Reg No.: _______________ Name: _______________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth Semester B.Tech Degree Supplementary Examination June 2023 (2019 Scheme)

Course Code: MAT 206

Course Name: GRAPH THEORY

Max. Marks: 100 Duration: 3 Hours

PART A

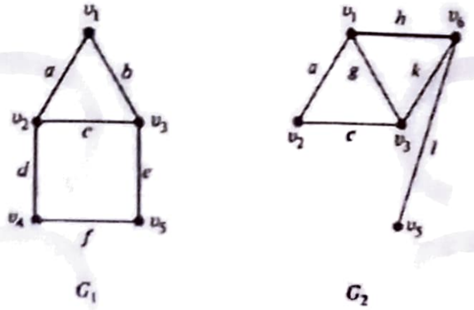
(Answer all questions; each question carries 3 marks) Marks

1 Prove that sum of the degrees of all vertices in G is twice the number of edges in G. 3

2 Define pendant vertex, isolated vertex, and null graph with an example. 3

3 Define Ring Sum of G_1 *and* G_2. 3

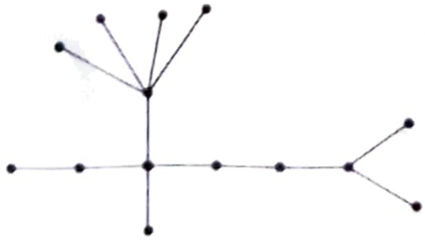
Find the ring sum of the following graphs



4 Define arbitrarily traceable graphs. Give an example. 3

5 Find the maximum number of vertices possible in a 3-level binary tree. Also find the maximum height possible in a binary tree with 11 vertices. 3

6 Label the vertices of the following graph with their eccentricities and hence find the diameter and centre of the graph. 3



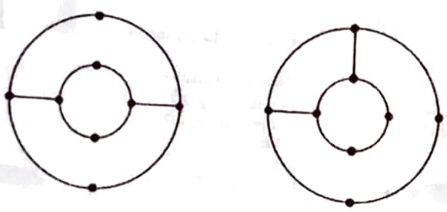
7		Show that every cut set in a connected graph must contain at least one branch of every spanning tree of G.	3
8		Show that K_5 is not planar.	3
9		Define Incidence Matrix. List four properties of it.	3
10		Show that every tree with two or more vertices is 2-chromatic.	3

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11	a)	Show that a simple graph with **n** vertices must be connected if it has more than $\frac{(n-1)(n-2)}{2}$ edges.	7
	b)	Are the following graphs isomorphic? Justify your answer.	7



12	a)	A simple graph with **n** vertices and **k** components can have at most $(n-k)(n-k+1)/2$ edges.	7
	b)	Show that in any group of two or more people, there are two always with exactly the same numbers of friends inside the group.	7

Module -2

13	a)	Prove that a given connected graph is an Euler graph if and only if all vertices of the graph are of even degree.	7
	b)	Prove that a connected graph is Euler if and only if it can be decomposed into circuits.	7
14	a)	Nine members of a new club meet each day for lunch at a round table. They decide to sit such that every member has different neighbours at each lunch. How many days can this arrangement last? Justify your answer	7

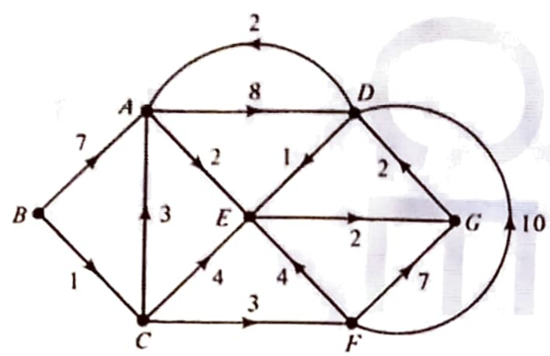
b) If G is a simple graph with **n** vertices and $d(v) \geq n/2$ for each v, then G is Hamiltonian. 7

Module -3

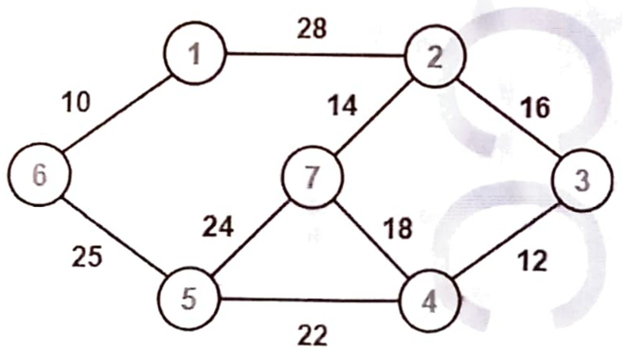
15 a) Show that a Tree with **n** vertices has exactly **n-1** edges. 7

b) Prove that every tree has either one or two centres. 7

16 a) Find the shortest distance from **B** to **G** using Dijkstra's Algorithm. 7



b) Find the minimum spanning tree using Prim's algorithm. 7



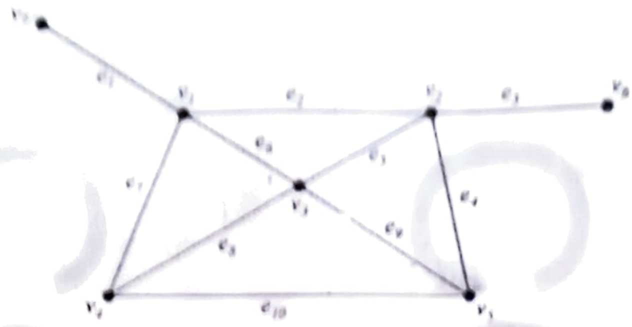
Module -4

17 a) A connected planar graph G with n vertices and e number of edges has $f = e - n + 2$ regions or faces. 7

b) For any graph G, prove that $vertex\ connectivity \leq edge\ connectivity \leq \frac{2e}{n}$ 7

18 a) Every circuit has an even number of edges in common with any cut set. 7

b) Find a spanning tree and hence find all fundamental cut sets, associated with it, of the following graph. 7

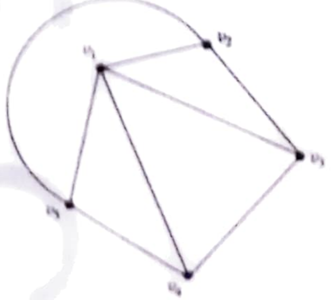


Module-5

19 a) If d_{max} is the maximum degree of vertices in a graph, then show that 7

$$chromatic\ number\ of\ G \leq d_{max} + 1$$

b) Find the chromatic polynomial of the following graph. 7



20 a) If A(G) is is an incidence matrix of a connected graph G with n vertices, then show that rank of A(G) is n-1. 7

b) Check whether the graph having following adjacency matrix is connected or not. 7

$$X = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

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