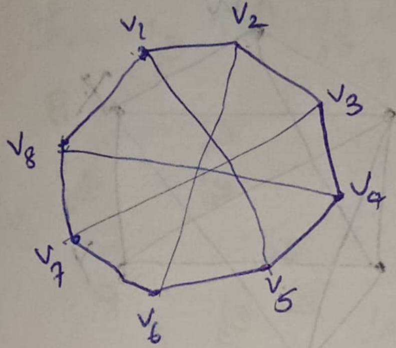
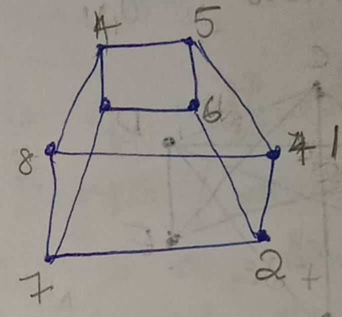
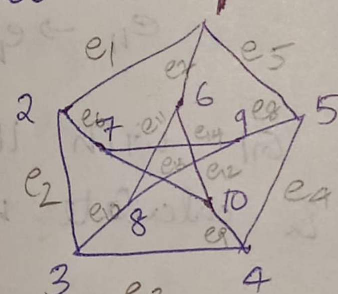
1. Are the two graphs isomorphic?

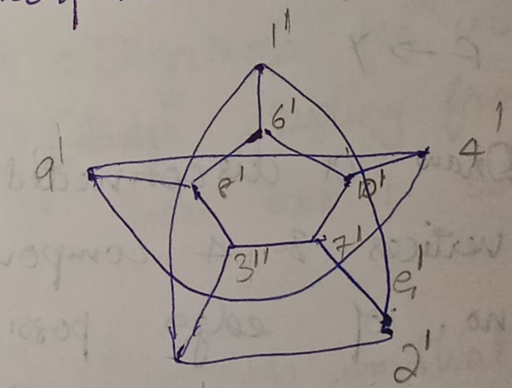




The graphs are not isomorphic as there is no edge correspondence of edges between. edge exist between u_8 and v_8 and v_1. No edge exist between u_8 and u_1 in the second graph.

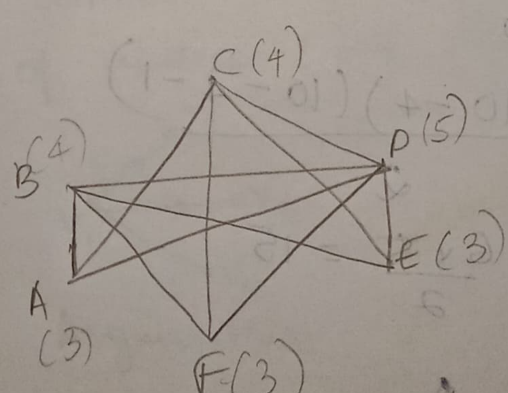
2. Determine whether isomorphic

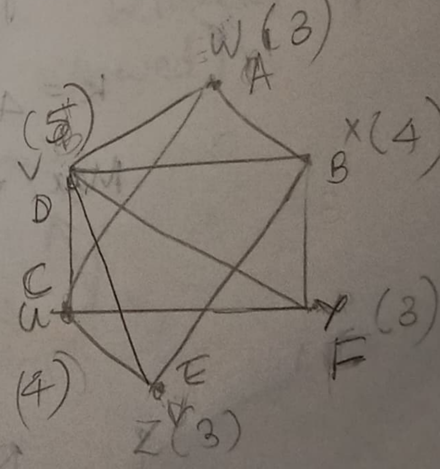




Not Isomorphic

3.





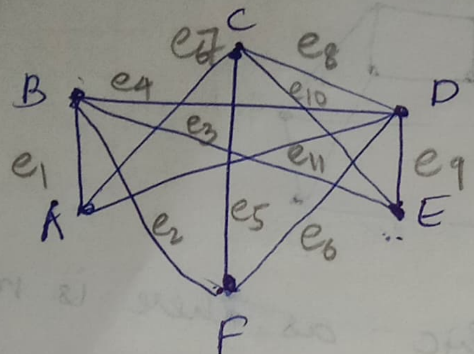
$D \Leftrightarrow V$

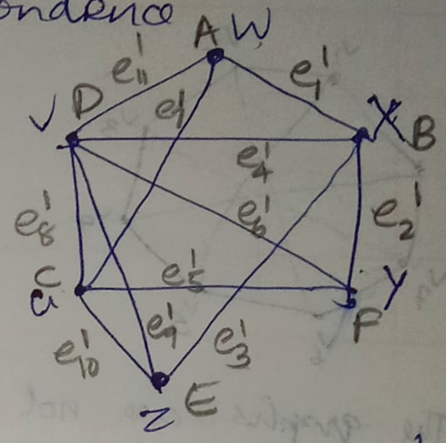
$D \to V$ $\quad$ $X \to B$

~~X~~ $\to B \to X$ $\quad$ $C \to u$

$A \to W$ $\quad$ $F \to Y$ $\quad$ ~~Z~~ $\to E \to Z$

The graphs are isomorphic ~~and~~ ~~of~~ as it has edge correspondence





A → W

B → X

C → u

D → V

E → Z

F → Y

$e_1 \to e_1'$	$e_6 \to e_6'$
$e_2 \to e_2'$	$e_7 \to e_7'$
$e_3 \to e_3'$	$e_8 \to e_8'$
$e_4 \to e_4'$	$e_9 \to e_9'$
$e_5 \to e_5'$	$e_{10} \to e_{10}'$
	$e_{11} \to e_{11}'$

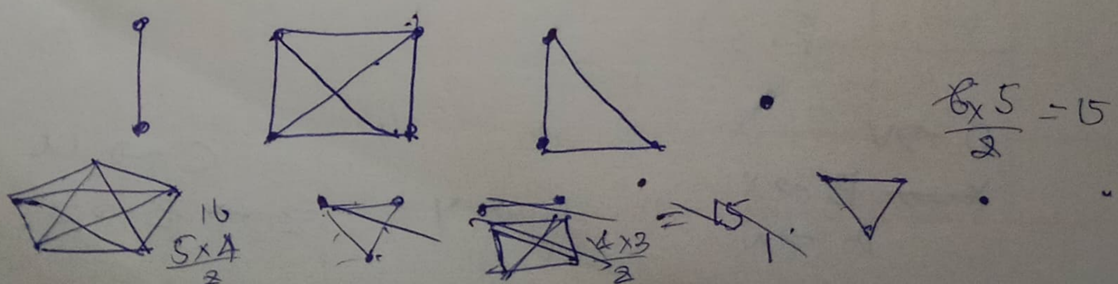
4. Draw a disconnected graph G1 with 10 vertices & 4 components & calculate max. no. of edges possible.

$$\text{Max edges} = \frac{(n-k)(n-k-1)}{2}$$

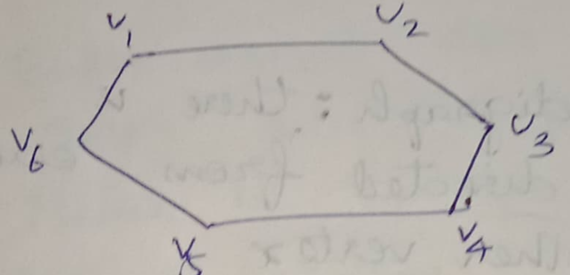
$k = 4 \quad n = 10$

$$\text{Max edge} = \frac{(10-4)(10-4-1)}{2}$$

$$= \frac{6 \times 5}{2} = 15$$



5. Why is hamiltonicity Dirac theorem not a necessary condition for a simple graph to have hamiltonian circuit

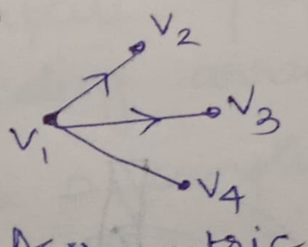


No. of vertices = 6
Degree of each vertex = 2
But ~~it~~ a hamiltonian path exsist even though.
No of verties > Degree of vertex.

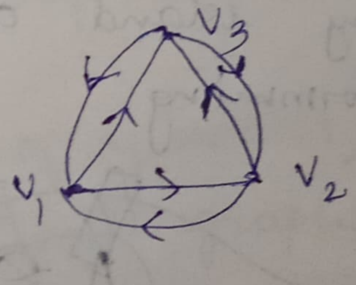
6. Differentiate symmetric & assymmetric digraph and draw a complete symmetric digraph with 4 verties

Symmetric digraph : Digraph ~~for~~ in which for every edge (a, b) (Edge starting from vertex a and going to b) there exsist an edge (b, a)

Assymmetric digraph : Digraph ~~to~~ that has atmost one directed edge between a pair of vertices (Self loops are allowed)

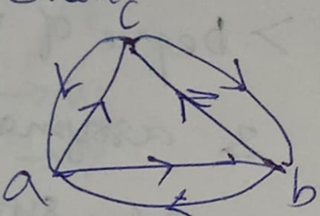


Asymmetric

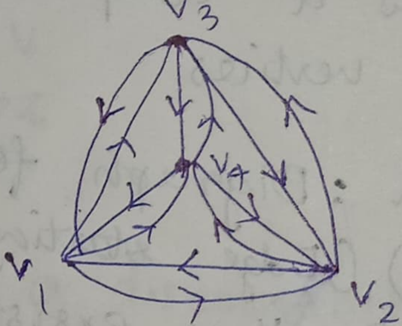


Complete assymetric : there is exactly one edge b/w every pair of vertices

Complete symmetric digraph : there is exactly one edge directed from every vertex to every other vertex



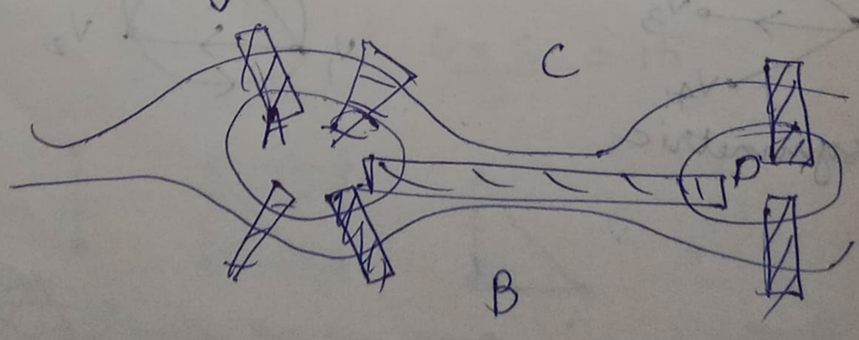
eg:

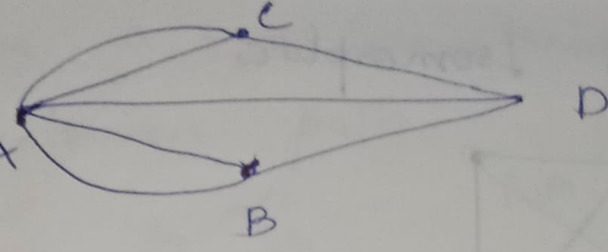


7. Write any 2 applications of graph.

i) Konisberg Bridge problem.

The problem is to ~~cross~~ travel along 7 bridges exactly once starting from any island and come back without swimming





Euler circuit does not exsist in the above graph since degree of the vertices are odd. Hence it is not possible.

(ii) Seating problem

9 members are to be seated around a round table such that no member sit next to the person he/she had already sat with. How many such seating arrange-ments are possible.

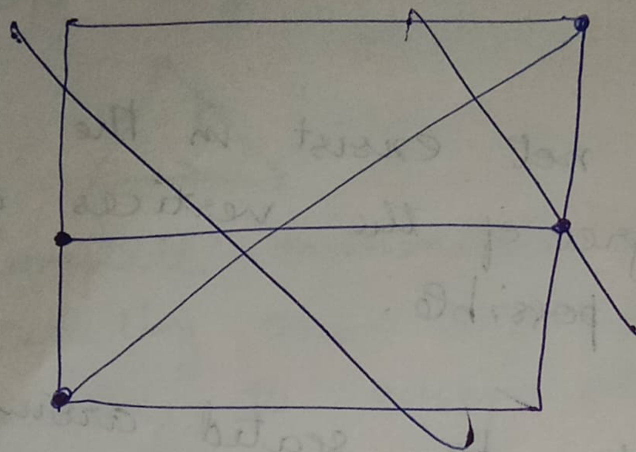
9 members are represented as vertices. An edge b/w 2 members represent them sitting together. Each member can sit next to 8 other members. Hence it forms a complete graph.

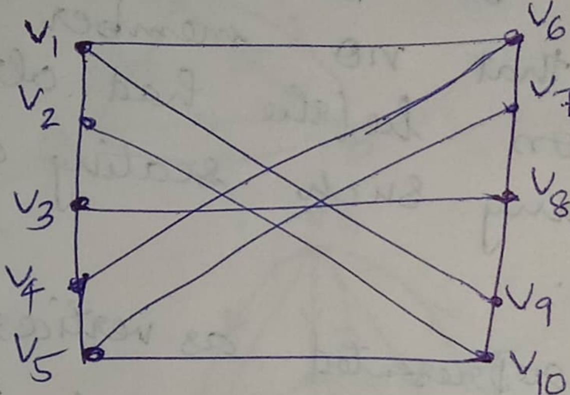
The various distinct ~~ha~~ edge-disjoint hamiltonian circuits possible in this complete graph will represent the ~~number~~ seating arrangement.

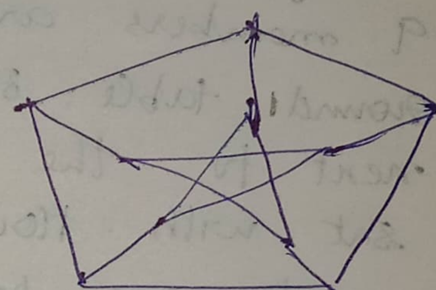
For a complete graph of n vertices, max number of edge disjoint hamiltonian graphs is given by $\frac{n-1}{2}$ if $n \geq 3$ and n is odd.

Hence, max no. of ~~ar~~ arrangements $= \frac{9-1}{2} = \underline{\underline{4}}$

8. Are the graphs Isomorphic

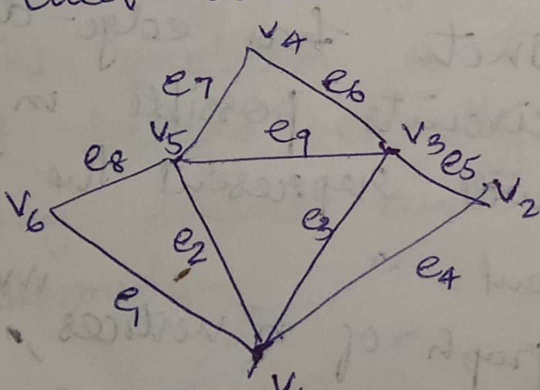






Not isomorphic as there is no edge correspondance

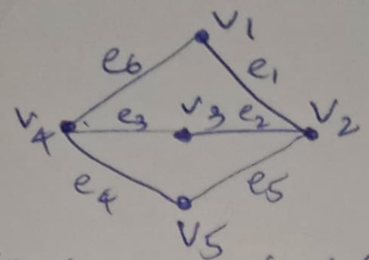
9. Find shortest path b/w v_1 to v_4. Also find a Euler circuit



Shortest path : $v_1 e_2 v_5 e_7 v_4$ OR $v_1 e_3 v_3 e_6 v_4$ } Length = 2

Euler circuit : $v_1 e_1 v_6 e_8 v_5 e_7 v_4 e_6 v_3 e_5$ –
– $v_2 e_4 v_1 e_2 v_5 e_9 e_3 v_1$

10. Draw a non-hamiltonian graph with a hamiltonian path.



Path: $v_1\ e_1\ v_2\ e_2\ v_3\ e_3\ v_4\ e_4\ v_5$

Hamiltonian path: a simple path that contains all the vertices exactly once

11. There are 37 telephones in a city. Is it possible to connect them with wires so that each one is connected to 7 others?

vertices → telephone

edges → connections.

Degree of each vertex = 7

$p = 37$

$d(v_i) \geq p/2$ by dirac theorem

$\frac{37}{2} = 18.5$

$d(v_i) \ngeq 18.5$

Hence this is not possible.

Sum of ~~verti~~ degree of vertex $= 37 \times 7$

No of edges $= \frac{37 \times 7}{2}$

$= 129.5$

Not possible as it has odd number of odd degree ~~edges~~ vertices.

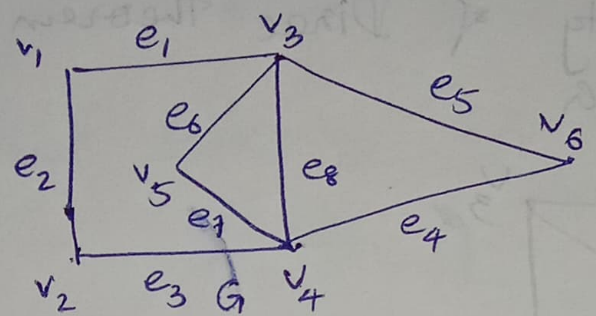
~~If a graph G has 8 vertices and it is Eulerian, then find maximum number of edges in G.~~

~~Max degree of a verte~~

12. Prove that a simple graph with n vertices must be connected, if it has more than $(n-1)(n-2)/2$ edges.

We know that a simple graph with n vertices and k connected components has atmost $\frac{(n-k)(n-k+1)}{2}$ edges.

~~The~~ If a simple graph with n vertices is not connected, it will contain atleast 2 connected components. The value of $k \geq 2$. There are atmost $(n-2)(n-2+1)/2$ edges in a graph, which contradicts ~~the~~ to the condition that graph has more than $(n-1)(n-2)/2$ edges. Hence graph is connected.

13. 

Is G an Euler graph? If yes, write an Euler line from G

Yes, it is an Euler graph as degree of all vertices is even.

$v_3\, \cancel{x}\, e_1\, v_1\, e_2\, v_2\, e_3\, v_4\, e_4\, v_6\, e_5\, v_3\, e_6\, v_5\, e_7\, v_4\, e_8\, v_3$

14. Find number of edge-disjoint Hamiltonian graph with 5 vertices (complete)

$\cancel{n(n-1}$ $\cancel{\frac{(n-1)!}{2} = \frac{4!}{2} = \frac{4\times3\times2}{2} = 12}$

$\frac{n(n-1)}{2} = \frac{5\times \cancel{4}^{2}}{\cancel{2}} = 10$

15. 19 students in a nursery school play a game each day, where they hold hands to form a circle. For how many days can they do this, with no student holding with the same playmates more than once?

~~Let each group~~

Let students be represented by vertices and holding hands be a by edge.

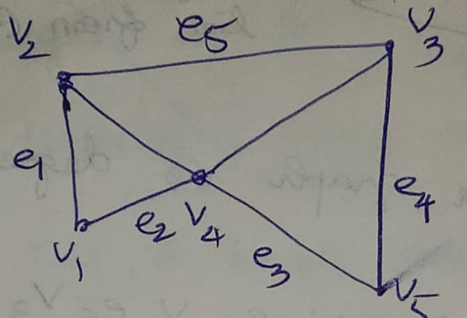
No. of edge disjoint hamiltonian circuits

$= \frac{n(n-1)}{2}$

$= \frac{19\times 18}{2} = \cancel{19\times 9 = 171}$

$= 9$

16. Check applicability of Dirac theorem in the following graph



No. of vertices = p = 5

$$\frac{p}{2} = \frac{5}{2} = 2.5$$

~~Deg~~ $d(v_1) = 2 < 2.5$

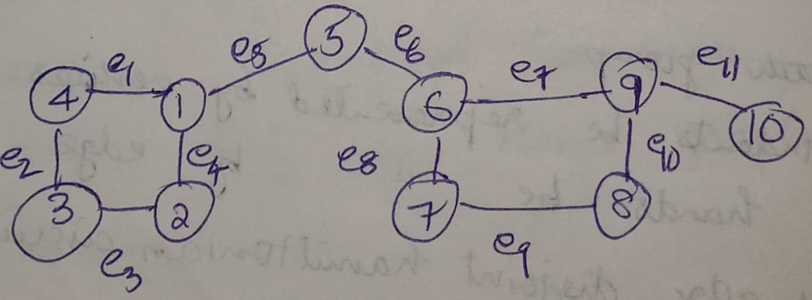
Hence by Dirac theorem Graph G is not hamiltonian.

But a hamiltonian path exists

~~v_2 e_1 v_1 v_4 e_3~~

v_2 e_1 v_1 e_2 v_4 e_3 v_5 e_4 v_3 e_5 v_2

17. Print a walk, trail, path and cycle on graphs



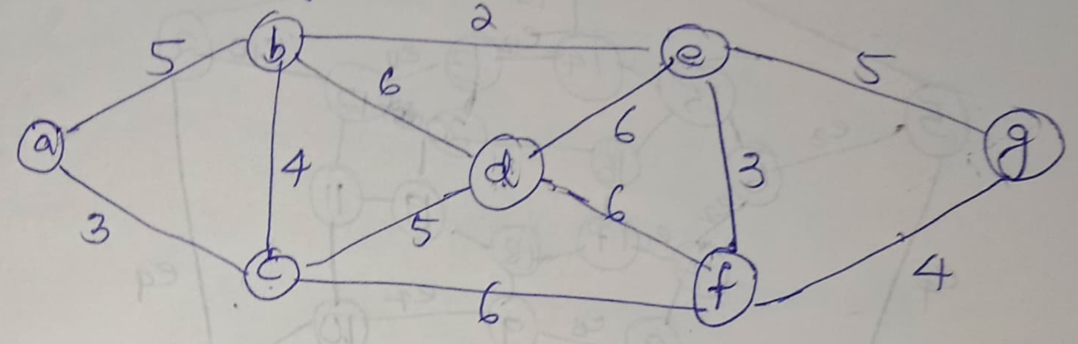
Walk: 4 e_1 1 e_5 5 e_6 6 e_7 9 e_{11} 10

Trail: 3 e_2 4 e_1 1 e_4 2 e_3

Path: 4 e_1 1 e_5 5 e_6 6 e_8 7 e_9 8 e_{10} 9

Cycle: 4 e_1 1 e_4 2 e_3 3 e_2 4

18. Print travelling salesman's tour on the graph



a - b - e - g - f - d - c - a

$5+2+5+4+6+5+3 = 30$

a - b - d - e - g - f - c - a $5+6+6+5+4+6+3$

$= 35$

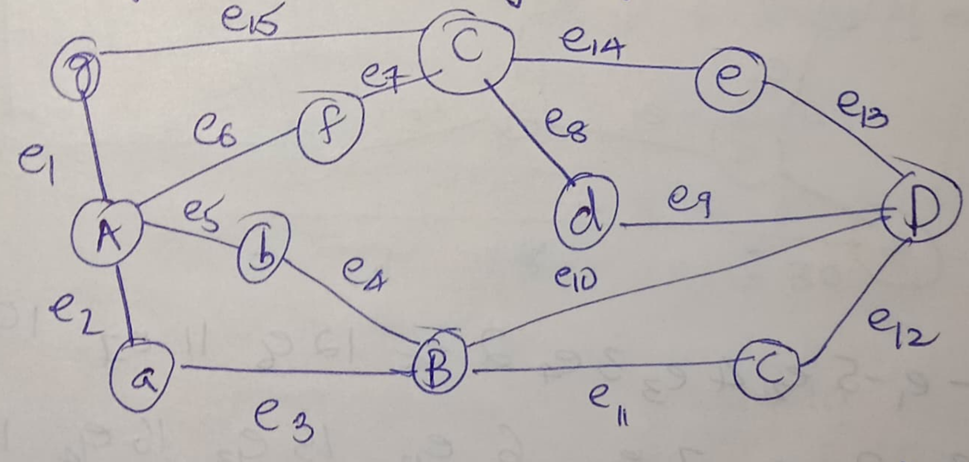
~~a - c - d - b - e - f~~

d - c - a - b - e - g - f - d

$5+3+5+2+5+4+6 = 30$

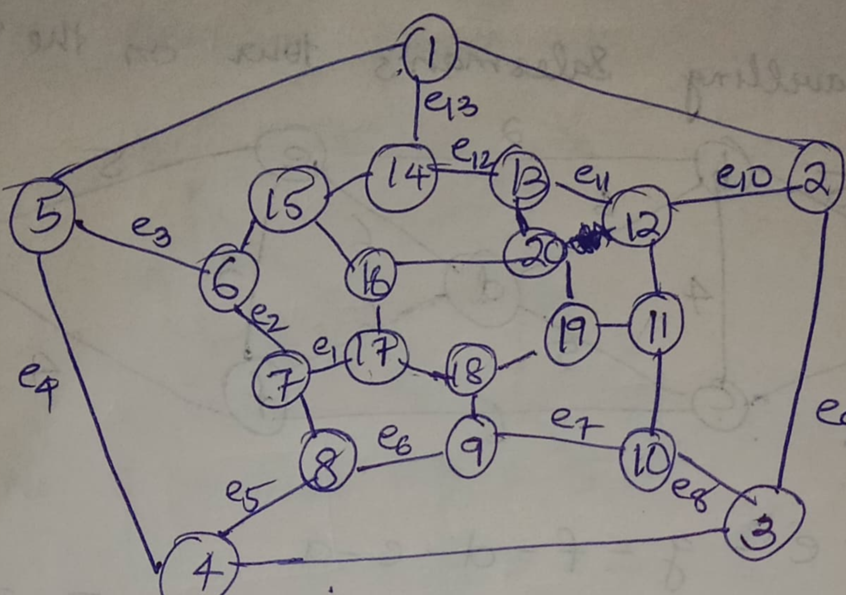
There are 2 minimum paths possible.

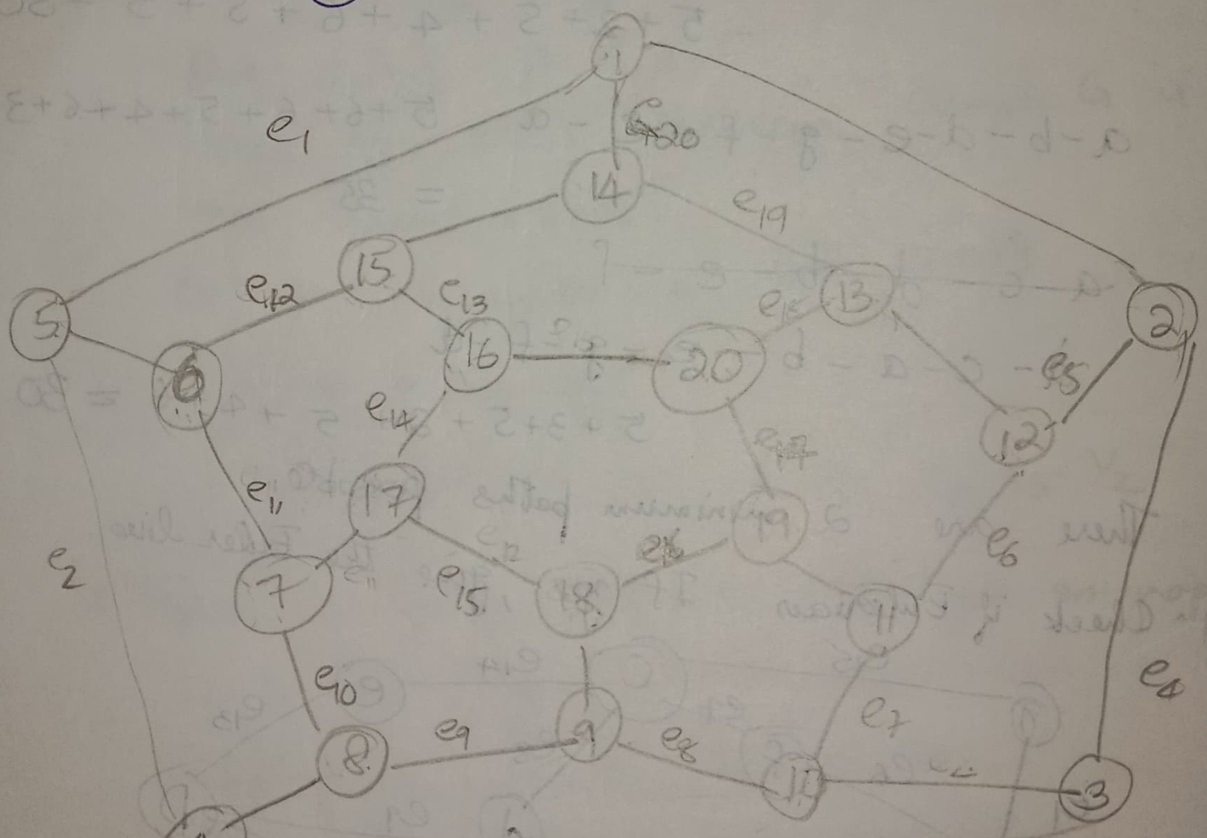
19. Check if Eulerian. If yes, give the Euler line.



Euler line: g e_1 A e_2 a e_3 B e_4 b e_5 A e_6 f e_7 C e_8 d e_9 D e_{10} B e_{11} C e_{12} D e_{13} e e_{14} c e_{15} g

20. Give hamiltonian circuit of graph G.





1 – e_1 – 5 e_2 4 e_3 3 e_4 2 e_5 12 e_6 11 e_7 10 e_8 9
e_9 8 e_{10} 7 e_{11} 6 e_{12} 15 e_{13} 16 e_{14} 17 e_{15} 18
e_{16} 19 e_{17} 20 . e_{18} 13 e_{19} 14 e_{20} 1

21) Let p1 and p2 be 2 different paths b/w 2 given vertices. Prove that ringsum of p1 and p2 is a circuit or a set of circuits.

22) What is the number of distinct hamiltonian ~~cir~~ circuits (not edge disjoint) in a complete graph of n vertices, n>=3.

$\frac{\cancel{n}(n-1)}{2}$, where n ~~= no~~ is odd

23) ~~In a gr~~ $\frac{\cancel{n}(n-2)}{2}$, if n is even

~~23) In a graph G let P~~

24) What is the number of vertices in an undirected graph with 27 edges, 6 vertices of degree 2, 3 vertices of degree 4 and remaining of degree 3?

Sum of degrees of vertices = 2 x no. of edges

$= 2 \times 27$

$6 \times 2 + 3 \times 4 + 3x$

$24 + 3x = 54$

$3x = 54 - 24 = 30$

$x = 10$

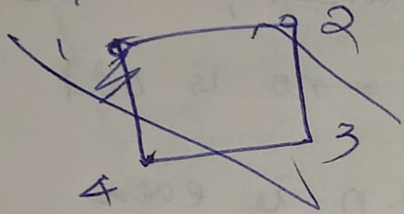
Total no. of vertices $= 6 + 3 + 10$

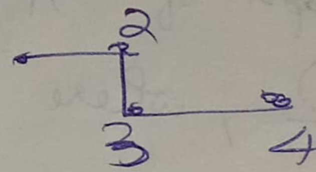
$= 19$

25) Consider an undirected graph G with 100 nodes. Find the minimum no. of edges to be included in G so that G is guaranteed to be connected.

~~Min no = 100~~ Min no. of edges = 99

eg: 4 vertices need 3 ~~4~~ edges in minimum





26) Suppose there are 20 people in a room. ~~Suppose~~ some pairs of the people shake hands and some don't. As the people leave the room, you ask each person whether they shook hands an odd number of times or even. Prove that number of people who answer "odd" is an even no.

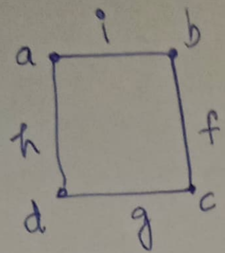
Let person be represented as vertices of Graph G and shaking hands as an edge.

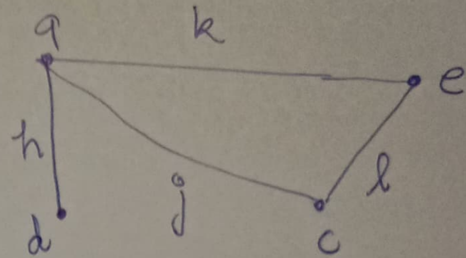
Degree of each vertex is the no. of times they shook their hands.

By theorem, no. of vertices of odd degree in a graph is always even.

~~Hence~~.

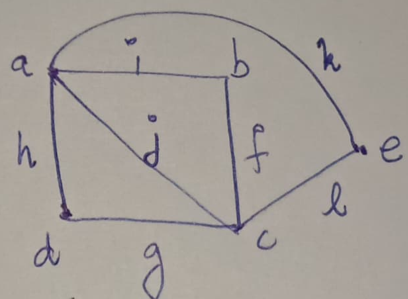
27)



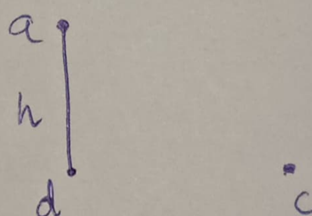


find union, intersection and ring sum.

Union



Intersection



Ring Sum