Module IV

Closure of an FD

Closure of an attribute

- Closure of an attribute X is represented as X+ is the set of attributes that can be derived using inference axioms.
- F={A$\rightarrow$B,B$\rightarrow$C,C$\rightarrow$D}
- Find A+
- Step 1:A+={A}
- consider left hand side of FD

- $F=\{A \rightarrow B, B \rightarrow C, C \rightarrow D\}$
- Step 1: $A+=\{A\}$
- consider $A \rightarrow B$
- $A+=\{AB\}$
- Consider $B \rightarrow C$
- $A+=\{ABC\}$
- Consider $C \rightarrow D$
- $A+=\{ABCD\}$
- Since no more FDs has to consider, $A+=\{ABCD\}$

Algorithm

- **Algorithm**: Determining X+, the Closure of : the set of attribute X under F

  X+= X;

  repeat

      oldX+= X+;

      for each functional dependency Y→ Z in F do

      If X+ ⊃ Y then X += X+ ∪Z;

  until (X+ = oldX+);

Equivalence of Functional Dependencies

- Let F and G are two FD sets for a relation R.
- If all FDs of F can be derived from FDs present in G, we can say that $G \supset F$ (G covers F) .
- If all FDs of G can be derived from FDs present in F, we can say that $F \supset G$ (F Covers G)
- If 1 and 2 both are true, F=G

- If F covers G and G covers F then F and G are equal

show the relationship between two FD sets. A relation R2(A,B,C,D) having two FD sets

FD1 = {A->B, B->C,A->C} and FD2 = {A->B, B->C, A->D}

- **Step 1.** Checking whether all FDs of FD1 are present in FD2
- A->B in set FD1 is present in set FD2.
- B->C in set FD1 is also present in set FD2.
- A->C is present in FD1 but not directly in FD2 but we will check whether we can derive it or not. For set FD2, $(A)^+ = \{A,B,C,D\}$. It means that A can functionally determine A, B, C and D. SO A->C will also hold in set FD2.
- As all FDs in set FD1 also hold in set FD2, FD2 ⊃ FD1 is true.

- **Step 2.** Checking whether all FDs of FD2 are present in FD1
- A->B in set FD2 is present in set FD1.
- B->C in set FD2 is also present in set FD1.
- A->D is present in FD2 but not directly in FD1 but we will check whether we can derive it or not. For set FD1, $(A)^+ = \{A,B,C\}$. It means that A can't functionally determine D. SO A->D will not hold in FD1.
- As all FDs in set FD2 do not hold in set FD1, FD2 $\not\subset$ FD1.

- In this case, FD2 $\supset$ FD1 and FD2 $\not\subset$ FD1, these two FD sets are not semantically equivalent.

Q.Show the relationship between two FD sets. A relation R(A,B,C,D) having two FD sets F = {A->B, B->C, AB->D} and G = {A->B, B->C, A->C, A->D}

- F = {A->B, B->C, AB->D}
  and G = {A->B, B->C, A->C, A->D}

- Check whether A→C and A→D can be derived from F
- A+={ABCD} using F
- Meaning of A+={ABCD}
- A→A
- A→B
- A→C
- A→D
- Now we can say that F covers G

- **F = {A->B, B->C, AB->D}**
  and G = {A->B, B->C, A->C, A->D}
- Check AB→D can be derived from G
- Find (AB)+ in G
- (AB)+={ABCD}
- AB→D is derived .F covers G
- Now F covers G and G covers F so F is equivalent to G