Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth Semester B.Tech Degree Examination June 2022 (2019 scheme)

Course Code: MAT206

Course Name: GRAPH THEORY

Max. Marks: 100 Duration: 3 Hours

PART A

(Answer all questions; each question carries 3 marks) Marks

1 Prove that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$. 3

2 Define walk, path and circuit with examples. 3

3 Draw a graph which is Eulerian but not Hamiltonian 3

4 Distinguish between strongly connected digraphs and weakly connected graphs with examples. 3

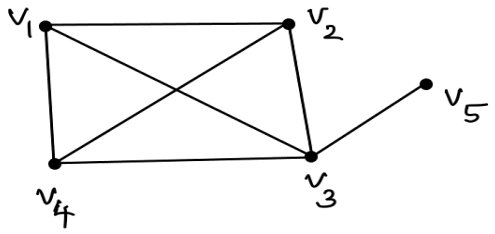
5 Prove that there is one and only one path between every pair of vertices in a tree. 3

6 Draw all unlabelled trees with 5 vertices. 3

7 Prove that the edge connectivity of a graph cannot exceed the degree of the vertex with the smallest degree in G. 3

8 Define planar graph and non-planar graph with examples. 3

9 Write the adjacency matrix for the following graph. 3



10 Prove that the chromatic polynomial of a complete graph with 4 vertices is $\lambda(\lambda-1)(\lambda-2)(\lambda-3)$. 3

PART B

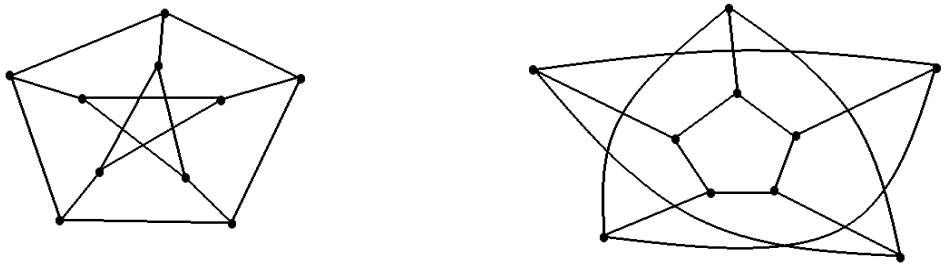
(Answer one full question from each module, each question carries 14 marks)

Module -1

11 a) Prove that the number of vertices of odd degree in a graph is always even 7

b) If a connected graph G is decomposed into two subgraphs g_1 and g_2, then prove that there must be at least one vertex common between g_1 and g_2 7

12 a) Determine whether the following graphs are isomorphic or not. 7



b) If a graph has exactly two vertices of odd degree, then prove that there must be a path joining these two vertices. 7

Module -2

13 a) In a complete graph with n vertices, prove that there are $\frac{n-1}{2}$ edge-disjoint Hamiltonian circuits, if n is an odd number ≥ 3. 7

b) 1)For a binary relation "is greater than" on the set X= {3.4,7,5,8} 7

i) Draw the digraph representing the above relation

ii) Write its relation matrix

2) Define equivalence digraph with an example

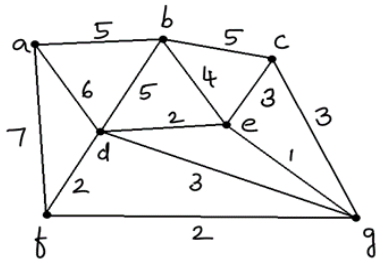
14 a) Prove that a connected graph G is an Euler graph if and only if all vertices of G are of even degree. 7

b) Define Hamiltonian circuit and Hamiltonian path. Give an example for each. Also draw a graph that has a Hamiltonian path but not a Hamiltonian circuit. 7

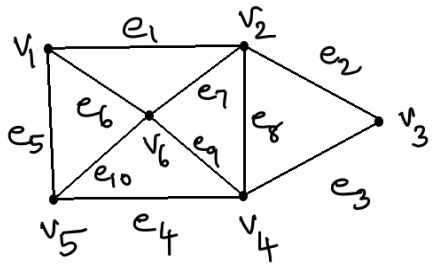
Module -3

15 a) Prove that every tree has either one or two centers 7

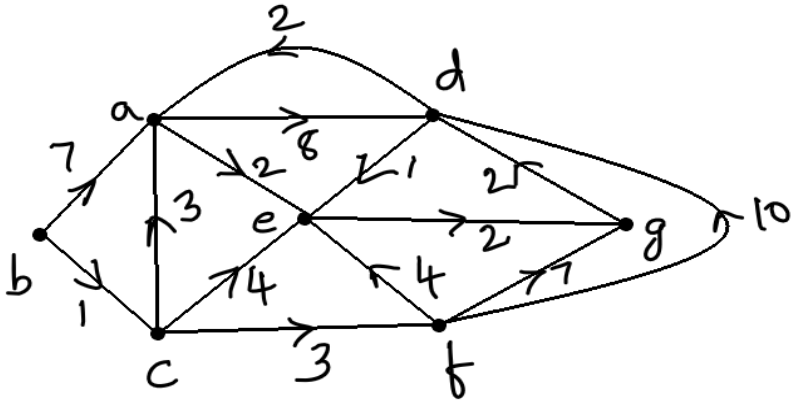
b) Apply Kruskal's algorithm to find the minimal spanning tree for the following weighted graph. 7



16 a) For any spanning tree of a connected graph with n vertices and e edges, prove that there are n-1 tree branches and e-n+1 chords. For the following graph find two spanning trees and hence show that an edge that is a branch of one spanning tree can be a chord with respect to another spanning tree of same graph. 7

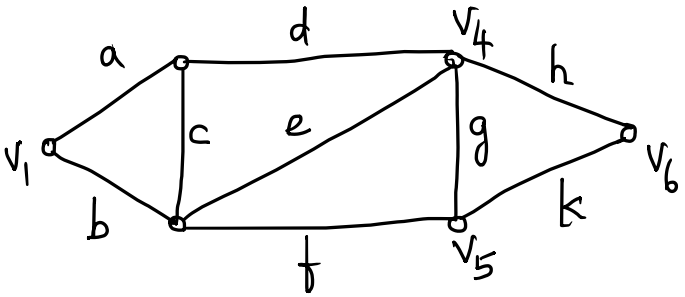


b) Use Dijkstra's algorithm to find the shortest path for the following weighted digraph and find the shortest distance from vertex a to other vertices. 7



Module -4

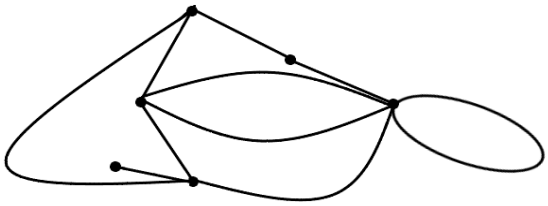
17 a) Illustrate the statement: "The ring sum of any two cut-sets in a graph is either a third cut-set or an edge disjoint union of cut-sets", in the following graph. 7



b) Define edge connectivity, vertex connectivity separable and non-separable graph. Give an example for each. 7

18 a) Prove that the complete graph on 5 vertices is non-planar 7

b) Draw the geometric dual of the following graph 7



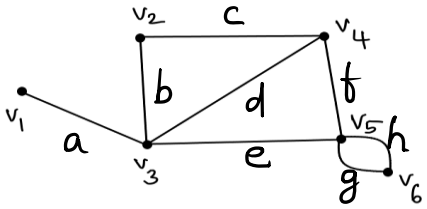
Module -5

19 a) For the following graph find the 7

i. Incidence matrix

ii. Path matrix between v_2 and v_5

iii. Circuit matrix



b) Draw a connected graph and show that the rank of its incidence matrix is one less than the number of vertices. 7

20 a) Prove that every tree with two or more vertices is 2-chromatic 7

b) Prove that a covering g of a graph is minimal if and only if g contains no path of length three or more. 7

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