Previous Year Questions
module 01 & 02

December 2017

01. Consider a graph G with 4 vertices V_1, V_2, V_3 and V_4 and degrees are 3 5 2 and 1 respectively. Is it possible to draw such a graph G. If not why?

Ans: According to theorem "The number of odd degree vertices in an un~~should~~ be ~~even~~ is always even"

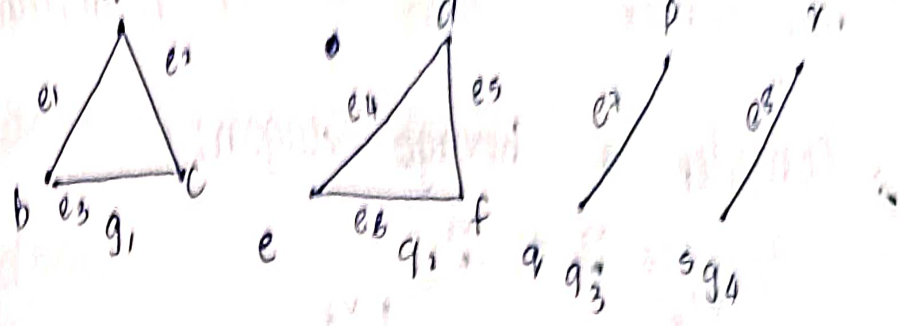
Here according to given degrees ~~are~~ 3 odd degree vertex are there contradicting the theorem. Hence no such graph exists.

also;

$$2e = 3+5+2+1$$

$$e = \frac{11}{2} \text{ is fraction.}$$

02. Draw a disconnected simple graph G_1 with 10 vertices & 4 components and also calculated maximum no. of edges possible in G_1



G_1.

For a simple graph with n vertices and K components it can have utmost $\frac{(n-K)(n-K+1)}{2}$ edges

ie; $\frac{(10-4)}{2}\frac{(10-5)}{2} = \frac{(6)(5)}{2} = 15$ edges.

$n = 10$; $K = 4$.

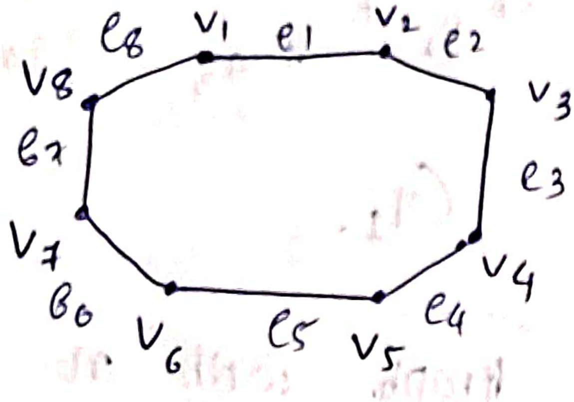
03. state Dirac theorem for hamiltonicity and why it is not a necessary condition for a simple graph to have hamiltonian circuit.

Ans Dirac theorem;

Let G be connected graph with total number of vertices and degree of $v \geq \frac{P}{2}$

any vertex v, then G is hamiltanian

⇒ Consider a ~~hexago~~ octagon;



no. of vertices. $p = 8$; $p \geq 3$

Here we can definetly find a hamiltanian circuit;

$v_1 \overset{e_1}{} v_2 \, e_2 \, v_3 \, e_3 \, v_4 \, e_4 \, v_5 \, e_5 \, v_6 \, e_6 \, v_7 \, e_7$

$v_8 \, e_8 \, v_1$

But ; $\frac{p}{2} = 4$ and

$d(v) = 2$.

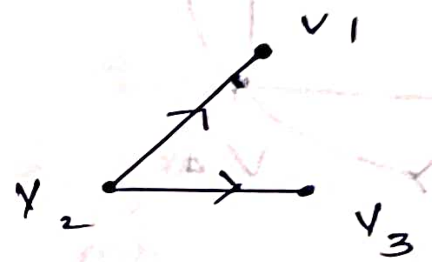
$$2 \not\geq \frac{p}{2}$$

So this is not a necessary condition

. Differentiate between Symmetric and asymmetric digraphs with examples and draw a Complete Symmetric digraph of 4 vertices.

Ans Asymmetric digraph :

- A digraph that have almost one directed edges b/w a pair of vertices.

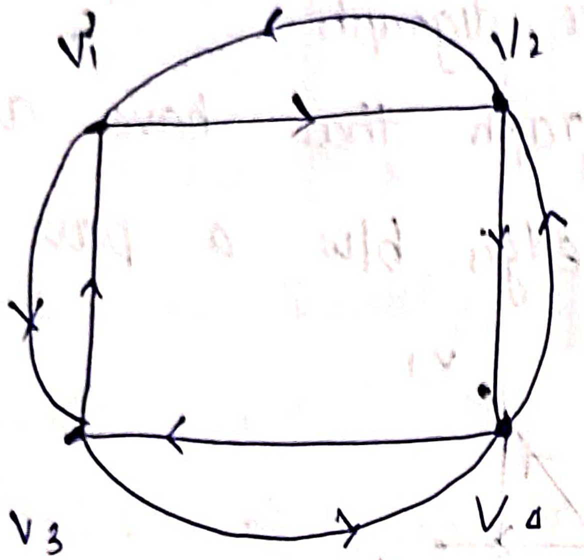


Symmetric digraph

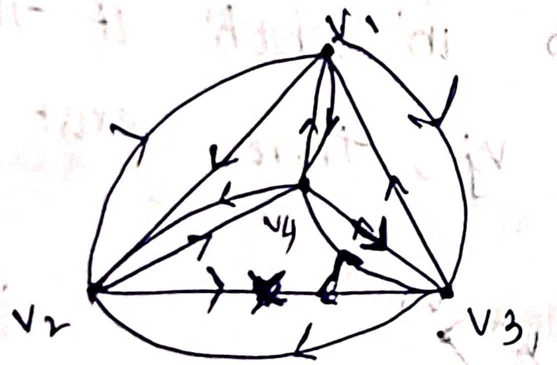
- A digraph in which if there is edge from v_i to v_j, there exist edge b/w v_j to v_i

complete symmetric:

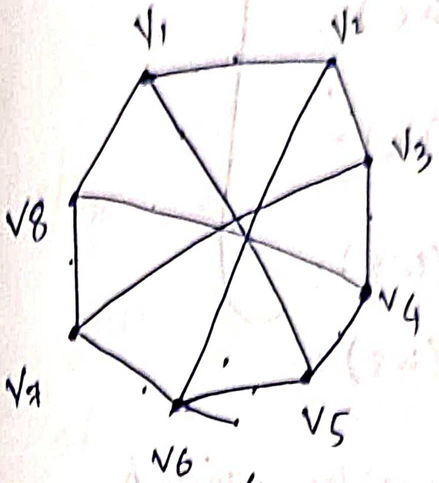
- A symmetric digraph in which there is exactly 1 edge directed from every vertex to every other vertex,



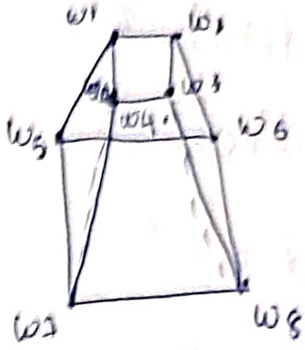
or



5. where the [illegible] [illegible] to be satisfied for 2 graphs to be isomorphic? Are the 2 graphs below isomorphic? explain with valid reasons.



G1



G2

Ans: For 2 graphs to be isomorphic:

→ no of vertices must be equal

→ no of edges must be equal

→ ~~degree of~~ equal no. of vertices with given degree.

~~no of edges~~ vertices = 8

~~no~~ G1	G2
no. of vertices : 8	no. of vertices : = 8.
edges: 12	no. of edges = 12.
~~Degrees~~	

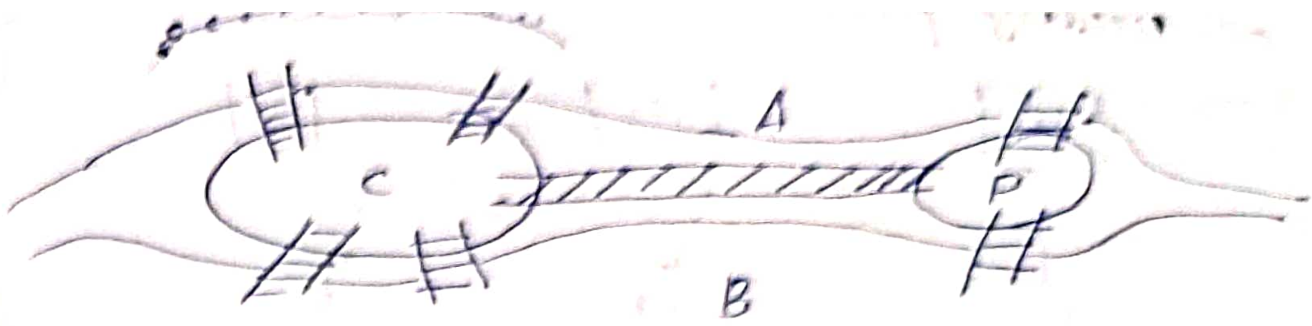
$d(v_1) = 3$
$d(v_2) = 3$
$d(v_3) = 3$
$d(v_4) = 3.$
$d(v_5) = 3.$
$d(v_6) = 3$
$d(v_7) = 3$
$d(v_8) = 3$

$\left.\begin{array}{l} d(w_2) = \\ d(w_3) = \\ d(w_4) = \\ d(w_5) = \\ d(w_6) = \\ d(w_7) = \end{array}\right\} = 3.$
$d(w_8) =$

since it is a 3 regular graph
every vertice in G_1 has maping in G_2.
making them ISOMORPHIC.

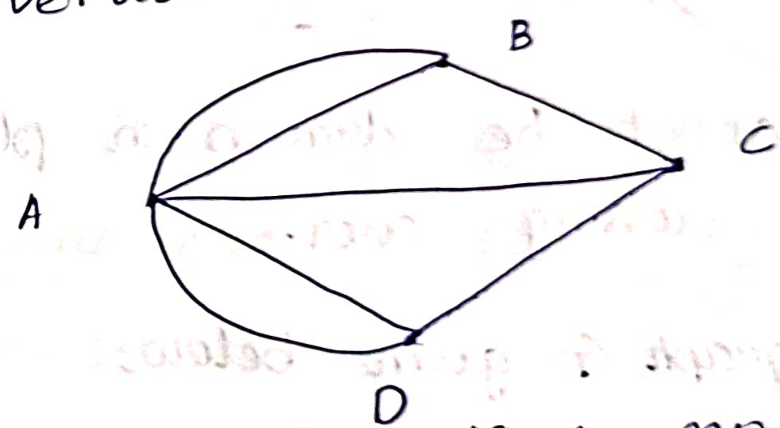
b) write 2 application of graphs with sufficient example:

Ans 01. Konigsberg bridge problem:
→ There are 2 island C & D formed by ~~preger~~ preger river. It is connected to each other and to banks A & B by 7 bridges as ~~follows~~ follows.



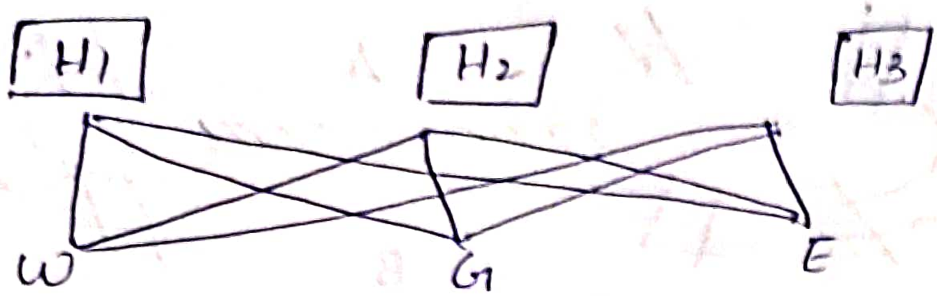
The problem was to start from any four land ABCD and walk over each of 7 bridges exactly once and to return back to starting point.

Euler represented this situation by mean of graph shown below with ABCD as vertices and bridges as edges



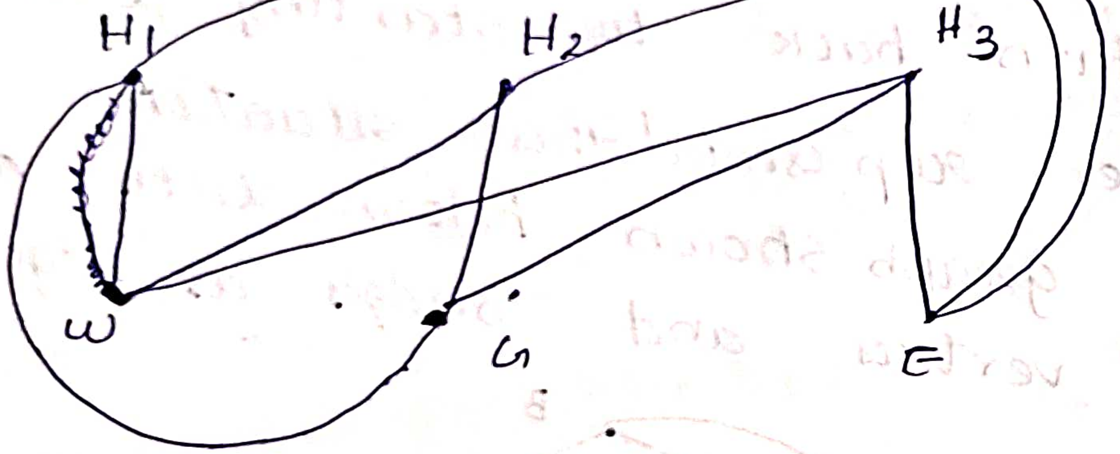
Euler proved that no solution is available for this problem.

02. utility problem:



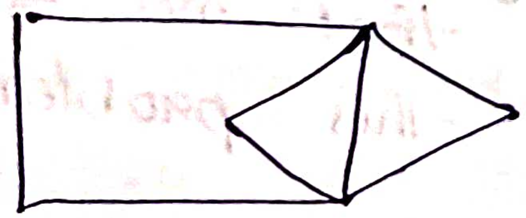
we have to provide 3 utilities water, Gas and electricity to each of 3 houses H_1, H_2, H_3 by means of conduits without crossover

Graphical Representation:



This graph cannot be drawn in plane without edges crossing over.

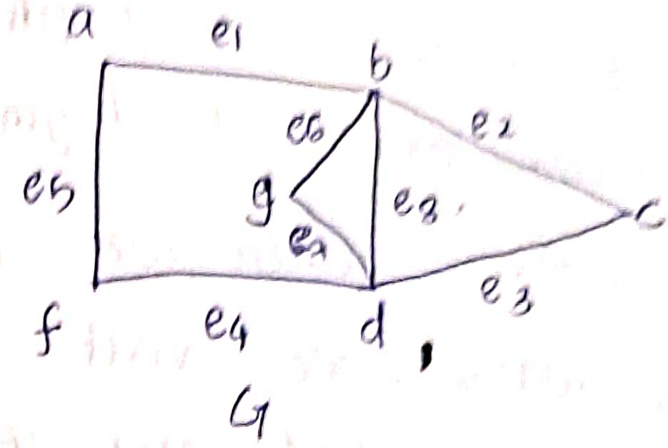
a) consider graph G given below:



Define Euler graph. Is G euler. If [illegible] write

Euler euler line from G

Ans:



euler graph

A graph with euler circuit [every edge exactly once]

Yes the given graph is euler by theorem below; (theorem stated in q (b))

" If degree of every vertex is even in a simple, connected graph it is is euler graph"

Here ~~vertex~~ $d(a) = 2$, $d(g) = 2$

$d(b) = 4$ $d(f) = 2$.

$d(c) = 2$

$d(d) = 4$

Euler line :

$a\, e_1\, b\, e_2\, c\, e_3\, d\, e_8\, b\, e_6\, g\, e_7\, d\, e_4\, f\, e_5\, a$

b) what is necessary & sufficient condition a graph to be Euler. Also prove it.

Ans: A connected graph is euler iff all vertices of G are of even degree.

PROOF :

Suppose G is an Euler graph. Thus it has a euler circuit. It begins and ends at vertex v. when we traverse through the walk, we visit the vertex v through 1 edge and exit it through another. ~~indicating that degree.~~ of ~~vertex v is 2. since~~ eulerian circuit consists of every edge so occurance of v contribute to 2 degree. It is also true of end vertices since its closed. hence proved.

To prove sufficiency of condition, assume degree of all vertex to be even. Now we start from an ~~vx~~ arbitary vertex to construct a walk. Since degree is even. we can enter & leave a vertex. we shall eventually ... when tracing come to an end. If the closed walk 'h' we traced contain all edges then it is eulerian circuit → euler graph.

$h \subset$ subgraph

If not:

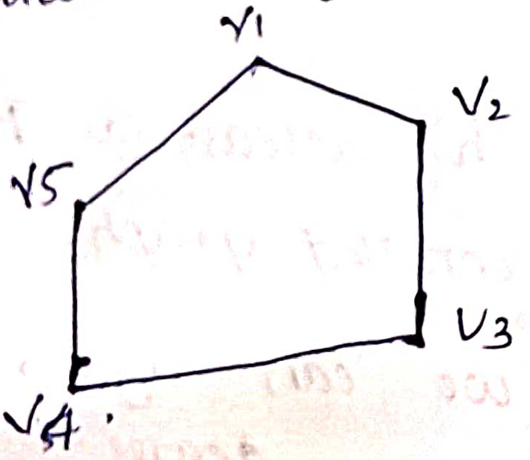
- we remove all edges in h from G
- h' with remaining ~~vertex~~ edges. (h' = subgraph)
- G, h - even degree vertices so ~~vertices~~ vertices of h' also even.
- h' must touch h atleast @ 1 vertex a bcz connected graph!
- starting frm a we can again construct ~~closed~~ walk ; since degree of vertex ~~d(a) also e~~ even we can ~~Ed~~ end at a.

- Thu walk in h' can be combined h to form new walk which star... v → has more edges than h
- re... till ...edges covered ⇒ so eu...

1. i) Define hamiltanian circuits & paths with example. Find out no. of edge-disjoint hamiltanian circuit possible in a complete graph with 5 vertices.

Ans Hamiltanian ~~circuit~~ path:
Simple path with all vertices exactly once.

Hamiltanian circuit:
circuit in G that contains all vertices exactly once except end vertices.



Hamiltanian circuit:
$V_1 V_2 V_3 V_4 V_5 V_1$
Hamiltanian path
$V_1 V_2 V_3 V_4 V_5$

ii) $n = 5$

$\frac{n-1}{2}$ edge disjont Hamitatina circut by theorem

$\frac{5-1}{2} = \frac{4}{2} = 2$

if $n \geq 3$ → odd.

b) state travelling sales man problem how its soln is related with hamiltonian circuit?

Ans TSP: It is a problem where salesman has to travel to many cities. To solve we need to find cheapest way for travelling salesman to ~~sto~~ visit every city & reach back to where he started. This is simply find hamiltanian circuit in a complete graph that has smallest overall cost.

x ——— x ——— x