and 8 respectively (2)
- b) Prove the statement, "Every circuit has an even number of edges in common with any cut-set" (4)
- c) Consider a binary tree with four weighted pendant vertices. Let their weights be 0.5, 0.12, 0.3 and 0.11. Construct a binary tree with minimum weighted path length. (3)
- 14 a) Define cut sets with an example. Give an application of finding cut-sets or edge (4)



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**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
V SEMESTER B.TECH DEGREE EXAMINATION(S), MAY 2010**

Course Code: CS309

Course Name: GRAPH THEORY AND COMBINATORICS

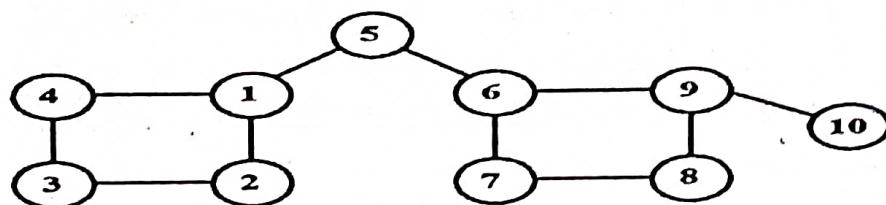
Max. Marks: 100

Duration: 3 Hours

PART A*Answer all questions, each carries 3 marks.*

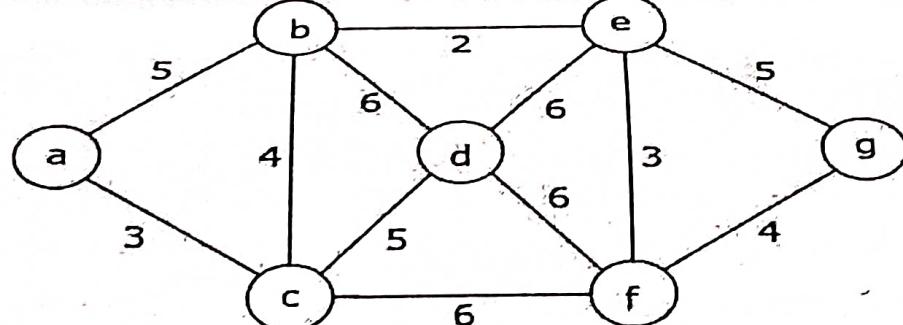
Marks

- 1 Print a Walk, trail, path and cycle on the graph below. 3



- 2 Define pendant vertex, isolated vertex and null graph with an example each. 3

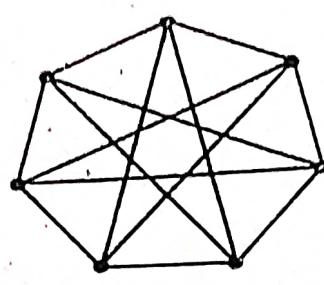
- 3 State travelling salesman problem. Print a travelling salesman's tour on the graph below. 3



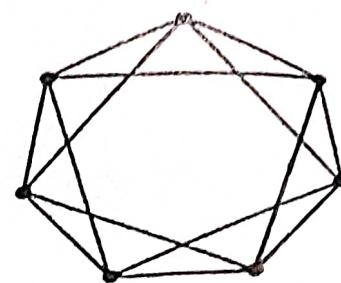
- 4 Prove Dirac's theorem for Hamiltonicity. 3

PART B*Answer any two full questions, each carries 9 marks.*

- 5 a) Define isomorphism of graphs. Show that the graphs (a) and (b) are isomorphic. 4



(a)



(b)

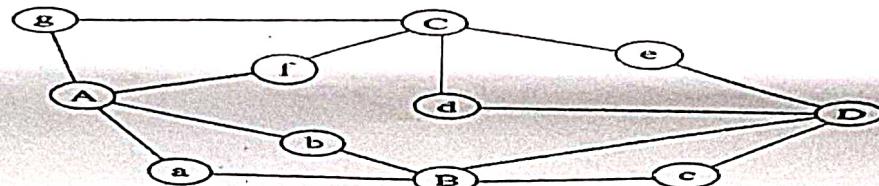
b) Define subgraph. Give two subgraphs of the above graph.(Fig. a)

c) Consider a complete graph G with 11 vertices.

1. Find the maximum number of edges possible in G

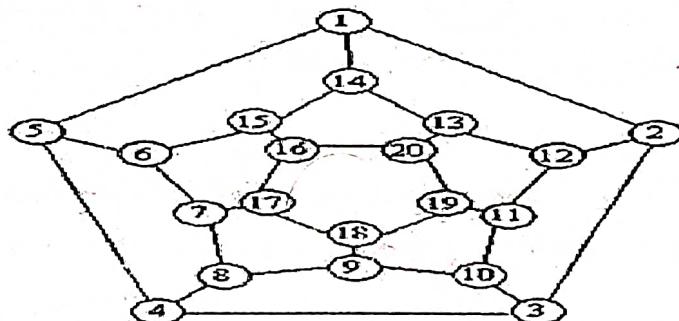
2. Find the number of edge-disjoint Hamiltonian circuits in G

- 6 a) Draw a simple disconnected graph with 10 vertices, 4 components and maximum number of edges.
- b) Explain any two applications of graphs.
- c) Check whether the given graph is an Euler graph and if yes, give the Euler line. Justify your answer.



- 7 a) Prove or disprove: If every vertex of a simple graph G has degree 2, then G is a cycle.

- b) Give Hamiltonian circuit of the following graph.



- c) In a graph G let p_1 and p_2 be two different paths between two given vertices. Prove that ringsum of p_1 and p_2 is a circuit or a set of circuits.

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fifth Semester B.Tech Degree Regular and Supplementary Examination December 2020

Course Code: CS309

Course Name: GRAPH THEORY AND COMBINATORICS

Duration: 3 Hours

Max. Marks: 100

PART A*Answer all questions, each carries 3 marks.*

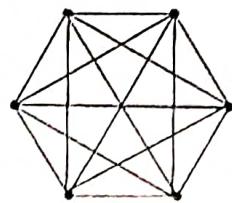
Marks

- 1) Prove that the number of vertices of odd degree in a graph is always even. (3)
- 2) 11 children in a nursery school stand next to each other in circle such that they have different friends each time. How much such different arrangements are there? Justify your answer. (3)
- 3) What is the number of vertices in an undirected connected graph with 27 edges, 6 vertices of degree 2, 3 vertices of degree 4 and remaining of degree 3? (3)
- 4) Find the number of distinct hamiltonian cycles (not edge disjoint) possible in a complete graph of n vertices, where $n \geq 3$. (3)

PART B*Answer any two full questions, each carries 9 marks.*

- 5) a) If graph G has 8 vertices and it is Eulerian (has Euler Circuit), then find the maximum number of edges in the Graph G . (4)
- b) Prove that "A connected graph is Eulerian if and only if every vertex has even degree". (5)
- 6) a) State and prove Dirac's theorem for hamiltonicity. (5)
- b) Consider an undirected graph G with 100 nodes. Find the minimum number of edges to be included in G so that the graph G is guaranteed to be connected. (4)

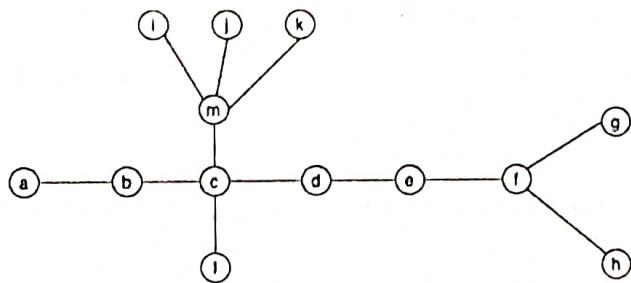
- 7) a) There are 20 people in a room. Suppose some pairs of the people shake hands and some don't. As the people leave the room you (who were not in the room) ask each person whether they shook hands an odd number of times or an even number of times. Prove that the number of people who answer "odd" is an even number. (4)
- b) Label the vertices and edges in the graph given below, find and separately draw any four different Hamiltonian circuits contained in the given graph. (Note: The Hamiltonian circuits need not be edge-disjoint). (5)



PART C

Answer all questions, each carries 3 marks.

- 8 Find the number of spanning trees in a complete graph of 4 labelled vertices. (3)
- 9 Prove the statement, "A graph with n vertices, $n-1$ edges and no circuits is connected". (3)
- 10 Prove that "Every cut set in a connected graph G must contain at least one branch of every spanning tree of G ". (3)
- 11 Find the center and radius and of the tree shown below: (3)



PART D

Answer any two full questions, each carries 9 marks.

- 12 a) Prove that a graph is a tree if and only if it is loop-less and has exactly one spanning tree (4)

301

Course Code: MAT206
Course Name: GRAPH THEORY

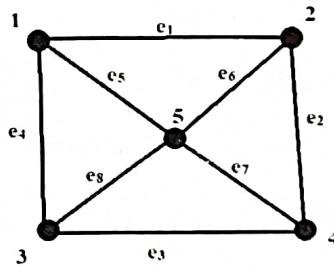
Max. Marks: 100

Duration: 3 Hours

PART A*(Answer all questions; each question carries 3 marks)*

Marks

- | | | |
|---|--|---|
| 1 | What is the maximum number of edges in a simple graph with n vertices?
Justify your answer. | 3 |
| 2 | There are 25 telephones in Metropolis. Is it possible to connect them with wires so that each telephone is connected with exactly 7 others? Why? | 3 |
| 3 | Show that all vertices of an Euler graph G are of even degree | 3 |
| 4 | Explain strongly connected and weakly connected graphs with the help of examples. | 3 |
| 5 | Prove that a connected graph G with n vertices and $n-1$ edges is a tree. | 3 |
| 6 | How many labelled trees are there with n vertices? Draw all labelled trees with 3 vertices. | 3 |
| 7 | Define planar graphs. Is K_4 , the complete graph with 4 vertices, a planar graph?
Justify. | 3 |
| 8 | Define fundamental circuits and fundamental cut-sets. | 3 |
| 9 | Construct the adjacency matrix and incidence matrix of the graph . | 3 |



- | | | |
|----|--|---|
| 10 | Define chromatic number. What is the chromatic number of a tree with two or more vertices? | 3 |
|----|--|---|

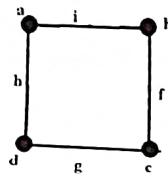
PART B

(Answer one full question from each module, each question carries 14 marks)

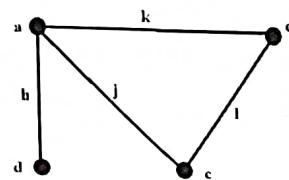
Module -1

- 11 a) Define complete graph and complete bipartite graph. Draw a graph which is a complete graph as well as a complete bipartite graph. 7
- b) Explain walks, paths and circuits with the help of examples. 7
- 12 a) Define isolated vertex, pendant vertex, even vertex and odd vertex. Draw a graph that contains all the above. 7
- b) Prove that simple graph with n vertices and k components can have at most $(n-k)(n-k+1)/2$ edges. 7

13 a)



Module -2



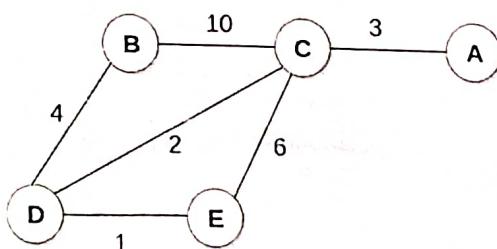
9

Find the union, intersection and ring sum of the above graphs.

- 14 b) State travelling salesman problem. How it is related to Hamiltonian circuits? 5
- a) Prove that in a complete graph with n vertices there are $(n-1)/2$ edge disjoint Hamiltonian circuits, if n is an odd number and $n \geq 3$. 7
- b) For which values of m, n is the complete graph $K_{m,n}$ an Euler graph? Justify your answer. 7

Module -3

- 15 a) Prove that a binary tree with n vertices has $(n+1)/2$ pendant vertices. 7
- b) Using Prims algorithm, find a minimal spanning tree for the following graph. 7



- 16 a) Write down Dijkstra's algorithm and use it to find the shortest path from s to t. 9

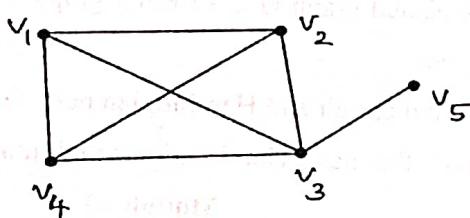
Reg No.: _____

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
 Fourth Semester B.Tech Degree Examination June 2022 (2019 scheme)

Course Code: MAT206**Course Name: GRAPH THEORY****Max. Marks: 100****Duration: 3 Hours****PART A***(Answer all questions; each question carries 3 marks)***Marks**

- | | | |
|---|--|---|
| 1 | Prove that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$. | 3 |
| 2 | Define walk, path and circuit with examples. | 3 |
| 3 | Draw a graph which is Eulerian but not Hamiltonian | 3 |
| 4 | Distinguish between strongly connected digraphs and weakly connected graphs with examples. | 3 |
| 5 | Prove that there is one and only one path between every pair of vertices in a tree. | 3 |
| 6 | Draw all unlabelled trees with 5 vertices. | 3 |
| 7 | Prove that the edge connectivity of a graph cannot exceed the degree of the vertex with the smallest degree in G . | 3 |
| 8 | Define planar graph and non-planar graph with examples. | 3 |
| 9 | Write the adjacency matrix for the following graph. | 3 |

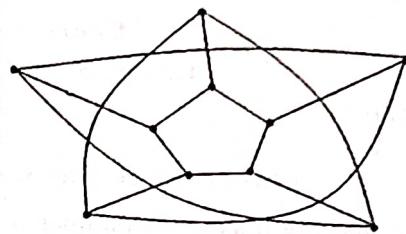
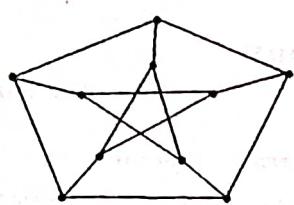


- | | | |
|----|---|---|
| 10 | Prove that the chromatic polynomial of a complete graph with 4 vertices is $\lambda(\lambda - 1)(\lambda - 2)(\lambda - 3)$. | 3 |
|----|---|---|

PART B*(Answer one full question from each module, each question carries 14 marks)***Module -1**

- | | | |
|--------|---|---|
| 11 (a) | Prove that the number of vertices of odd degree in a graph is always even | 7 |
|--------|---|---|

- b) If a connected graph G is decomposed into two subgraphs g_1 and g_2 , then prove that there must be at least one vertex common between g_1 and g_2 7
- 12 a) Determine whether the following graphs are isomorphic or not.



- b) If a graph has exactly two vertices of odd degree, then prove that there must be a path joining these two vertices. 7

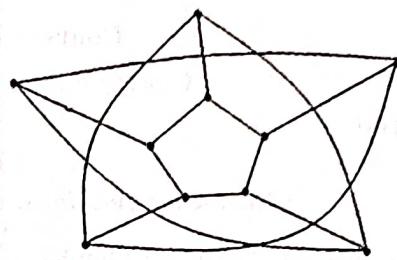
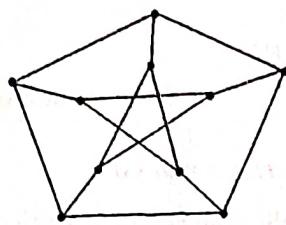
Module -2

- 13 a) In a complete graph with n vertices, prove that there are $\frac{n-1}{2}$ edge-disjoint Hamiltonian circuits, if n is an odd number ≥ 3 . 7
- b) 1) For a binary relation "is greater than" on the set $X = \{3, 4, 7, 5, 8\}$ 7
- i) Draw the digraph representing the above relation
 - ii) Write its relation matrix
- 2) Define equivalence digraph with an example
- 14 a) Prove that a connected graph G is an Euler graph if and only if all vertices of G are of even degree. 7
- b) Define Hamiltonian circuit and Hamiltonian path. Give an example for each. 7
Also draw a graph that has a Hamiltonian path but not a Hamiltonian circuit.

Module -3

- 15 a) Prove that every tree has either one or two centers 7
- b) Apply Kruskal's algorithm to find the minimal spanning tree for the following weighted graph. 7

- b) If a connected graph G is decomposed into two subgraphs g_1 and g_2 , then prove that there must be at least one vertex common between g_1 and g_2
- 12 a) Determine whether the following graphs are isomorphic or not.



- b) If a graph has exactly two vertices of odd degree, then prove that there must be a path joining these two vertices.

Module -2

- 13 a) In a complete graph with n vertices, prove that there are $\frac{n-1}{2}$ edge-disjoint Hamiltonian circuits, if n is an odd number ≥ 3 .
- b) i) For a binary relation "is greater than" on the set $X = \{3, 4, 7, 5, 8\}$
 ii) Draw the digraph representing the above relation
 iii) Write its relation matrix
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Module -3

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