

1. Consider the universal relation $R = \{A, B, C, D, E, F, G, H, I, J\}$ and the set of functional dependencies $F = \{AB \rightarrow C, A \rightarrow DE, B \rightarrow F, F \rightarrow GH, D \rightarrow IJ\}$. What is the key for R ? Decompose R into 2NF, then 3NF relation.

$AB \rightarrow C$	Full
$A \rightarrow D$	Partial
$A \rightarrow E$	"
$B \rightarrow F$	"
$F \rightarrow G$	Full
$F \rightarrow H$	"
$D \rightarrow I$	"
$D \rightarrow J$	"

$AB^+ = \{A, B, C, D, E, F, G, H, I, J\}$

AB is the only candidate key.

Non-prime attributes = $\{C, D, E, F, G, H, I, J\}$

Prime attributes = $\{A, B\}$

To convert to 2NF, decompose R as:

$R_1(A^*, B^*, C)$

$R_2(A^*, D, E, I, J)$

$R_3(B^*, F, G, H)$

There are transitive dependencies in the above relation. So to convert to 3NF, decompose as:

$R_1(A^*, B^*, C)$

$R_{22}(A^*, D, E)$

$R_{21}(A^*, D^*, I, J)$

$$R_{32} = \{B^*, F\}$$

$$R_{33} = \{F^*, G, H\}$$

2. Given the relation $R = \{A, B, C, D, E\}$ and
correspond set $F = \{A \rightarrow B, CD \rightarrow E,$
 $B \rightarrow D, E \rightarrow A\}$

Find out the candidate keys of relation, R.

$$A \rightarrow BC$$

$$CD \rightarrow E$$

$$B \rightarrow D$$

$$E \rightarrow A$$

$$A^+ = \{ABCDE\}$$

$$BC^+ = \{BCDEA\}$$

$$B^+ = \{BD\}$$

$$C^+ E^+ = \{ABCDE\}$$

$$CD^+ = \{CDEAB\}$$

$$BD^+ = \{BD\}$$

~~Candidate keys are $\{A^+, B^+, E^+, CD^+, BC\}$~~

Candidate keys = $\{A, E, CD, BC\}$

3. Consider the 2 sets of F.P

$$F = \{A \rightarrow C, AC \rightarrow D, E \rightarrow AD, E \rightarrow H\}$$

$$G = \{A \rightarrow CD, E \rightarrow AH\}$$

Check whether or not they are equivalent.

$$F = \{A \rightarrow C, AC \rightarrow D, E \rightarrow AD, E \rightarrow H\}$$

Find G^+ (\bar{G} , A^+ & E^+)

$$A^+ = \{ACD\}$$

$$E^+ = \{ADHCE\}$$

$$\begin{array}{l} \cancel{A \rightarrow C} \quad A \rightarrow CD \\ E \rightarrow AH \end{array}$$

So F covers G .

Find F^+ $G = \{A \rightarrow CD, E \rightarrow AH\}$

$$A^+ = \{ACD\}$$

$$AC^+ = \{ACD\}$$

$$E^+ = \{ADHCE\}$$

$$\begin{array}{l} \cancel{A \rightarrow C} \quad A \rightarrow AC \rightarrow D \\ E \rightarrow AD \quad E \rightarrow H \end{array}$$

So G covers F

As F covers G and G covers F , F and G are equivalent

4. Consider the relation

Stud (Sno , $Sname$, Cno , $Cname$)

Find closure of (Sno, Cno) . Given FD are:

a) $Sno \rightarrow Sname$

b) $Cno \rightarrow Cname$

$$(Sno, Cno)^+ = \{ Sno, Cno, Sname, Cname \}$$

It is a candidate key.

5. Let R be a relation

$R = (A, B, C, D, E, F)$ having FD:

$$A \rightarrow BC$$

$$B \rightarrow E$$

$$C \rightarrow EF$$

$$E \rightarrow CF$$

Find closure of (A, B)

$$(A, B)^+ = \{ ABC EF \}$$

6. Consider the relation:

$SP(Sno, Sname, Pno, Qty)$

FD of above relation:

$$(Sno, Pno) \rightarrow Qty$$

$$(Sname, Pno) \rightarrow Qty$$

$$Sno \rightarrow Sname$$

$$Sname \rightarrow Sno$$

Is this relation in 3NF. Check for it to be in BCNF. $Sname$ is considered unique for each Sno .

$$(Sno, ~~Sname~~, Pno)^+ = \{ Sno, Pno, Qty, Sname \}$$

$$Sno, (Sname, Pno)^+ = \{ Sname, Pno, Qty, Sno \}$$

Candidate keys: (Sno, Pno) & $(Sname, Pno)$

Prime attributes: $Sno, Pno, Sname$.

Non-prime attribute: Qty .

The relation is in 2NF as no non-prime attribute depends partially on a key.

Qty depends on a full key.

There is no transitive dependencies. So the relation is in 3NF.

It is not in BCNF as ~~$Sname$~~

$Sno \rightarrow Sname$

$Sname \rightarrow Sno$

Sno and $Sname$ are not superkeys.

To convert to BCNF, decompose as.

$R_1 (Sno^*, \text{~~Sname~~ } Pno^*, Qty)$

$R_2 (Sno^*, Sname)$

OR
 $R_1 (Sno^*, Pno^*, Qty)$

$R_2 (Sname^*, Sno)$

OR
 $R_1 (Sname^*, Pno, Qty)$

$R_2 (Sname^*, Sno)$

7. For the given relation R, which of the FDs are satisfied:

Relation R

X	Y	Z
1	4	2
1	5	3
1	6	3
3	2	2

a) $XY \rightarrow Z$ and $Z \rightarrow Y$

b) $YZ \rightarrow X$ and $Y \rightarrow Z$

c) $YZ \rightarrow X$ and $X \rightarrow Z$

d) $XZ \rightarrow Y$ & $Y \rightarrow Z$

a) $XY \rightarrow Z$ is satisfied but not $Z \rightarrow Y$

b) $YZ \rightarrow X$ is ~~is~~ ~~not~~ and $Y \rightarrow Z$ are

satisfied

c) $YZ \rightarrow X$ is satisfied but not $X \rightarrow Z$

d) $XZ \rightarrow Y$ is not satisfied. but $Y \rightarrow Z$ is satisfied.

8. Compute closure of F of FDs

$R = \{A, B, C, D, E\}$

$A \rightarrow B, C, CD \rightarrow E, B \rightarrow D, E \rightarrow A$

List candidate keys for R.

$$A^+ = \{A B C D E\}$$

$$B^+ = \{B D\}$$

$$~~C^+ = \{C\}~~ \quad CD^+ = \{C D E A B\}$$

$$E^+ = \{E A B C D\}$$

$$~~AB^+ = \{A B C\}~~$$

$$BC^+ = \{B C D E A\}$$

Candidate keys = $\{A, E, BC, CD\}$

9. Given two sets of FD F_1 and F_2 . Are they equivalent?

$$F_1: A \rightarrow B, AB \rightarrow C, D \rightarrow AC, D \rightarrow E$$

$$F_2: A \rightarrow BC, D \rightarrow AE$$

To check if F_1 cover F_2

$$~~A^+ = \{A\}~~ \quad A^+ \text{ with envt } F_1$$

$$A^+ = \{A B C\}$$

$$D^+ = \{D A C E B\}$$

$A \rightarrow BC$ and $D \rightarrow BC$ are derivable from A^+ and D^+ . Hence, F_1 covers F_2

To check if F_2 cover F_1

$$A^+ = \{A B C\}$$

$$AB^+ = \{A B C\}$$

$$D^+ = \{D A E B C\}$$

$$\textcircled{D} \quad A \rightarrow B$$

$$AB \rightarrow C$$

$$D \rightarrow AC$$

$D \rightarrow E$ are derivable.

Hence F_2 covers F_1 .

F_1 and F_2 are equivalent.

10. Consider the schema

$S = \{V, W, X, Y, Z\}$ say the FD

hold.

$Z \rightarrow V$

$W \rightarrow Y$

$XY \rightarrow Z$

$U \rightarrow \cancel{WZ} WX$

State whether the following decomposition of schema S is lossless join decomposition.

Justify the answer:

$S_1 = \{V, W, X\}$

$S_2 = \{V, Y, Z\}$

	$V(A_1)$	$W(A_2)$	$X(A_3)$	$Y(A_4)$	$Z(A_5)$
$S_1 = \{V, W, X\}$	a_1	a_2	a_3	b_{14}	b_{15}
$S_2 = \{V, Y, Z\}$	a_1	b_{22}	b_{23}	a_4	a_5

$Z \rightarrow V$

11. Consider the following 2 sets of FD
 $F = \{A \rightarrow C, AC \rightarrow D, E \rightarrow AD, E \rightarrow H\}$ and
 $G = \{A \rightarrow CD, E \rightarrow \dots\}$

11. Prove that any relation schema with two attributes is in BCNF.
 Consider the schema $R = \{A, B\}$ with 2 attributes.
 The possible non-trivial FDs are:

$$\{A\} \rightarrow B$$

and $B \rightarrow A$

There are 4 cases:

- i) No FD holds in R . Then, key is $\{A, B\}$ and relation satisfies BCNF
- ii) Only $A \rightarrow B$ holds, key is A and relation satisfies BCNF
- iii) Only $B \rightarrow A$ holds, key is B and relation satisfies BCNF
- iv) Both $A \rightarrow B$ and $B \rightarrow A$ hold. There are 2 keys A and B & relation satisfies BCNF.

Any relation with 2 attributes is in BCNF

12. For a relation, $R(A, B, C, D, E, F)$ the set of FDs, F is given as follows:

$$F = \{AB \rightarrow C, C \rightarrow A, BC \rightarrow D, ACD \rightarrow B, BE \rightarrow C, CE \rightarrow FA, CF \rightarrow BD, D \rightarrow E\}$$

Find a non-redundant cover for F . Is this the only non-redundant cover? Also find its canonical cover.

$$F' = F - \{AB \rightarrow C\} \rightarrow \{C\} \cup \{A \rightarrow B\}$$

$$A^+ = \{AC\}$$

$$F' = \{F - \{AB \rightarrow B\}\} \cup \{X - \{B\} \rightarrow A\}$$

$$Z = \{B \rightarrow C, \dots\}$$

$$B^+ = \{BCADEFA\}$$

$$A^+ =$$

Remove $ACD \rightarrow B$ from F and take

$$ACD^+ = \{ACD E F A B\}$$

So $ACD \rightarrow B$ is redundant

Non-redundant cover:

$$F = \{AB \rightarrow C, C \rightarrow A, BC \rightarrow D, \overline{ACD \rightarrow B}, BE \rightarrow C, CE \rightarrow FA, CF \rightarrow BD, D \rightarrow E\}$$

Canonical cover

To check if A is redundant in $AB \rightarrow C$

$$F = \{F - \{AB \rightarrow C\}\} + \{B \rightarrow C\}$$

$$= \{B \rightarrow C, C \rightarrow A, BC \rightarrow D, BE \rightarrow C, CE \rightarrow FA, CF \rightarrow BD, D \rightarrow E\}$$

Now take B^+ . if B^+ contains A then A is redundant

$$B^+ = \{BCDEFA\}$$

Hence B is ~~not~~

$$B \rightarrow A$$

$$\overline{AB \rightarrow C}$$

$$B \rightarrow C$$

B is redundant

$$F = \{ B \rightarrow C, C \rightarrow A, BC \rightarrow D, BE \rightarrow C, C \rightarrow F, CE \rightarrow A, CF \rightarrow B, CF \rightarrow D, D \rightarrow E \}$$

~~$C \rightarrow$~~ Check if B is redundant in $BC \rightarrow D$

$$C^+ = \{ C, A, D, E, F, B \}$$

$$C \rightarrow B$$

Hence B is redundant

$$F = \{ B \rightarrow C, C \rightarrow A, C \rightarrow D, BE \rightarrow C, CE \rightarrow F, CE \rightarrow A, CF \rightarrow B, CF \rightarrow D, D \rightarrow E \}$$

Check if B is redundant in $BE \rightarrow C$

$$E^+ = \{ E, C, A, F, B, D \}$$

Hence B is redundant

$$F = \{ B \rightarrow C, C \rightarrow A, C \rightarrow D, E \rightarrow C, CE \rightarrow F, CE \rightarrow A, CF \rightarrow B, CF \rightarrow D, D \rightarrow E \}$$

$$CE \rightarrow F$$

$$C^+ = \{ C, F, A, D, B, E \}$$

$$C, A, D, E, F$$

$$F = \{ B \rightarrow C, C \rightarrow A, C \rightarrow D, E \rightarrow C, C \rightarrow F, C \rightarrow A, C \rightarrow B, C \rightarrow D, D \rightarrow E \}$$

$$C \rightarrow D, D \rightarrow E$$

$$= \{ B \rightarrow C, C \rightarrow A, C \rightarrow D, E \rightarrow C, C \rightarrow F, C \rightarrow B, D \rightarrow E \}$$

is the minimal cover

13. Find BCNF Decomposition:

Shipping (Ship, Capacity, Date, Cargo, Value)

Ship \rightarrow Capacity

Ship, Date \rightarrow Cargo

Cargo, Capacity \rightarrow Value.

Specify key.

Key is Ship, Date.

R_1 (Ship*, Date*, Cargo)

R_2 (Ship*, Capacity)

R_3 (Cargo*, Capacity*, Value)

14. For $R = \{A, B,$