

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth Semester B.Tech Degree Supplementary Examination June 2023 (2019 Scheme)

Course Code: MAT 206

Course Name: GRAPH THEORY

Max. Marks: 100

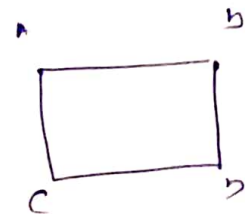
Duration: 3 Hours

PART A*(Answer all questions; each question carries 3 marks)*

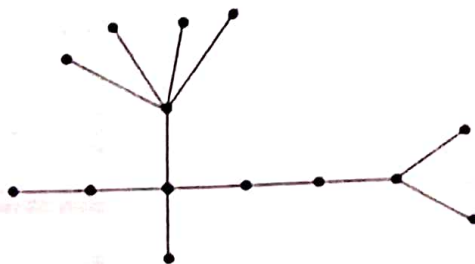
Marks

- 1 Prove that sum of the degrees of all vertices in G is twice the number of edges in G . 3
- 2 Define pendant vertex, isolated vertex, and null graph with an example. 3
- 3 Define Ring Sum of G_1 and G_2 . 3

Find the ring sum of the following graphs

 G_1  G_2 

- 4 Define arbitrarily traceable graphs. Give an example. 3
- 5 Find the maximum number of vertices possible in a 3-level binary tree. Also find the maximum height possible in a binary tree with 11 vertices. 3
- 6 Label the vertices of the following graph with their eccentricities and hence find the diameter and centre of the graph. 3



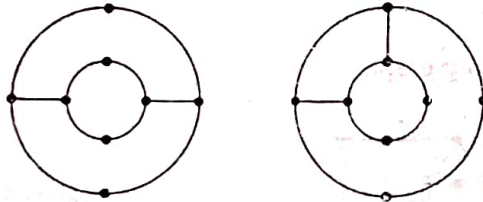
- 7 Show that every cut set in a connected graph must contain at least one branch of every spanning tree of G . 3
- 8 Show that K_5 is not planar. 3
- 9 Define Incidence Matrix. List four properties of it. 3
- 10 Show that every tree with two or more vertices is 2-chromatic. 3

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

- 11 a) Show that a simple graph with n vertices must be connected if it has more than $\frac{(n-1)(n-2)}{2}$ edges. 7
- b) Are the following graphs isomorphic? Justify your answer. 7



- 12 a) A simple graph with n vertices and k components can have at most $(n - k)(n - k + 1)/2$ edges. 7
- b) Show that in any group of two or more people, there are two always with exactly the same numbers of friends inside the group. 7

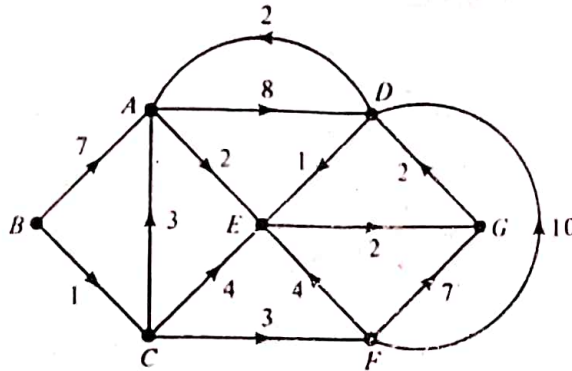
Module -2

- 13 a) Prove that a given connected graph is an Euler graph if and only if all vertices of the graph are of even degree. 7
- b) Prove that a connected graph is Euler if and only if it can be decomposed into circuits. 7
- 14 a) Nine members of a new club meet each day for lunch at a round table. They decide to sit such that every member has different neighbours at each lunch. How many days can this arrangement last? Justify your answer. 7

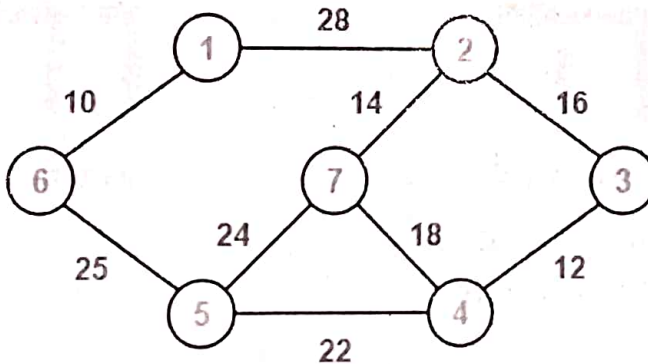
- b) If G is a simple graph with n vertices and $d(v) \geq n/2$ for each v , then G is Hamiltonian. 7

Module -3

- 15 a) Show that a Tree with n vertices has exactly $n-1$ edges. 7
 b) Prove that every tree has either one or two centres. 7
 16 a) Find the shortest distance from B to G using Dijkstra's Algorithm. 7

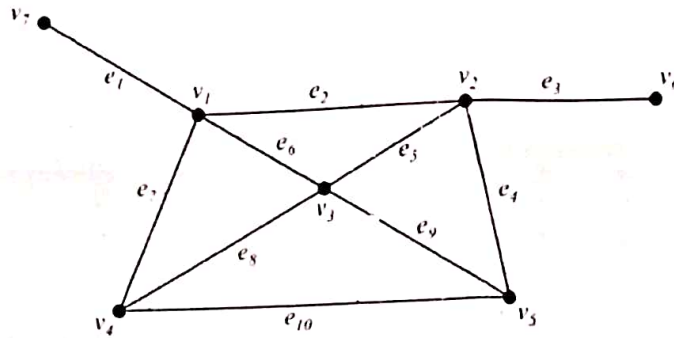


- b) Find the minimum spanning tree using Prim's algorithm. 7



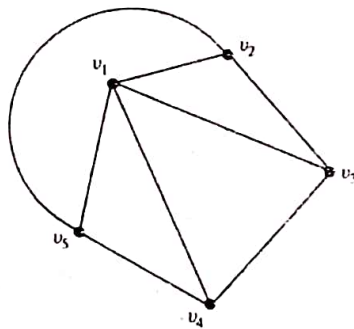
Module -4

- 17 a) A connected planar graph G with n vertices and e number of edges has $f = e - n + 2$ regions or faces. 7
 b) For any graph G , prove that $\text{vertex connectivity} \leq \text{edge connectivity} \leq \frac{2e}{n}$ 7
 18 a) Every circuit has an even number of edges in common with any cut set. 7
 b) Find a spanning tree and hence find all fundamental cut sets, associated with it, of the following graph. 7



Module -5

- 19 a) If d_{max} is the maximum degree of vertices in a graph, then show that
chromatic number of $G \leq d_{max} + 1$ 7
- b) Find the chromatic polynomial of the following graph. 7



- 20 a) If $A(G)$ is an incidence matrix of a connected graph G with n
 vertices, then show that rank of $A(G)$ is $n-1$. 7
- b) Check whether the graph having following adjacency matrix is connected or not. 7

$$X = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$
