# QUESTION BANK
# MAT 206 - GRAPH THEORY

| MODULE 1 | | | |
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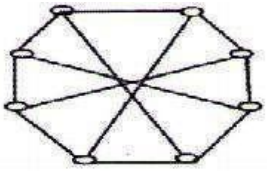
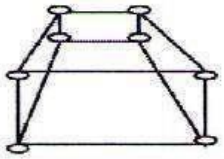
| **Sl. No** | **Questions** | **Marks** | **KTU/KU Month/Year** |
| 1 | Consider a graph G with 4 vertices: vl, v2, v3 and v4 and the degrees of vertices are 3,5,2 and 1 respectively.Is it possible to construct such a graph G? If not, why? | 3 | DEC17, DEC19 |
| 2 | Draw a disconnected simple graph Gl with 10 vertices and 4 components and also calculate the maximum number of edges possible in Gl. | 3 | DEC17 |
| 3 | What are the basic conditions to be satisfied for two graphs to be isomorphic? Are the two graphs below isomorphic? Explain with valid reasons.   | 6 | DEC17, DEC19 |
| 4 | Write any two applications of graphs with sufficient explanation. | 3 | DEC17 |
| 5 | Prove that the number of vertices of odd degree in a graph is always even. | 3 | DEC18 |
| 6 | Show that in a simple graph with n vertices, the maximum number of edges is n(n-l)/2 and the maximum degree of any vertex is n-1. | 3 | DEC18, SEP20 |
| 7 | Define isomorphism between graphs? Are the two graphs below isomorphic? Justify | 5 | DEC18 |

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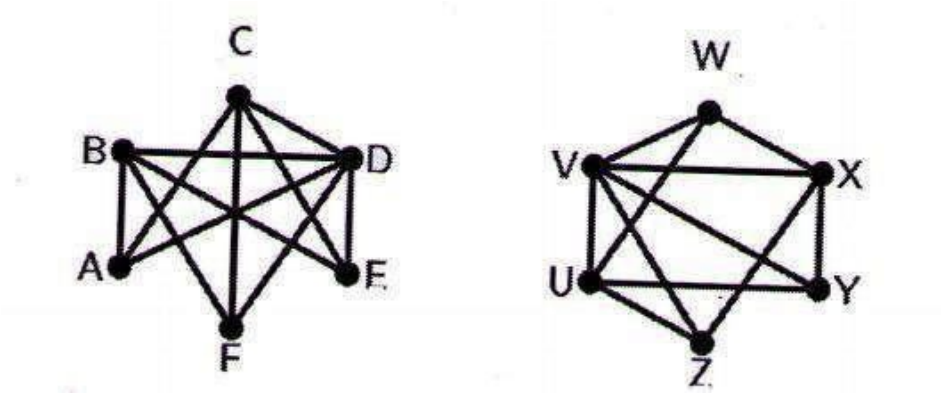
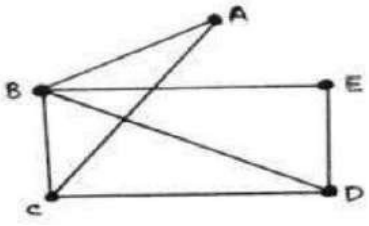
| |  | | |
| 8 | Prove that a simple with n vertices and K components can have atmost (n-k)(n-k+1)/2 edges. | 3 | SEP2020, JULY 2021 |
| 9 | If a connected graph G is decomposed into two subgraphs $g_1$ and $g_2$ , then prove that there must be at least one vertex common between $g_1$ and $g_2$. | 3 | DEC18, JUNE 2022 |
| 10 | Write a note on Konigsberg Bridge Problem | 3 | SEP2020 |
| 11 | A graph has exactly 10 vertices, 4 vertices of degree 3, 4 vertices of degree 2 and 2 isolated vertices. How many edges does the graph have? | 3 | Model Question |
| 12 | 19 students in a nursery school play a game each day, where they hold hands to form a circle. For how many days can they do this, with no students holding hands with the same playmates more than once? Substantiate your answer with graph theoretic concepts. | 4 | Model Question |
| 13 | Differentiate walk, path and circuit. | 3 | SEP2020 |
| 14 | Using the graph classify each sequence as a walk,a path or a circuit<br>1. E→C→D→E<br>2. A→C→D→E→B→A<br>3. B→D→E→B→C<br>4. A→B→C→D→B→A<br>  | 4.5 | SEP2020 |

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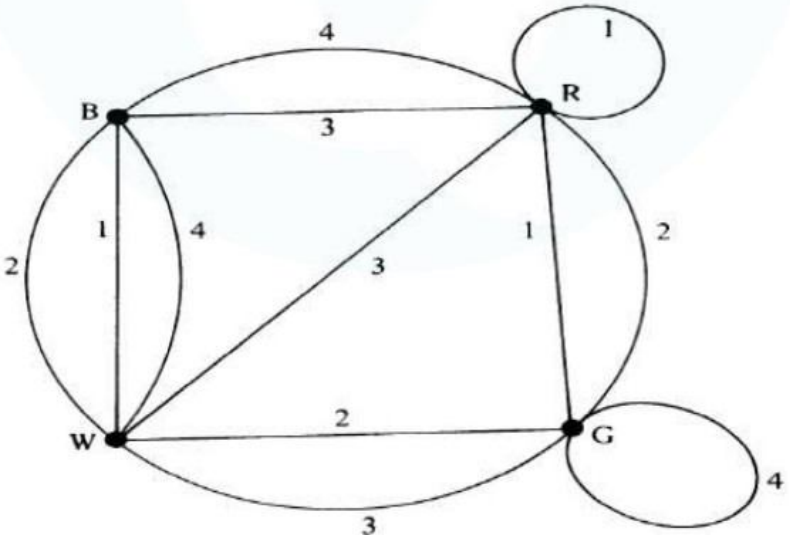
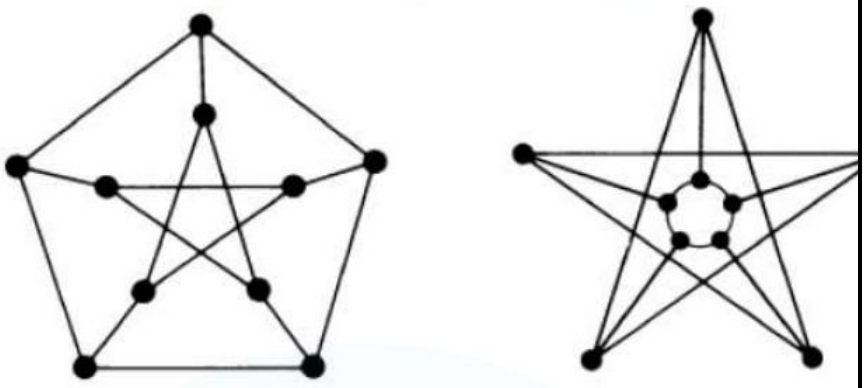
| 15 | a) Define subgraphs.What are edge disjoint and vertex disjoint subgraphs? Construc two edge disjoint subgraphs of the graph G.<br>  <br> | 4 | SEP 2020 |
| 16 | b) Check whether the two graphs are isomorphic or not. Justify your answer.<br>  | 7 | SEP2020, JUNE 2022 |
| 17 | Is it possible to have simple graphs with the following degree sequences? If yes, draw the graphs<br>a)2,3,3,3,3,3,4,5<br>b)1,3,3,4,5,6,6<br>c)1,2,3,3,4,5, | 5 | SEP2020 |
| 18 | Prove that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$<br>. | 3 | June 2022 |

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| 19 | Define walk, path and circuit with examples. | 3 | June 2022, July 2021 |
| 20 | Prove that the number of vertices of odd degree in a graph is always even. | 7 | June 2022 |
| 21 | If a graph has exactly two vertices of odd degree, then prove that there must be a path joining these two vertices. | 7 | June 2022 |
| 22 | What is the maximum number of edges in a simple graph with n vertices? Justify your answer. | 3 | JULY 2021 |
| 23 | There are 25 telephones in Metropolis. Is it possible to connect them with wires so that each telephone is connected with exactly 7 others? Why? | 3 | JULY 2021 |
| 24 | Define complete graph and complete bipartite graph. Draw a graph which is a complete graph as well as a complete bipartite graph. | 7 | JULY 2021 |
| 25 | Define isolated vertex, pendant vertex, even vertex and odd vertex. Draw a graph that contains all the above. | 7 | JULY 2021 |

**MODULE 2**

| Sl. No | Questions | Marks | KTU/KU Month/Year |
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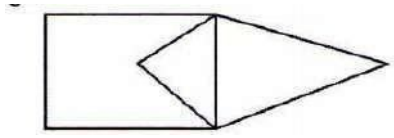
| 1 | Consider the graph G given below:  Define Euler graph. Is G an Euler? If yes, write an Euler line from G | 3 | DEC17 |
| 2 | What is the necessary and sufficient condition for a graph to be Euler? And also prove it. | 5 | DEC17 |
| 3 | Define Hamiltonian circuits and paths with examples. Find out thenumber of edge-disjoint Hamiltonian circuits possible in a completegraphwithfivevertices. | 5 | DEC 17SEP2020 |
| 4 | StateTravelling-Sales man Problem and how TSP solution is related with Hamiltonian Circuits? | 5 | SEP2020, JULY 2021 |
| 5 | Define Hamiltonian circuits and path with examples. Find out the number of edge disjoint Hamiltonian circuits possible in a complete graph with five vertices. | 3 | SEP2020 |

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| 6 | Differentiate between symmetric and asymmetric digraphs with examples and draw a complete symmetric digraph of four vertices. | 4 | DEC19 |
| 7 | Differentiate between complete symmetric and complete asymmetric graph with an example each. | 4 | DEC17 |
| 8 | Consider a complete graph G with l1 vertices.<br>a) Find the maximum number of edges possible in G.<br>b) Find the number of edge-disjoint Hamiltonian circuits in G | 4 | DEC18 |
| 9 | A connected graph G is a Euler graph if and only if it can be decomposed into circuits. | 6 | DEC18 |
| 10 | The total number of different ,not edge disjoint, Hamiltonian circuits in a complete graph of n vertices is (n-1)!/2. Prove. | 6 | DEC19 |
| 11 | Explain digraphs and binary relation on digraphs. | 4 | SEP2020 |
| 12 | Explain arbitrarily traceable graphs with suitable examples. | 4 | SEP2020 |
| 13 | Draw a graph which is Eulerian but not Hamiltonian | 3 | June 2022 |
| 14 | Distinguish between strongly connected digraphs and weakly connected graphs with examples. | 3 | June 2022 |
| 15 | In a complete graph with n vertices, prove that there are $\frac{n-1}{2}$ edge-disjoint Hamiltonian circuits, if n is an odd number $\geq 3$. | 7 | June 2022, July 2021 |
| 16 | 1)For a binary relation "is greater than" on the set X= {3.4,7,5,8}<br>i) Draw the digraph representing the above relation<br>ii) Write its relation matrix<br>2)Define equivalence digraph with an example | 7 | June 2022 |

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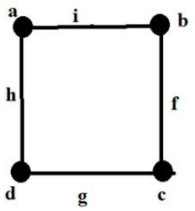
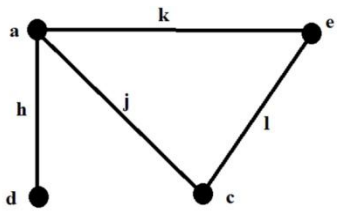
| 17 | Prove that a connected graph G is an Euler graph if and only if all vertices of G are of even degree. | 7 | June 2022 |
| 18 | Define Hamiltonian circuit and Hamiltonian path. Give an example for each. Also draw a graph that has a Hamiltonian path but not a Hamiltonian circuit. | 7 | June 2022 |
| 19 | Show that all vertices of an Euler graph G are of even degree | 3 | July 2021 |
| 20 | Explain strongly connected and weakly connected graphs with the help of examples. | 3 | July 2021 |
| 21 |   Find the union, intersection and ring sum of the above graphs. | 9 | July 2021 |
| 22 | For which values of m, n is the complete graph Km,n an Euler graph ? Justify your answer. | 7 | July 2021 |

## MODULE 3

| Sl. No | Questions | Marks | KTU/KU Month/Year |
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| 1 | Find the number of possible labeled trees that can be constructed with 50 vertices. | 2 | DEC18 |
| 2 | Consider a binary tree with four weighted pendent vertices. Let their weights be 0.5, 0.12, 0.13 and 0.11. Construct a binary tree with minimum weighted path length. | 3 | DEC18 |

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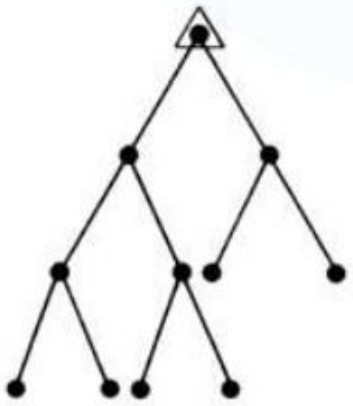
| 3 | Define spanning tree. Show that the edges forming a spanning tree in a planar graph G correspond to the edges forming a set of chords in the dual G*. | 5 | DEC18 |
| 4 | Draw the flow chart of the spanning tree algorithm and also clearly mark the five conditions to be tested in connection with the spanning tree construction in the flowchart. | 6 | DEC18 |
| 5 | Prove that all trees will have either one or more centers. | 3 | Model Question |
| 6 | What is the eccentricity of a node?How is it used in finding the center of a graph? Explain with examples. | 3 | Model Question |
| 7 | Show that a connected graph of n vertices and n edges has n-1tree branches and e-n+1 chords. | 5 | Model Question |
| 8 | Find the number of edges and vertices of a graph G if its rank and nullity are 6 and 8 respectively. Define rank and nullity of a graph G. | 6 | Model Question |
| 9 | Define a rooted binary tree with an example. Draw all trees of n labeled vertices forn = 3 and n=4. | 5 | SEP2020 |
| 10 | Define rooted binary tree. Find the path length of the following tre  | 4 | SEP2020 |

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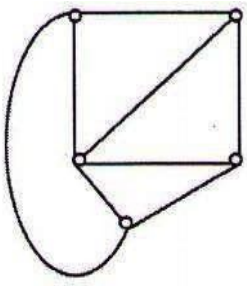
| 11 | Prove that a tree with n vertices has n-1edges. | 5 | Model Question |
| 12 | Prove that the distance between vertices of a connected graph is a metric. | 5 | Model Question |
| 13 | Write an algorithm for finding the shortest spanning tree(Kruskalalgorithm) | 6 | Model Question |
| 14 | List down any two properties of trees and also prove the theorem: A graph is a tree if and only if it is minimally connected. | 6 | Model Question |
| 15 | Let G= (V,E) be a connected graph, and let T:=(V, S) be a spanning tree of G. Let e = (a, b) be an edge of G not in T. Prove that, for any edge f on the path from a to b in T, (V, (Su{e}) -{f}) is another spanning tree for G. | 4 | DEC17 |
| 16 | Define spanning trees. Consider the graph G given below and obtainany three spanning trees from G. Calculate the number of distinct spanning trees possible from acomplete graph with n vertices.  | 5 | DEC17 |
| 17 | Prove that there is one and only one path between every pair of vertices in a tree. | 3 | June 2022 |
| 18 | Draw all unlabelled trees with 5 vertices. | 3 | June 2022 |
| 19 | Prove that every tree has either one or two centers | 7 | June 2022, July 2021 |

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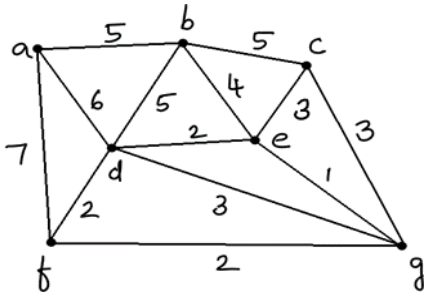
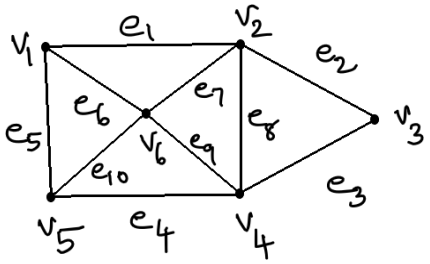
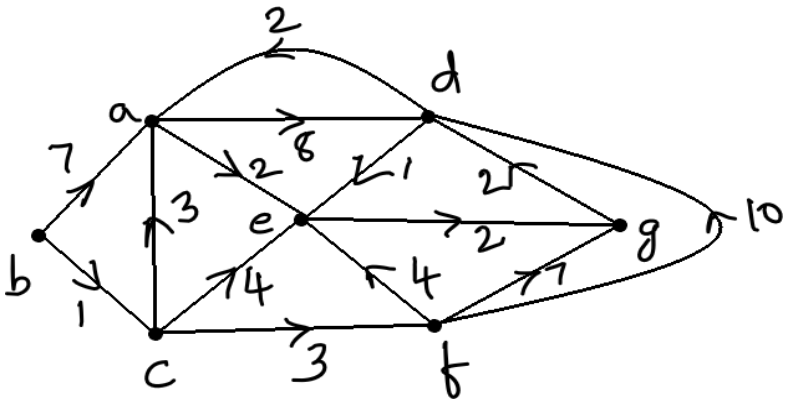
| 20 | Apply Kruskal's algorithm to find the minimal spanning tree for the following weighted graph.<br>  | 7 | June 2022 |
| 21 | For any spanning tree of a connected graph with n vertices and e edges, prove that there are n-1 tree branches and e-n+1 chords. For the following graph find two spanning trees and hence show that an edge that is a branch of one spanning treecan be a chord with respect to another spanning tree of same graph.<br>  | 7 | June 2022 |
| 22 | Use Dijkstra's algorithm to find the shortest path for the following weighted digraph and find the shortest distance from vertex a to other vertices.<br>  | 7 | June 2022,<br>July 2021 |

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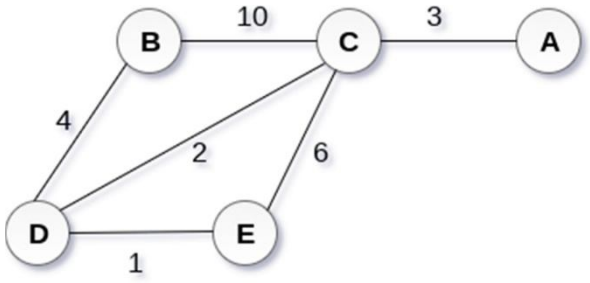
| 23 | Prove that a connected graph G with n vertices and n-1 edges is a tree. | 3 | July 2021 |
| 24 | How many labelled trees are there with n vertices? Draw all labelled trees with 3 vertices. | 3 | July 2021 |
| 25 | Prove that a binary tree with n vertices has (n+1)/2 pendant vertices. | 7 | July 2021 |
| 26 | Using Prims algorithm, find a minimal spanning tree for the following graph.<br>  | 7 | July 2021 |

## MODULE 4

| Sl. No | Questions | Marks | KTU/KU Month/Year |
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| 1 | Prove the statement: Every cut set in a connected graph G must also contain atleast one branch of every spanning tree of G | 3 | DEC17 |
| 2 | List down the properties stating the relationship between the edges of graph G and its dual G | 3 | DEC17 |
| 3 | Define a cutset. Find any four sets from graph G given below and also | 5 | DEC17 |

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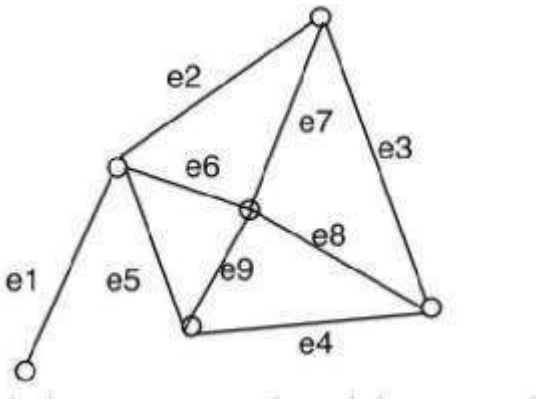
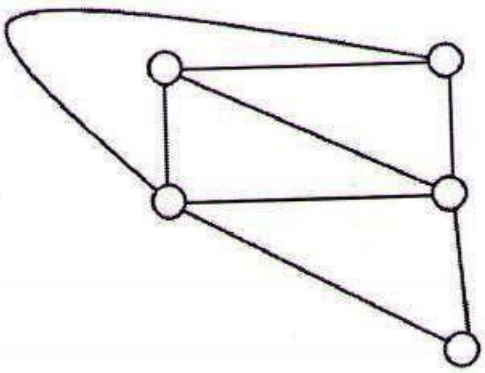
| | Find the edge connectivity of G.<br>  <br> | | |
| 4 | Draw two Kuratowski's graphs and also prove that Kuratowask's first the graph is nonplanar using appropriate inequality. | 4 | DEC17 |
| 5 | Draw a geometric dual(G*) of G given and also write about the relationship between a planar graph G and its dual G*<br>  | 6 | DEC18 |
| 6 | Prove the statement "Every circuit has an even number of edges in common with any cut-set". | 4 | DEC18 |
| 7 | Define edge connectivity and vertex connectivity. | 3 | Model Question |
| 8 | Show that a vertex v in a connected graph G is a cut vertex if there exist two vertices x and y in such that every path between x and y passes through v. | 5 | Model Question |
| 9 | State and prove Cayley's theorem. | 9 | Model Question |
| 10 | Prove that : A graph has a dual if and only if it is planar. | 7 | Model Question |

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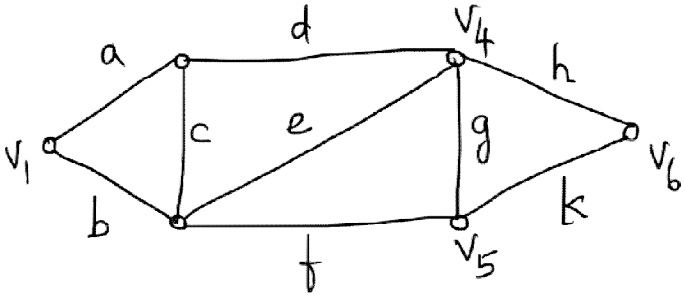
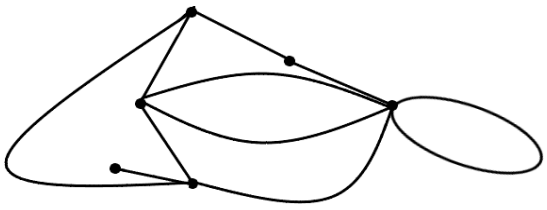
| 11 | Prove that: The complete graph of five vertices is nonplanar. | 6 | Model Question |
| 12 | Write a short note on Connectivity and separability. | 5 | Model Question |
| 13 | Prove that the edge connectivity of a graph cannot exceed the degree of the vertex with the smallest degree in G. | 3 | June 2022 |
| 14 | Define planar graph and non-planar graph with examples. | 3 | June 2022 |
| 15 | Illustrate the statement: "The ring sum of any two cut-sets in a graph is either a third cut-set or an edge disjoint union of cut-sets", in the following graph.<br>  | 7 | June 2022 |
| 16 | Define edge connectivity, vertex connectivity separable and non-separable graph.<br>Give an example for each. | 7 | June 2022 |
| 17 | Prove that the complete graph on 5 vertices is non-planar | 7 | June 2022 |
| 18 | Draw the geometric dual of the following graph<br>  | 7 | June 2022 |
| 19 | Define planar graphs. Is K4, the complete graph with 4 vertices, a planar graph? Justify | 3 | July 2021 |
| 20 | Define fundamental circuits and fundamental cut-sets. | 3 | July 2021 |

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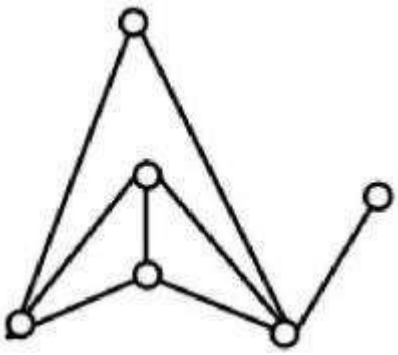
| 21 | Define cut-set. Prove that every circuit in G has an even number of edges in common with any cut-set. | 8 | July 2021 |
| 22 | Construct the geometric dual of the graph below | 6 | July 2021 |
| 23 | Prove that a connected planar graph with n vertices and e edges has e-n+2 regions. | 9 | July 2021 |
| 24 | Let G be a connected graph and e an edge of G. Show that e is a cut-edge if and only if e belongs to every spanning tree. | 5 | July 2021 |

## MODULE 5

| Sl. No | Questions | Marks | KTU/KU Month/Year |
|---|---|---|---|
| 1 | Derive the relationship between incidence matrix, fundamental circuit matrix and fundamental cut-set matrix representations of a graph. | 10 | Model Question |
| 2 | Write any two matrix representations of a graph. | 10 | Model Question |
| 3 | Prove that if B is a circuit matrix of a connected graph G with e edges and n vertices, rank of B = e-n+1 | 5 | Model Question |
| 4 | List down any four properties of the adjacency matrix. | 4 | DEC18 |
| 5 | Construct an adjacency matrix(x) for the following graph and also mention how the concept of edge sequences is described with $X^3$.  | 6 | DEC18 |

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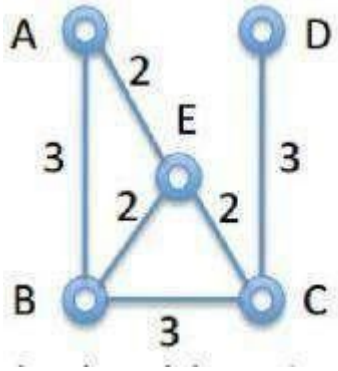
| 6 | Define vertex connectivity and Edge connectivity. | 4 | Model Question |
| 7 | Two graphs Gl and G2 are isomorphic if and only if their incidence matrices A(Gl) and A(G2) differ only by permutation of rows and Columns | 6 | DEC17 |
| 8 | Let A and B be, respectively, the circuit matrix and incidence matrix of a self-loop-free graph G. Prove that $AxB^T=0(mod2)$ | 4 | DEC17 |
| 9 | Explain circuit matrix and its properties. | 5 | Model Question |
| 10 | Let B and A be the circuit matrix and incidence matrix whose columns are arranged using the same order of edges. Show that every row of B is orthogonal to every row A. | 5 | Model Question |
| 11 | Write the properties of the incidence matrix. | 5 | Model Question |
| 12 | Explain cutest matrix and path matrix. | 5 | Model Question |
| 13 | Define adjacency matrix and construct a graph from thefollowing<br>adjacency matrix: $\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$ | 4 | DEC17 |
| |  | | |

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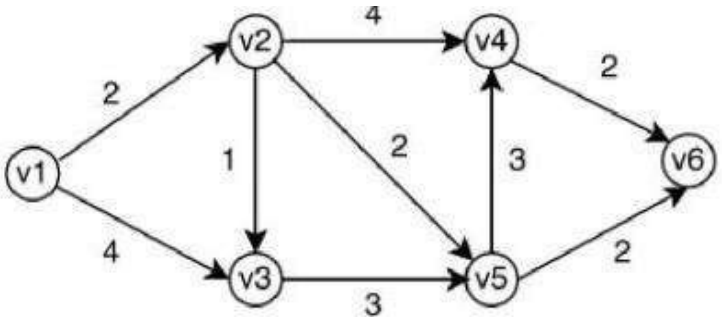
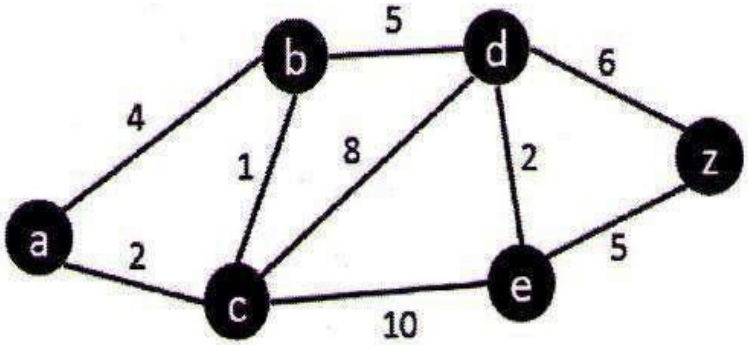
| 14 | Write the Dijkstra's shortest path algorithm. Apply this algorithm to find the shortes tpath between v1and v6.<br>  | 10 | DEC17 |
| 15 | Write the Dijkstra's Shortest Path Algorithm and apply this algorithm to find the shortest path between a and z.<br>  | 6 | DEC17 |
| 16 | Write an algorithm to find the connectedness and components of a graph and analyze the complexity of the algorithm | 10 | Model Question |
| 17 | Explain Floydwarshall algorithm with suitable examples. | 10 | SEP2020 |
| 18 | Prove that the chromatic polynomial of a complete graph with 4 vertices is $\lambda(\lambda-1)(\lambda-2)(\lambda-3)$. | 3 | June 2022 |
| 19 | For the following graph find the<br>i. Incidence matrix<br>ii. Path matrix between v2 and v5<br>iii. Circuit matrix | 7 | June 2022 |

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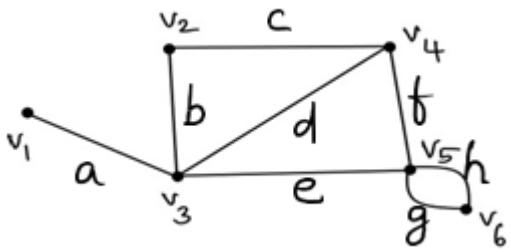
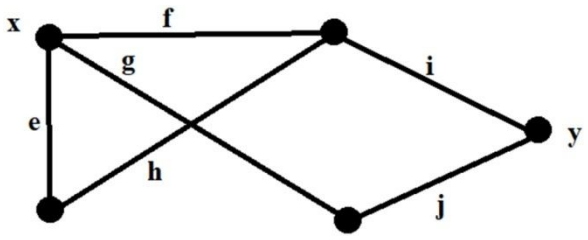
|---|---|---|---|
| |  | | |
| 20 | Draw a connected graph and show that the rank of its incidence matrix is one less<br>than the number of vertices. | 7 | June 2022 |
| 21 | Prove that every tree with two or more vertices is 2-chromatic | 7 | June 2022 |
| 22 | Prove that a covering g of a graph is minimal if and only if g contains no path of length three or more. | 7 | June 2022 |
| 23 | Construct the adjacency matrix and incidence matrix of the graph | 3 | July 2021 |
| 24 | Define chromatic number. What is the chromatic number of a tree with two or more vertices? | 3 | July 2021 |
| 25 | Explain four colour problem using the concept of chromatic number. | 5 | July 2021 |
| 26 | Let B and A be the circuit matrix and the incidence matrix of a graph G which is free from loops, whose columns are arranged using the same order of edges. Show that ABT=BAT=0 (mod 2). | 9 | July 2021 |
| 27 | Show that chromatic polynomial of a tree with n vertices is<br>$P_n(\lambda) = \lambda(\lambda - 1)^{n-1}$ | 7 | July 2021 |
| 28 | Define path matrix of a graph. Find the path matrix P(x, y) for the graph below.<br>  | 7 | July 2021 |

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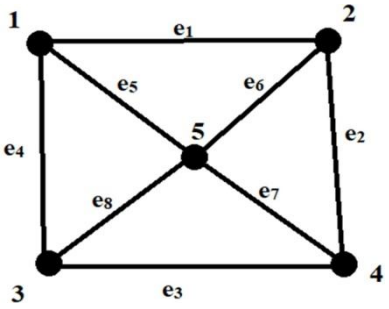
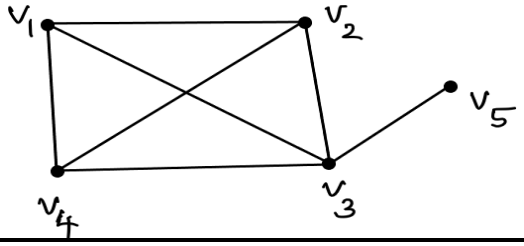
|---|---|---|---|
| 29 | Construct the adjacency matrix and incidence matrix of the graph .<br>  <br> | 3 | July 2021 |
| 30 | Write the adjacency matrix for the following graph.<br>  <br> | 3 | June 2022 |
| 31 | Write an algorithm for the shortest path between all pairs of vertices | 7 | SEP2020 |
| 32 | Write an algorithm for Depth-first search on a graph | 9 | Model Question |
| 33 | Explain planarity with examples. | 5 | SEP20 |